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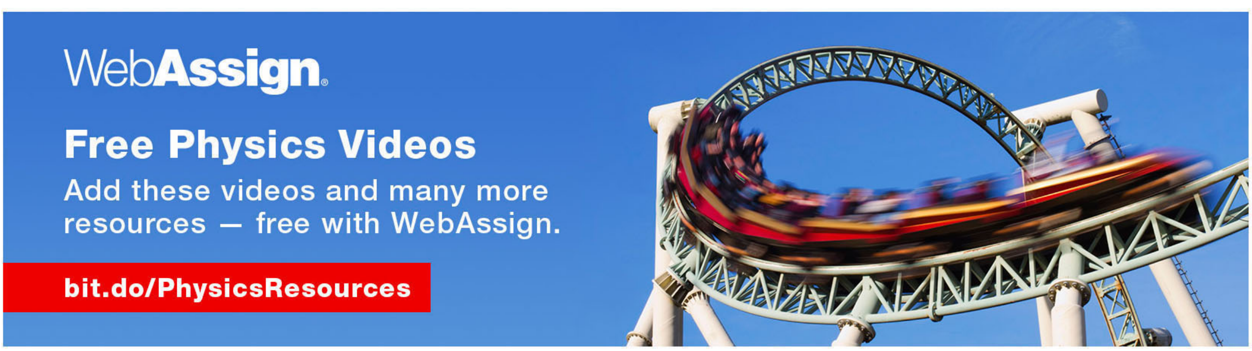

The Millikan oil-drop experiment: Making it worthwhile

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Improvements to the Millikan oil drop experiment as it is performed in teaching laboratories are described. Enhancement of the optics in the oil-drop apparatus greatly improves the visibility of the drops. Accurate timing of their motion becomes possible since they are now bright and sharply focused. With improved timing, the parameters such as microscope calibration and plate separation can become the principal sources of experimental error. Methods are described to accurately determine these experimental parameters. The workload for the experimenter is greatly reduced by using a computer to: act as a smart stopwatch, calculate the drop charge in real time, perform the statistics, and make records of the experiment. The convenience and speed offered by the computer, coupled with improved optics, relieves the eyestrain, fatigue, and frustration usually associated with this experiment. Nye's watch oil was used in this study. A return to Millikan's original iteration method for the correction to Stokes' law lowers the calculated charge for small drops by about 2%. With these improvements the calculated charges are sufficiently accurate that guesswork as to the multiplicity of their charge is essentially eliminated. A student typically obtains the value of the electronic charge accurate to about 1% with 1 h of experimentation. Student and instructor satisfaction are much improved. © *1995 American Association of Physics Teachers.*

I. INTRODUCTION

The Millikan oil-drop experiment is often included in physics laboratories. Apparently, many instructors still feel that it provides a valuable experience for the students. It has current importance as some Millikan-type experiments have recently been performed which searched for fractional charges.[1]

However, as a teaching-lab experiment it does not enjoy a good reputation for three principal reasons: eyestrain, tedium, and poor, unconvincing results. Part of the problem lies in the equipment. Even an expensive apparatus may give disappointing results because economies have been taken with critical elements. The experiment has languished as expectations for it have declined. The scarcity of articles on this experiment in the last two decades does not mean that there is no room for improvement. Quite the opposite is true.

Examples of attempts to remedy this situation are: sharing the workload among many students,[2] using plastic spheres of known radius instead of oil drops of random sizes,[3] developing a drop-selection and voltage-setting strategy to collect the best data,[4] and using laser illumination and a video camera in a demonstration-type experiment.[5] It is my impression that some instructors and equipment manufacturers have dealt with the problem by simply lowering their expectations and only attempt to obtain results which show the quantum nature of electric charge. They seem not to expect an accurate measurement of the charge of the electron. Computer software has even been written to simulate the experiment rather than perform it.

This paper reports on a different approach: (1) improving the quality of the optics in the apparatus so that the drops are easily visible and the timing of their motion can be made more accurate, (2) determining the experimental parameters (microscope calibration, plate separation, voltage, oil density, and air viscosity) as precisely as can reasonably be done, (3) using a computer to act as a smart stopwatch, automatic calculator, and data recorder, (4) applying the correction to Stokes' law in a more precise way than is often done, and (5) making it clear to the students that good results are possible and expected.

The outcome of the present work is that much data can be taken in a short time even by a person working alone. Results are generally very good, and student morale during and after the experiment is high.

For this study, a Model 0620B Hoag–Millikan apparatus available from Sargent–Welch Scientific Company[6] was used. However, many of the procedures described and recommendations made here should work for other models.[7]

II. CHARGE EQUATION

The experiment consists of measuring the terminal speeds of charged oil drops as they fall through air under the force of gravity, and then as they are pulled up by an applied electric field which exists between parallel plates. The analysis has been presented many times.[8,9] From Stokes' law, the radius, a, of the drop is given by

$$a=(9\eta v_g/2g\rho)^{1/2}, \tag{1}$$

where η is the viscosity of air, v_g is the terminal speed of the drop falling under gravity, g is the acceleration of gravity, and ρ is the mass density of the oil measured in air.

Stoke's law must be corrected for spheres that are comparable in size to the mean-free path of the air molecules by multiplying the viscosity by a correction factor:

$$\eta_{\text{eff}}=\eta[1+b/(pa)]^{-1}, \tag{2}$$

where η_{eff} is the effective viscosity of air, p is the absolute barometric pressure in cm Hg, and $b=6.17\times10^{-6}$ m cm Hg is a constant. This correction factor depends on the drop radius, and the calculated drop radius depends on this factor via Eq. (1) in the form of the corrected viscosity. It seems to be widely assumed that iteration between Eqs. (1) and (2) is not necessary. In his original work Millikan iterated twice so that this correction converged sufficiently.[10] Teaching versions of this experiment use lower voltages and thus often work with smaller drops than did Millikan. For a drop for which Eq. (1) gives an initial radius estimate of 5×10^{-7} m, multiple iteration lowers the charge calculated using Eq. (4) by about 2% compared with using the estimate from Eq. (1)

directly. Larger drops produce less change. In this study, the computer iterated six times for each timing of a drop. This produced complete convergence in single-precision computer arithmetic.

Alternately, Eq. (2) may be substituted into Eq. (1) and then solved for the radius as Hoag did.[11] This gives

$$a=[9\eta v_g/2\rho g+(b/2p)^2]^{1/2}-b/2p. \tag{3}$$

Both the iteration method and Eq. (3) give identical results.

The charge, q, on the oil drop is given by

$$q=(6\pi d/V)(9\eta^3/2\rho g)^{1/2}[1+(b/pa)]^{-3/2} \times(v_e+v_g)v_g^{1/2}, \tag{4}$$

where d is the separation of the plates, V is the potential difference between the plates, and v_e is the speed of the drop with the electric field applied.

III. OPTICS

The quality of the optics is the most important factor in getting good results from the oil-drop experiment. The optical system consists of a low-power microscope with a calibrated reticle and an illuminator. The drops must be easily visible and sharply focused so that the timing can be accurate. High visibility is also necessary to avoid eyestrain, fatigue, and confusion between drops.

A. The microscope

The oil drops were not highly visible in the apparatus as it existed when this study began.[12] Through the microscope, the image of the focusing pin showed the halos, colors, and indefinite focus which are characteristic of optical aberrations.

The microscope objective consisted of a simple, planoconvex lens. The convex side faced the oil drops rather than the more-distant reticle. This misapplication of the lens shape factor produced spherical aberration[13] which was largely eliminated by simply reversing the lens. With this simple change, the oil drops were much easier to see. Further improvement was obtained by replacing the objective lens with a 50 mm effective focal length (EFL), 12.5 mm diam achromat from Edmund Scientific[14] (Cat. No. 32,317). This lens is biconvex with the less convex side facing the chamber. Being 1.5 mm larger in diameter than the original, the objective-lens holder had to be bored slightly. The effort and expense were well worthwhile.

B. The illuminator

The illuminator consisted of a #44 incandescent lamp, a disk of heat-absorbing glass, and a simple symmetric biconvex condensing lens. With the lamp much nearer the lens than the oil drops, spherical as well as chromatic aberrations were evident. This lens was replaced with a 26 mm EFL, 18 mm diam achromat (Edmund Cat. No. 32,203) with the more-convex side facing the more-distant chamber. This lens did not completely rid the lamp image of aberrations, but more light was concentrated on the drops.

All #44 lamps are not identical. Some do not have the filament well centered, and some have thick deformed glass on the end of the envelope. Selecting a good lamp significantly improves the illumination. A #44 lamp has a stated lifetime of 3000 h, but its useful life in this experiment may be considerably shorter. With use, the filament may sag under gravity so that it is no longer straight enough for its (vertical) image to illuminate the drops along their entire travel. Defocusing the lamp image to cover a larger volume should not be considered as an option because a high light intensity is needed on the drops.

C. Background light

The plastic spacer ring which separates the plates had white paint as a viewing background. Presumably, this was to make the reticle more easily visible, but the illumination was excessive. A light trap was formed by placing a strip of flat black tape on the inner edge of the spacer opposite the illuminator. The edges around the viewing slot were blackened using a permanent transparency marker. The white viewing background was half-darkened with black vertical lines.

Further improvement in the drop visibility was obtained by extending the illuminator tube with a shade made of black paper. This reduced the scattered light from the illuminator that entered the microscope objective both directly and by reflection off the side of the chamber.

These changes made the oil drops stand out boldly with enough background light to see the reticle well. Drops that were large enough to be used could be seen with the room lights on. However, the room lights did reduce contrast and caused more fatigue for the experimenter since the unused eye had to be kept tightly closed. It is better to have the room lights subdued.

D. Drop visibility

After these optical improvements were completed, the oil drops could be seen as tiny bright dots rather than the former dim, fuzzy, colored blobs. Drops which carried a single charge and were small enough to be raised by the electric field were very easy to follow. Even the fine mist from the aspirator could be seen as individual drops. These were far too small to be used in the experiment because the fall times would be too long and the Brownian motion too large, not because of low visibility.

Oil drops which are not at the same distance from the microscope are seen to focus differently. This makes it easier to follow a particular drop if the field is crowded. Besides reducing eyestrain, the high visibility and sharp focus of the drops improves the accuracy of their timing since they briefly disappear behind the reticle lines.

E. Microscope calibration

The microscope was accurately calibrated by mounting it on an optical bench so that it directly viewed the cross hair in a micrometer eyepiece readable to 0.0025 mm, whose own calibration was verified with a precision traveling microscope. Repeated measurements produced and confirmed a calibration which is accurate to 0.2%. High accuracy is needed in the microscope calibration because it determines the distance over which the drop motion is timed, and the drop speeds enter Eq. (4) to the 3/2 power. If the microscope is out of calibration by 1%, a systematic 1.5% error is produced in the calculated charge. The oil drop apparatus used in this study was supplied with a calibration scale which was itself found to be accurate. However, I estimated that using this scale to calibrate the microscope could produce no better than about 1% accuracy.

To actually maintain high microscope accuracy while viewing the drops, the experimenter must focus on both the reticle and the drop simultaneously since the magnification[15] is dependent upon the object and image distances being equal to their values during calibration.[16] For this reason, if the drop should drift forward or backward, it should be kept in sharp focus (by moving the entire microscope) even if this is not necessary to see it well. Whether focusing should be done while a timing is in progress depends on how rigidly and horizontally the microscope is mounted. I recommend focusing before a timing begins.

IV. EXPERIMENTAL PARAMETERS

The improvement in the optics allows better determinations of the velocities of the drops. The velocities are still subject to the random effects of Brownian motion and human reaction time. However, the uncertainty in the velocities can be reduced to a small value by averaging. This means that the other quantities in Eq. (4) must be known accurately lest they become the principal contributors to the experimental error.

A. Plate separation

By sliding an accurate straight edge over the unmounted plates, raised rims could be felt on the holes near the plate edges and on the droplet hole. Raised rims or edge nicks which contact the plastic spacer make it difficult to reassemble the apparatus with the same plate separation each time. The rims were removed with a countersink tool and then inspected under a microscope to be sure there were no burrs.

Simply measuring the thickness of the plastic spacer would seem the most straightforward method of determining the plate separation. However, the spacer thickness in this particular apparatus was found to vary slightly from place to place, and it also had a slight wedge shape. Taking an average is not the best procedure since the plates will rest only on the thick spots. Also, the clamping force applied to the plates is rather small so any waviness the spacer may have might not be completely flattened. In an attempt to handle this situation I assumed that the plates were planes which rested on three points on the spacer (not necessarily the thickest three). The expression for the plate separation, d, at the center is[17]

$$d=t_1+\frac{(t_2-t_1)\sin\theta_3-(t_3-t_1)\sin\theta_2}{\sin\theta_3-\sin\theta_2-\sin(\theta_3-\theta_2)}, \tag{5}$$

where t_1, t_2, and t_3 are the spacer thicknesses at the points of contact, and θ_2 and θ_3 are the angles around the spacer (carrying their signs) measured from point 1 to points 2 and 3.

A second determination of the plate separation was made by measuring the thickness of each plate, then the thickness of the stack of the plates and the spacer all held together with screws and nuts similar to the ones in the apparatus. Because the top plate has a recess in its upper surface, these measurements were made at four points around the centers of the plates. The appropriate differences were taken, then averaged. This method yielded a plate separation which was 0.16% larger than the first method.[18]

The second determination was taken as the more accurate since the first method does not account for waviness of the spacer or unnoticed bumps.[19] The top plate was marked so that the chamber could be reassembled with the same orientation each time.

B. Oil density

I found the density of Nye's watch oil[20] to be 859.9 ± 0.2 kg/m^3 at 22 °C using a calibrated 10 ml specific gravity bottle. The instructions which came with the oil drop apparatus gave a value of 890 kg/m^3. To check if perhaps some volatile components of the oil might evaporate during the atomization process, I heated 11 ml of oil in an open beaker in a drying cabinet for one week with agitation every 12 h. Some oil did evaporate as indicated by a reduction in the mass. However, the density did not change measurably, so I concluded that the value in the instructions was either a misprint (890 vs 860), or perhaps the composition of the oil had changed. Hoag[21] reported a density of 920 kg/m^3 for Nye's watch oil in 1948.

C. Other parameters

The voltage used in this study was 405 V.[22] It was monitored using a $4\frac{1}{2}$ digit multimeter which was found to read 0.2 V high when compared to a recently calibrated $6\frac{1}{2}$ digit meter.[23] A regulated power supply[24] was used to supply an indicated 405.2 V.

The computer program written for this study corrects for the variations in the viscosity of air caused by temperature.[25] A brass thermometer well which accepts a Hg-glass thermometer bulb was fabricated and soldered to the upper plate. Vacuum grease was used for thermal contact between the well and the thermometer bulb. The temperature of the upper plate was checked to see if the illuminator raised the chamber temperature. No increase over room temperature was detected after leaving the (1.5 Watt) illuminator on for 1 h. The thermometer was left in place during the experiment.

V. COMPUTER PROGRAM

There is nothing to be gained by having the students manually perform all of the tedious calculations. Also, a computer can be used as a smart stopwatch so that data can be automatically input, processed immediately and output to a printer and a disk file for records of the experiment.

When the program starts, the voltage, barometric pressure, temperature, and number of reticle spaces used can be entered or updated.[26] The field-on and field-off times are measured by tapping the space bar as a stopwatch. All of the operations that may be required during the time that a drop is in view can be selected by three options, pressing: (1) the space bar, (2) any letter key, or (3) any number key. This means that the experimenter does not need to look away from the microscope. Beeps indicate the operation being performed. Once a drop has been timed, the results of the calculations are printed to the screen and to a line printer. Immediate charge calculation aids in drop selection. The sound of a dot-matrix printer provides additional feedback to the experimenter.[27] Rather than averaging the times and then computing the charge, the charge is computed from each pair of timings and the resultant charges are averaged.[28] Timings which are immediately known to be mistakes can be marked on the record and removed from the average by simple key pressing. This amount of computer assistance means that usually 50 to 200 pairs of timings can be made per hour, depending on the size and charge (hence speed) of the drops and also on the time spent selecting drops.

This program is written for BASIC.[29] It was used in a compiled form so there would be less potential for timing errors due to slow execution. However, no timing errors were detected during tests run on a 10 MHz, 286 PC–AT without compilation.[30] The program, along with other support programs and a more complete description, are available free of charge by contacting me by E-mail or by sending a formatted disk.[31]

VI. THE DROPLET HOLE AND DROPS

The experiment is performed with the small hole in the upper plate left open. This produces three possible problems. The electric field might be reduced in the region where the drops are timed. A gradient in the electric field might exist which could interact with the induced electric dipole moment of the drop. Also, air flow through the hole can blow the drops around.

A computer relaxation solution to Laplace's equation showed that for the comparatively large hole in this apparatus, the potential at the bottom center of the hole differed from the upper plate potential by 2.5% of the potential difference between the plates. However, about one hole diameter down from the upper plate the effect was negligible, and no gradient in the electric field was detectable in the region where the drops were timed.

A more important effect at my location is wind. On windy days, the drops showed beautiful laminar flow patterns into and out of the hole as the room barometric pressure changed slightly.[32] This flow can change the timing measurements of the drops. Plugging the oil-spray hole helped, but a better solution for this apparatus was found. Latex tile grout was used to plug the screw holes and the gap around the electrical cable which are located in the insulating base just under the bottom plate. The spray hole was then left open. It seems that the actual "wind effect" is breathing of the (larger) spray chamber through leaks into the experimental chamber, then through the droplet hole.

This solution seemed adequate for calculating the charges on the drops since averaging will give reliable values for the rise and fall times. However, the wind effect will increase the variance of the times, and the variance is used in estimating the value of Boltzmann's constant as described in the Sec. X below.

A solution for the wind effect was needed other than sealing the entire apparatus in a constant pressure vessel as Millikan did. A 5/64 in. diam hole was drilled through the upper plate over the viewing slot in the plastic spacer. The hole was then enlarged from the top almost through the plate. This location was chosen because there was already some air leak through the viewing slot in the spacer. Convection through the chamber, and subsequent drifting of the drops, might result if there were openings on opposite sides. Providing this less restricted air path bypassing the droplet hole further reduced the wind effect so that it was no longer visible except slightly on very windy days.

All of the possible droplet-hole problems are minimized by timing the drops nearer the bottom plate.

With the aspirator that came with this apparatus the atomization occurs within the oil bottle, and only the smaller drops are blown out of its neck. This keeps the apparatus cleaner and avoids plugging of the droplet hole. In our apparatus the beveled end of the aspirator neck almost sealed the opening of the spray chamber. When the aspirator bulb was squeezed, air and many drops were forced by pressure into the experimental chamber. Sometimes there were more drops visible than could be managed. Enlarging the spray hole so that air could escape around the aspirator neck solved the problem.

A small amount of debris was found in the aspirator after the data presented in Table I were gathered. Viewed under a microscope, this debris could be seen to be small pieces of grit, fibers, and even one small metallic particle. Presumably, there were also particles in the oil which were too small to be seen. The debris is probably dust drawn into the aspirator from the air. Some particles could be seen entering the aspirator tube when the bulb was squeezed. It seems unlikely to happen, but the inclusion of debris in a measured drop would alter its results. To minimize this problem, the oil can be changed more frequently. I also glued a disk of filter paper over the air-entrance hole in the aspirator bulb.

VII. STUDENT RESULTS

The improvements in this experiment evolved over an 8 week period. The first group of students had access to just the reduction in scattered light and a simple computer program to do the charge calculations. The second group had one session like the first, and a second session with the simple microscope objective lens turned properly to reduce spherical aberration. With this simple change one of the students remarked, "It's ten times better than it was."

The last four students had access to all the optical changes. The computer was used as the stopwatch as well as doing the computations. The parameters had also been measured accurately. Their data along with some I gathered are presented in Table I. For easier interpretation by the reader, the calculated charge, q, is presented in units of 1.5793×10^{-19} C. The reason for the choice of this particular constant is described in Sec. VIII.

It was obvious that student morale improved as the experiment did. The last four students were actually excited to get accurate results in so short a time. None complained of eyestrain. Possibly they were influenced by the more positive attitude of the instructor toward this experiment.

For these last students, the new experimental procedure was explained orally since no revised write-up was yet available. The student, working alone, then practiced for about 5 min before actually taking data. Each was asked to get good data for five different charges. Some interpreted this to mean "five drops." I felt that having a goal, rather than working for a certain time, would result in more careful experimentation. I did not anticipate that the students would get satisfactory data as quickly as they did. With the practice time, drop selection, timing of the drops, and breaks, each was finished in less than 1 h. It was recommended to Student No. 1 that slower drops should be used. This was not done with the others.

No data have been omitted from Table I. Timings that were immediately known to be mistakes could be canceled without looking away from the microscope unless another timing had already begun. This removal process was used only three times during the 209 pairs of timings. A more liberal use of this feature of the computer program might yield better results.

The student reported that drop No. 3 did not move steadily when the field was applied. This shows in the data and is assumed to be a switch problem.[33] The contacts were cleaned

Table I. Experimental oil-drop data[a] and results.

Drop	No. timings N	Duration[b] (min.) D	Mean times[c] (s) t_e	t_g	Mean radius[d] ($\times 10^{-6}$ m) a	Mean charge[e] ($\times 1.5793\times 10^{-19}$ C)[f] q
1	5	3	9.181±0.062[g]	17.490±0.182	0.895±0.005	5.0496±0.0439
2	4	7	22.436±0.289	52.182±0.338	0.501±0.002	1.0153±0.0066
3[h]	5	4	35.473±1.305	11.766±0.158	1.100±0.008	4.3075±0.0428
4	5	5	30.967±0.423	12.875±0.139	1.050±0.006	3.9749±0.0569
5	5	7	29.823±0.505	12.995±0.102	1.044±0.004	3.9723±0.0343
6	21	8	9.454±0.076	7.260±0.043	1.414±0.004	12.1297±0.0635
7	8	2	4.148±0.018	5.554±0.022	1.622±0.003	24.2212±0.1155
8	9	7	23.698±0.212	17.446±0.094	0.897±0.002	3.0432±0.0199
9	8	6	28.967±0.493	7.602±0.045	1.380±0.004	8.0625±0.0599
10	11	6	7.269±0.030	12.229±0.081	1.080±0.004	8.1930±0.0572
11	9	4	11.285±0.210	7.252±0.103	1.416±0.011	11.3222±0.1196
12[h]	3	2	7.417±0.054	15.982±0.198	0.940±0.006	6.3527±0.0540
12	4	3	7.198±0.067	17.095±0.127	0.907±0.004	6.1155±0.0167
13	5	6	15.170±0.133	34.503±0.514	0.627±0.005	1.9563±0.0255
14	7	10	28.977±0.332	45.775±0.330	0.539±0.002	0.9794±0.0095
15	8	11	26.561±0.215	46.392±0.463	0.535±0.003	1.0209±0.0132
15	11	12	10.375±0.054	47.318±0.570	0.529±0.003	2.0023±0.0198
16	4	1	10.588±0.200	2.939±0.027	2.250±0.010	35.3636±0.4051
17	8	3	6.675±0.041	10.481±0.099	1.172±0.006	10.0544±0.0696
18	9	10	14.269±0.081	11.695±0.099	1.108±0.005	6.0037±0.0470
19	5	10	24.748±0.184	50.816±1.697	0.511±0.009	0.9892±0.0326
20	5	7	23.838±0.194	51.498±0.808	0.507±0.004	0.9984±0.0107
21	13	5	4.848±0.050	7.251±0.035	1.418±0.004	17.2759±0.1262
22	11	6	8.753±0.047	15.073±0.111	0.971±0.004	6.0449±0.0337
23	3	3	18.803±0.374	28.796±0.445	0.691±0.006	2.0286±0.0132
23	9	21	88.926±1.512	30.198±0.318	0.674±0.004	0.9964±0.0122
24	11	11	36.984±0.781	12.453±0.072	1.072±0.003	4.0024±0.0236

[a]Drops 1–5 by student No. 1, drops 6–10 by student No. 2, drops 11–15 by the present author, drops 16–20 by student No. 3, drops 21–24 by student No. 4.
[b]Total time for observations, not including drop selection.
[c]t_e=rise time with the electric field on, t_g=fall time under gravity.
[d]Drop radius, calculated by iteration between Eqs. (1) and (2).
[e]From Eq. (4) with d=0.004 401 2 m, V=405 V, η corrected for chamber temp. (CRC, Ref. 25), ρ=859.9 kg/m^3, g=9.796 37 m/s^2 (Helmert's eq., CRC, Ref. 25, p. F-91), drop motion timed over 0.001 594 5 m, each pair of times were used, then the calculated charges were averaged.
[f]See Sec. VIII.
[g]Any value following ± is an error-of-the-mean=(std. dev.)/$N^{1/2}$.
[h]Data rejected for cause, see Sec. VIII.

and no reports of this behavior were made by the others.[34] This drop probably had five or more charges if the switch problems were the cause of the large variance in its field-on times. It is good training for students to learn when data are likely to be faulty for a cause and then to reject such data. This is a study to improve a teaching experiment, so I am reporting the incident for the reader's benefit, but rejecting it from further analysis.

On drop No. 12, I was distracted by a discussion with a student, and knew my timings were suspect. I averaged these data and started over with the same drop. I have reported these data to show that even with the experiment working well, very close attention to the timings is still critical. This set of data is also not included in further analysis.

VIII. INTERPRETATION

This study primarily attempts to show how it is possible to obtain high quality data in the student version of the oil drop experiment, not how to best use these data in a teaching lab. Individual instructors have different views on this subject. Some may want their students to partially reproduce Millikan's work. Others may choose a simpler approach. With quality data any method should be improved. One possible method of data analysis is described below.

As a starting point in analyzing the calculated charges it was first observed that there were six drops which all had approximately the same charge, which was the smallest value in the set. These were averaged to yield $1.5793\pm 0.0101\times 10^{-19}$ C. It was tentatively assumed that if there is a unit charge, then perhaps this is approximately its value. This is further justified by the fact that there are three drops with approximately twice, one with three times, three with four times, one with five times, three with six times this charge and none in between.[35] So that the reader can more easily see the quantization of the charges on the drops, the charges in Table I have been presented in multiples of this initial estimate. This is simply a convenient scaling factor and carries no assumptions.

The above procedure is further justified in another way. With the computer providing real time calculations of the drop charges, after a few small drops have been selected and timed, it becomes fairly evident to the experimenter that the only drops which carry less charge than these slow moving ones are completely unaffected by the electric field. Students seem to derive some satisfaction in observing these neutral drops.

If the data for drop No. 3 and the first set for No. 12 are eliminated, the remaining calculated charges in Table I show strong evidence of charge atomicity. This is especially true if one also considers the accuracy of the calculated charges as indicated by the errors of the mean (EOM). If only one pair of timings was made on a drop, then no confidence limits could be calculated. In this study multiple timings were always used.

For clarity in the discussions which follow, a short digression on statistics is in order. There is approximately a 32% probability that a mean value will be in error by more than one EOM. There is a 5% probability that a mean value will be in error by two EOM, and only a 1% probability of being in error by more than three EOM. Put another way, we can be 99% confident that the value of the parent population[36] (the answer we are seeking) is within three EOM of the obtained mean value.

Examination of Table I shows that 16 of the 17 drops whose charges are less than seven are within three EOM of being one, and only one, integer.[37] For a refinement in the assumed unit charge, these charges were all divided by this assumed (integer) charge multiplicity and the results were then averaged. This gives $1.5823\pm0.0050\times10^{-19}$ C, an increase of 0.19%. This new value of the assumed unit charge was divided into each of the charges to obtain the assumed charge multiplicity of the remaining drops. Even drop No. 7 with a (now estimated) charge of 24.1751 ± 0.1152 assumed charges was within three EOM of only one integer. We are thus more than 99% confident of the correct charge multiplicity of each drop except for drop No. 16 with an indicated 35.30 ± 0.40 unit charges. We cannot be even 95% confident that the correct charge multiplicity is 35 so this drop was not included in the determination of the final average unit charge. More than 1 min should be spent timing these fast, highly charged drops.

Continuing the same method to include the remainder of the acceptable drops, we obtain $1.5882\pm0.0043\times10^{-19}$ C (0.9913 ± 0.0027 of the accepted value of e) for the final average value of the unit charge. This is quite close by the usual standards for student versions of this experiment. The fact that it is approximately three EOM from the accepted value indicates the probability that systematic errors are still present in the experiment.

In units of the accepted charge of the electron, the five experimenters' results are: 0.9887 ± 0.0056, 0.9988 ± 0.0029, 0.9903 ± 0.0089, 0.9841 ± 0.0033, and 0.9926 ± 0.0038. All are within 1.6%, and three are within 1% of the accepted value.

Students will know the expected answer for the unit charge. Unless the instructor wants to "hide" the answer from them, there is little reason not to have the computer program print out the drop charge multiplicity rather than having the students waste time and energy using their calculators and possibly losing the drop in the process. In the student version of the computer program, an option for the instructor is to select that the charge multiplicity not be printed out, and the calculated charges be multiplied by some constant which only the instructor knows. An approximate value of the unit charge might be supplied to the student. After the student has decided on her value of the unit charge, she could then be told the value of her constant. This is one way to simulate Millikan's situation.

IX. TIMING CONSIDERATIONS

There is possibly a problem in timing large, highly charged drops. For a timing, the drop is first positioned slightly on one side of the start line. It then moves across to the stop line. It is quickly turned around and then timed as it moves in the other direction. The process is then repeated for averaging. The computer records the times immediately, so very little time is usually spent turning the drop around. When the movement of the drop is well established in the experimenter's mind, there is a tendency to anticipate the line crossing rather than waiting for the drop to reach the line before reacting. This is more likely at the stop line than the start line and results in slightly reduced measured time intervals, therefore higher calculated speeds, thus higher calculated charges. This will have a larger relative effect for the shorter times than for the longer times.[38] In my experience, I was also more anxious when the drop moved fast and was more likely to anticipate. Two of the students admitted that they also might have been subject to this type of behavior. As can be seen in Table I, the more highly charged drops do indicate a slightly larger value for the unit charge than the lesser charged drops. This is possibly statistically significant.

To check this hypothesis, I performed a simulation. An electric clock with a large, smoothly moving, sweep second hand was connected to power via a switch. Drop timing was simulated by use of the clock switch and the computer program developed for this study. Various timing options were simulated. The method of waiting for the event, then reacting, was the most accurate for me. This type of activity improves with practice as evidenced by decreasing variances in my timings.

Perhaps the best viewing method is to avert one's vision slightly as the drop approaches the line. With a sharply focused drop in an uncrowded field, the blinking out of the light is easily detected, and there is less of a tendency to anticipate. The same method should be used at both the start and stop lines since there is the possibility of differing reaction times with different methods.

All of these timing studies were done after the students had taken their data, so none of this advice was given to them. However, some data which I have taken subsequent to that reported in Table I do appear to give more accurate results.

X. BOLTZMANN'S CONSTANT

An estimate of Boltzmann's constant can be made from the variance in the timings caused by Brownian motion via the methods of Hoag,[39] and Wall and Christensen.[40] In the present study these methods were extended by performing a least-squares fit to a straight line. This is necessary to include the various sizes of drops as well as variations in the experimental conditions. Adapting the method of Wall and Christensen, the variance of the times may be written as $\sigma_t^2=\alpha+\gamma x$, where σ_t^2 is the variance in times for a set of timings, α is the zero point variance due to human and instrumental effects,[41] $\gamma=k/(3\pi)$, k is Boltzmann's constant, $x=T/(\eta s^2)t_{\mathrm{av}}^3[1+b/(pa)]/a$, T is the absolute temperature, s is the distance over which the drop motion is timed, t_{av} is the average time for the set, a is the mean drop radius for this particular drop, and the other quantities are same as in Eq. (4). Both rise and fall times were used as separate points in the curve-fitting process. The quantities σ_t^2 and x contain the variables which depend on the drop, its timing and the experimental conditions.

The extra hole had not yet been drilled in the upper plate to reduce the wind effect when the data reported in Table I was taken. From these data we obtain $k = 2.4 \pm 0.3 \times 10^{-23}$ J/K. The fact that this value is slightly large is probably partially due to the wind effect.

The mean values of the times are used to calculate the charge on the drop, but Boltzmann's constant also requires their variance. More observations must be made to obtain a reliable estimate of a variance than a mean. Unless a large amount of data is taken, the oil drop experiment is not an accurate method for determining Boltzmann's constant, but its estimation provides an instructive bonus and turns the troublesome Brownian motion into a benefit. In the future, I intend to perform the Boltzmann calculation as a group project after all of the students have completed the experiment. The computer program used while taking data provides rough estimates of Boltzmann's constant for each drop assuming no zero-point contribution to the variance.

XI. CONCLUSIONS

This classic experiment can be transformed into a rewarding experience for both student and instructor. Upgrading the optics is the most-needed modification. The microscope objective lens has been sadly neglected by equipment manufacturers, yet it is perhaps the most critical element in the entire apparatus.

Also important are accurately measuring the microscope calibration, the plate separation, and the voltage. If one assumes *a priori* that results will not be accurate, then the temptation is to measure parameters poorly, or just take the manufacturer's word. To state the obvious, unless each number that goes into Eq. (4) is accurate, then accurate results are impossible.

It is not the usual intent of student experiments to determine fundamental constants to very high precision. However, once a threshold of accuracy has been reached, students feel good about the experiment, and there is a desire in some of them to measure even more accurately as long as it is not too much trouble. For this study I did have the computer display the charge multiplicity. There is an advantage in this. Students can see that they are getting really good data and are then freed from the pressure to get good data to please the instructor. They all shifted their emphasis to trying to measure a drop with only one charge. Four of the five experimenters did succeed in this.

It is good for students to develop the attitude that experimental accuracy is fundamentally important. An experiment that gives inexplicably poor results may create the opposite impression. An experimental setup which measures some parameters with obvious imprecision, such as using an analog panel meter to measure the voltage, might have the same negative effect.

There are pedagogical advantages to using oil drops rather than plastic spheres in this experiment. One is that all but two of the variables and parameters that go into the calculation of the charge can be accurately measured locally. The exceptions are the viscosity of the air, which is a handbook value, and the correction constant, b. With enough data on differing sizes of drops, even the value of b could be roughly determined as part of a group experiment. Neither the density nor the radius of the plastic spheres are likely to be measured directly as part of the experiment.[42] By contrast, the oil density can be measured by a standard method, and the same oil used for this drops. Second, using the uniform-size plastic spheres one could interpret the quantized charges as quantization for the particular size object. Energy quantization can certainly depend on the size of the container. By using different size oil drops the true quantum nature of electric charge is better demonstrated. There is a practical advantage also. With the proper aspirator the oil actually produces less of a cleanup problem than the spheres and does not dry out. The oil buildup in the apparatus at the conclusion of this study was just barely visible as a slight sheen.

By using a computer as an assistant, much of the work load can be relieved. More time is then available for timing drops, and the timings go faster. Students can get an accurate determination of the electronic charge and convincing evidence of its atomicity. In this study, three of the five experimenters obtained values for the electronic charge within 1% of the accepted value. There is essentially no guesswork in assigning the multiplicity of the charge if the data is of the quality that can be obtained with proper equipment and technique.

I did not have the students use the radioactive source to change the charge on the drops. I wanted to obtain data on as many different drops as possible. There were two incidents where the drop changed charge spontaneously.

For this study I made the measurements of the parameters. A beneficial small project would be to have the students examine Eq. (4) to discover which quantities are the most critical. The viscosity, microscope calibration, and timings are raised to the 3/2 power. The plate separation and voltage are to the first power, while oil density and gravity only to the 1/2 power. The barometric pressure enters only weakly. It is an important lesson for the students to realize that, for this particular experiment, it is better to exert more effort in calibrating the microscope than in correcting the barometer for the temperature of the mercury. After this analysis has been done, they could then find ways to measure the most important parameters to sufficient accuracy before performing the remainder of the experiment.

If the students make a calculated estimate of the number of electrons in a typical drop (about 10^{12}), they should be impressed to see that an imbalance of just one charge can be seen and measured accurately. Another striking calculation is finding the mass of a typical drop from its density and radius (about 10^{-15} kg). The drops are actually weighed in the experiment.

With accurate and easily obtained results, this experiment offers additional opportunities for learning and is a rewarding experience for both students and instructors.

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[1]C. D. Hendricks *et al.*, "Efficient bulk search for fractional charge with multiplexed Millikan chambers," Meas. Sci. Technol. **5**, 337–347 (1994). This is a feasibility study. It has a bibliography of fractional charge studies.

[2]Mark A. Heald, "Millikan oil-drop experiment in the introductory laboratory," Am. J. Phys. **42**(3), 244–246 (1974).
[3]C. N. Wall and F. E. Christensen, "Dual-purpose Millikan experiment with polystyrene spheres," Am. J. Phys. **43**(5), 408–413 (1975).
[4]J. I. Kapusta, "Best measuring time for a Millikan oil drop experiment," Am. J. Phys. **43**(9), 799–800 (1975).
[5]Steve Brehmer, "Millikan without the eyestrain," Phys. Teach. **29**(5), 310 (1991).
[6]Sargent-Welch Scientific Co., 911 Commerce Ct., Buffalo Grove, IL 60089-2375. Those interested in receiving further suggestions relevant to this specific model of apparatus should contact me.
[7]The SWOSU Physics Department has three different models of oil-drop apparatus. I replaced the simple, symmetric objective lens in a Pasco Model 250A oil drop apparatus with an Edmund Cat. No. 32,720 achromat (Ref. 14) with the flatter side facing the chamber. The oil drops were then much more visible. This apparatus does not have a provision for focusing on the drops during the experiment, so the accuracy of data is limited. Its illuminator does not lend itself to easy enhancement. This is still a worthwhile modification for anyone with only this old apparatus. I attempted no modifications on a Griffin L89-959 apparatus (Griffin & George Ltd., 285 Ealing Rd., Alperton Wimbley, Middlesex, UK HAO 1HJ) since I deemed its microscope mounting to be insufficiently rigid.
[8]J. B. Hoag, *Electron and Nulear Physics* (D. Van Nostrand, Princeton, NJ, 1948), 3rd ed. pp. 2–6. The experiment is described on pp. 14–17.
[9]A. C. Melissinos, *Experiments in Modern Physics* (Academic, New York, 1966), pp. 2–8; *The Taylor Manual*, edited by T. B. Brown (Addison-Wesley, Reading, MA, 1959), pp. 392–394; Henry Semat, *Introduction to Atomic and Modern Physics* (Holt, Rinehart, and Winston, New York, 1972), 5th ed., pp. 14–18. Other modern physics texts treat the experiment, but many do not include a discussion on the correction to Stoke's law.
[10]R. A. Millikan, *Electrons (+ and −), Protons, Neutrons, and Cosmic Rays* (University of Chicago, Chicago, IL, 1935), p. 102.
[11]J. B. Hoag, Ref. 8, p. 11.
[12]The modifications to the optics are listed in order of decreasing importance, not in chronological order.
[13]See, e.g., F. A. Jenkins and H. E. White, *Fundamentals of Optics* (McGraw-Hill, New York, 1976), 4th ed., pp. 153–157.
[14]Edmund Scientific, 101 East Gloucester Pike, Barrington, NJ 08007-1380.
[15]Familiar examples of the magnification changing are focusing a camera or a photographic enlarger. As one focuses, the image size changes and is not a stationary value at proper focus.
[16]If the microscope has aberrations, then it is difficult to focus on a drop well enough to meet these requirements. Another problem might arise for people whose eyes will still accommodate. Their eye might accommodate to bring the drop into focus at the expense of the reticle. To avoid this, they should focus on the drop while it is near a reticle line.
[17]For a particular spacer, the situation in theory may be statically indeterminate, depending on how the nuts are tightened. Assuming that the nuts are tightened gradually and alternately, this should not be a problem. It should be noticed that all three points of contact would not normally be on any one semicircle. The exception is if two of the points of contact are π rad. apart. In this case, let point 1 be at one of these points, and $\theta_2=\pi$. Equation (5) then reduces to $d=(t_1+t_2)/2$, as it must.
[18]All of these measurements were made using a machinist's micrometer which could be read to 0.0001 in. The micrometer calibration was checked using gauge blocks.
[19]For some models of apparatus one or both of these methods may not be practical.
[20]Sargent–Welch Scientific Co., Ref. 6, Catalog No. WL0620J.
[21]J. B. Hoag, Ref. 8, p. 16.
[22]It might be desirable to use a different voltage for a particular drop. In this study the experimenter was working alone and might lose the drop while entering another voltage into the computer. An obvious solution is to interface the voltmeter to the computer. In the spirit of keeping the experiment as simple as possible, while still producing good results, interfacing was not done for this study.
[23]Model 179A ($4\frac{1}{2}$ digit, with an LED display for use in the dark) and Model 2000 ($6\frac{1}{2}$ digit), Keithley Instruments, Inc., 28775 Aurora Rd., Cleveland, OH 44139.
[24]Model IP-17, Heath Company, Benton Harbor, MI 49022. This power supply needed to warm up for about 15 min to stabilize sufficiently. A 0–10 kΩ variable resistor was inserted at the high end of the voltage-setting potentiometer as a fine voltage control.
[25]*CRC Handbook of Chemistry and Physics* (CRC, West Palm Beach, FL, 1977), 58th ed. p. F-58. A systematic error of 0.3%/°C in the calculated charge will occur if the air viscosity is not corrected for temperature.
[26]Fixed parameters such as the microscope calibration, plate separation, oil density, and gravity are stored in a data file.
[27]If one line is printed then the last pair of timings has been removed. If a few lines are printed, then the current pair of timings is complete. If many lines are printed, then the data have been averaged, and the program is ready for the next drop or a new charge on the same drop.
[28]It makes little difference which of these methods is used to calculate the average charge if the variances in the measurements are small. This particular method was chosen because it still yields easily useful data if a drop should change charge during a series of timings. Also, it is the preferred method if there is any evaporation of the oil drop or if the drop should grow in mass by merging with any of the fine oil mist.
[29]QUICKBASIC 4.5, Microsoft Corporation, One Microsoft Way, Redmond, WA 98052-6399. A few modifications may be required for other forms of BASIC.
[30]Tested by timing a digital and an analog clock with the program.
[31]Available for a PC computer. Executable versions of the programs are included so the BASIC language is only necessary if modifications are desired.
[32]Air conditioning systems can cause the pressure in a building to be different from outside. If this is the case, then opening and closing of exterior doors can have the same effect as wind.
[33]I had observed this behavior earlier and attempted to repair the switch. The polarity of the field was not recorded, but the student thought that this was his only drop which used the "upper plate positive" switch position.
[34]Students observed that this switch sparked when changed. Apparently, it was momentarily shorting the power supply and possibly eroding the contacts. Connecting a 12 kΩ resistor between the power supply and the unit relieved this problem.
[35]No amount of data can ever completely eliminate the possibility that the unit charge on the drops might be $\frac{1}{2}$, $\frac{1}{3}$, etc. of the value obtained in the experiment. However, it does become very improbable as more data are amassed.
[36]This value might still be shifted from the true value by systematic effects.
[37]Drop No. 12 has only four timings and a very small EOM. Many readings are necessary to obtain accurate estimates of the scatter in the data.
[38]Although not done in this study, the rise times can be controlled with the voltage. Both rise and fall times can be controlled by selecting drops of the proper size. This is an advantage of using oil drops rather than plastic spheres. Kapusta (Ref. 4) considered both reaction time and Brownian motion and concluded that the best accuracy can be obtained when the rise and fall times are each of the order of 10 s. However, for drops with small charges, longer rise times are necessary unless the voltage can be increased.
[39]J. B. Hoag, Ref. 8, pp. 9–12, 17–18.
[40]C. N. Wall and F. E. Christensen, Ref. 3, 411–412.
[41]The timer resolution of a PC computer is approximately $\Delta t=0.055$ s. This resolution at both the beginning and end of a time interval will produce an expected variance in the measured times of $2(1/\Delta t)\int_0^{\Delta t}(t-t_{\rm av})^2\,dt=(\Delta t)^2/6=0.0005$ s^2, since $t_{\rm av}=\Delta t/2$ in this instance. A test program was run to determine the variance in the times caused by program interruptions for memory refresh and any other system effects. This varied slightly between computers but had a typical value of 0.000 04 s^2. This must be doubled to include both end effects. The keyboard scan rate of 60 Hz will produce a variance of 0.000 09 s^2. Variances of independent effects may be added. The sum of all of these gives an overall expected system variance for the times of 0.0007 s^2. This is a small effect and should not noticeably increase the variance in the calculated charges. This effect is already included in the zero-point variance α.
[42]Given their density, the radius of the plastic spheres can be determined via the same method as used for the oil drops. However, avoiding this step seems to be the principal rationale for using the spheres.