Some results for a flat plate boundary layer

0.1 Flat plate boundary layer

In this section we consider the computation of the flow over a flat plate. The plate is horizontal and starts at $(x, y) = (0, 0)$. The boundary layer solution is an approximate solution for the problem of laminar flow past a flat plate. The solution is given by (derived by Prandtl's student Blasius)

$$u = U f'(\eta),$$
$$v = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\Big(\eta f' - f\Big),$$

where the similiarity variable η is defined as

$$\eta = y\sqrt{\frac{U}{\nu x}},$$

and where f satisfies the 3rd order ODE boundary value problem,

$$f f'' + 2f''' = 0, \qquad f(0) = 0,\ f'(0) = 0,\ f'(\infty) = 1.$$

This problem can be solved as a shooting problem with initial condition

$$f''(0) \approx 0.3320573362151946.$$

Note that v only makes sense if $\sqrt{\frac{\nu U}{x}}$ is small which implies ν is small and x is not too small (i.e. we cannot evaluate the solution too close to the leading edge). We thus start the computation at some offset value $x = x_0$. The approximate thickness of the boundary layer is

$$\delta(x) \approx C_\delta\sqrt{\frac{\nu x}{U}},$$

where $C_\delta \approx 5$ for $u \approx .99U$ on the edge of the boundary layer. The thickness of the boundary layer at inflow will thus be $\delta(x_0)$ and we should therefore have enough grid points to resolve this inflow profile.

The boundary layer solution is evaluated in the class BoundaryLayerProfile (see `cg/common/src/BoundaryLayerProfile.C`). Since the solution is only approximate the errors will not go zero as the mesh is refined. The errors should become smaller, however, as $\sqrt{\frac{\nu U}{x_0}} \to 0$, e.g. if $\nu \to 0$ or $x_0 \to \infty$.

The Cgins script `flatPlate.cmd` (in `cg/ins/runs/flatPlate`) can be used to solve for the flow past a flat plate.

Figure 1 shows results for the flat plate boundary layer for the case $\nu = 10^{-3}$, $x_0 = 5$. These results were computed using grid `flatPlate4.order4.dy.01.ml2.hdf` using the IM24 scheme (2nd-order in time, fourth-order in space).

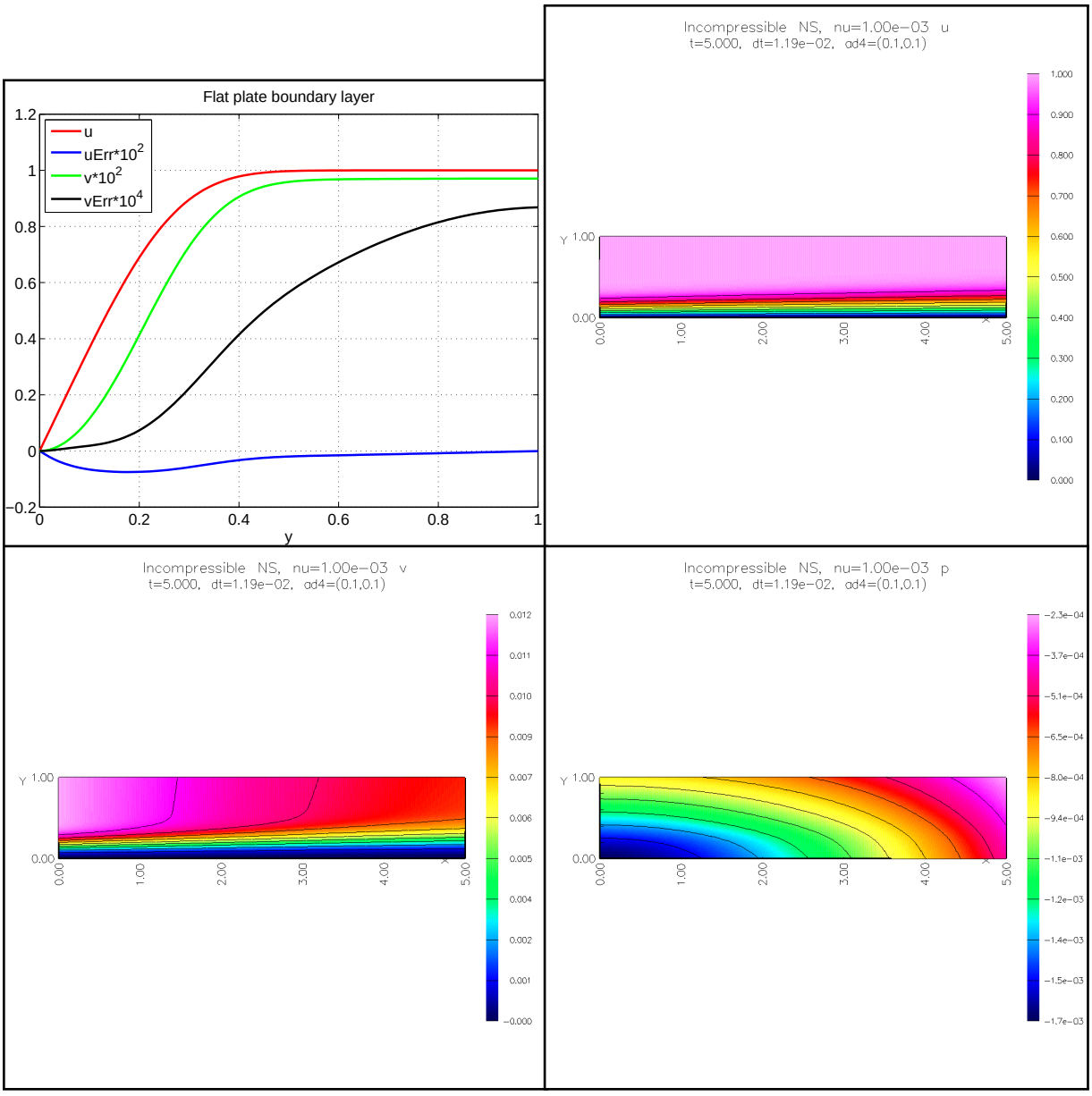

Figure 1: Flat plate boundary layer. Results from IM24, $\nu = 10^{-3}$, grid $\mathcal{G}_{fp}^{(4)}$. To left: solution profiles and errors at $x = 3$.

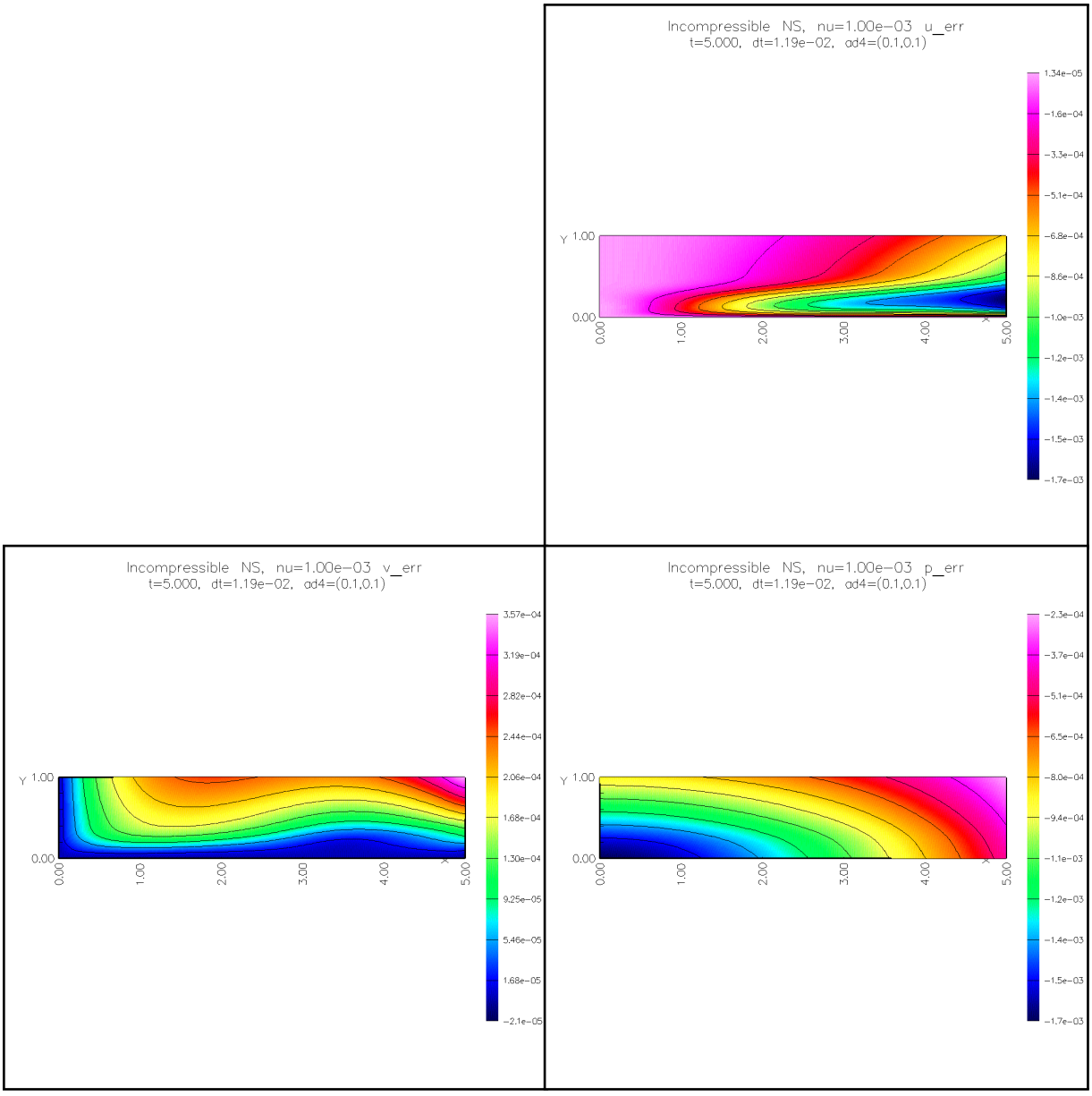

Figure 2: Flat plate boundary layer. Results from IM24, $\nu = 10^{-3}$, grid $\mathcal{G}_{fp}^{(4)}$. Solution errors.