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Tarefa 6 = Sistemas lineares

1. Tarefa Básica: Regre de Cramer

1.

a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$ $m = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} = 2 \cdot 3 - (-1) \cdot (-1) = 6 - 1 = 5$

$$m_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} \quad D_x = 2 \cdot 3 - (-1) \cdot (-3) = 6 - 3 = 3$$

$$m_y = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \quad D_y = 1 \cdot (-3) - 2 \cdot (-1) = -3 - (-2) = -1$$

$$x = \frac{3}{5} \quad y = \frac{-1}{5}$$

$$\text{b)} \begin{cases} 3x - y + 2 = 1 \\ 2x + 3y = -1 \\ 4x + y - 2z = 7 \end{cases}$$

$$m = \left[\begin{array}{ccc|cc} 3 & -1 & 1 & 3 & -1 \\ 2 & 0 & 3 & 2 & 0 \\ 4 & 1 & -2 & 4 & 1 \end{array} \right]$$

$$3 \cdot 0 \cdot (-2) + (-1) \cdot 3 \cdot 2 + 1 \cdot 2 \cdot 5 - 1 \cdot 0 \cdot 4 - 3 \cdot 3 \cdot 1 \cdot (-1) \cdot 2 \cdot (-2) = -23$$

$$m_x = \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 1 & -1 \\ -1 & 0 & 3 & -1 & 0 \\ 7 & 1 & -2 & 7 & 1 \end{array} \right] \Delta_2 = -23 \quad m_y = \left[\begin{array}{ccc|cc} 3 & -1 & 1 & 3 & -1 \\ 2 & 0 & -1 & 2 & 0 \\ 4 & 1 & 7 & 4 & 1 \end{array} \right] \Delta_2 = 23$$

$$m_{xy} = \left[\begin{array}{ccc|cc} 3 & 1 & 1 & 3 & 1 \\ 2 & -1 & 3 & 3 & -1 \\ 4 & 7 & 2 & 4 & 7 \end{array} \right] \Delta_2 = 23 \quad x = \frac{-23}{-23} = 1 \quad y = \frac{-23}{-23} = 1$$

$$z = \frac{23}{-23} = -1$$

2.

$$\begin{cases} 3x + 4y - z = 1 & 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 & \underline{x - 2y + 3z = 8} \\ x - 2y + 3z = 8 & 4x + 2y + 2z = 9 \end{cases}$$

$$4x + 5y + 2z = 12 \cdot (-1) = -4x - 5y - 2z = -12$$

$$\underline{-4x - 5y - 2z = -12}$$

$$4x + 2y + 2z = 9$$

$$-3y = -3$$

$$y = \frac{-3}{-3}$$

$$y = 1$$

Ri. a) J

$$3. \begin{cases} x + 2y + z = 1 \\ 3x + y - z = -2 \\ 2x + 3y - z = 1 \end{cases} \quad m = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -1 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{bmatrix} \quad D = 1$$

$$m_x = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & -1 \\ 3 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 1 & 3 \end{bmatrix} \quad D_x = -1 \quad m_y = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -2 & -1 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & -2 \\ 2 & 1 \end{bmatrix} \quad D_y = 1$$

$$m_z = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{bmatrix} \quad D_z = 0 \quad A + B + C = 0 - 1 + 1 = 0$$

$$x = \frac{-1}{-1} = -1 \quad y = \frac{1}{1} = 1 \quad z = \frac{0}{1} = 0 \quad R.C. 0$$

4.

$$\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases}$$

$$m = \begin{bmatrix} 1 & 2 & -3 & | & 29 \\ 1 & 3 & 2 & | & 4 \\ 1 & -1 & -2 & | & 8 \end{bmatrix} \quad D = 16$$

$$m_x = \begin{bmatrix} 29 & 2 & -3 & | & 29 & 2 \\ 4 & 3 & 2 & | & 4 & 3 \\ 8 & -1 & -2 & | & 8 & -1 \end{bmatrix} \quad 10x = 16 \quad x = \frac{16}{16} = 1$$

$$m_y = \begin{bmatrix} 1 & 29 & -3 & | & 1 & 29 \\ 1 & 4 & 2 & | & 1 & 4 \\ 3 & 8 & -2 & | & 1 & 8 \end{bmatrix} \quad 10y = 80 \quad y = \frac{80}{16} = 5$$

$$m_z = \begin{bmatrix} 1 & 2 & 29 & | & 1 & 2 \\ 1 & 3 & 4 & | & 1 & 3 \\ 1 & -1 & 8 & | & 1 & -1 \end{bmatrix} \quad 10z = -96 \quad z = \frac{-96}{16} = -6$$

$$x + y + z = 1 + 5 + (-6) = 0$$

R: a) 0

$$5. \begin{cases} 2x + y = 5 \\ 2y + z = 3 \end{cases}$$

$$\begin{cases} 2x + 2y + z = 7 \end{cases}$$

$$m = \left[\begin{array}{ccc|cc} 2 & 1 & 0 & 2 & 1 \\ 0 & 2 & 1 & 0 & 2 \\ 3 & 2 & 1 & 3 & 2 \end{array} \right] \quad 10 = 3$$

$$m_x = \left[\begin{array}{ccc|cc} 5 & 1 & 0 & 5 & 1 \\ 3 & 2 & 1 & 3 & 2 \\ 7 & 1 & 1 & 7 & 1 \end{array} \right]$$

$$20x = 4 \quad x = \frac{4}{3}$$

$$m_y = \left[\begin{array}{ccc|cc} 2 & 1 & 5 & 2 & 1 \\ 0 & 2 & 3 & 0 & 2 \\ 3 & 2 & 7 & 3 & 2 \end{array} \right]$$

$$m_y = \left[\begin{array}{ccc|cc} 2 & 5 & 0 & 2 & 5 \\ 0 & 3 & 1 & 0 & 3 \\ 3 & 7 & 1 & 3 & 7 \end{array} \right] \quad 10y = 5 \quad y = \frac{5}{3}$$

$$20y = 5 \quad y = \frac{5}{3}$$

6.

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + 0y + 0z \\ 2x + y + 0z \\ -x + 2y + 2z \end{bmatrix} = \begin{bmatrix} x \\ 2x + y \\ -x + 2y + 2z \end{bmatrix}$$

$$\begin{bmatrix} x + 0y + 0z \\ 2x + y + 0z \\ -x + 2y + 2z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

$$\begin{cases} x = 3 \\ 2x + y = 7 \\ -x + 2y + 2z = -1 \end{cases}$$

$$20x = 6$$

$$m = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 7 & 1 \\ -1 & 2 \end{bmatrix} \quad 10 = 2 \quad mx = \begin{bmatrix} 3 & 0 & 0 \\ 7 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 7 & 1 \\ -1 & 2 \end{bmatrix} \quad z = \frac{6}{2} = 3$$

$$mxy = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 7 & 0 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 7 & 1 \\ -1 & 2 \end{bmatrix} \quad 20y = 2$$

$$y = \frac{2}{2} = 1 \quad mxy = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 7 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 21 & 1 \\ -5 & 2 \end{bmatrix} \quad 20y = 0$$

3: e) $y = 0$

Tarea Básica - Examenante

1

$$S = \begin{cases} 2x - y - 3y = -5 \\ x + 3y - 2y = 11 \\ x - 5y = 3 \end{cases}$$

-1 - 2

$$\left(\begin{array}{ccc|c} 1 & 0 & -5 & 3 \\ 2 & -1 & -3 & 15 \\ 1 & 3 & -5 & 11 \end{array} \right)$$

\rightarrow 3x

$$\left(\begin{array}{ccc|c} 0 & -1 & 7 & 11 \\ 0 & 3 & 4 & 8 \end{array} \right)$$

$$-y + 7 - 1 = -11$$
$$-y = 4$$

$$(0 \ 0 \ 25 : -25)$$

$$x - 5 - 1 = 3$$
$$x = -2$$

$$25y = -25$$
$$y = -1$$

2.

$$\begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases} \quad y = \frac{x}{2} \quad z = \frac{x}{3}$$

$$x + \frac{x}{2} + \frac{x}{3} = 11 \quad \text{MKG} = 6$$

$$x \cdot 6 + x \cdot 6 + x \cdot 6 = 11 \cdot 6$$

$$x \cdot 6 + 3x + 2x = 66 \rightarrow 11x = 66$$

$$x = 6$$

$$6 + x + y =$$

$$6 + 6 + 6 = 18$$

$$R: b) 18$$

3.

$$\begin{cases} x+y+z=0 \\ 2x-y-2z=1 \\ 6y+3z=-12 \end{cases}$$

$$x + (-9) + 2 = 0$$

$$x = 7$$

$$y + (-1) \cdot 2 = 3$$

$$y = 9$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & -2 & 1 \\ 0 & 6 & 3 & 4 \end{array} \right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -4 & 1 \\ 0 & 6 & 3 & 4 \end{array} \right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & -5 & -10 \end{array} \right]$$

$$\begin{aligned} -5y &= -10 \\ y &= 2 \end{aligned}$$

R: d) 2

4.

$$A, B, C \text{ für } R\$ 68,00$$

$$B + 20, C - A \rightarrow \frac{B+C}{5} = a \rightarrow 5B + C = 5a \rightarrow 5A - 5B - C = 0$$

$$\frac{C+20}{100} = A - B, B \rightarrow \frac{C+A}{5} = 3B \rightarrow 5C + A = 15B \rightarrow A - 15B + 5C = 0$$

$$\begin{cases} A + B + C = 68 \\ 5A - 5B - C = 0 \\ A - 15B + 5C = 0 \end{cases}$$

$$D \left| \begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 5 & -5 & -1 & 5-5 = -10 \\ 1 & -15 & 5 & 1-15 \end{array} \right. \quad -5+15+25 = \\ -25+(-1)+(-75) =$$

$$0+1020+0=$$

$$DA = \left| \begin{array}{ccc|c} 68 & 1 & 1 & 68 \\ 0 & -5 & -1 & 0-5 = -1 \\ 0 & -15 & 5 & 0-15 \end{array} \right. \quad -1700+1020 = -680 \quad A = \frac{DA}{D} = \frac{-680}{-136} = 20$$

$$DB = \left| \begin{array}{ccc|c} 1 & 68 & 1 & 1 \\ 5 & 0 & -1 & 5-5 = -68-1700 = -1768 \\ 1 & 0 & 5 & 1-15 \end{array} \right. \quad 0+0+(500) =$$

$$B = \frac{DB}{D} = \frac{-1768}{-136} = 13$$

$$DC = \left| \begin{array}{ccc|c} 1 & 1 & 68 & 1 \\ 5 & -5 & 0 & 5-5 = -500-(-340) = -460 \\ 1 & -15 & 0 & 1-15 \end{array} \right. \quad 0+0+(-500)$$

$$C = \frac{DC}{D} = \frac{-460}{-136} = 35$$

$$A = R\$ 20,00$$

$$C-A$$

R: a)

$$B = R\$ 13,00$$

$$A-B = 20-13 = 7$$

$$C = R\$ 35,00$$

$$5. \quad A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 5 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \begin{cases} 3x + 4y = 134 \\ x + 5y = 94 \\ 2x + 5y = 148 \end{cases}$$

$$\text{Ges. : R\$ 134,00} \quad D = \begin{bmatrix} 0 & 3 & 4 & 0 \\ 1 & 0 & 5 & 10 = 34 \\ 2 & 5 & 0 & 210 \end{bmatrix}$$

$$D_A = \begin{vmatrix} 134 & 3 & 4 \\ 115 & 0 & 5 \\ 48 & 1 & 0 \end{vmatrix} \quad 0 + 670 + 0 = 670 \quad x = D_{3A} = 510 = 19$$

$$0 + 1180 - 670 = 510 \quad D = 34$$

$$0 + 350 - 460 = 1120$$

$$D_B = \begin{vmatrix} 0 & 134 & 4 \\ 1 & 115 & 5 \\ 2 & 48 & 0 \end{vmatrix} \quad 220 + 0 + 0 = 920 \quad y = D_{2B} = 612 = 18$$

$$0 + 1532 - 920 = 612 \quad 18 \quad 34$$

$$0 + 1340 + 192 = 1532$$

$$D_C = \begin{vmatrix} 0 & 3 & 134 & 0 \\ 1 & 0 & 115 & 1 \\ 2 & 1 & 48 & 2 \end{vmatrix} \quad 0 + 0 + 144 = 144 \quad z = D_{3C} = 680 = 20$$

$$0 + 690 - 134 = 680 \quad 20 \quad 34$$

$$15 + 18 + 20 = 53 \quad 0 \text{ hたる } \times \text{ R\$ 53,00}$$

R: a)