Nome: Rodrigo Moreira da Silva Data: CT II - 317

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Tarefa Básica - Matriz Inversa

1.

$A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$ $\begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{cases} 3x + y = 1 \\ 15 + 3y = 0 \end{cases}$ $\begin{cases} -x + 2 = 0 \\ -5 + 6 = 1 \end{cases}$

$3x + y = 1$
$3x + (-5) = 1$
$3x - 5 = 1$
$3x = 1 + 5$
$3x = 6$
$\boxed{x = 2}$

$15 + 3y = 0$
$y = \frac{-15}{3}$
$y = -5$

$-x + 2 = 0$
$x = 2$

$x + y$
$\boxed{2 - 5 = -3}$

2.

$A = \begin{pmatrix} 1 & 0 & 1 \\ K & 1 & 3 \\ 1 & K & 3 \end{pmatrix}$ $\begin{matrix} 1 & 0 \\ K & 1 \\ 1 & K \end{matrix}$

$1 + 3K + 0$

$3 + 0 + K^2$

$3 + K^2 - 3K - 1 = 0$
$K^2 - 3K + 2 = 0$

$\Delta = (-3)^2 - 4 \cdot 1 \cdot 2$
$\Delta = 9 - 8$
$\Delta = 1$

$x = \frac{-(\pm 3) \pm \sqrt{1}}{2 \cdot 1} = \frac{3 + 1}{2} = \frac{4}{2} = 2$

$\frac{3 - 1}{2} = \frac{2}{2} = 1$

A. c)

3.

B inversa de $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ $\det A = 12 - 10 = 2 \neq 0$

$B = A^{-1} \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \cdot \frac{1}{2} = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$ R: c)

4.

$20 + 2x + 3x - 5x + 20$

$\begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix} \begin{matrix} x & 1 \\ 3 & 1 \\ 10 & 1 \end{matrix} = 26 + x^2 - 5x + 20 = x^2 - 5x + 6 \neq 0$

$x^2 + 20 + 6 = 26 + x^2$

$\Delta = (5)^2 - 4 \cdot 1 \cdot 6$

$\Delta = 25 - 24$

$x = -(-5) \pm \frac{\sqrt{1}}{2 \cdot 1} = \frac{5 \pm 1}{2} = \frac{6}{2}$ $\sqrt{1}$ $\Delta = 1$

$\frac{5-1}{2} = \frac{4}{2} = 2$ $\{x+3, x+2\}$

R: a)

5.

$2 + 2 + 2 = 6$

$A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{matrix} -1 & 1 \\ 2 & 1 \\ 1 & 1 \end{matrix}$ $\det = 7 - 6 = 1$

$1 + 2 + 4 = 7$

$\begin{vmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix} \begin{matrix} 1 & 1 \\ 2 & 1 \\ 1 & 1 \end{matrix}$ $A^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$ -1

$-1 \quad -1 \quad 2 \quad -1 \quad 1$

$2 \quad 1 \quad -2 \quad 2 \quad 1$

a) $\begin{pmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{pmatrix} \cdot A^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$ $A \cdot A^{-1} = \begin{pmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{pmatrix}$

6.

$(X \cdot A)^t = B$

$X \cdot A \cdot A^{-1} = B^t \cdot A^{-1}$

$X = B^t \cdot A^{-1}$

7.

$B = \begin{bmatrix} x \\ y \end{bmatrix}$, $C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$

$A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} \cdot B = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x & 5y \\ 5x & 6y \end{bmatrix}$

$\det: 24 - 25 = -1$

$A = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} = -1 \rightarrow A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}$

8.

$A = \begin{bmatrix} 2 & K \\ -2 & 1 \end{bmatrix}$ $\quad 2 \cdot 1 - (-2)K$ $\quad 2K + 2 = \det A$

$2K + 2 = -1$

$2K = -3$

$K = \frac{-3}{2}$

$2K_1 + 2 = 1$

$2K_1 = -1$

$K_1 = \frac{-1}{2}$

$\frac{-1}{2} + \left(\frac{-3}{2}\right) = \frac{-4}{2} = -2$

9.

a) $(A+B) \cdot (A-B)$

$A^2 = AB + BA - B^2 \neq 0$

b) $(A+B)^2 =$

$a^2 + ab + ba + b^2$

$ab = ba$

c) $\frac{\det(A)}{\det(-A)} = \frac{1}{-1} = 1$

d) $B = A^{-1}$

$B = \frac{1}{\det A}$