

Final Project

Due date: December 20, 2018

Seismic tomography using ray theory

The objective of this assignment is to design a synthetic seismic tomography experiment and to familiarize yourselves with the fundamental aspects of a tomographic inversion: model parameterization, construction of the design matrix for the forward problem, and solving the least-squares problem using damping. The practical aspects of the assignment are entirely based on Matlab, and several scripts are provided in the package, `tomography.tar.gz`. Extract the package using following command in the terminal:

```
tar -zxvf FinalProject.tar.gz
```

or

use any unzip applications.

The phase velocity map and the source-receiver geometry for this problem are shown in Figure 1.

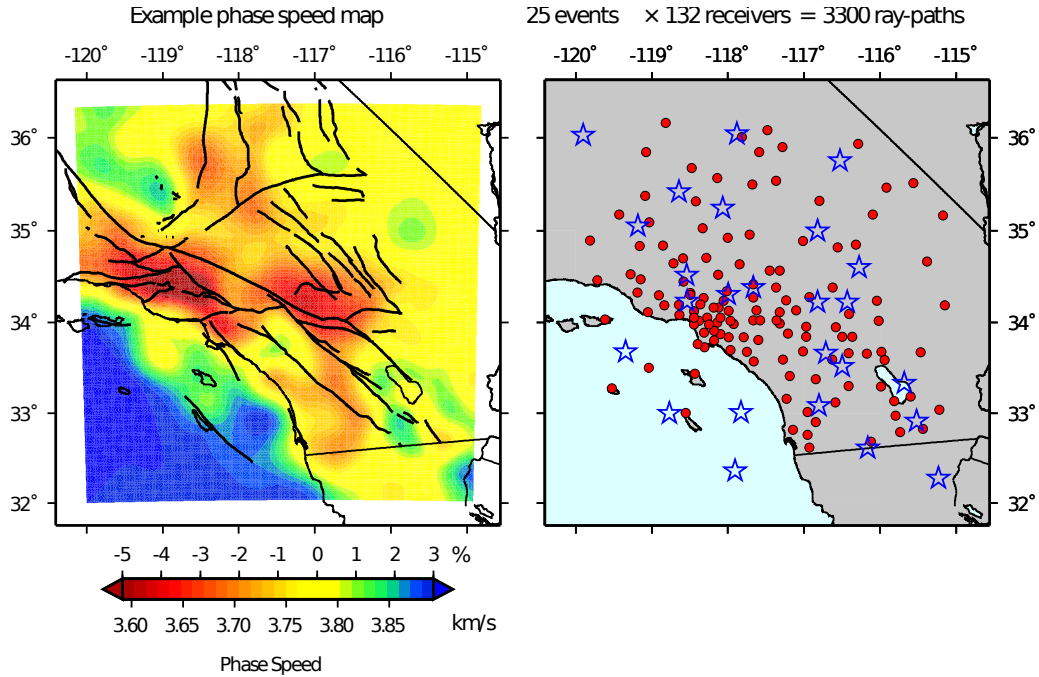


Figure 1: *Left*: Example phase velocity map and faults for Southern California. The phase velocity map is for Rayleigh waves with periods of 20 seconds. *Right*: Source-receiver geometry for the problem in this homework. The \star symbols denote the locations of 25 earthquakes; the \circ symbols denote the locations of 132 broadband receivers in the Southern California Seismic Network.

Some relevant references are listed at the end for additional information.

1. Forward problem: constructing the design matrix.

- (a) *Model parameterization.* A tomographic model is typically described in terms of a perturbation in wave speed structure from some reference model, i.e., $\delta \ln c = \frac{\delta c}{c}$. This function may be expanded in terms of a set of M basis functions:

$$\delta \ln c(\mathbf{x}) = \sum_{k=1}^M \delta m_k B_k(\mathbf{x}) \quad , \quad (1)$$

where, δm_k , $k = 1, \dots, M$, represent the perturbed model coefficients. Here we choose to use the spherical spline basis functions [4, 5]. First, familiarize yourself with these basis functions by running the Matlab script `test_spline_vals.m`, which generates a randomly chosen location for a spherical spline basis function of grid order q . Change the value for q and see what happens. An example of a $q = 8$ basis function is shown in Figure 2b. What is the value of the $q = 8$ spline function that is centered at (longitude, latitude) = $(-117^\circ, 34^\circ)$, and evaluated at the point $(-117.05^\circ, 34.25^\circ)$?

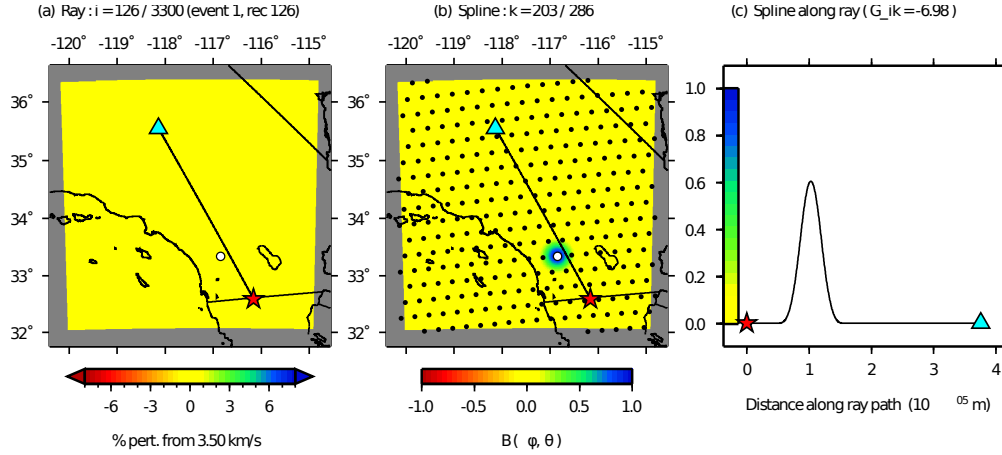


Figure 2: Example computation for an element, G_{ik} , of the design matrix \mathbf{G} , using rays. The row index i is the source-receiver combination, the column index k is the basis function index. The source is denoted by the \star , the receiver is denoted by the Δ , and the \circ shows the center-point of the spherical spline in (b). (a) Ray path for event number 1 and receiver number 126, corresponding to the $i = 126$ index of the $N = 3300$ ray paths. (b) $B_{203}(\mathbf{x})$, the spherical spline basis function for index $k = 203$. Also shown are the center-points of the $M = 286$ spherical splines. (c) Spline B_{203} evaluated along the ray path. The value of the phase speed for the reference model is constant, so $G_{ik} = (-1/c) \int_{\text{ray}_i} B_k ds$. In this example $G_{ik} = 1/(3500 \text{ m s}^{-1})(2.45 \times 10^4 \text{ m}) = -6.98 \text{ s}$.

- (b) *Formulate an entry of the design matrix.* From the class notes, the ray theory traveltime perturbation for the i th source-receiver combination is given by

$$\delta T_i = - \int_{\text{ray}_i} c^{-1} \delta \ln c ds \quad , \quad (2)$$

where, ds denotes a segment of the i th ray. Show how this leads to $\mathbf{d} = \mathbf{G} \delta \mathbf{m}$, where the data vector is given by

$$\mathbf{d} = (\delta T_1, \dots, \delta T_i, \dots, T_N)^T \quad , \quad (3)$$

and write the expression for the G_{ik} element of the design matrix \mathbf{G} .

- (c) *Compute an entry of the design matrix.* Now compute the $N \times M$ design matrix \mathbf{G} in Matlab. A template program `compute_Gik_ray.m` should help you to start with. Take into account the following details:

- Assume a homogeneous reference phase speed model with $c_0 = 3500 \text{ m/s}$.

- Because the reference model is homogeneous, the ray paths are simply great circles. Use the built-in Matlab function `gcwaypts` to compute the ray paths, and use approximately a few hundred points to effectively describe the ray path. The function `arcdist.m` computes the great-circle distance (in degrees) between two points on a sphere.
- Use the source-receiver ordering (see also Figure 1b) given by

$$i = (\text{isrc} - 1) * \text{nrec} + \text{irec} \quad .$$

Here index i is the row index (or measurement index), isrc is the source index, and irec is the receiver index. For example, row $i = 3293$ of \mathbf{G} (and \mathbf{d}) corresponds to the pairing of source 25 with receiver 125.

- Use $M = 286$ spherical spline basis functions of order $q = 8$; the center-points are in the data file `con_lonlat_q08.dat`.

What is the value of G_{ik} with $i = 126$ and $k = 204$? What does each row of \mathbf{G} correspond to? What does each column of \mathbf{G} correspond to?

- Save the design matrix (e.g., save G_{ik}), so that you can readily load it (e.g., load G_{ik}) for the next problem.

2. Inverse problem: solving for the unknown velocity structure.

At this point, you should have saved your design matrix. Now open the template file `tomography_hw.m`.

- Write the symbolic expression for the solution $\delta\mathbf{m}$ to the least squares problem $\mathbf{G}\delta\mathbf{m} = \mathbf{d}$ in the case where $\mathbf{G}^T\mathbf{G}$ is full rank. List the dimensions of $\delta\mathbf{m}$, \mathbf{d} , \mathbf{G} , $\mathbf{G}^T\mathbf{G}$, and $\mathbf{G}^T\mathbf{d}$.
- In order to do the inverse problem, you will need the data vector of traveltime measurements (Equation 3). Load the column vector `measure_vec.dat` into Matlab, and check that it has dimension 3300×1 .
- Compute $\delta\mathbf{m}$ using the formula above. What do you get, and why is this the case?
- If $\mathbf{G}^T\mathbf{G}$ is not full rank, then a damping parameter λ may be introduced to stabilize the inversion. One possible solution to the problem is:

$$\delta\mathbf{m} = (\mathbf{G}^T\mathbf{G} + \lambda^2\mathbf{I})\mathbf{G}^T\mathbf{d} \quad . \quad (4)$$

Compute $\delta\mathbf{m}$ for a range of λ values. For example, try

```
lamvec = 10.^linspace( log10(minlam), log10(maxlam), numlam);
with  $\lambda_{\min} = 0.1$  and  $\lambda_{\max} = 40.0$ .
```

- Plot some of these model vectors by expanding them in the spherical spline basis functions, use the same color scale for each plot (type `colormap(seis)` after each figure is done, if you want the `seis` color scale from GMT), and also include the locations of the sources in the plots. The necessary matrix, \mathbf{B} is pre-computed, in `tomography_hw.m`, such that the function value $\delta \ln c(\phi_i, \theta_i)$ is computed by the dot product $\mathbf{B}(i,:) \cdot \mathbf{m}$. Comment on any trends that you see in phase speed maps that you produce.
- Now, perform an inversion using only the first five events; this requires using a reduced form of your original design matrix. Use the same damping parameters as before, and show and discuss your results.

- (g) What are two ways to stabilize the inverse problem, given total control over the experimental design? (Hint: How can we construct a matrix $\mathbf{G}^T \mathbf{G}$ with fewer zeros?)

Note: This package needs a Mapping Toolbox. If your Matlab does not have Mapping Toolbox license, please use the Matlab installed in the `nobel.princeton.edu`. You may use `ssh -Y YourUniversityID@nobel.princeton.edu`

References

- [1] W. Menke. *Geophysical Data Analysis: Discrete Inverse Theory*. Academic Press, San Diego, California, 1989.
- [2] C. Tape, Q. Liu, and J. Tromp. Finite-frequency tomography using adjoint methods — methodology and examples using membrane surface waves. *Geophysical Journal International*, 168:1105–1129, 2007.
- [3] A. Tarantola. *Inverse Problem Theory and Methods for Model Parameter Estimation*. SIAM, Philadelphia, Pennsylvania, 2005.
- [4] Z. Wang and F. A. Dahlen. Spherical-spline parameterization of three-dimensional earth. *Geophysical Research Letter*, 22:3099–3102, 1995.
- [5] Z. Wang, J. Tromp, and G. Ekström. Global and regional surface-wave inversion: A spherical-spline parameterization. *Geophysical Research Letter*, 25:207–210, 1998.