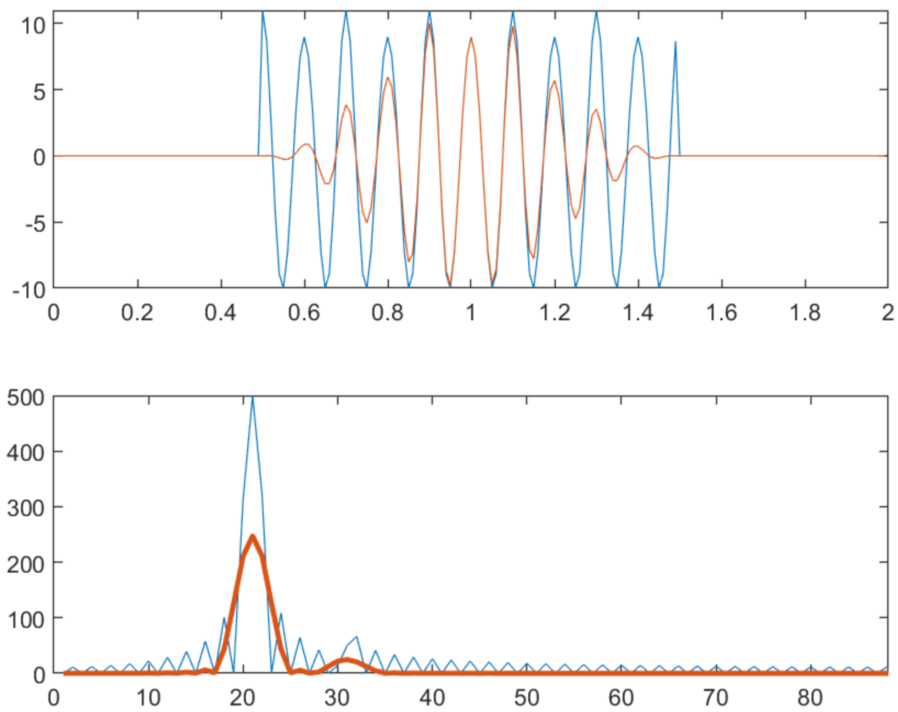
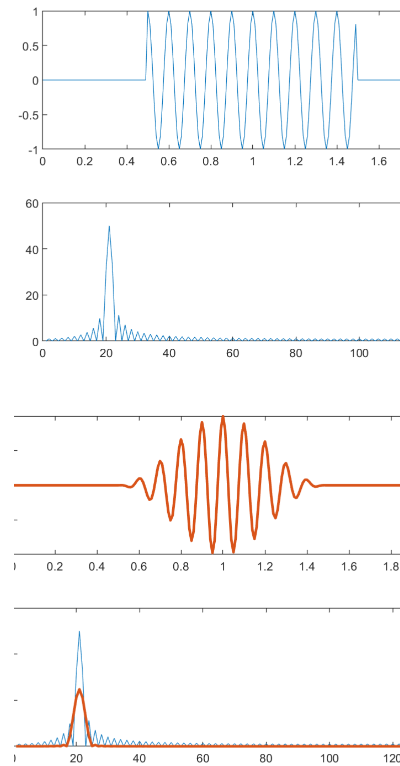

10
5
0
-5
-10
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
500
400
300
200
100
0
0 10 20 30 40 50 60 70 80



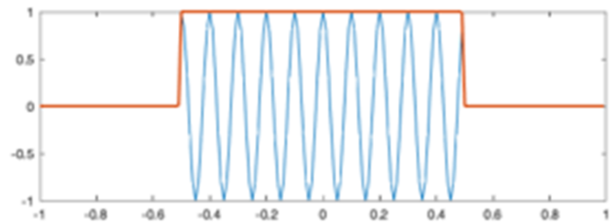
Windowing

Windowing

- Application of convolution theorem in the reverse direction
- Example: Windowed sinusoid

$$s(t) = \begin{cases} \cos(2\pi f_0 t)\,;\, t \in [-T/2, T/2] \\ 0; \quad \text{otherwise} \end{cases}$$

Box car window



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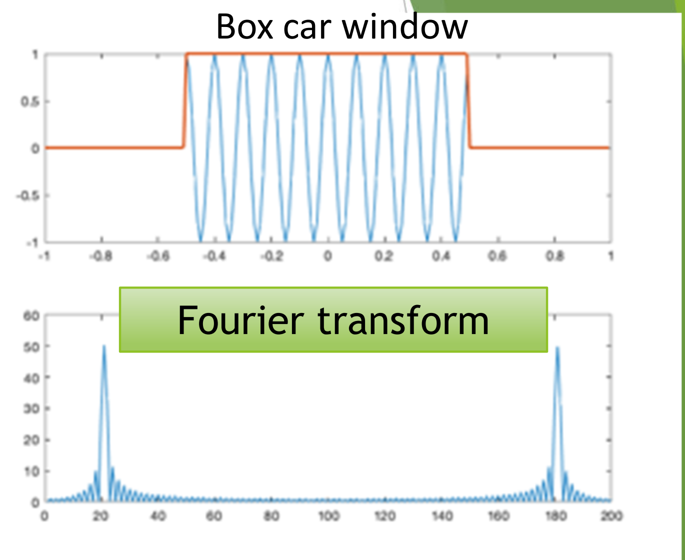

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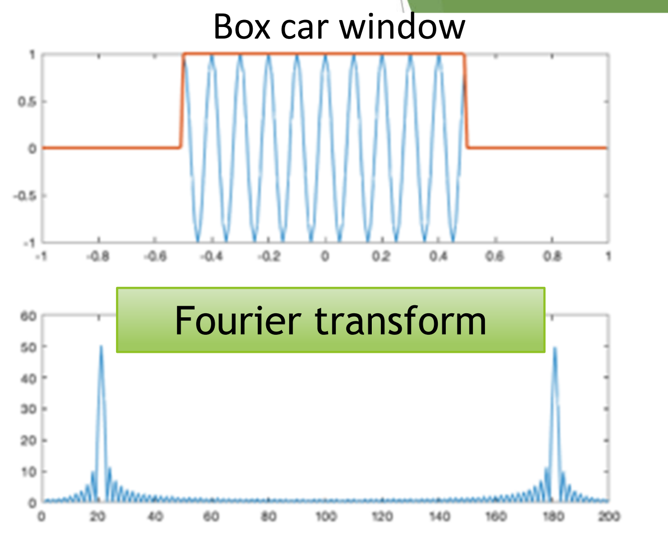

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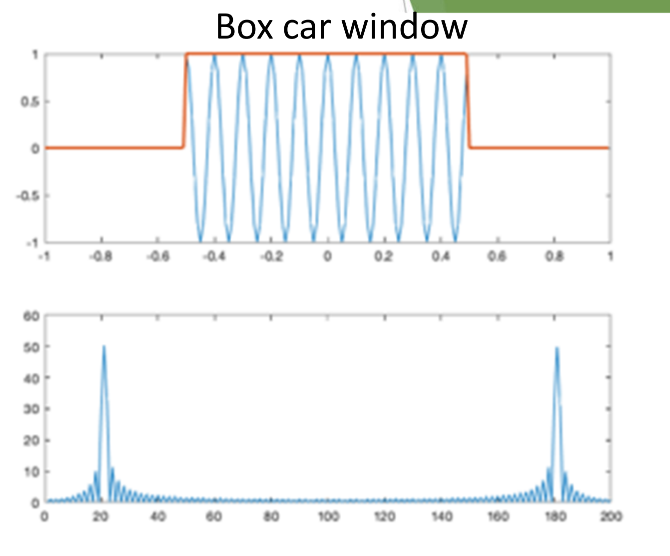
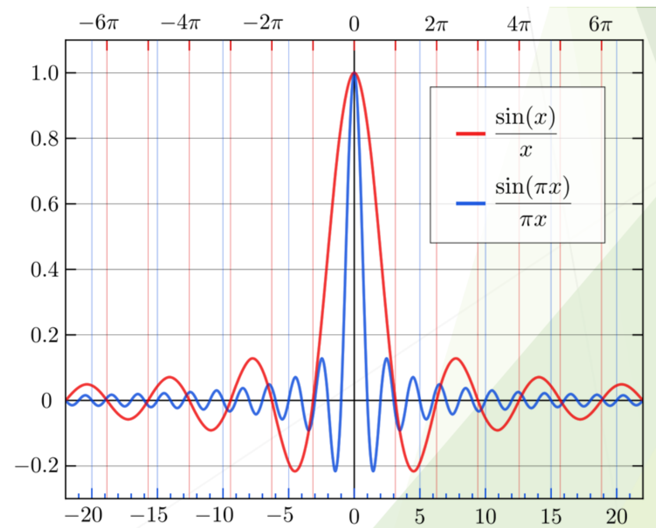
- (Prove) Fourier transform of Box car window:

$$\tilde{\Pi}(\mathrm{f}) = T\mathrm{sinc}(fT) = T\sin(\pi f T)/(\pi f T)$$





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$$\tilde{\Pi}(\mathrm{f}) = T\mathrm{sinc}(fT) = T\sin(\pi f T)/(\pi f T)$$

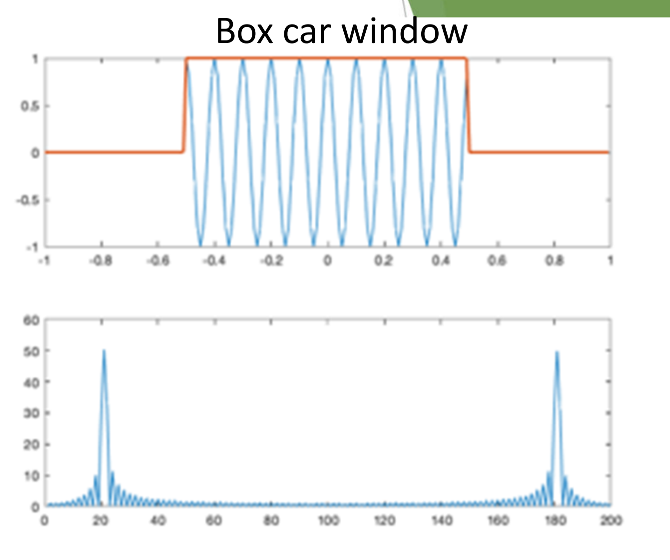
- Fourier transform of cosine (Lecture 3):

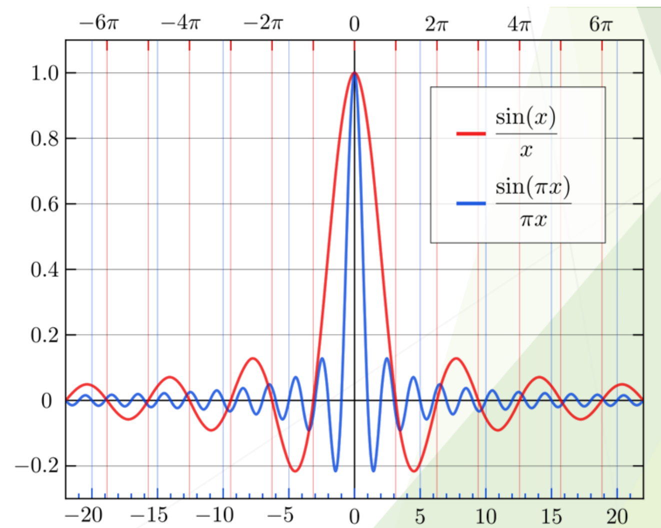
$$\frac{1}{2}(\delta(f - f_0) + \delta(f + f_0))$$

- Convolution theorem:

$$\tilde{s}(f) = \int_{-\infty}^{\infty} df' \, T\mathrm{sinc}((f - f')T)\left[\frac{1}{2}(\delta(f' - f_0) + \delta(f' + f_0))\right]$$

$$= \frac{T}{2}\left[\mathrm{sinc}\big((f - f_0)T\big) + \mathrm{sinc}((f + f_0)T)\right]$$





Windowing

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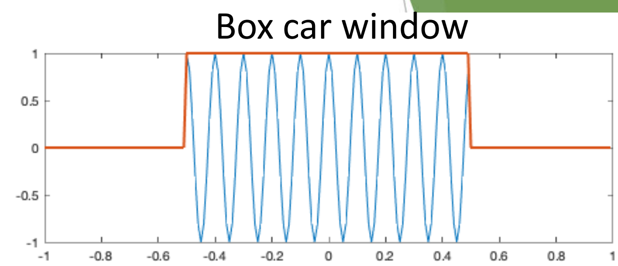

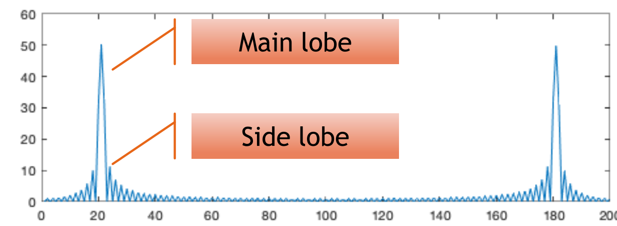

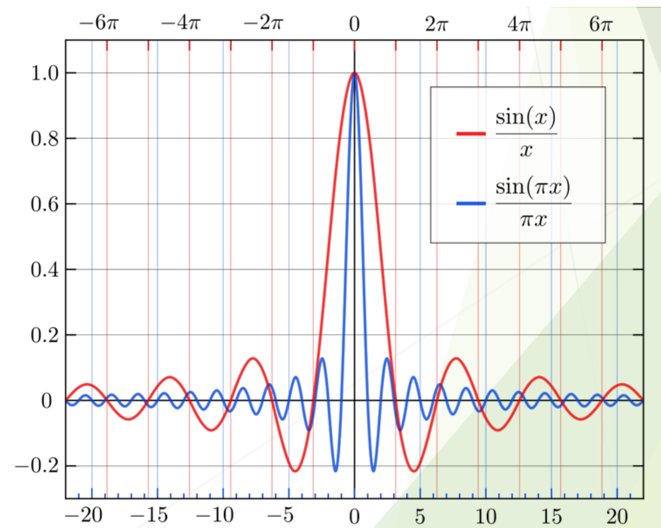

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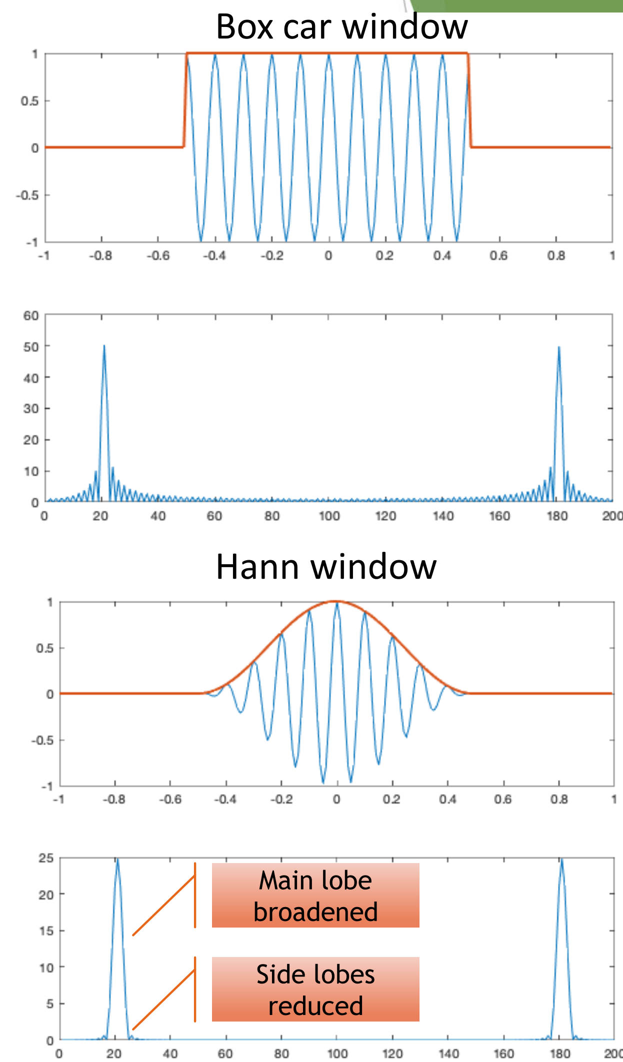
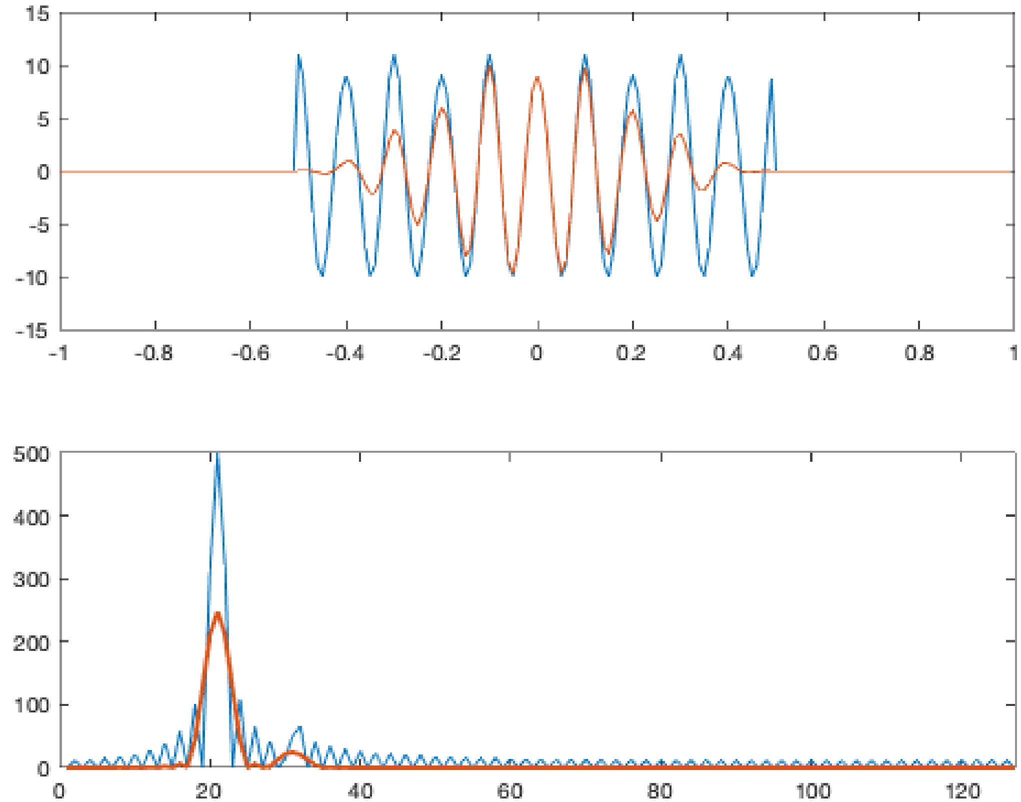
- Using a different window (e.g., Hann) results in a different trade-off between the main lobe width and side lobe height



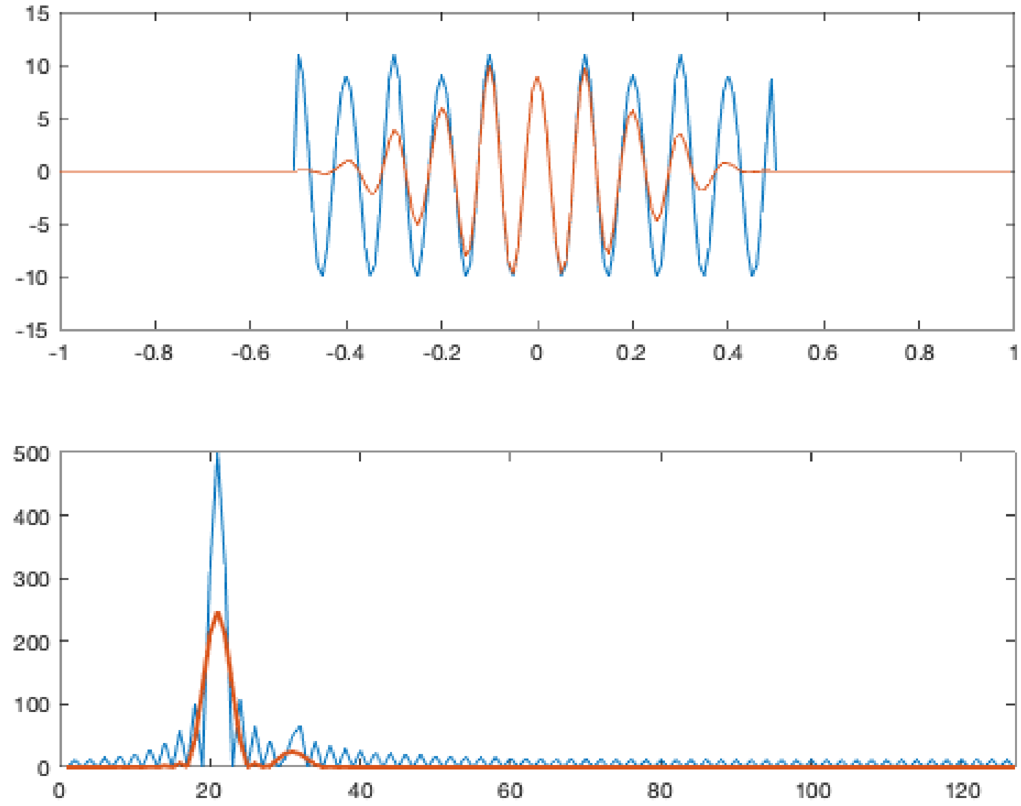
Windowing

- Different window choices have different trade-offs
- The figure shows the sum of two sinusoids and the Fourier transform under different window choices (boxcar and Hann)



Windowing

- Different window choices have different trade-offs
- The figure shows the sum of two sinusoids and the Fourier transform under different window choices (boxcar and Hann)
- Note that it is easier to see the second, weaker, signal with Hann windowing compared to boxcar, where the side lobe is higher than the signal peak
- On the other hand, the peaks are broadened: In the presence of noise, it would be harder to estimate the true signal frequency from the Hann windowed data due to the much broader width of the peaks



Run DATASCIENCE_COURSE / DSP / WindowExample.m