Jump Conditions

Let (n_1, n_2) be the normal to the interface in the physical space.

Kinematic $\Rightarrow$

$$n_1 \bar{V}_{1I} + n_2 \bar{V}_{2I} = n_1 V_{1I} + n_2 V_{2I}$$

Dynamic $\Rightarrow$

Solid traction $= \underline{t}^T = \beta \underline{n}^T \cdot \underline{\underline{P}}$

$$\bar{t}_1 = \beta \frac{r_x P_{11} + r_y P_{21}}{(r_x^2 + r_y^2)^{1/2}} \quad \text{case 2}$$

$$= \frac{\beta}{(\quad)^{1/2}} \tilde{P}_{11}$$

$$\bar{t}_2 = \frac{\beta}{(\quad)^{1/2}} \tilde{P}_{12}$$

or $$\bar{\bar{t}}_1 = -\frac{p}{(\ \ )^{1/2}} \tilde{P}_{11} \qquad \text{case } 1$$

$$\bar{t}_2 = -\frac{p}{(\ \ )^{1/2}} \tilde{P}_{12}$$

~~gas~~ fluid traction

$$\underline{t} = -\underline{n}\, p \qquad \text{(outward normal for fluid)}$$

So,

$$-p = n_1 \bar{t}_1 + n_2 \bar{t}_2$$

$$0 = n_2 \bar{t}_1 - n_1 \bar{t}_2$$

Case 1

$$n_1\left(\bar{V}_1 - \xi_p Z_{1p} - \xi_s Z_{1s}\right) + n_2\left(\bar{V}_2 - \xi_p Z_{2p} - \xi_s Z_{2s}\right)$$

$$= n_1\left(V_1 + n_1 \xi\right) + n_2\left(V_2 + n_2 \xi\right).$$

$$n_1\left(\bar{V}_1 - V_1\right) + n_2\left(\bar{V}_2 - V_2\right)$$

(1) $\Rightarrow$

$$= \xi + \left(n_1 Z_{1p} + n_2 Z_{2p}\right)\xi_p + \left(n_1 Z_{1s} + n_2 Z_{2s}\right)\xi_s$$

$$-p_3 = \frac{-\beta}{(c_x^2 + c_y^2)^{1/2}}\left[n_1\left(\tilde{P}_{11} + \rho_0 C_p Z_{1p}\xi_p + \rho_0 C_s Z_{1s}\xi_s\right)\right.$$

$$\left. + n_2\left(\tilde{P}_{12} + \rho_0 C_p Z_{2p}\xi_p + \rho_0 C_s Z_{1s}\xi_s\right)\right]$$

$$= -\left(p - \rho a \xi\right)$$

$$-\frac{\beta}{(\ )^{1/2}}\left(n_1\tilde{P}_{11} + n_2\tilde{P}_{12}\right) + p = \rho a \xi$$

(2) $\Rightarrow$

$$+\frac{\beta}{(\ )^{1/2}}\left[\rho_0 C_p\left(n_1 Z_{1p} + n_2 Z_{2p}\right)\xi_p\right.$$

$$\left. + \rho_0 C_s\left(n_1 Z_{1s} + n_2 Z_{2s}\right)\xi_s\right]$$

$$0 = \frac{-\beta}{(r_x^2 + r_y^2)^{1/2}} \Big[n_2 \left(\tilde{P}_{11} + \rho_0 c_p z_{1p} \xi_p + \rho_0 c_s z_{1s} \xi_s \right) - n_1 \left(\tilde{P}_{12} + \rho_0 c_p z_{2p} \xi_p + \rho_0 c_s z_{2s} \xi_s \right) \Big]$$

$$(3) \Rightarrow \quad -n_2 \tilde{P}_{11} + n_1 \tilde{P}_{12} = \rho_0 c_p (n_2 z_{1p} - n_1 z_{2p}) \xi_p + \rho_0 c_s (n_2 z_{1s} - n_1 z_{2s}) \xi_s$$

Matrix form

$$\begin{bmatrix} 1 & z_{np} & z_{ns} \\ \rho a & \alpha \rho_0 c_p z_{np} & \alpha \rho_0 c_s z_{ns} \\ 0 & \rho_0 c_p z_{tp} & \rho_0 c_s z_{ts} \end{bmatrix} \begin{bmatrix} \xi \\ \xi_p \\ \xi_s \end{bmatrix} = \begin{bmatrix} \Delta v \\ \Delta p \\ 0 \end{bmatrix}$$

Maple gives

$$\xi = \frac{\alpha\, \omega_p\, \omega_s \left(Z_{pn} Z_{st} - Z_{sn} Z_{pt} \right) \Delta V}{\alpha \omega_p \omega_s (\quad) - \omega\, \omega_s\, Z_{pn} Z_{st} + \omega\, \omega_p\, Z_{sn} Z_{pt}}$$

$$+ \quad \frac{\left(Z_{sn} Z_{pt}\, \omega_p - Z_{pn} Z_{st}\, \omega_s \right) \Delta P}{\text{same denom.}}$$

where

$$\omega = \rho a \quad , \omega_p = \rho_0 c_p \,, \quad \omega_s = \rho_0 c_s$$

$$\alpha = \frac{\beta}{(\quad)^{1/2}}$$

$$Z_{pn} = n_1 Z_{1p} + n_2 Z_{2p}$$

$$Z_{pt} = n_2 Z_{1p} - n_1 Z_{2p}$$

~~etc.~~ $\Delta V = n_1 (\bar{V}_1 - V_1) + n_2 (\bar{V}_2 - V_2)$

$$\Delta p = -\alpha \left(n_1 \tilde{P}_{11} + n_2 \tilde{P}_{12} \right) + p$$

Case 2.

$$n_1\left(\bar{v}_1 + \xi_p z_{1p} + \xi_s z_{1s}\right) + n_2\left(\bar{v}_2 + \xi_p z_{2p} + \xi_s z_{2s}\right)$$

$$= \text{same}$$

$$n_1(\bar{v}_1 - v_1) + n_2(\bar{v}_2 - v_2)$$

(1) $\Rightarrow$ $$= \xi - (n_1 z_{1p} + n_2 z_{2p})\xi_p - (n_1 z_{1s} + n_2 z_{2s})\xi_s$$

$$-p_I = \frac{\beta}{(\ )^{1/2}}\left[\text{same}\right] = -(p - \rho a \xi)$$

$$\frac{\beta}{(\ )^{1/2}}\left(n_1 \tilde{P}_{11} + n_2 \tilde{P}_{12}\right) + p = \rho a \xi$$

(2) $\Rightarrow$ $$-\frac{\beta}{(\ )^{1/2}}\left[\text{same}\right]$$

(3) $\Rightarrow$ same.