Phase Velocity Tomography

Reporter: Lun Zhang **Date:** Dec. 26, 2018

1. Method (Least square inversion)

(1) Design Matrix Computation (compute_Gik_ray.m)

$$G_{ik} = (-1/c) \int_{\mathrm{ray}_i} B_k \, \mathrm{d}s.$$

Note that in the integration process I sampled 1000 points at which Bk was evaluated on the path evenly.

(2) Inversion and Iteration (tomography_hw.m)

$$\delta \mathbf{m} = (\mathbf{G}^T \mathbf{G} + \lambda^2 \mathbf{I}) \mathbf{G}^T \mathbf{d}$$

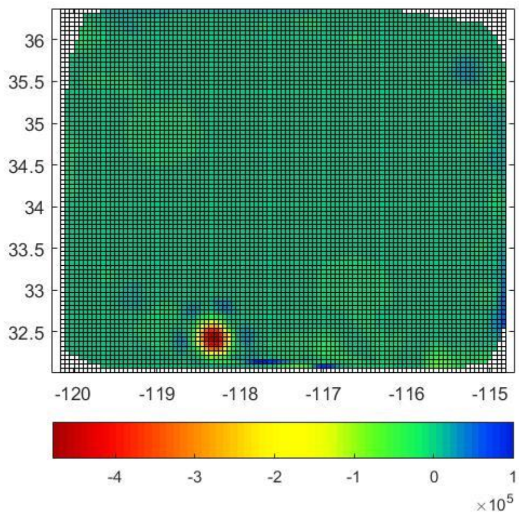
$$\delta \ln c(\mathbf{x}) = \sum_{k=1}^{M} \delta m_k \, B_k(\mathbf{x})$$

Iteration for **5** times

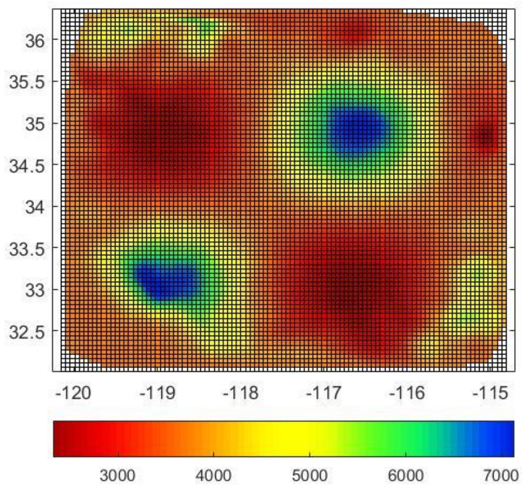
In the iteration process I used cubic interpolation for producing dense grids, which were used for c0 evaluation in next integration process.

2. Results

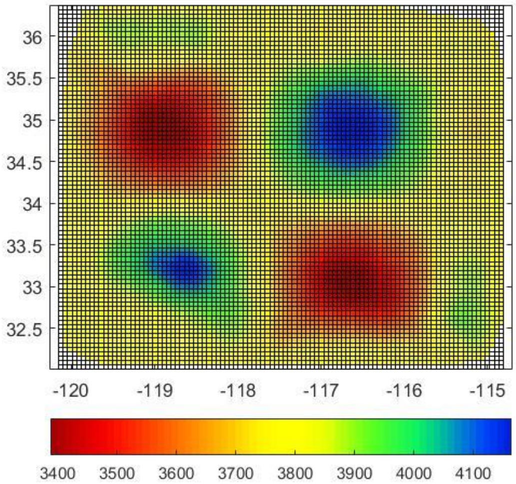
(1) Lamda = 0.1



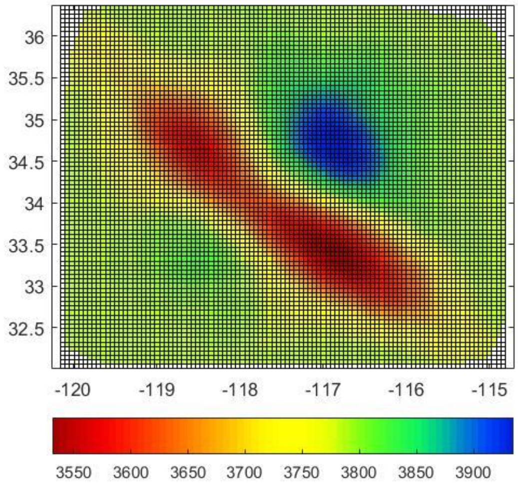
(2) Lamda = 1



(3) Lamda = 40



(4) Lamda = 500



3. Discussion

(1) Rms trend

Lamda = 0.1:
2.0388 -> 2.0388 -> 2.0387 -> 2.0384 -> 2.0383
Lamda = 1:
2.0387 -> 2.0387 -> 2.0387 -> 2.0386 -> 2.0386
Lamda = 40:
2.004 -> 1.9737 -> 1.9457 -> 1.919 -> 1.8928
Lamda = 500:
0.79231 -> 0.39679 -> 0.22698 -> 0.13776 -> 0.08573

As lamda (>1) increases, rms decreases.

(2) Eigenvalue of G^TG+lamda^{2}I

While lamda is small, the variance of eigenvalues seems to be huge (the dispersion order can reach to 10^6 in ‘Lamda = 0.1’ case.

In contrast, lager lamda produces lower variance of eigenvalues, the order of them seems homogenous.

(3) Resolution Matrix

While lamda is small, some diagonal elements of resolution matrix seem to be normal (near 1), others approach 0, the variance is large, the result resolution is high but the distribution is not so reasonable.

In contrast, larger lamda produces relatively homogenous diagonal elements but with low values (near 10^{-6} order in 'Lamda = 500' case), so the resolution reduces as a result of smoothing. The result seems relatively reasonable with proper lamda like 10.

(4) Velocity variance

As lamda increases, the variance and dispersion of result phase velocity turns to decrease.

4. Complemental questions in assignment

(1) What is the value of the q = 8 spline function that is centered at (longitude, latitude) = (−117°; 34°), and evaluated at the point (−117.05°; 34.25°) ?

```
>> spline_vals(-117,34,8,-117.05,34.25,{1})
ans =
    0.2334
```

(2) write the expression for the G_{ik} element of the design matrix G.

G_{ik} = sum(spline_vals(B_lon(k),B_lat(k),q,line_lon,line_lat))*ds*1000/c0

(3) What is the value of G_{ik} with i = 126 and k = 204? What does each row of G correspond to? What does each column of G correspond to?

$G_{126,204}$ = -9.5944s
Row -> integration of 268 B spline functions respectively on a given ray path
Column -> integration of 1 B spline function on each ray path respectively

(4) Write the symbolic expression for the solution m to the least squares problem Gm = d in the case where G^TG is full rank. List the dimensions of m, d, G, G^TG, and G^Td.

$m = (G^TG)^{-1}G^Td$
m: 268X1
d: 3300X1
G: 3300X286
G^TG: 286X286
G^Td: 286X1

(5) Compute m using the formula above. What do you get, and why is this the case?

Many elements in deltam is Nan, because the G^TG is near singular matrix, whose generalized inverse matrix has large variance, which weakened the reasonability of deltam.

(6) What are two ways to stabilize the inverse problem, given total control over the experimental design? (Hint: How can we construct a matrix G^TG with fewer zeros?)

Expand the cell size and reduce the number of cells.

Collect more events with various propagation direction for better coverage, then add them to original data.