

# EE 615

## Experiment No. 3

Aero 2-DoF: PID Controller for Pitch and Yaw Control

Lab report submitted by

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# 1 Introduction

The Aero 2 Degree of Freedom (DoF) system is a widely used setup to understand the control dynamics of aerospace vehicles. It enables the study of pitch and yaw movements and the implementation of control strategies to achieve desired performance. This experiment involves designing and implementing PID controllers in SIMULINK to regulate the pitch and yaw movements of the system.

## 2 Aim

The aim of this experiment is to design and implement two PID controllers in **SIMULINK** to separately control the **pitch** and **yaw** of the Aero 2-DOF system.

## 3 Objectives

- Analyze the dynamics of the Aero 2-DOF system and derive its mathematical model.
- Design and implement PID controllers for independent pitch and yaw control.
- Tune the controller gains to achieve the required system performance:
  - Peak time:  $t_p \leq 2.5s$  for pitch and  $t_p \leq 3.5s$  for yaw.
  - Percent Overshoot:  $PO \leq 5\%$  for both axes.
- Validate the system response through simulations and real-time hardware implementation.

## 4 Control Algorithm

The **control algorithm flowchart** is as follows:

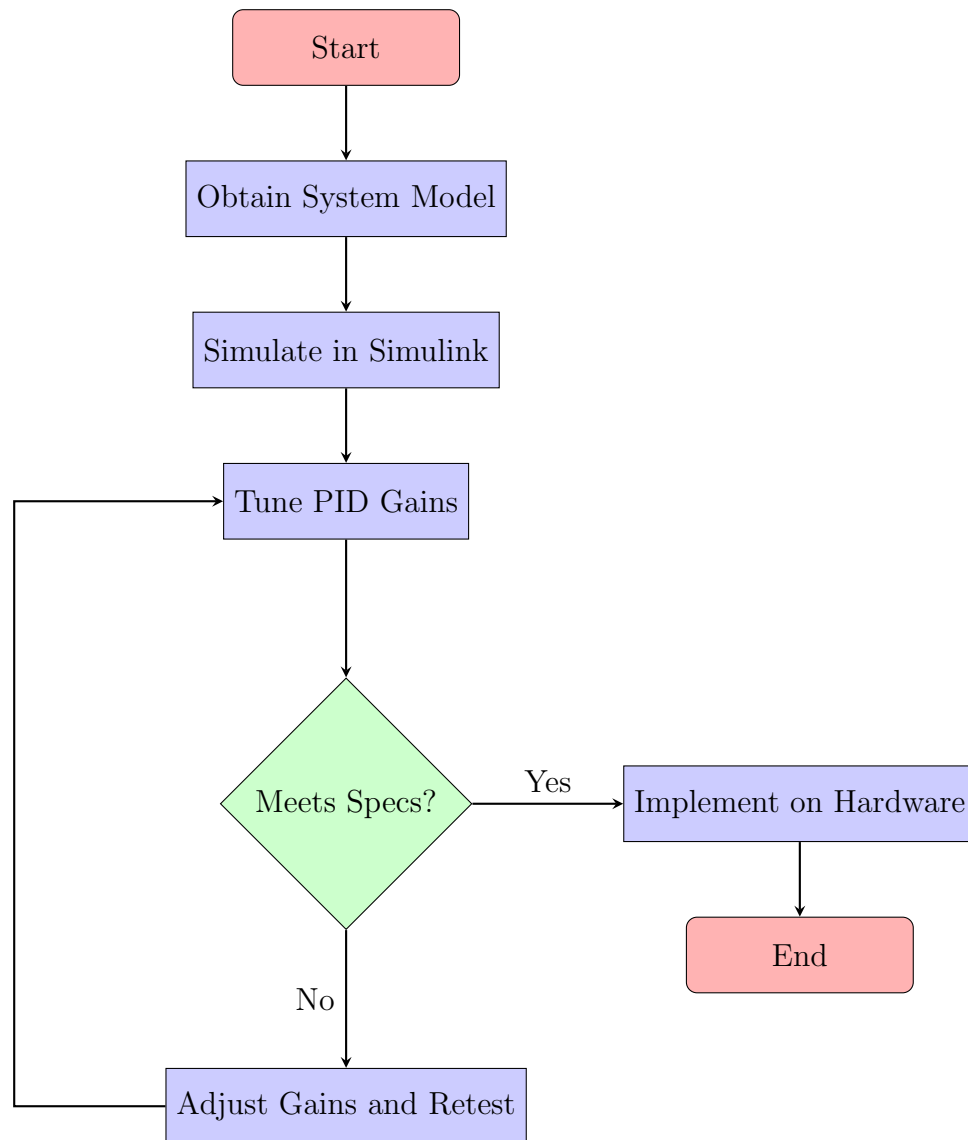


Figure 1: Control Algorithm Flowchart

## 5 Mathematical Model and Transfer Function

The equations of motion governing the system are:

$$J_p \ddot{\theta} + D_p \dot{\theta} = K_{pp} V_p D_t + K_{py} V_y D_t \quad (1)$$

$$J_y \ddot{\psi} + D_y \dot{\psi} = K_{yp} V_p D_t + K_{yy} V_y D_t \quad (2)$$

Taking the Laplace transform of the equations:

$$J_p s^2 \Theta(s) + D_p s \Theta(s) = K_{pp} V_p D_t + K_{py} V_y D_t \quad (3)$$

$$J_y s^2 \Psi(s) + D_y s \Psi(s) = K_{yp} V_p D_t + K_{yy} V_y D_t \quad (4)$$

Rearranging for the transfer functions:

$$\Theta(s) = \frac{K_{pp} D_t}{J_p s^2 + D_p s + K_{sp}} V_p + \frac{K_{py} D_t}{J_p s^2 + D_p s + K_{sp}} V_y \quad (5)$$

$$\Psi(s) = \frac{K_{yp} D_t}{J_y s^2 + D_y s} V_p + \frac{K_{yy} D_t}{J_y s^2 + D_y s} V_y \quad (6)$$

Given the parameter values:

- $J_p = 0.0232$  (total moment of inertia about pitch axis)
- $D_p = 0.0020$  (damping about pitch axis)
- $K_{sp} = 0.0074$  (stiffness about pitch axis)
- $J_y = 0.0238$  (total moment of inertia about yaw axis)
- $D_y = 0.0019$  (damping about yaw axis)
- $K_{pp} = 0.0032$  (thrust force gain acting on the pitch axis from the pitch/front rotor)
- $K_{py} = 0.0014$  (thrust force gain acting on the pitch axis from the yaw/rear rotor)
- $K_{yp} = -0.0032$  (cross-torque thrust gain acting on yaw from pitch rotor)
- $K_{yy} = 0.0061$  (thrust force gain acting on the yaw axis from the yaw/rear rotor)
- $D_t = 0.1674$

## 6 Designing PD Controller

In this experiment, a proportional-derivative (PD) control is used to control the pitch and yaw axes to a desired angle.

### 6.1 Pitch and Yaw Models

The pitch and yaw can be modeled as separate transfer functions.

For Pitch:

$$\Theta(s) = \frac{K_{pp}D_t}{J_p s^2 + D_p s + K_{sp}} V_p \quad (7)$$

For Yaw:

$$\Psi(s) = \frac{K_{yy}D_t}{J_y s^2 + D_y s} V_y \quad (8)$$

## 7 Control Design

The PD control follows the structure:

$$u = K_p[r(t) - y(t)] - K_d \dot{y}(t) \quad (9)$$

Taking the Laplace transform:

$$Y(s) = \alpha[K_p(R(s) - Y(s)) - K_d s Y(s)] \quad (10)$$

$$\frac{Y(s)}{R(s)} = \frac{\alpha K_p}{1 + \alpha(K_p + K_d s)} \quad (11)$$

## 8 PD Controller for Pitch

For pitch,

$$\alpha = \frac{\Theta(s)}{V_p} = \frac{K_{pp}D_t}{J_p s^2 + D_p s + K_{sp}} \quad (12)$$

Thus, the closed-loop transfer function is:

$$\frac{Y(s)}{R(s)} = \frac{K_p D_t K_{pp}}{J_p s^2 + (D_p + K_{pp} D_t K_d) s + (K_{sp} + K_{pp} D_t K_p) / J_p} \quad (13)$$

Comparing with the standard second-order system:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (14)$$

We get:

$$\omega_n^2 = \frac{K_{sp} + K_{pp} D_t K_p}{J_p} \quad (15)$$

$$2\zeta\omega_n = \frac{D_p + K_{pp} D_t K_d}{J_p} \quad (16)$$

Solving for  $K_p$  and  $K_d$ :

$$K_p = \frac{J_p \omega_n^2 - K_{sp}}{K_{pp} D_t} \quad (17)$$

$$K_d = \frac{2\zeta \omega_n - D_p}{K_{pp} D_t} \quad (18)$$

## 9 PD Controller for Yaw

For yaw,

$$\alpha = \frac{K_{yy} D_t}{J_y s^2 + D_y s} \quad (19)$$

Thus, the closed-loop transfer function is:

$$\frac{Y(s)}{R(s)} = \frac{K_{yy} D_t K_p}{J_y s^2 + (D_y + K_{yy} D_t K_d) s + \frac{K_{yy} D_t K_p}{J_y}} \quad (20)$$

By comparing:

$$K_p = \frac{J_y \omega_n^2}{K_{yy} D_t} \quad (21)$$

$$K_d = \frac{2\zeta \omega_n J_y - D_y}{K_{yy} D_t} \quad (22)$$

## 10 Calculating $\omega_n$ and $\zeta$ using PO and Peak Time

The overshoot percentage and peak time are given by:

$$\% \text{ Overshoot} = e^{-\zeta \pi / \sqrt{1 - \zeta^2}} \quad (23)$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} \quad (24)$$

Given:

- % Overshoot = 5%
- $T_p = 2.5$  sec (for Pitch)
- $T_p = 3.5$  sec (for Yaw)

Solving, For Pitch:

$$\zeta = 0.6901, \quad \omega_n = 1.7363 \quad (25)$$

$$K_p = 116.7073 \quad K_d = 100.022 \quad (26)$$

For Yaw:

$$\zeta = 0.6901, \quad \omega_n = 1.7363 \quad (27)$$

$$K_p = 35.8373, \quad K_d = 38.0220 \quad (28)$$

## 11 Simulation Block Diagram

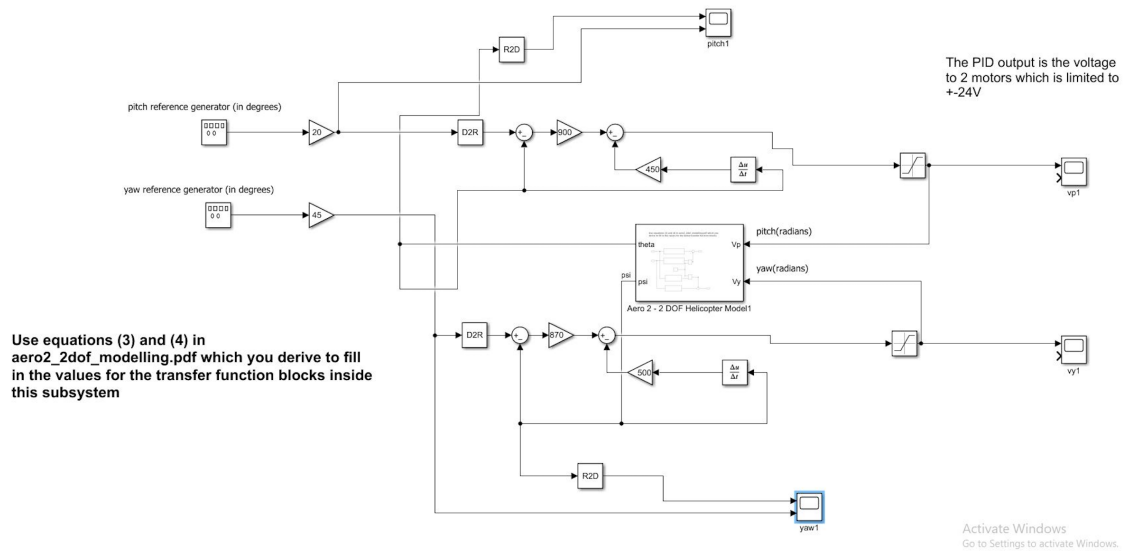


Figure 2: Simulation Block Diagram

## 12 Hardware Block Diagram

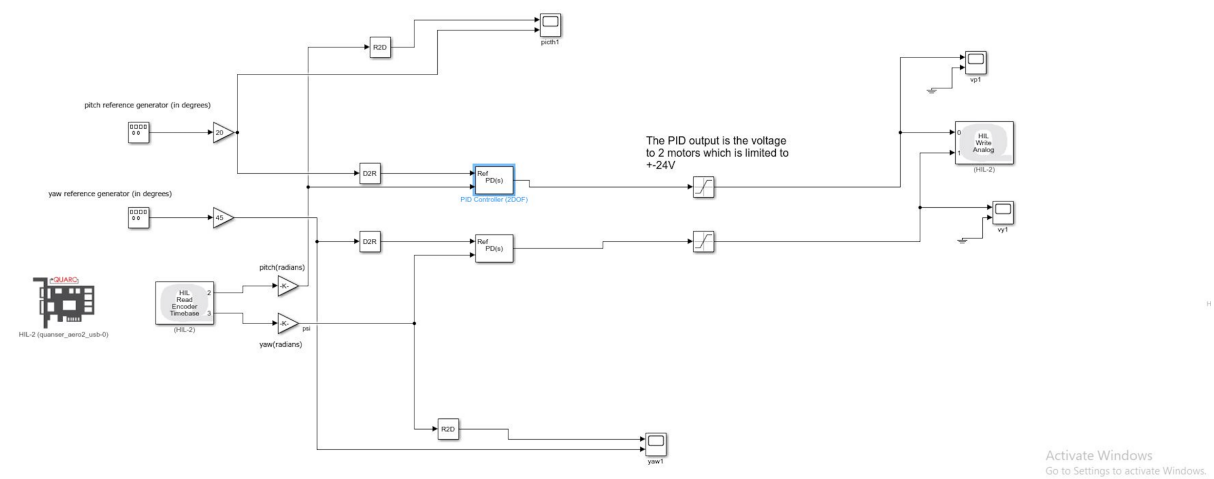


Figure 3: Hardware Block Diagram



## 13 Results

Hardware:

For Pitch:

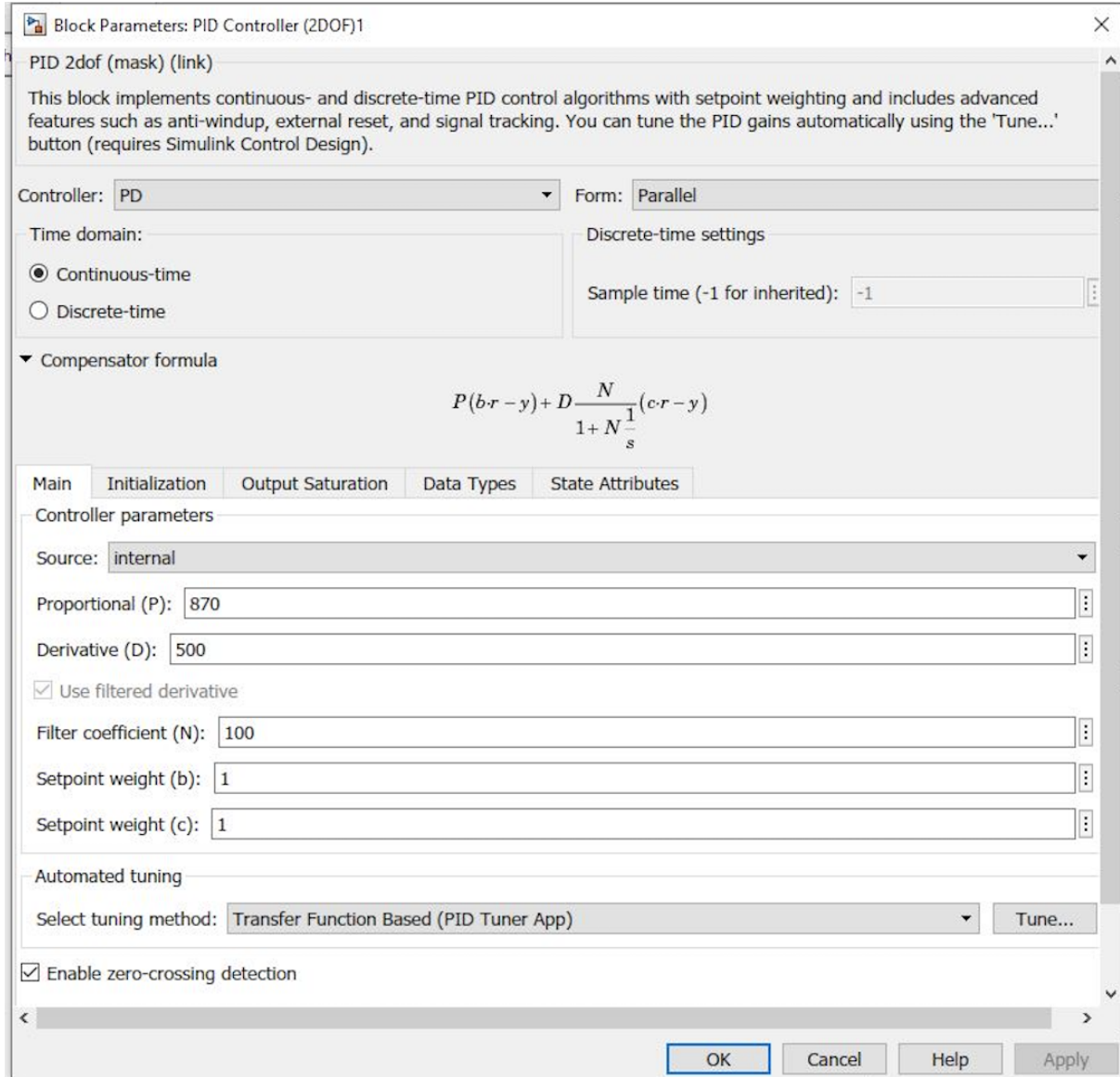


Figure 4: Pitch Parameters in Hardware

$$K_p = 920, \quad K_d = 480 \quad (29)$$

For Yaw:

$$K_p = 870, \quad K_d = 500 \quad (30)$$

Block Parameters: PID Controller (2DOF)

PID 2dof (mask) (link)

This block implements continuous- and discrete-time PID control algorithms with setpoint weighting and includes advanced features such as anti-windup, external reset, and signal tracking. You can tune the PID gains automatically using the 'Tune...' button (requires Simulink Control Design).

Controller: **PD** Form: **Parallel**

Time domain:  
☒ Continuous-time  
☐ Discrete-time

Discrete-time settings  
 Sample time (-1 for inherited): **-1**

▼ Compensator formula

$$P(b-r-y) + D \frac{N}{1 + N \frac{1}{s}} (c-r-y)$$

Main Initialization Output Saturation Data Types State Attributes

Controller parameters

Source: **internal**

Proportional (P): **920**

Derivative (D): **480**

☒ Use filtered derivative

Filter coefficient (N): **100**

Setpoint weight (b): **1**

Setpoint weight (c): **1**

Automated tuning

Select tuning method: **Transfer Function Based (PID Tuner App)** **Tune...**

☒ Enable zero-crossing detection

OK Cancel Help Apply

Figure 5: Yaw Parameters in Hardware

## 14 Challenges Faced

The main challenge was tuning. We were obtaining good results in simulation with certain parameter values, but the same performance was not observed in the hardware implementation. The smoothness of the response can be further improved by implementing an integrator.

## 15 Conclusion

The designed controllers successfully achieved the desired performance specifications. The results were validated through both simulation and hardware testing. Future improvements can include adaptive control techniques to enhance robustness.

Parameters	Pitch	Yaw
Peak Time (s)	0.934	2.8
Peak Overshoot (%)	3.1	3.9

Table 1: Simulation Response of Pitch and Yaw

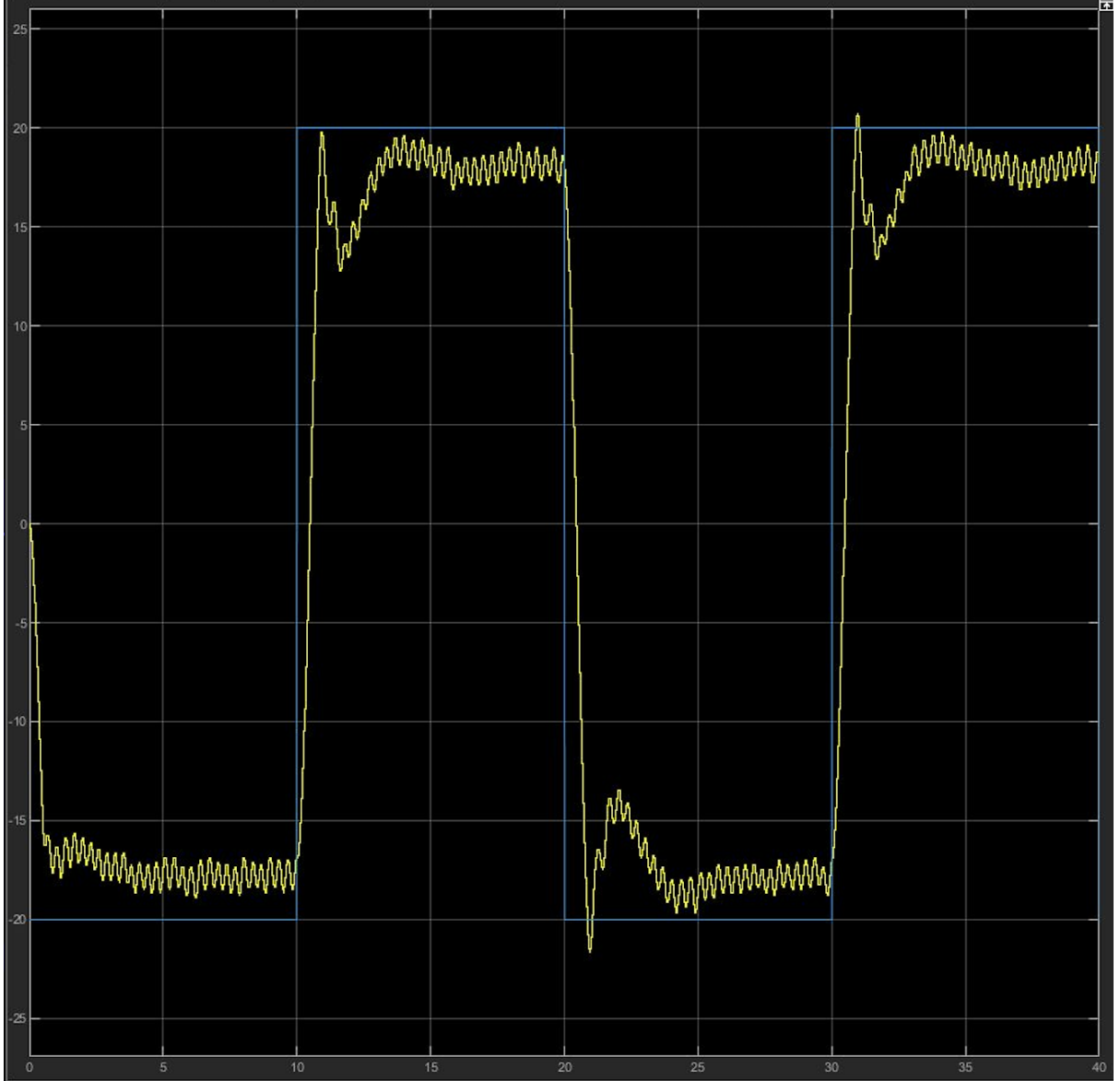


Figure 6: Pitch Response in Hardware

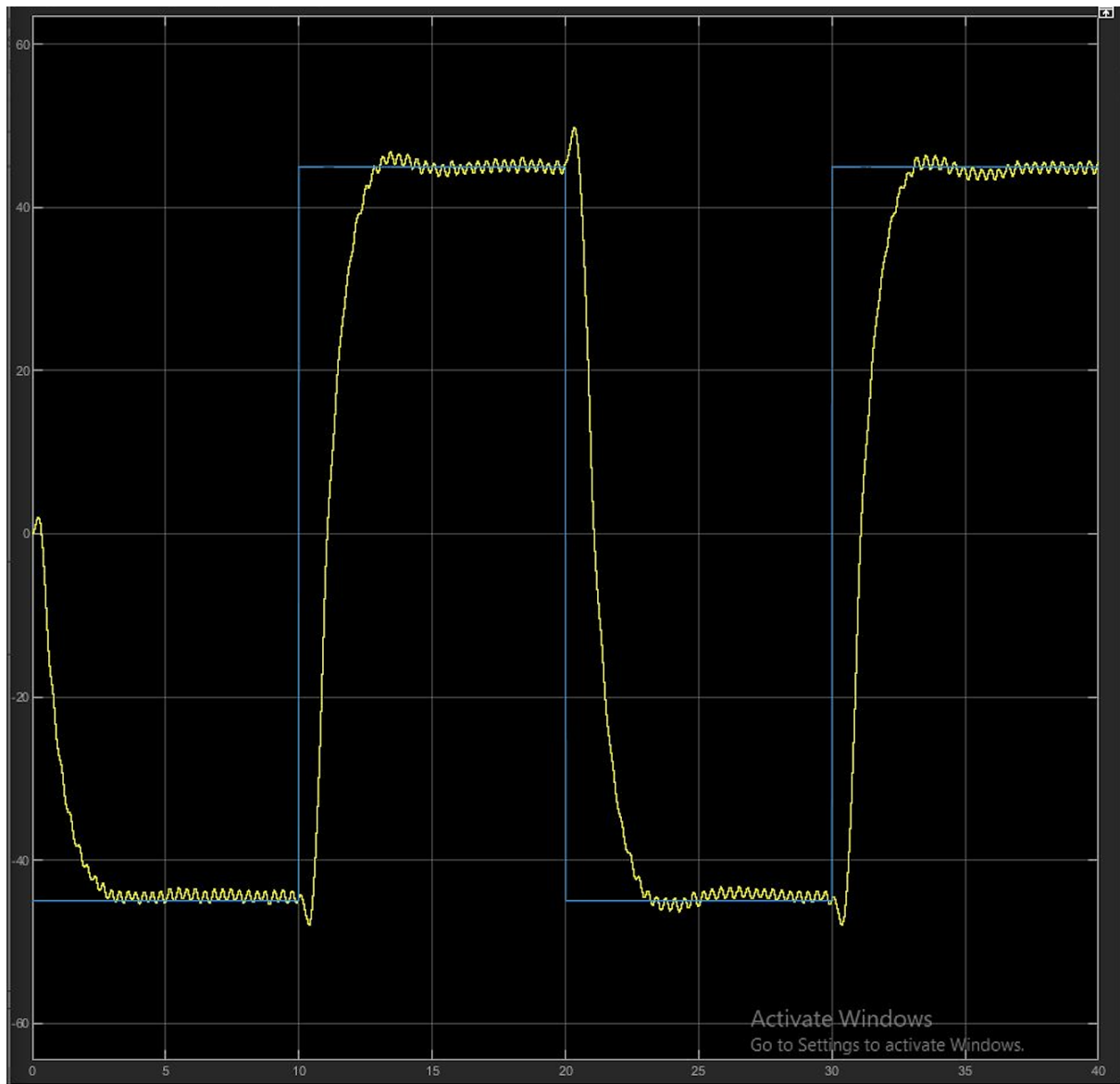


Figure 7: Yaw Response in Hardware

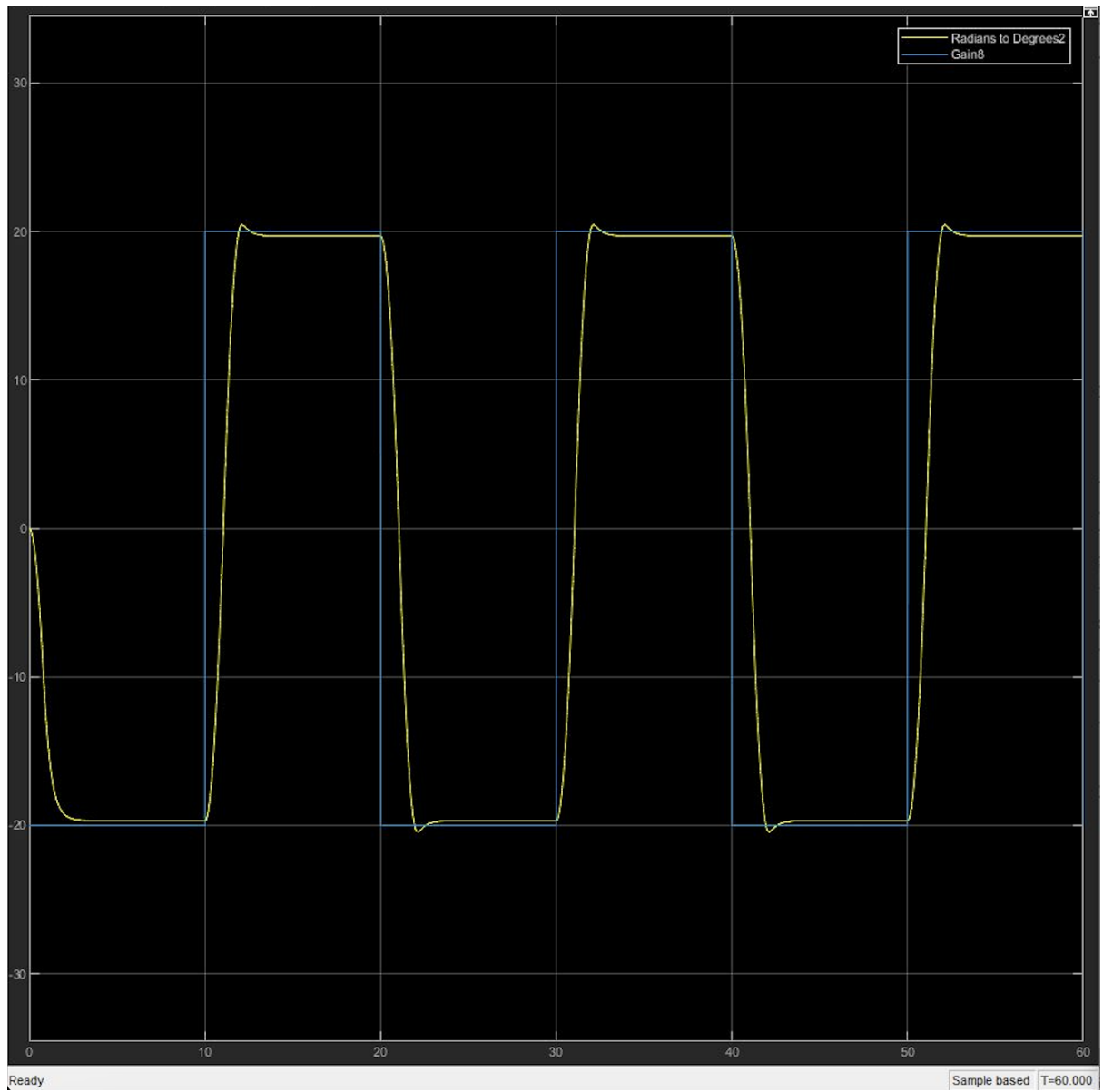


Figure 8: Pitch Response in Simulation

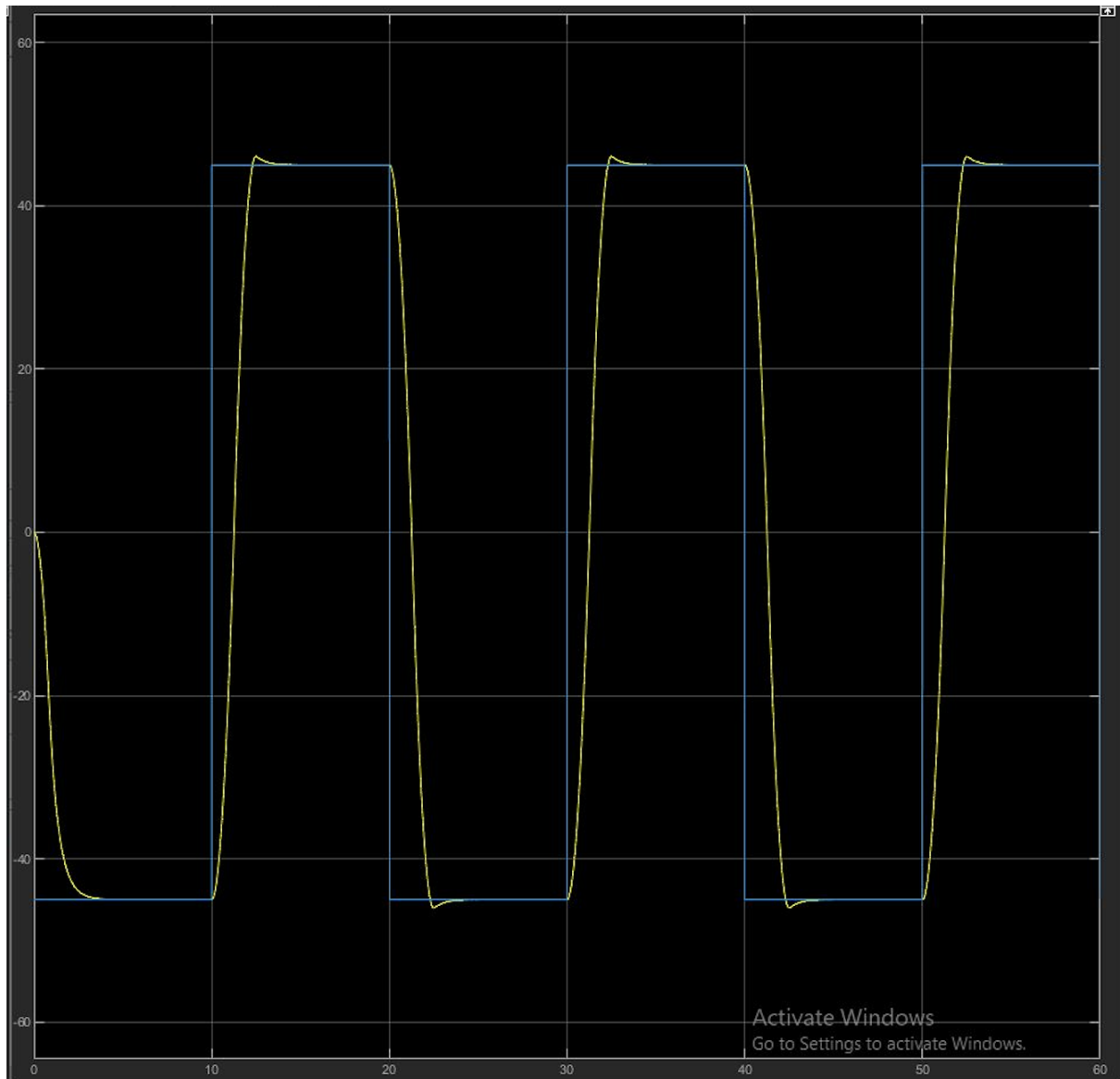


Figure 9: Yaw Response in Simulation