Some results for a flat plate boundary layer

0.1 Flat plate boundary layer

In this section we consider the computation of the flow over a flat plate. The plate is horizontal and starts at $(x, y) = (0, 0)$. The boundary layer solution is an approximation solution to the laminar flow past a flat plate. The solution is given by (derived by Prandtl's student Blasius)

$$
\begin{aligned}
u &= U f'(\eta), \\
v &= \frac{1}{2}\sqrt{\frac{\nu U}{x}}\Big(\eta f' - f\Big),
\end{aligned}
$$

where the similiarity variable η is defined as

$$
\eta = y\sqrt{\frac{U}{\nu x}},
$$

and where f satisfies the 3rd order ODE:

$$
ff'' + 2f''' = 0, \qquad f(0) = 0,\ f'(0) = 0,\ f'(\infty) = 1.
$$

This problem can be solved as a shooting problem with initial condition

$$
f''(0) \approx 0.3320573362151946
$$

Note that v only makes sense if $\sqrt{\frac{\nu U}{x}}$ is small which implies ν is small and x is not too small (i.e. we cannot evaluate the solution too close to the leading edge). We thus start the computation at some offset value $x = x_0$

The thickness of the boundary layer is

$$
\delta(x) \approx C_\delta \sqrt{\frac{\nu x}{U}},
$$

where $C_\delta \approx 5$ for $u \approx .99U$ on the edge of the boundary layer. The thickness of the boundary layer at inflow will this be $\delta(x_0)$ and we should therefore have enough grid points to resolve this layer.

This boundary solution is evaluated in the class `BoundaryLayerProfile`. Since the solution is only approximate the errors will not go zero as the mesh is refined. The errors should become smaller, however, as $\sqrt{\frac{\nu U}{x_0}} \to 0$, e.g. if $\nu \to 0$ or $x_0 \to \infty$.

The cgins script `flatPlate.cmd` can be used to solve for the flow past a flat plate.

Figure 1 shows results for the flat plate boundary layer for the case $\nu = 10^{-3}$, $x_0 = 5$.

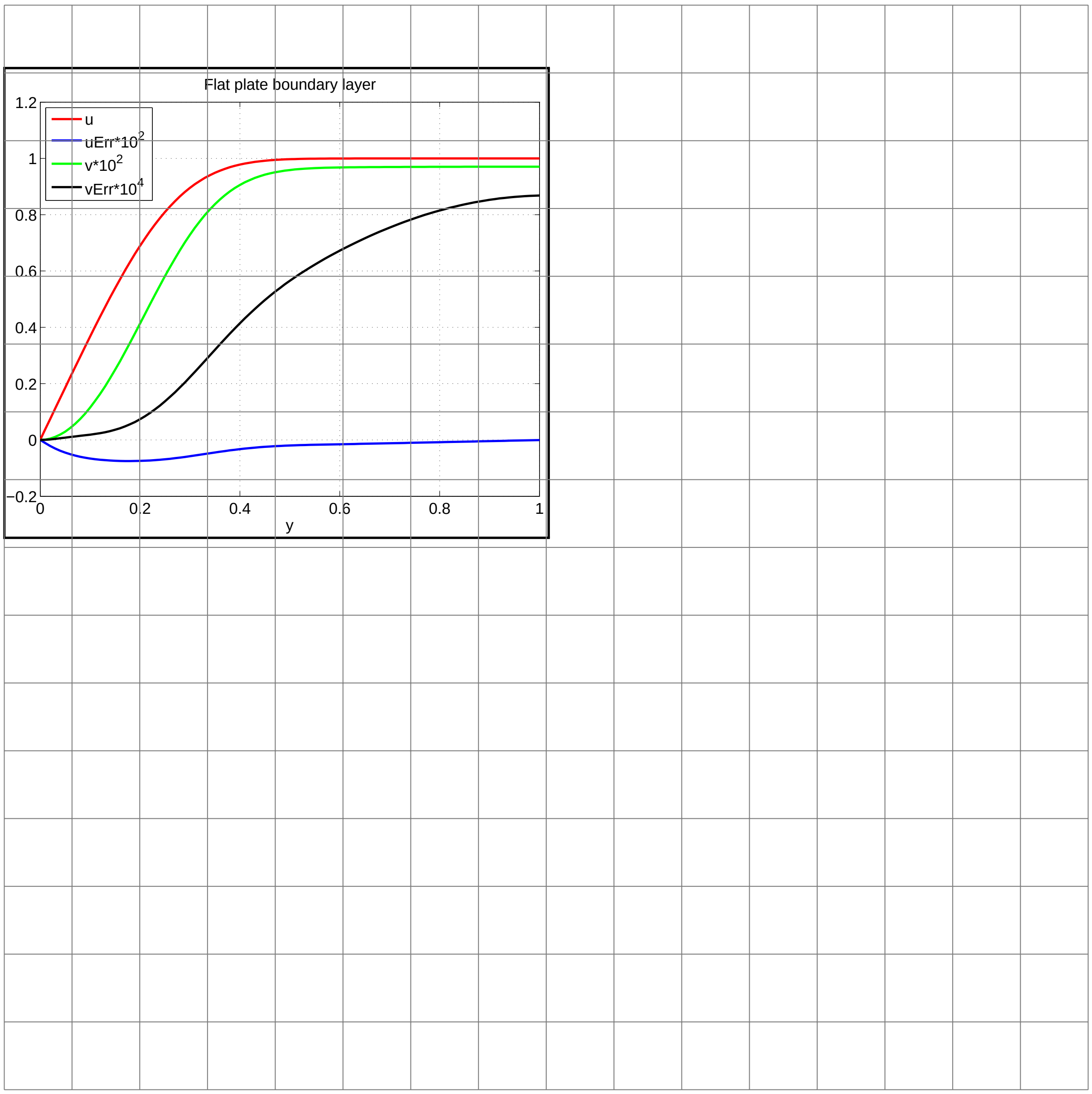

Figure 1: Flat plate boundary layer. Top left: Results from IM24, $\nu = 10^{-3}$, grid $\mathcal{G}_{fp}^{(4)}$, profiles at $x = 3$.