APPENDIX: THE FABRY-PEROT INTERFEROMETER

INTRODUCTION

Two plates of flat, transparent material, aligned mutually parallel, may give rise to important interference effects. In particular, if the inner surfaces of the two plates have a high reflectivity, the cavity can sustain multiple reflections of light if the wavelength and the angle satisfy certain conditions, which we shall derive shortly. If the spacing of the plates is adjustable, the device is called a Fabry-Perot *interferometer*, and if the spacing of the plates is fixed, the device is called a Fabry-Perot *étalon*. Our étalon consists of two flat, transparent discs, with the inner surfaces coated with a highly reflecting material, separated by a precision spacer. Another version of the étalon is a solid transparent rod of index of refraction n, reflectively coated at each end.

THEORY

If a plane wave of wavelength λ is incident upon the étalon at incident angle θ, the light will enter the cavity and bounce back and forth, with some light eventually transmitted and the remainder back-reflected.

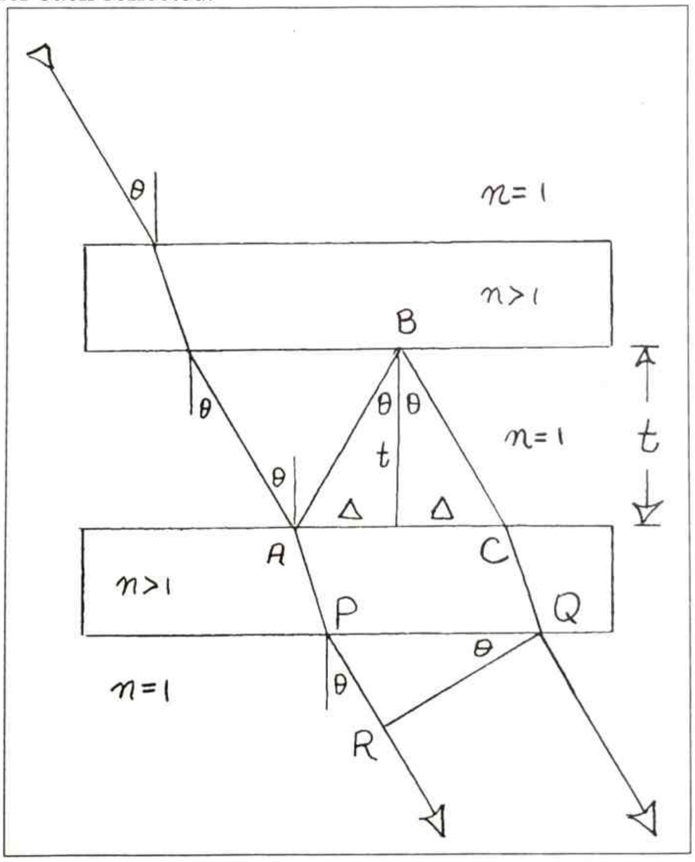

Figure 20. Ray diagram of the Fabry-Perot Interferometer

In order for the primary ray to interfere constructively with the reflected ray, the extra phase advance caused by the extra distance traveled, and by the phase shift φ at each surface, must be a multiple of the wavelength. Referencing Figure 6, the extra distance is just AB + BC – PR. Simple trigonometry tells us that AB = BC = $t/\cos(\theta)$, and PR = PQ$\sin(\theta)$. But PQ = AC = $2\Delta = 2t\tan(\theta)$, so PR = $2t\tan(\theta)\sin(\theta)$. Thus the extra distance is $2t/\cos(\theta) - 2t\tan(\theta)\sin(\theta) = 2t\cos(\theta)$ (please verify!). Hence we have the requirement for constructive interference

$$\boxed{\frac{2t\cos(\theta)}{\lambda} + 2\frac{\phi}{2\pi} = m}. \tag{93}$$

At this angle and for this wavelength, the incident wave will be transmitted with 100% efficiency. If, for a given wavelength, the angle does not exactly satisfy equation (93), but is close, then the transmission coefficient will be less than 1, and will be given by[51]

$$\begin{aligned} T &= \frac{1}{1+\frac{4R}{(1-R)^2}\sin^2(\frac{\delta}{2})} \\ \frac{\delta}{2} &= \frac{2\pi t}{\lambda}\cos\theta + \phi \end{aligned} \tag{94}$$

To get a feel for equation (93), take λ=546.07 nm (the green line of mercury); t = 0.500 cm. (not quite our value), and $\phi/\pi = 0$.[52] Then

$$\cos(\theta_m) = \frac{m}{18312.536} \tag{95}$$

The smallest angle satisfying this equation will be when the right hand side is less than or equal to 1; *i.e.* for m = 18312, 18311, 18310, ... $= m_{\max}, m_{\max}-1, m_{\max}-2, \ldots$. To each of these decreasing values of m corresponds a small but increasing angle θ_m where the light is transmitted with unity probability. For the angles θ_m close to 0, we can set $\cos(\theta_m) \simeq 1$, yielding from (93) the following useful approximation:

$$\frac{1}{\lambda} \simeq \frac{1}{2t}(m - \phi/\pi) \tag{96}$$

We now carry out the analysis of equation (93) to higher order. We will consider only angles small compared to unity; in this cases we can use the small angle approximation $\cos\theta \simeq 1-\theta^2/2$ and express Eq. (93) as

$$\frac{2t}{\lambda}(1-\frac{\theta_m^2}{2}) + \frac{\phi}{\pi} = m \tag{97}$$

[51] See, e.g. Born and Wolf, *Principles of Optics*, or Jenkins and White, *Fundamentals of Optics*.

[52] Since $0<\varphi<\pi$, this qualitative argument is not changed by setting $\varphi=0$.

Solving this equation for θ_m, we get

$$\theta_m^2 = 2 + \frac{\lambda}{t}\left[\frac{\phi}{\pi} - m\right] = \frac{\lambda}{t}\left[\frac{2t}{\lambda} + \frac{\phi}{\pi} - m\right] \tag{98}$$

Recalling that the allowable values of m are a set of very large, but descending, integers $m = m_{\max}, m_{\max}-1, m_{\max}-2, \ldots$, we associate with them the respective ascending sequence of integers $p = 1, 2, 3, \ldots$, corresponding to increasing angles of incidence θ_p. One will then observe unity transmission at angles given by

$$\boxed{\theta_p^2 = \frac{\lambda_0}{t}(p - e(\lambda_0)); \quad p = 1, 2, 3, \ldots} \tag{99}$$

Where $0 \le e(\lambda_0) \le 1$ and is given by

$$e(\lambda_0) \equiv \mathrm{mod}\left(\frac{2t}{\lambda_0} + \frac{\phi}{\pi}\right) \tag{100}$$

We see that $e(\lambda_0)$ is a wavelength-dependent residual phase number arising from the fact that t is not an integer multiple of λ_0. For our purposes $e(\lambda_0)$ does not play a significant role.

With monochromatic incident radiation, we can measure the angles of maximum intensity, and plot θ_p^2 versus p. From (99) we should get a straight line, with slope λ / t. Figure 7 shows an example of such a plot for $t = 0.5$ cm. and $\lambda = 546.06$ nm.

In practice, we cannot predict beforehand the precise values of θ_p because we do not know the phase shift ϕ, nor do we know the value of $2t/\lambda$ to six significant figures. However, the usefulness of the Fabry-Perot étalon lies in our ability to determine wavelength *differences* to very high precision. In what follows we will show how this is accomplished.

Suppose two closely spaced wavelengths of light, λ_a and λ_b, are incident upon the étalon. Let us compare the angular separation of the rings *that occur for the same order m*. From equation (98) we have

$$\begin{aligned}(\theta_m^{(a)})^2 &= 2 + \frac{\lambda_a}{t}\left(\frac{\phi}{\pi} - m\right) \\ (\theta_m^{(b)})^2 &= 2 + \frac{\lambda_b}{t}\left(\frac{\phi}{\pi} - m\right)\end{aligned} \tag{101}$$

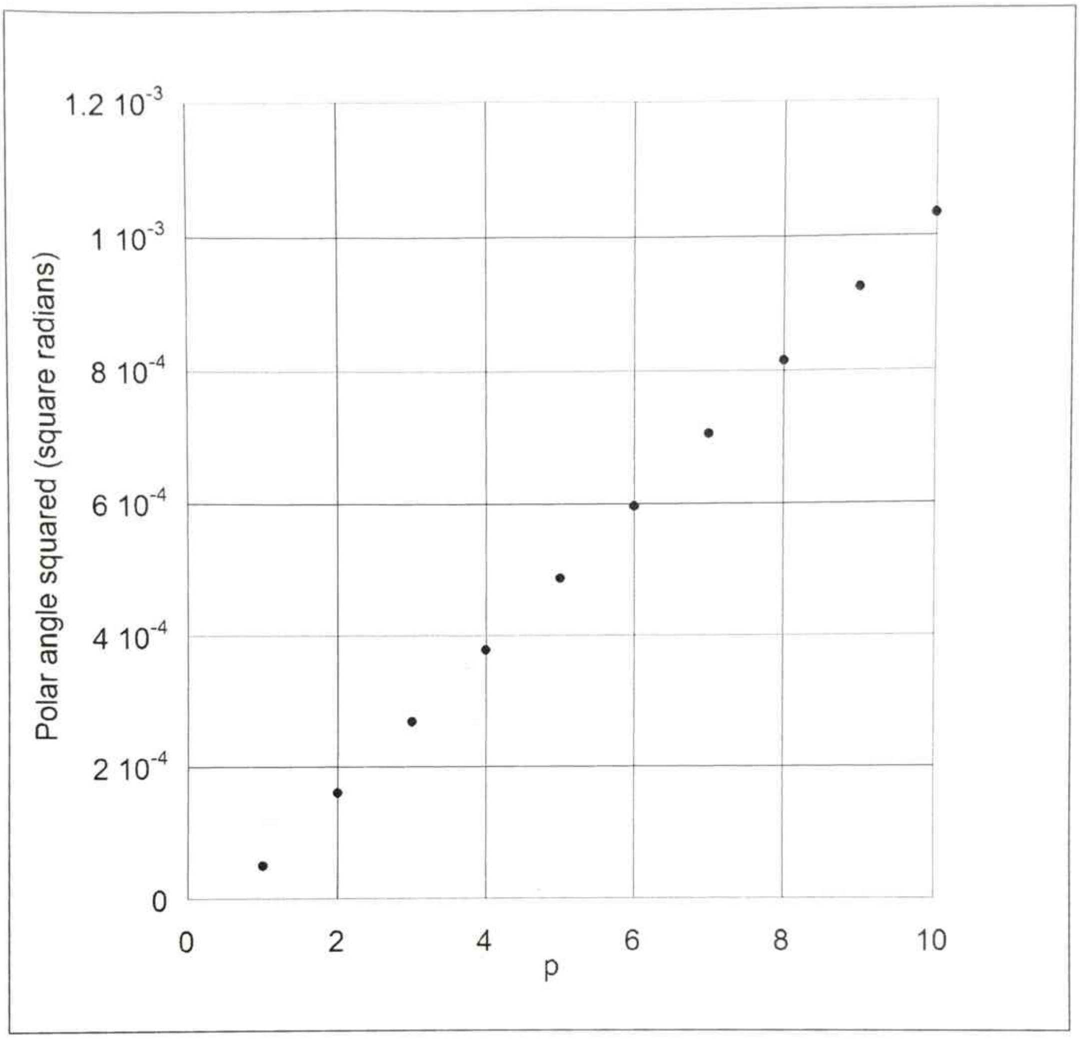

Figure 21. A plot of θ_p^2 versus p, for $\lambda = 546.074$ nm, $t = 0.500$ cm, and $\phi = 0$.

Subtracting the two equations,

$$(\theta_m^{(a)})^2 - (\theta_m^{(b)})^2 = \frac{\phi / \pi - m}{t}(\lambda_a - \lambda_b) \tag{102}$$

Using the approximation given in equation (96), we may write

$$\frac{m - \phi / \pi}{2t} \approx 1/\lambda_a \approx 1/\lambda_b \approx 1/\bar{\lambda}\,. \tag{103}$$

We therefore obtain the very simple and elegant result[53]

$$\boxed{(\theta_m^{(a)})^2 - (\theta_m^{(b)})^2 = 2\frac{\lambda_b - \lambda_a}{\bar{\lambda}} \simeq 2\bar{\lambda}\left(\frac{1}{\lambda_a} - \frac{1}{\lambda_b}\right) \simeq 2\frac{E_a - E_b}{\bar{E}}} \tag{104}$$

In our case, the application of a magnetic field to our sample will split a line $\bar{\lambda}$ into two or more lines λ_a and λ_b; the difference in the square of their angles will be given by (104). This result is central to many kinds of high resolution spectroscopy, including Zeeman spectroscopy.

In practice, a Fabry-Perot étalon is usually *deliberately* illuminated by an extended, uncollimated source. Contrary to one's intuition, and contrary to the situation with grating spectrometers, this is not only *easy*, it is also a *good* thing.[54] Then, if the étalon is illuminated with monochromatic light and then viewed with an astronomical telescope or a camera focused at infinity, one will observe sharp rings at successively larger polar angles.

RESOLUTION OF A FABRY-PEROT INTERFEROMETER

The resolving power of any spectrometer is usually specified in terms of the full-width at half maximum (FWHM) of the spectral lines. If you analyze Equation (94) in detail, you will find that, for an otherwise perfect Fabry-Perot étalon, the FWHM is given by a very simple expression,

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda}{2t}\frac{1-R}{\pi\sqrt{R}} \tag{105}$$

where the variables have been defined earlier. For our étalon, $t = 0.6499$ cm and $R \approx 0.9$.[55] For our experiment, $\lambda = 546$ nm. This leads to $\Delta\lambda / \lambda = 1.4\text{x}10^{-6}$!

[53] Inexplicably, this simple expression is neither derived in Born and Wolf, nor in Melissinos.

[54] A Fabry-Perot étalon is, in effect, a filter; a plane wave incident with a certain direction results in a plane wave transmitted with *exactly* the same direction, but with transmittance T. Therefore, there *must* be an angular spread of the source comparable to the angular range of the rings that one wishes to observe. However, the transverse *size* of the beam does not matter, unless the étalon has inhomogeneities.

[55] You might think that the resolution should go to zero as R goes to 1. Theoretically this is the case; however, for values of R greater than about 0.9, surface imperfections dominate and larger values of R do not help.