Image Processing*

L. S. G. KOVÁSZNAY†, SENIOR MEMBER, IRE, AND H. M. JOSEPH‡, SENIOR MEMBER, IRE

***Summary*—A scalar function of two independent variables can be visualized as an image. All mathematical operations can be conceived as a modification or processing of the original image. An important class of modifying operators can be realized by special scanning techniques without using a rapid access memory storage device. It was found that the two important operators so far explored may have practical importance. One is contour enhancement which has a "deblurring" effects akin to aperture correction and "crispening" in television practice; the other is contour outlining that produces a line drawing from a picture with continuous tones. The general concepts developed may also permit extension of the method to analog computers for certain classes of partial differential equations. The flexibility and adaptability of the system offer practical application whenever some predetermined operation is required on picture material.**

Introduction

A BLACK AND WHITE continuous picture or a black line drawing can be regarded as a communication channel, i.e. as a carrier of information. Before the advent of modern electrical communication techniques the most common means of transmitting information over great distances was to physically transport a sheet of paper with drawings or symbols on it. The electronic techniques, the applicable circuit theory, and correspondingly the concepts of the supporting theory of communication, all revolve about functions of one independent variable, namely, time.

An interesting thought arises. The techniques and the explicit findings of communication theory can be generalized to functions of two independent variables. The optical reflectivity of a paper print for a given angle of illumination is a function of two co-ordinates and therefore is a function of two independent variables. Operations or functional relationships can be visualized in this case as obtaining a new picture from an old one by a prescribed process.

The major objective of the present study was the development of a method for processing pictures by electronic techniques, similar to those used in television with emphasis upon the realization of a few important operators.

The attention of the authors was attracted to this problem when thinking about the following intriguing question: How does one perceive, identify, and recollect pictures? Naturally the solution of the mystery of visual perception is far beyond our objectives. However, the attempts reported here may contribute some insight into particular aspects of the general problem.

It does appear that the process of visual perception is homogeneous and isotropic, in the sense that translation or rotation of a viewed scene produces no change in the character of the result except for the positional change itself. In consequence prime emphasis was placed upon isotropic operators.

The first operator explored was named contour enhancement. It is worthwhile to note that a similar phenomenon occurs in visual perception and is called different names by the biologist and psychologist. One such is "brightness contrast effect" (Fig. 1). It involves a sharpening of the diffuse boundaries of adjacent domains of different intensities.[1] Later it will be shown that it can be accomplished by a linear operator involving the Laplacian of the picture function.

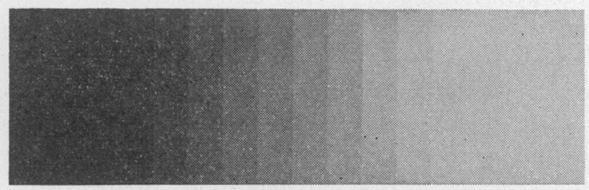

Fig. 1—Step tablet illustrating contour enhancement in the eye. Each step is a uniform shade of gray, although it appears lighter near the darker step and darker near the lighter one. The uniformity of each area can be checked by masking all steps except one.

The second operator developed was named contour outlining and it is probably an inherent process in humans,[2,3] since even the prehistoric caves exhibit drawings that are essentially the outline sketches of animals. It seems that it is a rather simple and convenient method for "stripping-down" a picture of its unnecessary details.[4] Broadly speaking it appears that human visual perception somehow reduces the amount of picture information by outlining the picture before storing it or before comparing it with other stored images. The fact that cartoons and caricatures can be recognized and understood by most people suggests that there might be processes which are more of a "mechanical" nature than a "mental" one, meaning that they do not rely upon extensive memory storage, since the sig-

* Original manuscript received by the IRE, August 22, 1954; revised manuscript received January 31, 1955. This work was done at the National Bureau of Standards under a program of Basic Instrumentation, Research, and Development sponsored by the Office of Naval Research, the Air Research and Development Command, and the Atomic Energy Commission.

† Johns Hopkins University, Baltimore, Md., also Consultant to the National Bureau of Standards.

‡ ACF Electronics, a division of ACF Industries, Inc., Alexandria, Va.; formerly with the National Bureau of Standards.

[1] George M. Bryam, "The physical and photochemical basis of visual resolving power," *Jour. Opt. Soc. Amer.*, vol. 34, pp. 718–738; December, 1944.

[2] K. S. Lashley, "The problem of cerebral organization in vision," *Biol. Symp.*, vol. VII, (Visual Mechanisms), Heinrich Klüver, ed., The Jacques Cattell Press, Lancaster, Pennsylvania; 1942.

[3] K. Koffka, "Principles of Gestalt Psychology," Harcourt, Brace and Co., New York; 1935.

[4] F. Attneave, "Some Information Aspects of Visual Perception," *Psych. Rev.*, vol. 61, pp. 183–193; May, 1954.

nificance of cartoons seems to be the same for everyone irrespective of diverse individual experiences.

The process by which "stripped down" pictures are identified by comparing them with the stored ones is the so called "Gestalt" problem which puzzles many students of the life sciences. This more general problem, namely, how outlines (black line pictures) can be further processed, clarified, and identified has not been attacked in the present study. The authors believe, however, that exploration along these lines will inevitably occur, perhaps as a result of stimulation from the present work.

List of Symbols

x, y, x', y' Cartesian co-ordinates
t Time
ξ, η Displacement in coordinates
$f(x, y)$ Function describing original picture
$F(x, y)$ Function describing resulting picture
Ω Functional operator
θ Angle of rotation
A, B, C, C_1, C_2 Arbitrary constants
$\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ Laplacian operator
$g(x, y)$ Function describing perfect picture (before blurring)
τ Time interval, or time constant (e.g. $\tau = RC$)
k Diffusion coefficient
ψ Rescaling operator
$F^*(x, y)$ Outline picture
U Speed of scanning spot
T Period or repetition in scan (frame period)
f_0 Frequency of scan pattern repetition
f_1 Frequency of horizontal scan
f_2 Frequency of vertical scan
n_1 Number of horizontal scan periods during a full frame period
n_2 Number of vertical scan periods during a full frame period
D Photographic density (logarithmic definition)
r Transmittance
I, I_0, I_1 Luminous intensity of cathode-ray spots
e Voltage delivered by feedback amplifier
E_g Intensity grid voltage measured from cut-off
E_0 Intensity grid bias in absence of video signal
K Feedback ratio
e_0 Output voltage
ϵ Base of natural logarithms
$(\dot{\ })$ Dot indicates differentiation with respect to time, $d(\ )/dt$
$(\ )_x, (\ )_{xx}, (\ )_{xy}$ Subscripts x and y indicate differentiation with respect to x and y
P Distribution of scanning spot intensity
P^* Modified function of spot intensity

Classes of Operations in Two Dimensions

The mathematical formulation of the problem is the following: The image that is being processed is called the "original" picture; the image that results from processing is called the "resulting" picture. The process is completely defined if we know the resulting picture for every conceivable original. Mathematically the process can be characterized by an operator. If the original picture is given as $f(x, y)$ and the resulting picture as $F(x, y)$, where f and F may represent reflective (or transmittance),

$$F(x, y) = \Omega[f(x, y)]$$

Ω is an operator that represents the processing. Ω can be linear or nonlinear, and it can involve derivatives or integrals of the function.

Let us introduce the following definitions:

Definition 1

An operator is said to be homogeneous if the operation is independent with respect to translation of the coordinate system.
If

$$x' = x + \xi$$
$$y' = y + \eta,$$

then the operator is homogeneous if

$$F(x, y) = \Omega[f(x, y)]$$

and

$$F(x', y') = \Omega[f(x', y')].$$

Definition 2

An operator is said to be isotropic if it is homogeneous and the operation is independent with respect to rotation and reflection of the coordinate system.
If

$$x' = x \cos\theta \pm y \sin\theta + \xi$$
$$y' = x \sin\theta \mp y \cos\theta + \eta,$$

then Ω is isotropic if

$$F(x, y) = \Omega[f(x, y)]$$

and

$$F(x', y') = \Omega[f(x', y')].$$

Definition 3

An operator is said to be linear if the operation is distributive with respect to summation and commutative with respect to multiplication by a constant.

Let c_1 and c_2 be arbitrary constants and f_1 and f_2 be arbitrary functions: then Ω is linear if

$$\Omega(c_1f_1 + c_2f_2) = c_1\Omega(f_1) + c_2\Omega(f_2).$$

Although isotropy implies homogeneity, the reverse is not true. Linearity, however, is an independent property and may or may not coincide with homogeneity or isotropy.

Another classification of operators can be made if we consider whether derivatives or integrals are used. This classification, however, is somewhat misleading since with the inclusions of the Dirac delta function, $\delta(x)$ and its derivatives, as possible kernels, even differential operators can be written in disguise of integral operators.

The operators involved in our present study are generally homogeneous and isotropic operators. This is natural since it corresponds to the properties of vision in that the resulting picture remains the same when we shift or turn the original picture except for the shifting and turning.

If the resulting picture F depends only on the behavior of f in the infinitesimal neighborhood of the point (x, y) we can restrict our attention to differential operators. The most general homogeneous linear differential operator is a linear expansion of the function and its derivatives at the point (x, y) with constant coefficients. (If the homogeneity is not required the coefficients may be functions of x and y.) The space derivatives of order n form a symmetrical tensor of rank n. If isotropy of the operator is also required, the tensors must be isotropic tensors. It is possible to show[5] that only even order derivatives occur in isotropic linear differential operators and they form the Laplacian operator and its repeated forms. Thus the most general linear isotropic operator is given by

$$F(x, y) = Af(x, y) + B\nabla^2 f(x, y) + C\nabla^4 f(x, y) + \cdots$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

If only the first two terms are kept, we have the simplest nontrivial isotropic linear differential operator. With a convenient choice of A and B, we obtain

$$F(x, y) = f(x, y) - \gamma^2\left[\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right]. \tag{1}$$

We shall call this operation "contour enhancement."

We can also form nonlinear differential operators. The isotropic condition permits odd derivatives (e.g. first) only if their even functions occur. Keeping only first derivatives and their simplest even function (square), we obtain the simplest nontrivial essentially nonlinear isotropic differential operator

$$F(x, y) = \text{constant} \times \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]. \tag{2}$$

[5] H. Jeffreys and B. S. Jeffreys, "Mathematical Methods of Physics," Cambridge University Press, Cambridge, Mass., p. 87; 1950.

The resulting picture is the square of the absolute magnitude of the gradient vector of the original $f(x, y)$ and as such its value is invariant with respect to translation or rotation of the co-ordinate system. Since we are dealing with nonlinear operators, we can also have an arbitrary rescaling of the square of the gradient.

Using the operator given by (2), the resulting function is always positive or zero, the latter for areas of constant intensity. High values of F indicate high gradients, and therefore indicate contour regions. The operator can be modified into a genuine outlining operator by clipping the function F so that

$$F^*(x, y) = \begin{cases} 1 & \text{if} \quad F(x, y) > c \\ 0 & \text{if} \quad F(x, y) < c. \end{cases}$$

$F^*(x, y)$ will be a line drawing, indicating the zones of intense gradient. There will be no intermediate shades. Further circuit combinations can produce outlines with constant line thickness that suggest pencil or ink drawings. A quantization of values can be introduced in a similar manner. The contour enhancing operator

$$\Omega = 1 - \gamma^2\nabla^2$$

is a first approximation to an "anti-diffusion" or "deblurring" operator. Let us assume that the original picture $f(x, y)$ is really the result of a degrading process obeying the diffusion equation

$$\frac{\partial f(x, y, t)}{\partial t} = k\nabla^2 f(x, y, t), \tag{3}$$

where k is the (positive) diffusion coefficient and f is also a function of time as well as of the space coordinates. The initial condition is given by

$$f(x, y, 0) = g(x, y),$$

where $g(x, y)$ is the perfect picture. Experimentally we have at our disposal $f(x, y, \tau)$ where τ is the length of diffusion time interval. Expanding $f(x, y, t)$ into a Taylor series around $t = \tau$ we obtain

$$g(x, y) = f - \tau\frac{\partial f}{\partial t} + \frac{\tau^2}{2}\frac{\partial^2 f}{\partial t^2} - \cdots (-1)^n \frac{\tau^n}{n!}\frac{\partial^n f}{\partial t^n} + \cdots. \tag{4}$$

Substituting the Laplacian operator for the time derivatives according to (3), we obtain

$$g(x, y) = f - k\tau\nabla^2 f + \frac{k^2\tau^2}{2}\nabla^4 f \cdots - \frac{\tau^n k^n}{n!}\nabla^{2n} f \cdots. \tag{5}$$

If the series converges (and it certainly does if $g(x, y)$ is a continuous function) the perfect picture can be recovered. The first two terms give the contour enhancing operator (1) with

$$h\tau = \gamma^2.$$

Diffusion blurs a point into a spot with a Gaussian distribution having a standard deviation proportional to

$\gamma=\sqrt{k\tau}$. If blurred spots corresponding to original pin points are known in $f(x, y)$, then γ can be determined. An alternate practical method is to vary γ and find the best enhancement by human judgment.

Conversion of Space Domain to Time Signals by Scanning

It is quite conceivable that an operation can be performed readily if the entire function $f(x, y)$ representing the original picture is stored and is available at all times. However, if one desires to dispense with the storage system and realize at least a class of operators, one may still succeed by selecting an appropriate scanning system.

The scan can be defined in the following manner:

$$x = x(t) \tag{6a}$$

$$y = y(t) \tag{6b}$$

$$f(x, y) = f[x(t), y(t)] = \phi(t) \tag{6c}$$

If the scanning raster is sufficiently dense, and $f(x, y)$ is free of singularities, the function $\phi(t)$ represents the picture.

The following equations apply:

$$\dot{\phi} = \dot{x}f_x + \dot{y}f_y \tag{7a}$$

and

$$\ddot{\phi} = (\dot{x})^2 f_{xx} + \ddot{x}f_x + 2\dot{x}\dot{y}f_{xy} + (\dot{y})^2 f_{yy} + \ddot{y}f_y. \tag{7b}$$

If a scan is chosen so that in four subsequent strokes, denoted A, B, C, and D, a spot on the image is scanned linearly in all four directions—forward and return on both x and y axes—then during

$$\begin{aligned} &\text{A: } \dot{x} = U \quad \text{and} \quad \dot{y} = 0 \\ &\text{B: } \dot{x} = 0 \quad \text{and} \quad \dot{y} = U \\ &\text{C: } \dot{x} = -U \quad \text{and} \quad \dot{y} = 0 \\ &\text{D: } \dot{x} = 0 \quad \text{and} \quad \dot{y} = -U, \end{aligned}$$

where U is a constant velocity. Hence

$$\ddot{x} = \ddot{y} = 0.$$

Taking the average of the four strokes,

$$\ddot{\phi}_{av.} = \frac{U^2}{2} f_{xx} + \frac{U^2}{2} f_{yy} = \frac{U^2}{2} \nabla^2 f. \tag{8}$$

The average of the second derivative with respect to time is proportional to the Laplacian of the function $f(x, y)$.

Similarly, for the squared time derivative,

$$(\dot{\phi})^2 = (\dot{x})^2(f_x)^2 + (\dot{y})^2(f_y)^2 + 2\dot{x}\dot{y}f_x f_y.$$

With above type of scan the average of four strokes is

$$[(\dot{\phi})^2]_{av.} = \frac{U^2}{2}(f_x^2 + f_y^2) = \frac{U^2}{2}(\nabla f)^2. \tag{9}$$

The average squared time derivative of $\phi(t)$ is proportional to the squared gradient of $f(x, y)$.

The above analysis has demonstrated that a linear scan that sweeps any picture element in turn from all four directions with identical velocity can be used to realize certain isotropic operators without the need for rapid access memory storage of the entire picture. The averaging of the four strokes occurs automatically if the scan is sufficiently fast so that the persistence of the phosphor, the time lag of the retina, or the retentivity of the photographic emulsion, provides the averaging.

The velocities of the different strokes and the scanning program may be further modified to obtain the higher order isotropic operators. This is discussed in Appendix III. Of course, repeated processing using intermediate pictures may also be used to realize these operators.

Definition 4:

A scan is isotropic at a point if certain averages formed over all crossings at the point during the entire scan sequence obey the following three conditions:

1. The average first time derivative is zero,
2. The average of the squared first time derivative is proportional to the square of the gradient,
3. The average second derivative is proportional to the Laplacian.

A scan may be isotropic in one region but not everywhere (e.g., a Lissajous figure formed by two sine waves of different frequencies is isotropic at the center but not elsewhere).

Generation of the Special Scan

An isotropic scan, by which the realization of certain important isotropic operators becomes simple, can be performed in several ways.

1. The central portion of a Lissajous figure formed by two sine waves may be employed. However the requirements are approximately satisfied over only a small central portion.
2. An interlacing raster of curves such as cycloids or spirals.[6]
3. A conventional television scan rotated 90 degrees after each frame is completed.
4. Symmetrical triangular waves of slightly different frequencies for the horizontal and vertical deflections. The resulting Lissajous figure consists of straight lines and, if operated slowly, has the appearance of a slowly varying rectangle.

The repetition rate of the raster is held constant by synchronizing the difference frequency of the two waves to the frequency of one of the waves. In this way slow drifts in the oscillator frequency do not cause relatively large changes in the density of scan.

Arithmetic relations between the two scanning frequencies follow. If the respective frequencies of the two triangular waves are f_1 and f_2 and they are commensurable, then at one instant their zero crossings occur simul-

[6] K. R. Wende, RCA, U. S. Patent 2531544, November 28, 1950. A diagonal interlacing scan is described in this paper.

taneously and after a time interval T, the zero crossings again occur simultaneously. Accordingly,

$$T = \frac{n_1}{f_1} = \frac{n_2}{f_2},$$

where n_1 and n_2 are integral numbers and each is the number of full scan cycles during a complete frame cycle. The repetition rate of the entire pattern will be

$$f_0 = \frac{1}{T} = \frac{f_1}{n_1} = \frac{f_2}{n_2} = \frac{f_1 - f_2}{n_1 - n_2}. \tag{10}$$

The difference $n_1 - n_2$ is the number of frame interlaces and this can arbitrarily be made unity. Then

$$n_1 = n_2 + 1$$

and

$$f_0 = f_1 - f_2.$$

A scan in which $n_1 = 5$ and $n_2 = 4$ is illustrated in Fig. 2.

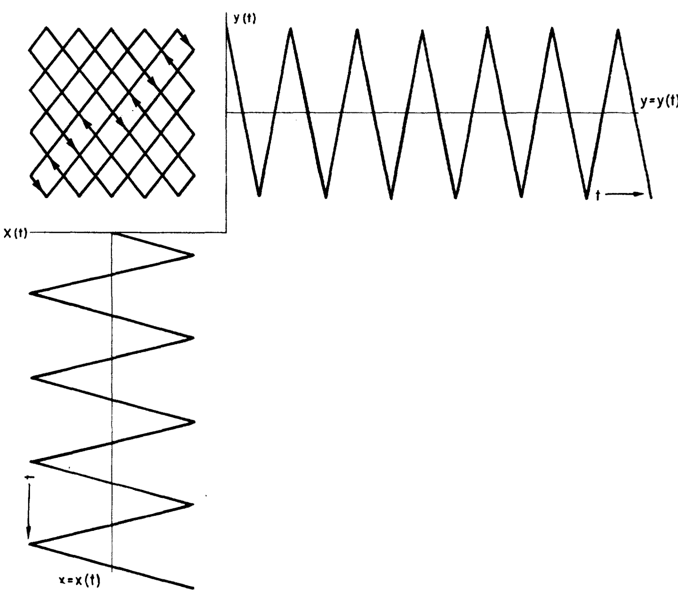

Fig. 2—Isotropic scan formed by triangular waves. Both the horizontal and vertical sweeps are triangular waves. In the present example there are five full waves in the vertical direction for every four full waves in the horizontal direction.

The number of traverses within a fundamental period T should be large and hence both n_1 and n_2 should be comparatively large. In our experiments, for convenient use of binary circuits, the following values were chosen.

$$f_1 = 256 \text{ cps}$$
$$n_2 = 512 = 2^9$$
$$n_1 = 513 = 2^9 + 1$$
$$f_0 = .5 \text{ cps}$$

Experiments

Equipment

A block diagram of the basic equipment used to carry out the experiments is shown in Fig. 3.

The picture generating equipment consists of a flying spot scanner that utilizes feedback around fluorescent screen. Its mode of operation follows.

Corresponding to an instantaneous position of the luminous spot on the scanning tube, a real image of the spot is formed on the transparency. A portion of the light is transmitted through the transparency and condenser lens to the phototube. The amount of light and

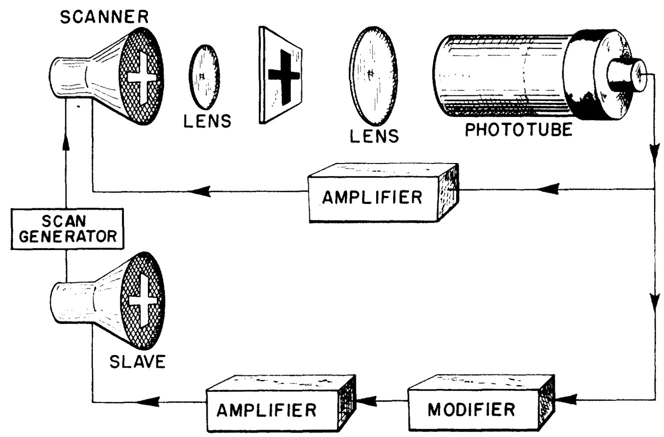

Fig. 3—Block diagram of experimental image processing system. Feedback around the fluorescent screen of the scanning cathode ray tube is accomplished by feeding the amplified phototube signal to the scanner. This signal is also modified and the resulting picture is displayed on the slave cathode-ray tube screen.

hence the phototube signal depends upon the local transmittance of the transparency. The phototube signal is amplified and applied to the scanner tube intensity grid so that it reduces the intensity of the scanning spot.[7] The equilibrium produced is such that for high values of transmittance the spot intensity is much reduced and for completely opaque regions there is no reduction of intensity. The resulting intensity pattern on the scanner face is a negative of the transparency. It is shown in Appendix II that the resulting negative has the tone gradations of a true photographic negative if the amplifier has a high gain (feedback ratio is high). This feedback has advantages similar to negative feedback in amplifiers. The nonlinearity is reduced, the bandwidth is increased, and the effect of phosphor persistence is reduced. The disadvantages of reduced signal amplitude and increased possibilities of oscillation common to systems that employ inverse feedback also occur.

The scan is provided by a scan generator (see block diagram, Fig. 4, page 565). Since two different frequencies, having a well-regulated difference, are required, a two-phase rotating transformer is used to give a uniformly increasing phase lag. As the shaft is rotated, the frequency of rotation is added to the frequency of the input sine wave. In order to reduce resolution errors, the rotating transformer is operated at a frequency 16 times that of the final scan frequency. The master oscillator is usually set at 4 kc, resulting in a scan frequency of about 250 cps. The rotating transformer is driven by a

[7] R. Theile and H. McGhee, "The application of negative feedback to flying spot scanners," *Jour. Brit. IRE*, June, 1952.

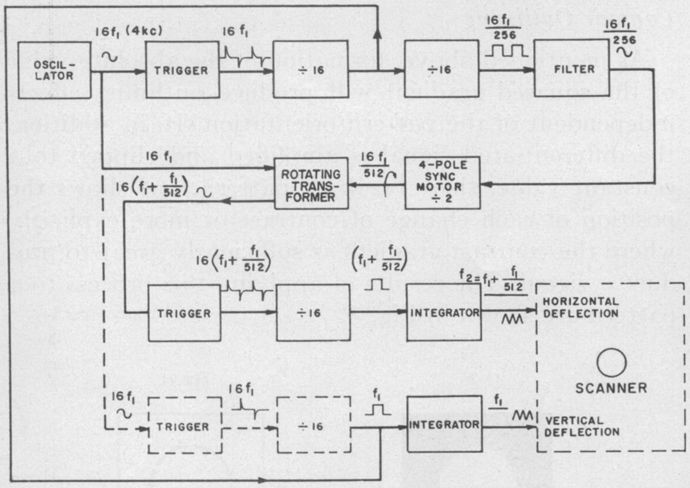

Fig. 4—Block diagram of the scan generator. The dotted line shows alternative connection

synchronous motor, whose input is in the form of a sine wave with a frequency $\frac{1}{16}$ that of the scan frequency. In this manner, the integer ratio relation between the two scan frequencies is maintained. Two square waves, having a small difference in frequency, are formed and the triangular waves are obtained by integration.

The phototube signal is fed through the modifier and an amplifier identical to the scanner amplifier, to a monitor or "slave" cathode-ray tube whose sweep is identical to that of the scanner. If the signal is fed to the monitor without modification, the monitor screen pattern will be identical to that of the scanner.

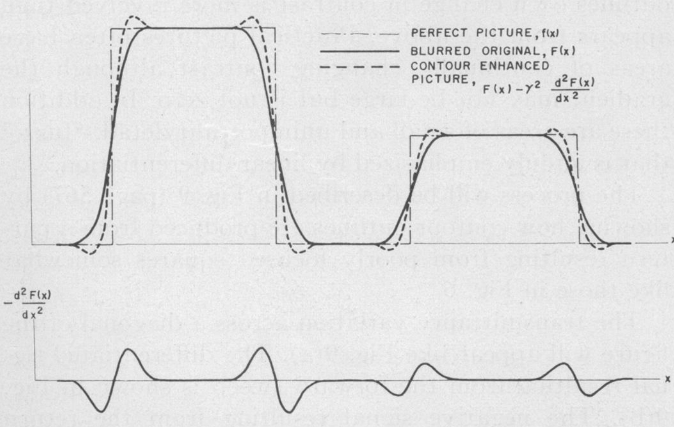

Fig. 5—Principle of contour enhancement. The upper part shows three pictures functions: one is the perfect picture signal, the second is the picture signal from the degraded original, and the third shows a partial restoration towards the perfect picture signal. The lower figure shows the negative second derivative of the original signal that was added as a correction signal to the signal from the degraded original.

Contour Enhancement

The deblurring operator was given before, in (1). With the use of the isotropic scan, the operation involves adding a certain amount of negative second time derivative to the picture signal [see (8)]. A diagrammatic explanation of contour enhancement is given in Fig. 5.

The results of applying contour enhancement are shown in two sets of pictures shown in Fig. 6. Fig. 6(a) and 6(d) show the pictures reproduced by the video signal. Fig. 6(b) and 6(e) show the pictures produced by the negative of the doubly differentiated signal. Fig. 6(c) and 6(f) are the result of subtracting a portion of the doubly differentiated signal from the video signal.

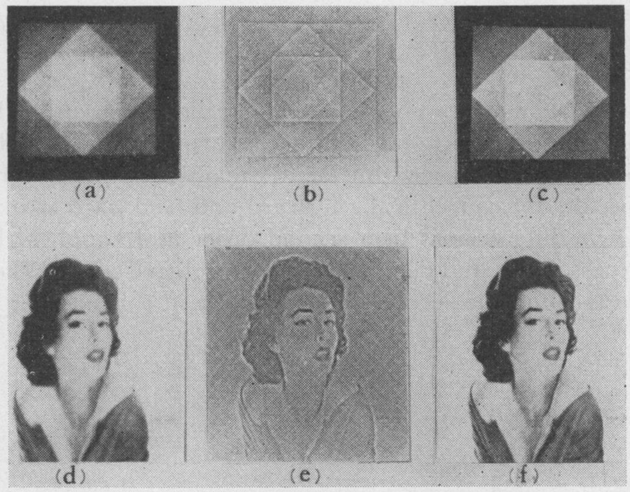

Fig. 6—Experiments in contour enhancement. (a) Blurred original picture, (b) correcting signal (negative Laplacian), (c) enhanced resulting picture, (d) blurred original picture, (e) correcting signal (negative Laplacian), (f) enhanced resulting picture.

The special scan offers some saving in circuits for obtaining the second derivative. When every point is scanned both in the positive and negative directions the resulting picture is formed by the arithmetic average of the two video signals in the two strokes. The sign of the first derivative (and every odd derivative) changes with the reversal of scan direction (Fig. 7, next page), but the second derivative (and every even derivative) remains the same for both scan directions. In practical application the differentiating circuit has a time constant $\tau = RC$. The operators to be considered follow:

(a) Perfect first derivative: $j\omega$

(b) Perfect second derivative: $-\omega^2$

(c) Practical first derivative (by $RC = \tau$ circuit)

$$\frac{i\omega\tau}{1 + j\omega\tau}$$

(d) Practical second derivative (by cascading two differentiating circuits)

$$\frac{-\omega^2\tau^2}{1 - \omega^2\tau^2 + 2j\omega\tau}$$

[Average of two opposite scan strokes eliminate the out-of-phase (imaginary) components]

(e) Average perfect first derivative: 0

(f) Average perfect second derivative: $-\omega^2$

(g) Average practical second derivative by cascading two differentiating circuits

$$\frac{-\omega^2\tau^2(1 - \omega^2\tau^2)}{(1 + \omega^2\tau^2)^2}$$

(h) Practical second derivative by averaging the practical first derivatives

$$\frac{\omega^2\tau^2}{1+\omega^2\tau^2}$$

It is plain that (g) is inferior to (h) by a factor

$$\frac{1-\omega^2\tau^2}{1+\omega^2\tau^2},$$

and (h) involves only one differentiating circuit. The physical explanation is simple: the "practical" first derivatives have some time lag of the order of τ and therefore do not cancel in the forward and backward stroke. Their difference is of a second order and supplies the necessary second derivative (with negative sign). When performing the second differentiation in this manner in two mutually perpendicular directions we obtain the Laplacian operator. However, this short cut method for obtaining the second spatial derivatives does not exist for the conventional television scan.

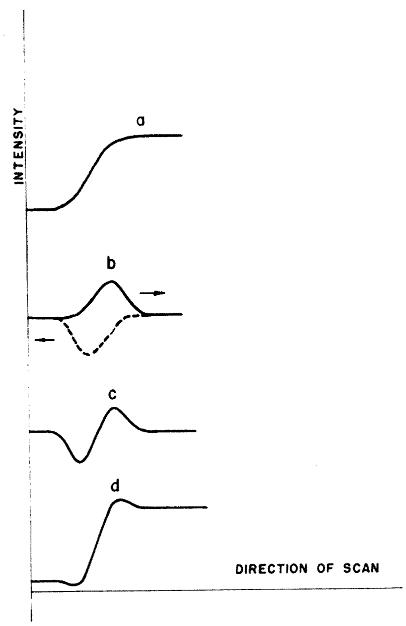

Fig. 7—Practical realization of contour enhancement by one differentiation only. (a) A somewhat degraded picture signal of a transition between two levels. (b) The derivative of this signal, but delayed slightly in the direction of scan. The dotted line refers to the return signal. (c) The algebraic sum of the two signals shown in (b). This is approximately equal to an undelayed negative second derivative signal. (d) The sum of the signal shown in (c) and that shown in (a).

The actual contour enhanced pictures (Fig. 6), were obtained by this latter method since it required less gain in the system and therefore fewer stages.

There were previous attempts to improve a degraded television picture by adding a suitable compensating signal by such methods as "crispening."[8] However the application of these methods, is limited by the unidirectional character of the conventional television scan.

[8] P. C. Goldmark and J. M. Hollywood, "A new technique for improving the sharpness of pictures," PROC. I.R.E., vol. 39, p. 1314; October, 1951.

Contour Outlining

As mentioned above, formation of the absolute value of the squared gradient will produce outlining effects independent of the pattern orientation. If, in addition, the differentiated signal is amplified and clipped to a constant value, there results a pattern that shows the position of each change of contrast or more explicitly where the contrast gradient is sufficiently great to produce a signal. The results of applying this process to a pattern are shown in Fig. 8.

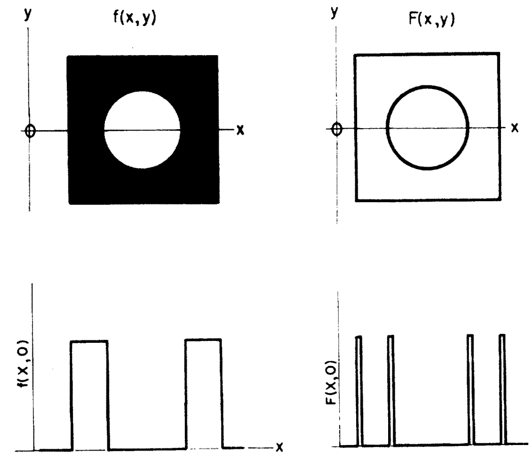

Fig. 8—Principle of contour outlining. The upper part shows a picture and its derived outline. The lower part shows the corresponding video signals across a single scan ($y = 0$).

The practical procedure for producing a signal that outlines each change in contrast is more involved than appears from the above. Practical pictures often have areas of continually changing contrast although the gradient may not be large but is not zero. In addition there are areas of small and unimportant detail, "fuzz," that is unduly emphasized by linear differentiation.

The process will be described in Fig. 9 (page 567) by showing how contour outlines are produced from a pattern resulting from poorly focused squares somewhat like those in Fig. 6.

The transmittance variation across a diagonal of the figure will appear like Fig. 9(a). The differentiated signal resulting from the forward sweep is shown in Fig. 9(b). The negative signal resulting from the return sweep is suppressed by a half-wave rectifier; also the threshold of the rectifier is set so as to suppress the small signal "fuzz."

The signal of Fig. 9(c) is differentiated again to form the doubly differentiated signal shown in Fig. 9(d). This signal is applied in turn to a trigger circuit (which responds to positive signals) to form the result shown in Fig. 9(e).

It will be noted that the trailing edges of the square waves correspond to the appropriate position on the center of the changes in Fig. 9(a). These square waves are differentiated, rectified, and then inverted, to form the result shown in Fig. 9(f). The position of the pulse is indicated by the pulse on the baseline of Fig. 9(a).

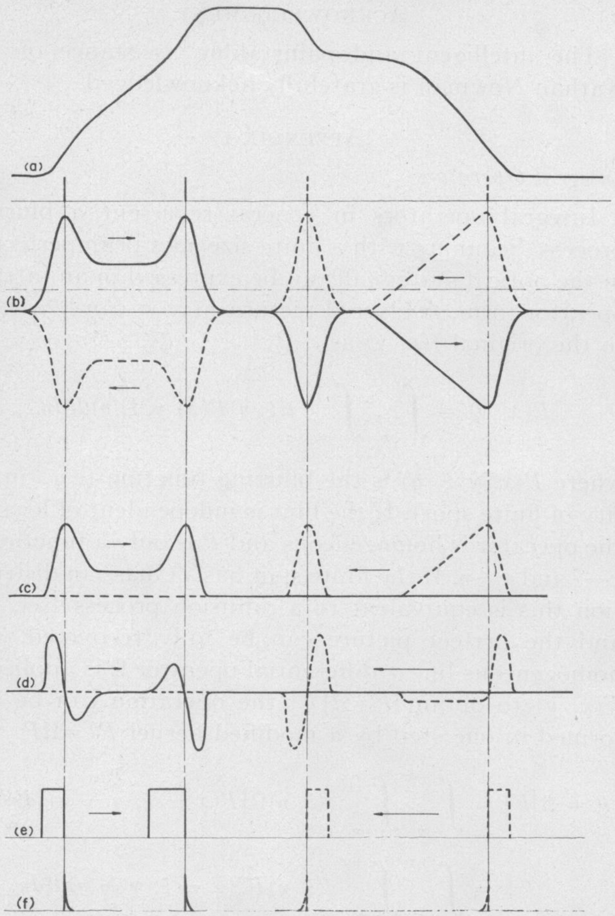

Fig. 9—Practical contour outlining procedure. The procedure consists of obtaining an outline at the points where the slope of the video signal has a local maximum exceeding a threshold value. The steps are indicated below. The forward scan is shown in solid lines and the return traces with dotted lines. (a) Original picture signal, (b) first derivative, (c) rectification with bias, (d) second derivative (when first derivative exceeds threshold), (e) trigger driven by (d), (f) pulses, differentiated and rectified (e).

The picture that is obtained appears to be an outline drawing drawn with a pencil whose width is determined by the pulse width that appears on the screen. This can be varied by changing the time constant and amplitude of the last differentiating stage. Fig. 10 shows outlines made from the same originals as in Fig. 6.

There are some errors, due to the combined delays of the differentiating circuits, in the time taken for the pulse to rise to the triggering value, and for the delay in the trigger circuit itself. This last depends upon the amplitude of the trigger signal and hence is not constant, but it can be minimized. All the delays but the last could be compensated for by delaying the monitor sweep for the appropriate time interval.[8] This sweep delay may permit the use of larger values of differentiating time constants (τ) and hence require less amplification after differentiation. Therefore greater freedom from noise may be obtained.

The effect of these delays upon the resulting picture is determined by the experimental procedure. In the processes indicated in Fig. 9 delays in the differentiating will have the effect of shifting the outlining pulses toward lighter or darker regions, depending upon the polarity of the picture signal. Therefore, using a positive or a negative transparency as the original will result in contours of somewhat different location. A calibration of the system can be performed by outlining both a positive and a negative picture. When identical results occur the system is in proper alignment.

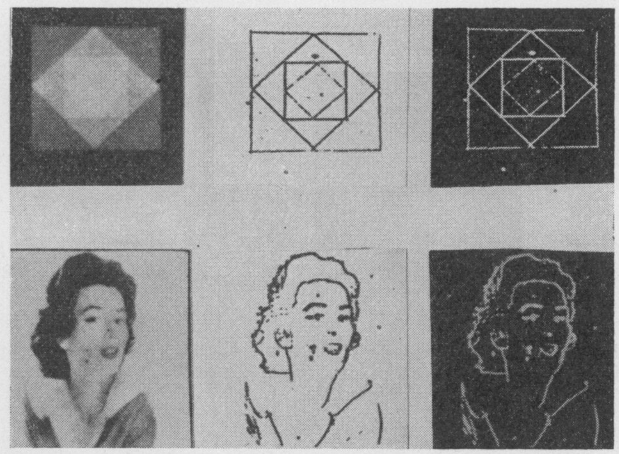

Fig. 10—Experiments in contour outlining. The originals have been processed to make the outlines. The monitor shows the outlines as light lines on a dark background. The dark line pictures are obtained by a photographic reversal.

Further Applications

The investigation so far has covered the possibilities of some elementary processes applied to an image.

Conversion of the image to a time-varying signal permits considerable flexibility in the reformation of the pattern. The co-ordinate measure numbers of the two-dimensional, monitor screen picture can be varied at will within fairly wide limits. The resulting pattern can be stretched, warped, changed in position, rotated with respect to the original pattern, or interchanged about an arbitrary axis by using negative measure numbers.

This suggests that recognition by matching to a known pattern or to a given mathematical formula may be possible since distortion can be tried easily. The Gestalt problem may be illuminated by such a method. Possibly automatic devices for recognizing patterns or shapes can be constructed.

The potential uses of the contour enhancement process may include transmission of an image with less impairment through a lesser bandwidth channel or to pre-emphasize a picture that is to be sent through a low definition channel, in a manner analogous to pre-emphasis in an audio amplifier for the purpose of improving the response at higher frequencies. This apparent improvement may be made on inherently low definition patterns such as an X-ray picture of soft tissues to make the detail more apparent, that is, more quickly visible. An X-ray picture, Fig. 11(a), processed in this manner is shown in Fig. 11(b) on the next page.

The contour outlining may be a means of making a sketch map of terrain, or it may be used as a means of reducing the information content to lower the bandwidth requirements for transmission or for the recognition mentioned above. In addition this method may be useful for establishing the position of the contour. For instance, the contour of the heart may be set by the contour outline process without human error, a matter of some importance in measuring the heart volume. This may be especially significant when large amounts of data must be processed.

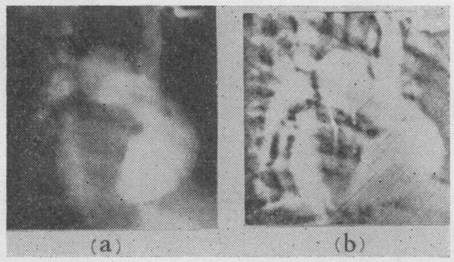

Fig. 11—Contour enhanced by X-ray picture. Left—photographic copy of original heart X-ray. Right—contour enhanced picture of the same.

An interesting further application is the use of the scan lines in order to simulate the method of characteristics used for solving hyperbolic partial differential equations. By using the special scan, but only two perpendicular strokes without their return sweeps, we obtain two sets of characteristics

$$x + y = \text{constant}$$

and

$$x - y = \text{constant}.$$

A solution of the hyperbolic differential equation

$$\frac{\partial^2 F}{\partial x^2} - \frac{\partial^2 F}{\partial y^2} = 0$$

is

$$F(x, y) = A(x + y) + B(x - y),$$

where A and B are arbitrary functions of their arguments.

If the video signal is provided by integration, F will be a constant along characteristics as long as the integrand is zero. This fact may be used to generate the functions $A(x+y)$ and $B(x-y)$ by integrating an original cutout picture that will impose the boundary conditions. Away from the boundaries the integral will be constant at the value it attained at the boundary. Thus it represents a solution of the form given above. Naturally a reset or clamp is necessary so that the integral value may be reset when the pattern is scanned again.

The method may be extended to nonlinear hyperbolic differential equations if the scan angle is modified by the video signal itself. These further lines of research will be reported later.

Acknowledgment

The intelligent and painstaking assistance of Mr. Nathan Newman is gratefully acknowledged.

Appendix I

Integral Operators

Integral operators in general represent a blurring process. Scanning with a finite size spot or imperfection in the optical system all can be expressed in an integral operator form. A blurred picture $F(x, y)$ can be related to the original $f(x, y)$ as

$$F(x, y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(\xi, \eta) P(x, y, \xi, \eta) d\xi d\eta, \tag{11}$$

where $P(x, y, \xi, \eta)$ is the blurring function (e.g., intensity of finite spot). If the blur is independent of location the operator is homogeneous and P is only a function of $x-\xi$ and $y-\eta$. If the finite spot has a Guassian distribution this is equivalent to a diffusion process [see (3)] and the perfect picture can be fully recovered. If a homogeneous linear differential operator Ω is applied to $F(x, y)$ to obtain $F = \Omega[F]$ the operation can be performed in one step by a modified kernel $P^* = \Omega P$

$$\begin{aligned} \widehat{F} = \Omega[F] &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(\xi, \eta) \Omega[P(x - \xi, y - \eta)] d\xi d\eta \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(\xi, \eta) P^*(x - \xi, y - \eta) d\xi d\eta \end{aligned} \tag{12}$$

with

$$P^*(x - \xi, y - \eta) = \Omega[P(x - \xi, y - \eta)].$$

Knowing the general nature of the blur, P, one can always decide how much improvement can be obtained by applying a contour enhancing operator. A generalization of filter concepts applied to blurring and aperture correction has been made.[9]

A special case occurs when P^* becomes a delta function.

$$\Omega P = P^* = \delta(x - \xi)\delta(y - \eta)$$

then

$$\widehat{F} = \Omega F = f(x, y),$$

and the picture is exactly deblurred. In such a case, P must be an elementary solution of differential equation

$$\Omega P = 0. \tag{13}$$

For the contour enhancing operator in one dimension, as in (13),

$$P(x) - \gamma^2 \frac{\partial^2 P}{\partial x^2} = 0,$$

the special solution for $P(x)$ is

$$P(x) = \tfrac{1}{2}\epsilon^{-1x-\xi/\gamma}.$$

[9] Otto Schade, "Electro-optical characteristics of television systems," *RCA Rev.*, vol. 9, pp. 5–37, 245–286, 490–530, 653–686; 1948.

Naturally this is not a very realistic blur distribution, it is only indicative of the feasibility of perfect deblurring. (But in two dimensions operator $1-\gamma^2\nabla^2$ corresponds to a P with logarithmic singularity at origin.)

If, however, the contour-enhancing operator is extended to include the bi-Laplacian, more realistic blur distributions can be found (such that the original can be more nearly recovered), than with a blur distribution that uses just the Laplacian.

Appendix II

Negative Feedback Analysis on a Flying Spot Scanner

It is assumed that the cathode ray tube intensity grid voltage E_g is measured from the point of extinction (cutoff) and that the luminous intensity I is either proportional to the grid voltage (linear assumption) or to the square of the grid voltage (square law assumption). The former is a better approximation for high, the latter for low intensities.

For the linear assumption,

$$\frac{I}{I_0} = \frac{E_g}{E_0}, \tag{14}$$

For the square law assumption,

$$\frac{I}{I_0} = \frac{E_g{}^2}{E_0{}^2}. \tag{15}$$

where I_0 and E_0 are corresponding values of I and E_g when no feedback signal is applied. The feedback voltage e is obtained from the photo multiplier through the amplifier.

$$-e = \text{constant} \times rI \quad 0 < r < 1, \tag{16}$$

where $r(x, y)$ is the transmittance of the inserted (negative) slide. For convenience introduce the feedback ratio K by rewriting (16) as follows:

$$-e = \frac{KrI}{I_0} E_0. \tag{16a}$$

The grid voltage is originally set at E_0 and with feedback it now becomes

$$E_g = E_0 - e. \tag{16b}$$

Using (14), (15), (16a) and (16b) to solve for I/I_0, we obtain the following: with linear assumption

$$\frac{I(x, y)}{I_0} = \frac{1}{Kr(x, y)} \cdot \frac{Kr(x, y)}{1 + Kr(x, y)}; \tag{17}$$

with square law assumption,

$$\frac{I(x, y)}{I_0} = \frac{1}{Kr(x, y)}\left[1 - \frac{1 - \sqrt{4Kr(x, y) + 1}}{2Kr(x, y)}\right]; \tag{18}$$

in both cases,

$$\frac{I}{I_0} \to \frac{1}{Kr} \quad \text{when } Kr \to \infty.$$

The image appearing on the screen and the original negative placed into the focal plane of the scanner have a relationship very similar to that of a positive transparency and its original negative. This indicates that the function $f(x, y)$ has to be fed into the equipment in the form of a negative (with "gamma" equal unity). This is not an inconvenience. On the contrary, it makes recycling, for instance, much easier. Using photographic convention, we can state that the equipment behaves like a negative emulsion characterized by the density vs illumination curve

$$D = -\log \frac{I}{I_0} = \phi(\log r).$$

It can be plotted using (17) or (18). The contrast is measured by

$$\text{"gamma"} = -\frac{\partial \log I}{\partial \log r}.$$

For square law assumption,

$$\frac{\partial \log I}{\partial \log r} = -\left(1 - \frac{1}{\sqrt{1 + 4Kr}}\right) = -\left(1 - \sqrt{\frac{I}{I_0}}\right); \tag{19}$$

or, for linear assumption,

$$\frac{\partial \log I}{\partial \log r} = -\frac{Kr}{1 + Kr} = -\left(1 - \frac{I}{I_0}\right). \tag{20}$$

The negative sign indicates a negative process and for large feedback ratios the gamma becomes unity.

Contrast can be even boosted at the dark end of scale by operating monitor at lower (dc) intensity.

Let

I_m = the luminous intensity on the monitor,

and

μE_0 = the voltage setting of the monitor with no video signal.

Then, using the square law assumption, the ratio of intensities on the monitor and scanner becomes

$$\frac{I_m}{I_0} = \left(\frac{\mu E_0 - e}{E_0}\right) = \left[\sqrt{\frac{I}{I_0}} - (1 - \mu)\right]^2, \tag{21}$$

and the gamma of the modification becomes

$$\frac{\partial \log I_m}{\partial \log I} = \sqrt{\frac{I}{I_m}} = \frac{1}{1 - (1 - \mu)\sqrt{\frac{I_0}{I}}}. \tag{22}$$

For $\mu < 1$, the contrast increases and at the point of extinction becomes infinite. Naturally, with the linear assumption, similar results can be obtained.

Appendix III

The method of forming the Laplacian [see (8)], by averaging the second derivative can be extended to the bi-Laplacian operator

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}. \tag{23}$$

Using a scan pattern that consists of the present scan pattern (Fig. 2) but applying it twice: the first time as it is and the second time rotated by a 45 degree angle, we obtain a scan raster that consists of eight strokes symmetrically distributed. Using eight frames of conventional television scan each rotated by 45 degrees with respect to the previous one would be equally acceptable.

If the scanning spot moves with a uniform velocity in the direction forming an angle θ with the x-axis.

$$\dot{x} = U\cos\theta; \quad \dot{y} = U\sin\theta; \quad \ddot{x} = \ddot{y} = 0.$$

The fourth time derivative of the video signal becomes

$$\ddddot{\phi} = U^4\left[\frac{\partial^4 f}{\partial x^4}\cos^4\theta + 4\frac{\partial^4 f}{\partial x^3 \partial y}\cos^3\theta\sin\theta + 6\frac{\partial^4 f}{\partial x^2 \partial y^2}\cos^2\theta\sin^2\theta + 4\frac{\partial^4 f}{\partial x \partial y^3}\cos\theta\sin^3\theta + \frac{\partial^4 f}{\partial y^4}\sin^4\theta\right]. \tag{24}$$

If we take eight strokes so that in turn

$$\theta = 0,\ \frac{\pi}{4},\ \frac{2\pi}{4},\ \frac{3\pi}{4},\ \frac{4\pi}{4},\ \frac{5\pi}{4},\ \frac{6\pi}{4},\ \frac{7\pi}{4};$$

the average fourth time derivative becomes

$$\langle\ddddot{\phi}\rangle_{av} = \frac{3}{8}U^4\left[\frac{\partial^4 f}{\partial x^4} + 2\frac{\partial^4 f}{\partial x^2 \partial y^2} + \frac{\partial^4 f}{\partial y^4}\right] = \frac{3}{8}U^4\nabla^4 f. \tag{25}$$

Naturally a continuously rotating scan pattern (either conventional television or our special scan) would also be acceptable for both the Laplacian and bi-Laplacian if the average occurs over an entire cycle. It is conceivable that by similar methods, higher order Laplacian operators may be obtained from the corresponding orders of time derivative.

Pulse-Switching Circuits Using Magnetic Cores*

M. KARNAUGH†

Summary—**The synthesis of a large class of pulse-operated magnetic logic and switching circuits is developed from basic principles. Simple design methods, as well as circuit organization and logic, are treated in detail. Some new circuit types are presented along with innovations in notation and design procedure. However, material from other sources is included for the sake of completeness. A bibliography of related works is appended.**

Introduction

MAGNETIC CORES having nearly rectangular hysteresis loops seem destined to become important components of digital computers. They offer unique reliability, versatility, and economy, for applications which do not require extremes of high speed or low-power dissipation.

Considerable work has already been done in building both large scale and small scale memories with such cores. They have also been employed in logic and switching circuits, but a great deal remains to be accomplished in this field.

The material that follows deals with the latter use. The treatment is intended to be fairly general, and certain special techniques, dealt with elsewhere in the literature, will be omitted. The most notable omissions are the use of nonmagnetic interstage delays,[1] the standardized magnetic decision elements,[2] and the coincident-current selection schemes.[3]

Also, the design of the necessary pulse generating circuits will not be discussed. It will be assumed that sources of current pulses are available to power the circuits and that the current waveforms are rectangular unless otherwise specified. While rounded pulses may also be used, this assumption will simplify the exposition.

The Switching Process

The magnetic cores currently available which have the most desirable characteristics are toroidal in form, consisting either of ceramic ferrite material or of ultrathin ferromagnetic alloy tape wound on a nonferromag-

* Original manuscript received by the IRE, January 19, 1955.
† Bell Telephone Labs., Murray Hill, N. J.

[1] See bibl. ref. 14, 15.
[2] See bibl. ref. 18.
[3] See bibl. ref. 19–29.