Homework 2

Due: 2024 Oct. 21

1. Exercise 1 on page 36.

2. Exercise 2 on page 46. Questions vi and vii are bonus questions.

3. Consider the boundary-value problem below,

$$u_{,xx} + f = 0,$$
$$u(1) = g,$$
$$-u_{,x}(0) = h.$$

Assume that $f = \sin(x)$.

a. Employing the linear finite element space with equally spaced nodes, set up the stiffness matrix and load vector using 3 elements. Solve the matrix problem. Use $\hat{f} \approx \sum_{B=1}^{n+1} f_B N_B$ in the assembly of the load vector. Do you get nodally exact solution here?

b. Repeat the calculation without invoking the approximation $\hat{f}$. In other words, calculate (N_A, f) rather than $\left(N_A, \hat{f}\right)$ in the load vector. What do you observe? Notice that the following identity can be helpful.

$$\int x sin(x) dx = -\int x d cos(x) = -x cos(x) + \int \cos(x)\, dx = -x cos(x) + \sin(x).$$

4. A quadrature rule is often used for the calculation of integrals on computers. It involves the quadrature points ξ_l and their weights w_l. It approximates an integral as follows,

$$\int_{-1}^{1} g(\xi) d\xi \approx \sum_{l=1}^{n_{\text{int}}} w_l g(\xi_l),$$

where n_{int} is the number of quadrature points. For the Gaussian quadrature rule with $n_{\text{int}} = 2$, we have $\xi_1 = -1/\sqrt{3}$, $w_1 = 1$ and $\xi_2 = 1/\sqrt{3}$, $w_2 = 1$. Verify that the two-point Gaussian rule can exactly integrate the monomials 1, ξ^2, ξ^3 but not ξ^4.