Millikan Oil Drop Experiment
Physics 134

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Abstract

Robert. A Millikan was the first to measure the fundamental charge e within 1% of today's standards, and we aim to replicate- to a lesser extent- this process. The charge e and the accuracy of its value are paramount in several discoveries, such as the photoelectric effect, the Bohr Model of a hydrogen atom, and several other significant particles. Moreover, it is essential for all modern precision measurements related to chemistry, spectra, high energy, etc. This paper will outline the history leading to Millikan's research, use Newton's and Stokes' Laws to obtain our measurements, frame our procedure, and finalize our analysis. Our closing statements cover error propagation, drop selection, results, and critical issues contributing to error.

Contents

1 Introduction 4

2 Physics 6

2.1 Derivation of oil drop radius a . . . 8

2.2 Derivation of oil drop charge q . . . 11

3 Methods 12

4 Analysis 17

4.1 Equations . . . 18

4.2 Additional Error . . . 20

4.3 Data Tables . . . 20

4.4 Final Results . . . 23

5 Additional Images 25

List of Figures

1 Artist rendition of Millikan Oil Drop Experiment (Unknown) . . . 4

2 Coordinate System of Neutral Chamber . . . 6

3 Electric Field Generated by Applied Voltage . . . 6

4 Free-body diagram of an oil droplet in free fall. (Bishop,2019) . . . 7

5 Free-body diagram of the oil droplet rising in the electric field. (Bishop, 2019) . . . 7

6 Droplet video from Applied Vision 4 viewed on Filmora . . . 12

7 Millikan Oil Drop Set-up . . . 13

8 Apparatus Housing . . . 13

9 Extended Set-up . . . 14

10 Histogram of charge values from Table (7) . . . 23

11 Histogram of e values from Table (7) . . . 24

12 Apparatus Platform (PASCO) . . . 25

13 Thermistor Resistance Table . . . 26

List of Tables

1 Table of Constants used in Analysis . . . 8

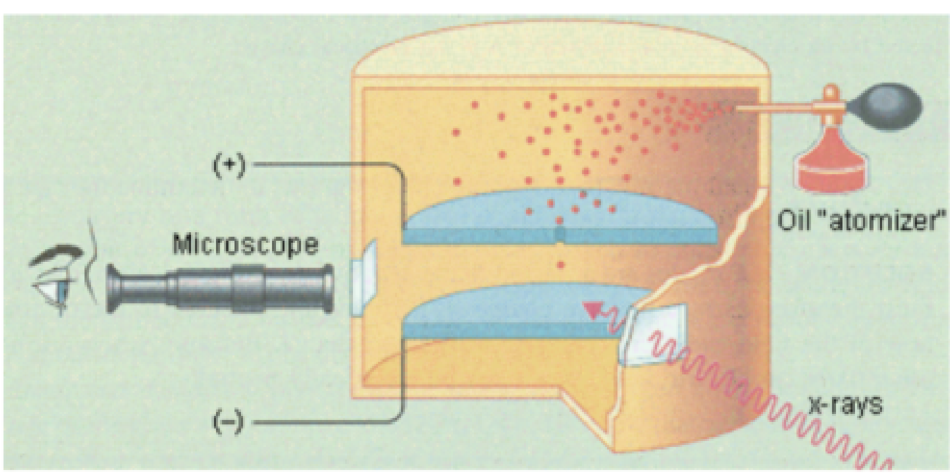
2 Table of Items and their Potential Uncertainties 16
3 Table of Error Contributions 17
4 Sample Table of 14 Measured values. Distance in meters and time in seconds. . . . 21
5 Sample Table of 14 calculated Velocities. Rise and fall velocities in m/s. 21
6 Sample Table of 14 calculated Variances. Variance calculated from Table.5 values. 22
7 Sample Table of 14 calculated charges, standard deviations, their associated e values, and assumed integer multiple. 22
8 Table of Final Values 23

1 Introduction

Electricity and its properties have been a hot commodity as early as 600 BC. Discovered first by the Greeks (UCSC, 2022), they noticed rubbing fur on amber would attract light objects. Today we know this as static electricity. It wasn't until a little over 2000 years later, between the 17th-19th century, that an influx of theories began to emerge. Many theorists embedded their work around the idea of fluid electricity. Benjamin Franklin, the most notable at the time, postulated electrical fluid or fire existed in all matter, and an excess or deficiency of it would produce a net positive or negative charge(UCSC, 2022). Franklin was also the first to reason the existence of an even smaller particle composing the fluid electricity. Michael Faraday later supported his work in his experiments with electrolysis, the process by which electric current is passed through a substance to effect a chemical change. In 1891, Dr.G Johnstone Stoney coined the term "electron," shortly thereafter, Faraday, Townsend, H.S Wilson, and J.J Thomson attempted to measure the infamous fundamental charge e = 1.602E-19 Coulombs (UCSC, 2022).

At the turn of the century, several new controversial theories involving the wave-particle duality of light surfaced. Notable scientists such as Max Planck, Albert Einstein, and Robert A. Millikan began to have their work in electricity noticed. Both Planck and Einstein utilized the existence of the electron, but it was Millikan that paved the way for precision measurements of e. Millikan employed simple, ingenious reasoning in measuring the charge of a charged oil droplet. His apparatus can be simplified into three pieces: precision optical equipment, parallel plates to create an electric field, and an ionizer to ionize the droplets. An artist's rendition shows the process in which the droplets are introduced in **Figure.1**.

Figure 1: Artist rendition of Millikan Oil Drop Experiment (Unknown)



Millikan's oil-drop apparatus

Millikan uses Newton's 1st law on a droplet moving at a constant velocity, i.e., terminal velocity, to obtain an equation for the charge. He also uses Stokes law for the force of viscous drag, which is proportional to the radius and speed of the droplet. The subsequent two sections dive deeper into these subjects. In performing this experiment, Millikan discovered two things. First, the droplets held charge in integer multiples of a specific value, 1.602E-19C. This recurring constant was indivisible, the smallest value a droplet could have, and thus fully realized as the fundamental constant. Millikan's discovery supported a plethora of ideas and findings, such as Einstein's photoelectric effect, Neil Bohr's Model of the Hydrogen atom, and several others. As new particles were discovered, each one depended on the fundamental charge. Quarks, a smaller particle theorized to compose other particles, were said to be precisely e, but quarks are never found isolated. As an example, protons are composed of three quarks, resulting in one integer value of e, further supporting the indivisible nature of the fundamental charge.

In this paper, we wish to rediscover this value due to its high importance in physics as it is today.

2 Physics

We show our interpretation and, to an extent, a simplification of the physics involved in the Millikan Oil Drop Experiment. The following will outline our use of Newton's 1st Law, Stoke's Law of viscous drag, their relationship to our setup, and finalize with a derivation of several equations important to our analysis.

To better understand the following calculations, we being with referencing two figures, **Figure.2** and **Figure.3**. In **Figure.2**, we highlight the orientation of our coordinate system within the chamber. To reduce the potential of losing a negative sign, we assign positive values moving to the right and down in our system. Resulting in any falling velocities being positive and rising velocities being negative. This nuance is accounted for in our calculations further in the reading. With this understood, we can now consider **Figure.3**. Here, we use Faraday's integral formula of electrostatics to determine the electric field between two parallel plates found within our chamber. We presume the system is ideal where the electric field, the distance between the two plates, and the voltage applied remains constant. All resulting values will be positive due to our coordinate system.

Figure 2: Coordinate System of Neutral Chamber

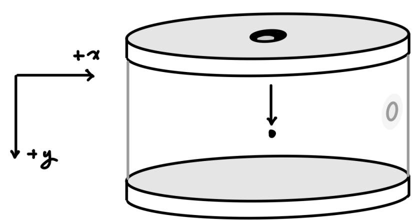
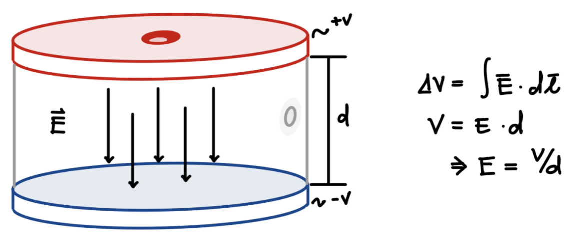


Figure 3: Electric Field Generated by Applied Voltage



With the foreground set, we bring a droplet into our chamber and examine the acting forces contributing to our radius and charge calculation. Since the droplet is tiny compared to the distance it will travel, we assume those viewed by the eyepiece have reached terminal velocity. Next, consider **Figure.4** and **Figure.5**, both illustrating Newton's 1st Law or rather the droplet moving at a constant velocity, i.e., terminal. In **Figure.4**, we only consider the force of gravity F_g and Stokes' Law for the force of viscous drag F_v.

Stokes' Law of viscous fluids:

$$\vec{F}_v = -6\pi a\eta_{eff}(a)\vec{v}$$

Before we continue, it's worth speaking briefly about Stokes' Law concerning our experiment, but no derivation is provided. Viscosity acts as a frictional force to water flowing through a pipe

Figure 4: Free-body diagram of an oil droplet in free fall. (Bishop,2019)

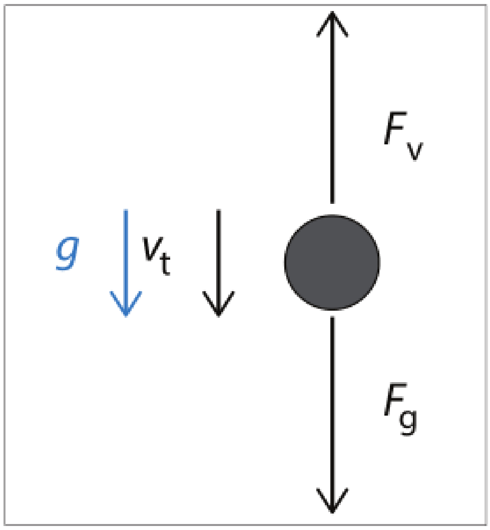
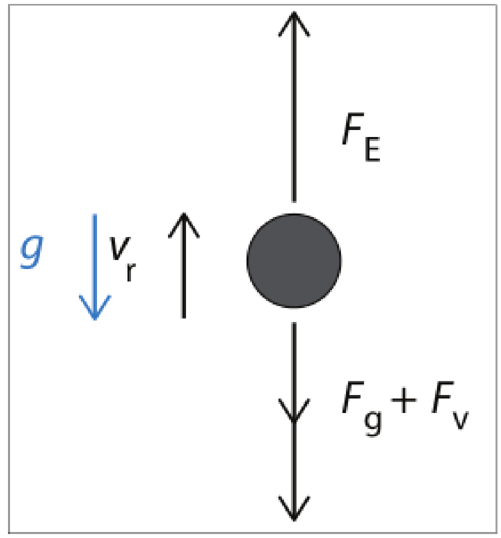


Figure 5: Free-body diagram of the oil droplet rising in the electric field. (Bishop, 2019)



or, in our case, an object falling in a viscous medium. Our viscous medium will be air, and our object is the droplet. To properly utilize this law, Millikan and, by extension, we will assume the following: a perfectly spherical droplet, the fluid flow is laminar, and the friction is only between the air and itself, not the oil of the droplet. Additionally, we acknowledge that if any radii approach the free mean path of air 68 nm under 1atm and 23 degrees Celsius, then our equation of viscous force will fail. Note this force depends on three key variables: the radius of our drop, the velocity of the droplet, and $\eta_{eff}(a)$ a viscosity value which corrects for the radius of the droplet. We simplify it further but using its 1st order approximation shown below.

Viscosity of Air corrected for the radius of the droplet

$$\eta_{eff}(a) = \frac{\eta_0}{1 + \frac{b}{p \cdot a}}$$

See **Table (1)** for a description of the constants found in the equation above. We return to why **Figure.4** is necessary with this information in mind. Note in this figure the droplet is not under the influence of an electric field. This distinction allows us to work in one dimension, i.e., the y/vertical dimension, and solve for the droplet's radius. With the radius known, we can then place a positive voltage on our parallel plates and induce an electric field. Assuming our droplet is not neutral, it will be propelled upward but still at a terminal velocity speed. We see this in **Figure.5**. This arrangement will allow us to use all our known variables to solve for the charge on the droplet.

To better demonstrate this, we have a series of derivations below. Note, a detailed one will be for the radius (a), but then it is assumed the process is understood and shorted for the charge (q).

Table 1: Table of Constants used in Analysis

Symbol	Value	Units	Description
b	8.20E-3	N/m	Viscosity Correction Factor
g	9.80	m/s^2	Acceleration of Gravity
p	1.013E5	N/m^2	Atmospheric Pressure
V	501	V	Voltage across plates
η_0	1.824E-5	$N-s/m^2$	Viscosity of air, uncorrected for radius (tabulated)
ρ	8.86E2	kg/m^3	Density of oil droplet
ℓ	68	nm	Mean free path of air

2.1 Derivation of oil drop radius a

First we recite our conditions for obtaining the radius. The electric field must be off i.e. $E = 0$, our direction is positive for downward in y, movement is at terminal velocity v_{y0} (not under the influence of an electric field). At terminal velocity we exercises Newton's 1st law the sum of forces equaling zero. Recall F_g is the force of gravity and F_v the viscous force which always points opposite to the drops velocity.

$$0 = F_{yg} + F_{yv}$$

Next, we replace them with their equivalent values.

$$0 = mg - 6\pi a\eta_{eff}(a)v_{y0}$$

We can replace mass of the droplet with its equivalent $m = V\rho$.

$$0 = V\rho \cdot g - 6\pi a\eta_{eff}(a)v_{y0}$$

Finally, using the volume of the droplet we solve for a in terms of η_{eff} and v_y0.

$$\begin{aligned} 0 &= \frac{4}{3}\pi a^3\rho g - 6\pi a\eta_{eff}v_{y0} \\ \frac{4}{3}\pi a^3\rho g &= 6\pi a\eta_{eff}v_{y0} \\ a^2 &= \frac{9\eta_{eff}v_{y0}}{2\rho g} \end{aligned}$$

Since the radius is always a positive value we take the principle root.

$$a = \sqrt{\frac{9\eta_{eff}(a) \cdot v_{y0}}{2\rho g}}$$

We simplify this further by substituting in $\eta_{eff}(a) = \frac{\eta_0}{1+\frac{b}{pa}} = \frac{\eta_0 pa}{pa+b}$.

Staring at a^2,

$$\begin{aligned}
a^2 &= \frac{9v_{y0} \cdot \eta_{eff}(a)}{2\rho g} \\
&= \frac{9v_{y0}}{2\rho g} \cdot \frac{\eta_0 pa}{pa+b} \\
a^2 \cdot (2\rho g(pa+b)) &= 9v_{y0}\eta_0 pa \\
a^3 2\rho gp + a^2 2\rho gb &= 9v_{y0}\eta_0 pa \\
0 &= a^2 2\rho gp + a2\rho gb - 9v_{y0}\eta_0 p
\end{aligned}$$

Apply the quadratic formula:

$$a = \frac{-\beta}{2\alpha} \pm \sqrt{\frac{\beta^2 - 4\alpha\gamma}{4\alpha^2}}$$

Where $\alpha = 2\rho gp$, $\beta = 2\rho gb$, and $\gamma = -9v_{y0}\eta_0 p$

Term 1:

$$\begin{aligned}
\frac{-\beta}{2\alpha} &= \frac{-2\rho gb}{2 \cdot 2\rho gp} \\
&= -\left(\frac{b}{2p}\right)
\end{aligned}$$

Term 2 (inside the root):

$$\begin{aligned}
\frac{\beta^2 - 4\alpha\gamma}{4\alpha^2} &= \frac{\beta^2}{4\alpha^2} - \frac{4\alpha\gamma}{4\alpha^2} \\
&= \frac{4\rho^2 g^2 b^2}{4 \cdot 4\rho^2 g^2 p^2} + \frac{4 \cdot 9v_{y0}\eta_0 p}{4 \cdot 2\rho gp} \\
&= \left(\frac{b}{2p}\right)^2 + \frac{9v_{y0}\eta_0}{2\rho g}
\end{aligned}$$

Notice the term $\frac{b}{2p}$ occurs in both terms 1 and 2, this can be used to determine the sign of the root. Additionally, the values within the root are all positive, hence larger than $-\frac{b}{2p}$. Therefore we choose the positive root and our final equation for the radius a is as follows.

$$a = \sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9v_{y0}\eta_0}{2\rho g}} - \left(\frac{b}{2p}\right)$$

$\square$

2.2 Derivation of oil drop charge q

Similar steps are taken here but simplified. We now have to include the Coulomb Force F_E.

$$\begin{aligned} 0 &= F_{yg} + F_{yv} + F_{yE} \\ 0 &= mg - 6\pi a \eta_{eff}(a) v_{y0} + q\frac{V}{d} \end{aligned}$$

We will also utilize a system of equations to simplify our work. Where $k = 6\pi a \eta_{eff}(a)$ and $v_y E$ is terminal velocity for a droplet under the influence of an electric field.

$$\begin{aligned} mg - kv_{y0} &= 0 \\ mg - kv_{yE} + q\frac{V}{d} \end{aligned}$$

Eliminating k we obtain q.

$$q = \frac{4}{3}\pi a^3 \rho g \frac{d}{V} \frac{v_{yE} - v_{y0}}{v_{y0}}$$

$\square$

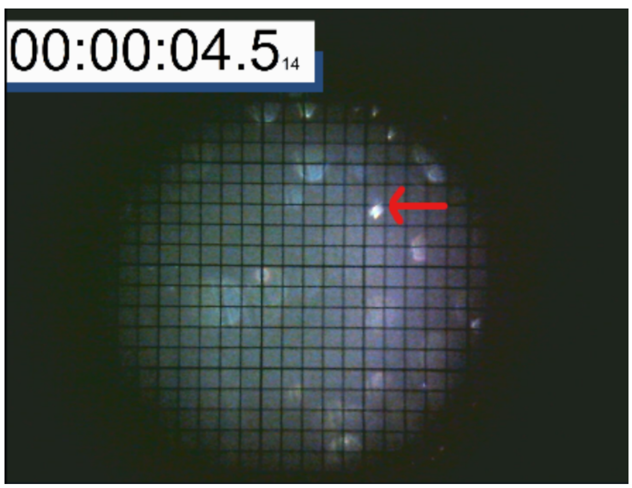
3 Methods

The following information is pulled from the *Millikan Oil Drop Apparatus Instruction Manual* and first-hand experience. We cover calibration, troubleshooting, drop selection, and a brief overview of the steps taken for data collection. **Table (2)** outlines all the equipment and software used.

Calibration of recording software, Applied Vision 4, is helpful but not required. The process is found in its documentation and outlines how to adjust the measurement reader, a line tool, to measure pixels and return a relevant distance value. An example would be 5 pixels correlating to 0.5mm. Most of our collection was done off-site, meaning we did not have access to this software.

To mediate this, we installed a separate software program, Wondershare Filmora, on our local computers to process the videos. Filmora allows the user to scroll through each recording frame by frame, allowing for more accurate distance measurements. Time is then accounted for by utilizing another feature by Filmora; we superimposed a digital stopwatch recording with fractional seconds onto our videos. See **Figure.6** for a better understanding of how this approach works.

Figure 6: Droplet video from Applied Vision 4 viewed on Filmora



Stopwatch superimposed onto the our droplet video

Time, distance, and processing are now accounted for. We move into the calibration of our major equipment, the Millikan Oil Drop apparatus and power supply.

Our Millikan Optical System is tightly integrated into the Millikan Oil Drop apparatus see **Figure.7**. Before performing delicate adjustments to the optical system, it is vital to have the platform perfectly level. We can do this by adjusting several screws found on either side of the platform and observing the bubble level, also seen in **Figure. 7**. We dismantle the Droplet Viewing chamber to extract the plastic divider and measure its thickness see **Figure. 8**. This value will serve as our plate separation (d). Measurement was done with a caliper and found to be 7.753E-3m or 7.753mm with an uncertainty of 0.0127mm. As a note, the power should not be on during the dismantling of any equipment. This experiment works with a device that can generate up to 10 mA a min. Anything above 1mA has the potential to do serious harm. With that said, we can transition to checking the voltage with the multi-meter and checking the temperature within the chamber.

Figure 7: Millikan Oil Drop Set-up

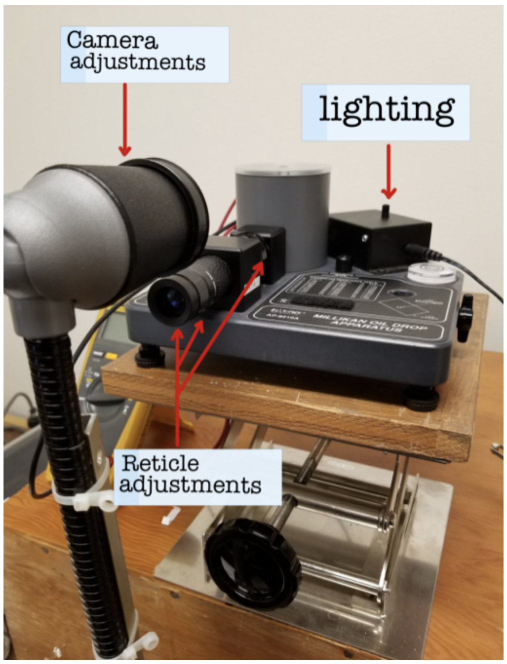
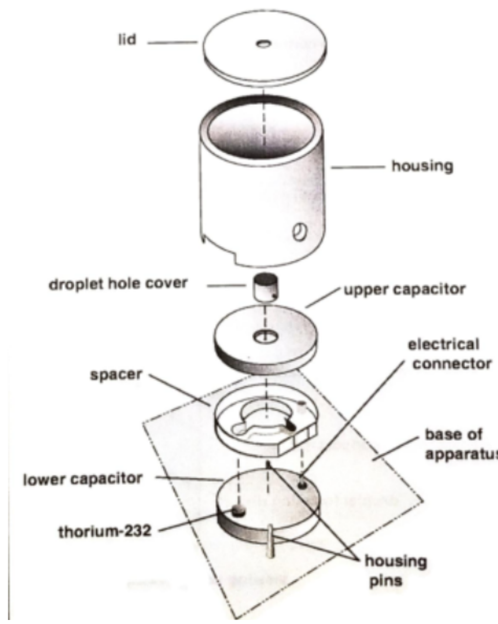


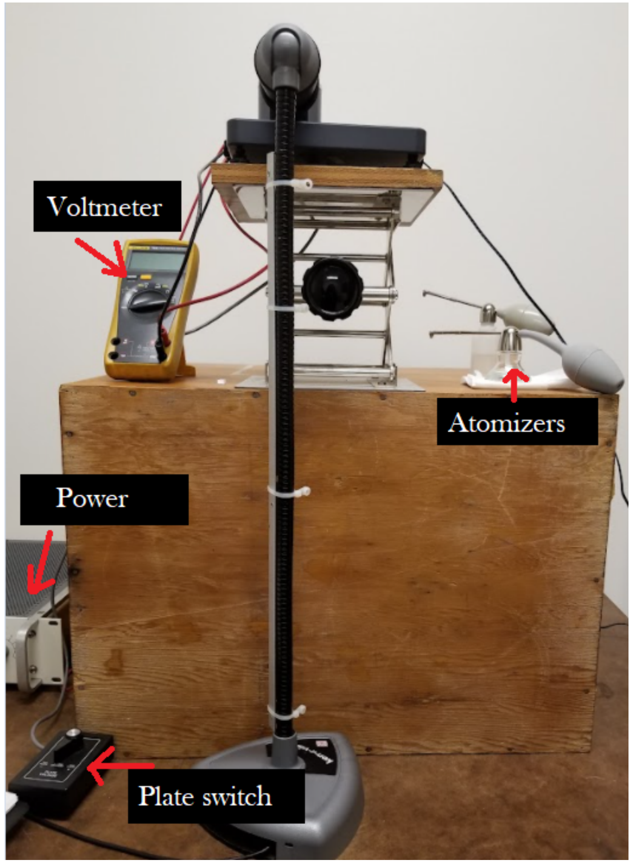
Figure 8: Apparatus Housing



Prior to any experiment or calibration, we checked that our power supply was operational and measured the voltage applied see **Figure.9** for a clear image. Throughout the experiment, we checked this value and noticed it remained constant during our lab period and each day thereafter. Due to this, we took this voltage as constant and removed the uncertainty. However, we will bring it up again in the analysis. Once we recorded the voltage, we turned the supply off, took our multi-meter leads, and placed them in the thermistor connectors to measure resistance **Figure.7** and **Figure.12**. In **Figure.13**, we see a table on the Millikan platform that indicates what temperature is related to our measured resistance. Typically this value would be room temperature 22-23 deg Celsius. This value is not used in the calculation. Still, it notes to the experimenter that if the temperature increases, the viscosity of our droplet will be affected and potentially cause evaporation. Both ultimately contribute to potential errors in data collection. The power should be turned off to mediate any temperature fluctuations and data collection paused between runs. Though, this was not done in practice and is advised to future experimentalists.

The last details of calibration lie with the Optical system. This is a very delicate and involved process, and it will be left to the reader to follow the steps outlined in the manual. Instead, I will reference what ideally is viewed by the camera, assuming all steps were correctly performed. In **Figure.6**, we see a shower of droplets falling into view of the camera. Key features to note are

Figure 9: Extended Set-up



the sharpness of the reticle lines, or rather the grid, the brightness of the bubbles, and the number of droplets. At the onset of experimentation, we did several runs with only one or two droplets appearing, if at all, because we could not produce a shower. It was later discovered one atomizer produced more adequate showers, and a method was developed to squeeze or push the droplets into view properly. We found that one sharp squeeze and release followed by a prolonged squeeze pushes the droplets into view. Although it is advised not to release the second squeeze while the atomizer is in the lid. If released, the atomizer will suck droplets out of view.

The most tedious and, at times, the frustrating portion of the lab is drop selection. We will describe best practices in obtaining a decent droplet, but in truth, we measured over 70 droplets, and each droplet gave some amount of grief which will be described in the analysis. First and foremost, the environment in which the experiment is done must be ideal. We require a dark room to clearly view the droplet and preferably draft and vibration-free to avoid disturbing its motion. Once achieved, set the ionization lever to "Spray Droplet Position" this allows air to escape as droplets enter the housing, maintaining the pressure within the chamber. Perform the aforementioned method of producing a shower of droplets. As soon as the shower is in view, feel free to either set the Ionization lever to "off" or for about 5 seconds to "on," but set it to "off" when complete.

Now we are ready to observe and select our droplet. Presumably, the user has the voltage on the plates set to ground and allows the droplets to free fall without the influence of an external eclectic field see **Figure.9**. From the Millikan Instruction Manual, they indicate we should search for droplets falling at a rate of 15 seconds per major reticle and for ones that resemble bright lights. This would be best seen after allowing some of the faster moving drops to move out of the viewing screen. Once they've vacated, use the plate charging switch to place a positive voltage on the top plate generating an electric field. At this point, certain drops will do one of the following: speed up their descent, cease movement, or propel upward. We are not interested in the motionless drops. Also, caution is advised when setting a voltage onto the plate for too long- it will clear the screen; this is also a method used if the viewing screen is too cluttered. Essentially, we will turn the field on and off to select the best behaving drop or set of drops, if lucky.

Now a brief account of data collection. Once a droplet is selected, we set our Applied Vision program to record. If possible, we capture up to three falls and three rises. If the drop behaves well enough, we start the setup for a new recording of an ionization run. This process involves the ionization lever to be set to "on" for 5 seconds then "off" when complete, which allows alpha particles to impact the droplets, allowing more charge to accumulate. Again we record up to three jumps. A run can run up to two minutes long in selected drops and then about two minutes per set of falls. Over three weeks, we collected and analysed for **(40)** droplets. The data was kept in a CSV file and analyzed in a file or in a Juypter notebook.

Table 2: Table of Items and their Potential Uncertainties

Item	Description	Uncertainty Associated
Atomizers (2)	Sprays very fine liquid droplets	n/a
Camera: Ken-a-vision	Records droplets in viewing chamber	Restricted to 5fps
Multi-meter: Fluke	Measured voltage applied by external power source	Although this measure was assumed to be exact. Due to the value remaining constant over each day we took measurements. **Basic Accuracy: 0.3%**
Power Supply	Supplies 500V, all measured values were taken at 501V	Image of supply is provided but company name was not found. This value was measured with the multimeter
Mineral Oil	Supply of droplets used in atomizers	Fixed to the value in the manual. The oil we used was 20 years old.
Millikan Oil Experiment Apparatus	Contains two parallel plates, ionizer,etc. (PASCO)	**Reticle measure error: 0.01mm**
Applied Vision 4 Software	Processes the video and saves it onto an external source.	n/a
Wondershare Filmora Software	Processes the videos from Applied Vision and allows the user to slowly scroll frame by frame to measure distance and time.	n/a
Online Stopwatch	Superimposed onto the video's using Filmora to obtain a more accurate time measure. www.timeanddate.com/stopwatch	**Error: 00:00.1s**
Plastic Separator	Measure (d) distance between plates	**d = 0.0127mm**
Jupyter Notebook	Software to run python, analyze data, and create Figures	User error can occur: erroneous values calculated, misprint, etc.
Excel	Software to record and store data	User error can occur: erroneous values calculated, misprint, ect.

4 Analysis

Several sources of error were identified and placed in **Table (3)** to highlight and summarize each one. Three of which are reticle measure, time measure, and plate separation. These were quantified and used to propagate errors in our calculations. In the following section, we show the necessary equations for our error propagation, identify some downfalls, and conclude with figures of our results.

Table 3: Table of Error Contributions

Item	Description	Error description/Value
Camera: Ken-a-vision	Fps is too low	5fps cannot monitor faster moving droplets. This error was allocated to the reticle measure and stopwatch measure.
Mineral Oil	20 years old	Error was not included in calculation but should be acknowledged.
Environment	Small room shared with other experimenters	Inadequate darkness, table holding apparatus was bumped on occasion due to shared space causing videos to have issues adding error to data collection.
Individual	Two data collectors	Neither discussed how selecting a droplet should be done and each collected and shared values. Good faith was used, but acknowledged as a source of error.
Optical System	Reticle measure of the distance traveled by droplet	x = 0.01mm
Stopwatch	Measure of time interval	t = 00: 00: 0.1
Plastic Separator	Measure (d) distance between plates	d = 0.0127mm
Evaporation	Droplets observed would only be readable for about 2 falls before evaporating	We assume over the entire lab period the temperature within the chamber increased. It was only reasoned at our last run that perhaps the droplets were evaporating too quickly since we did not turn it off in between runs to cool off. We also would only check the temperature at the start of the day.

4.1 Equations

First, we introduce our propagation and weighted sum formulas.

Variance of function f with two sources of error:

$$Var(f) = \sigma_f^2 = (\frac{\partial f}{\partial x})^2 \cdot (\sigma_x)^2 + (\frac{\partial f}{\partial t})^2 \cdot (\sigma_t)^2 \tag{1}$$

Variance weighted sum:

$$\sigma_f^2 = \frac{1}{\Sigma_i^3(\frac{1}{\sigma_{fi}^2})} \tag{2}$$

Variable weighted sum:

$$\bar{q} = \frac{\Sigma_i^3(\frac{q_i}{\sigma_i^2})}{\Sigma_i^3(\frac{1}{\sigma_{qi}^2})} \tag{3}$$

Standard deviation or error:

$$\sigma_f = \sqrt{Var(f)} \tag{4}$$

Next, to reduce confusion we define these variables: $\alpha = \frac{b}{2p}$, $\gamma = \frac{9\eta_0}{2\rho g}$, and

$$\phi = \frac{18\pi}{V} \cdot \sqrt{\frac{\eta_0^3}{2gp}} \cdot \left(\frac{1}{(1+\frac{b}{pa}}\right)^{3/2}$$

Recall the equations for the radius a and charge q,

$$a = \sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9v_{y0}\eta_0}{2\rho g}} - \left(\frac{b}{2p}\right)$$

$$q = \frac{18\pi}{V} \cdot \sqrt{\frac{\eta_0^3}{2gp}} \cdot \left(\frac{1}{(1+\frac{b}{pa})}\right)^{\frac{3}{2}} \cdot d \cdot \sqrt{v_{y0}} \cdot (v_{yE} - v_{y0})$$

After our substitutions we obtain the following,

$$a(v_{y0}) = \sqrt{(\alpha)^2 + \gamma \cdot v_{y0}} - (\alpha) \tag{5}$$

$$q(v_{y0}, v_{yE}, d) = \phi \cdot d \cdot \sqrt{v_{y0}} \cdot (v_{yE} - v_{y0}) \tag{6}$$

Applying equation (1) to equations (4) and (6),
Variance of radius a:

$$\sigma_a^2 = (\frac{\partial a}{\partial v_{y0}})^2 \cdot (\sigma_{v_{y0}})^2$$

$$\sigma_a^2 = \left(\frac{\gamma}{2\sqrt{\alpha^2 - \gamma \cdot v_{y0}}}\right)^2 \cdot (\sigma_{v_{y0}})^2 \tag{7}$$

Variance of charge q:

$$\sigma_q^2 = (\frac{\partial q}{\partial v_{y0}})^2 \cdot (\sigma_{v_{y0}})^2 + (\frac{\partial q}{\partial v_{yE}})^2 \cdot (\sigma_{v_{yE}})^2 + (\frac{\partial q}{\partial d})^2 \cdot (\sigma_d)^2$$

$$\sigma_q^2 = \left(\frac{\phi \cdot d \cdot (v_{yE} - 3v_{y0})}{2\sqrt{v_{y0}}}\right)^2 \cdot (\sigma_{v_{y0}})^2 + (\phi \cdot d \cdot \sqrt{v_{y0}})^2 \cdot (\sigma_{v_{yE}})^2 + (\phi \cdot \sqrt{v_{y0}} \cdot (v_{yE} - v_{y0}))^2 \cdot (\sigma_d)^2 \tag{8}$$

Notice both the radius and the charge depend on the velocities, but in collection we measure distance and time. So the next two equations illustrate our process where v_f and v_r are the fall and rise velocities respectively.

$$v_f = v_r = v = \frac{x}{t}$$

$$\sigma_v^2 = (\frac{\partial v}{\partial x})^2 \cdot (\sigma_x)^2 + (\frac{\partial v}{\partial t})^2 \cdot (\sigma_t)^2$$

$$\sigma_{vf}^2 = \sigma_{vr}^2 = (\frac{1}{t})^2 \cdot (\sigma_x)^2 + (\frac{x}{t^2})^2 \cdot (\sigma_t)^2$$

4.2 Additional Error

We must clear up a few things regarding the equations we have and the data collected in the following section. First, we acknowledge the formula for the charge q should contain error for the radius and voltage values. Due to time constraints, this was not incorporated, meaning this will inevitably remove error which should be present. Next, consider **Table (4)** where three coupled sets of distance and time are shown. Each line represents one droplet with three falls and three rises. To obtain **Table (5)**'s values we preform $V = x/t$ and apply equation (1) to the values from **Table (4)**. From here we take each coupled pair e.g. $(VF1, Vr1, varVf1, varVf1)$ and use equation (6) and (8) to find the charge and variance of each set. To illustrate this,

$$
\begin{aligned}
d1 &= [q_i, q_{ii}, q_{iii}, \sigma_{qi}, \sigma_{qii}, \sigma_{qiii}] \\
d2 &= [q_i, q_{ii}, q_{iii}, \sigma_{qi}, \sigma_{qii}, \sigma_{qiii}] \\
&. \\
&. \\
&. \\
d40 &= ...
\end{aligned}
$$

To reduce our triplicate to a single charge value, we take each droplets set of values and calculated a weighted sum, equation's (2) and (3). This operation will give us the first two columns of **Table (7)**. To obtain the last two columns we created a histogram of our data which revealed a very small accumulation of charges about integer values. Again, many of these values were not centered directly on an integer and it would benefit us to have more data, but due to time constraints this was not done. Resulting in an executive decision to divide each charge by the value e and estimate the integer associated. Quantifying this error was lost on us, but acknowledged as a large assumption on our part which will reduce error in our final analysis. Once we had our assumed integer value we returned to our original data set and divided each by their respective integer. This was also done to their respective variance values.

4.3 Data Tables

A set of tables containing measurements and calculations for the first 14 droplets obtained. We examined 40 in total, but to save space only 14 are shown. The final graphs are processed with all 40 droplets. The tables are listed in order of their measure/calculation.

Table 4: Sample Table of 14 Measured values. Distance in meters and time in seconds.

1Fm	**1Fs**	1Rm	1Rs	**2Fm**	**2Fs**	2Rm	2Rs	**3Fm**	**3Fs**	3Rm	3Rs
5.00E-04	13.7	5.00E-04	2	5.00E-04	10.6	5.00E-04	1.9	5.00E-04	13.5	5.00E-04	3.8
5.00E-04	4.2	5.00E-04	11.6	5.00E-04	4.3	5.00E-04	11.4	5.00E-04	4.4	5.00E-04	2.5
5.00E-04	4	5.00E-04	2.8	5.00E-04	4.4	5.00E-04	2.4	5.00E-04	4	5.00E-04	2.5
5.00E-04	5.7	5.00E-04	0.9	5.00E-04	4	5.00E-04	0.8	5.00E-04	4.4	5.00E-04	0.9
5.00E-04	4.5	5.00E-04	0.7	5.00E-04	4.2	5.00E-04	0.6	5.00E-04	4.1	5.00E-04	0.6
5.00E-04	13.3	5.00E-04	2.7	5.00E-04	15.5	5.00E-04	2.3	5.00E-04	11.8	5.00E-04	2.9
5.00E-04	16.6	5.00E-04	1.2	5.00E-04	18.5	5.00E-04	1.5	5.00E-04	20.5	5.00E-04	1.4
5.00E-04	17.9	5.00E-04	1.4	5.00E-04	15.1	5.00E-04	1.9	5.00E-04	17.1	5.00E-04	1.6
5.00E-04	11	5.00E-04	1.2	5.00E-04	16.8	5.00E-04	2.8	5.00E-04	19.8	5.00E-04	2.7
5.00E-04	11	5.00E-04	1.9	5.00E-04	13.8	5.00E-04	2	5.00E-04	21.5	5.00E-04	2
5.00E-04	19.4	5.00E-04	2	5.00E-04	21.2	5.00E-04	2.2	5.00E-04	16.5	5.00E-04	1.6
5.00E-04	8.99	5.00E-04	3.02	5.00E-04	8.96	5.00E-04	3.98	5.00E-04	8.98	5.00E-04	3.5
5.00E-04	11.6	5.00E-04	1.6	5.00E-04	10.6	5.00E-04	1.6	5.00E-04	12.4	5.00E-04	1.7
4.00E-04	8.6	5.00E-04	2.8	5.00E-04	9.4	3.00E-04	1.8	5.00E-04	10.3	5.00E-04	1.7

Table 5: Sample Table of 14 calculated Velocities. Rise and fall velocities in m/s.

Vf1	Vr1	Vf2	Vr2	Vf3	Vr3
3.65E-05	2.50E-04	4.72E-05	2.63E-04	3.70E-05	1.32E-04
1.19E-04	4.31E-05	1.16E-04	4.39E-05	1.14E-04	2.00E-04
1.25E-04	1.79E-04	1.14E-04	2.08E-04	1.25E-04	2.00E-04
8.77E-05	5.56E-04	1.25E-04	6.25E-04	1.14E-04	5.56E-04
1.11E-04	7.14E-04	1.19E-04	8.33E-04	1.22E-04	8.33E-04
3.76E-05	1.85E-04	3.23E-05	2.17E-04	4.24E-05	1.72E-04
3.01E-05	4.17E-04	2.70E-05	3.33E-04	2.44E-05	3.57E-04
2.79E-05	3.57E-04	3.31E-05	2.63E-04	2.92E-05	3.13E-04
4.55E-05	4.17E-04	2.98E-05	1.79E-04	2.53E-05	1.85E-04
4.55E-05	2.63E-04	3.62E-05	2.50E-04	2.33E-05	2.50E-04
2.58E-05	2.50E-04	2.36E-05	2.27E-04	3.03E-05	3.13E-04
5.56E-05	1.66E-04	5.58E-05	1.26E-04	5.57E-05	1.43E-04
4.31E-05	3.13E-04	4.72E-05	3.13E-04	4.03E-05	2.94E-04
4.65E-05	1.79E-04	5.32E-05	1.67E-04	4.85E-05	2.94E-04

Table 6: Sample Table of 14 calculated Variances. Variance calculated from Table.5 values.

varVf1	varVr1	varVf2	varVr2	varVf3	varVr3
6.04E-13	1.81E-10	1.09E-12	2.20E-10	6.24E-13	1.89E-11
1.37E-11	8.81E-13	1.27E-11	9.17E-13	1.18E-11	8.00E-11
1.60E-11	5.34E-11	1.18E-11	9.27E-11	1.60E-11	8.00E-11
5.45E-12	3.93E-09	1.60E-11	6.26E-09	1.18E-11	3.93E-09
1.10E-11	1.06E-08	1.37E-11	1.96E-08	1.48E-11	1.96E-08
6.45E-13	6.08E-11	4.60E-13	1.08E-10	8.47E-13	4.72E-11
3.96E-13	1.28E-09	3.14E-13	5.38E-10	2.52E-13	7.02E-10
3.36E-13	7.02E-10	4.87E-13	2.20E-10	3.71E-13	4.21E-10
9.97E-13	1.28E-09	3.86E-13	5.34E-11	2.71E-13	6.08E-11
9.97E-13	2.20E-10	5.94E-13	1.81E-10	2.28E-13	1.81E-10
2.83E-13	1.81E-10	2.35E-13	1.27E-10	4.01E-13	4.21E-10
1.62E-12	4.10E-11	1.63E-12	1.63E-11	1.62E-12	2.48E-11
8.81E-13	4.21E-10	1.09E-12	4.21E-10	7.56E-13	3.34E-10
1.64E-12	5.34E-11	1.45E-12	1.17E-10	1.16E-12	3.34E-10

Table 7: Sample Table of 14 calculated charges, standard deviations, their associated e values, and assumed integer multiple.

Charge q (C)	σ_q	e (C)	σ_e	n
-5.30E-19	3.21E-23	-1.77E-19	1.86E-23	3
-1.01E-18	3.40E-23	-1.68E-19	1.39E-23	6
-1.60E-18	3.12E-23	-1.60E-19	9.86E-24	10
-3.20E-18	1.95E-22	-1.60E-19	4.37E-23	20
-4.47E-18	2.94E-22	-1.60E-19	5.56E-23	28
-5.87E-19	3.44E-23	-1.47E-19	1.72E-23	4
-8.14E-19	6.26E-23	-1.63E-19	2.80E-23	5
-7.56E-19	5.49E-23	-1.51E-19	2.45E-23	5
-4.87E-19	3.49E-23	-1.62E-19	2.01E-23	3
-6.70E-19	4.58E-23	-1.68E-19	2.29E-23	4
-5.67E-19	4.16E-23	-1.89E-19	2.40E-23	3
-6.37E-19	2.45E-23	-1.59E-19	1.23E-23	4
-9.82E-19	6.54E-23	-1.64E-19	2.67E-23	6
-7.22E-19	3.82E-23	-1.81E-19	1.91E-23	4

4.4 Final Results

Unable to find conclusive evidence of our analysis being the correct method we conclude our results and add that this section may be wildly incorrect. Once we obtained our 40 values of e and plotted them in a histogram it was our goal to find the Z score, p-values, and two tailed values.

Z Score equation:

$$Z_{score} = \frac{x - \mu}{\sigma}$$

Presumably we take all the charges we have and preform another weighted sum using equations (2) and (3). We found our averaged value to be $e_{measured}$ = -1.58E-19 C which is about 1.4% away from e = 1.602E-19 C. **Table (8)** has all our values obtained.

Table 8: Table of Final Values

Item (C)	Value
$e_{measured}$	-1.58E-19 C
μ_e	8.169E-27 C
Z_{score}	1.380
CDF	0.916
2-Tail	0.167
p-value	0.915

Figure 10: Histogram of charge values from Table (7)

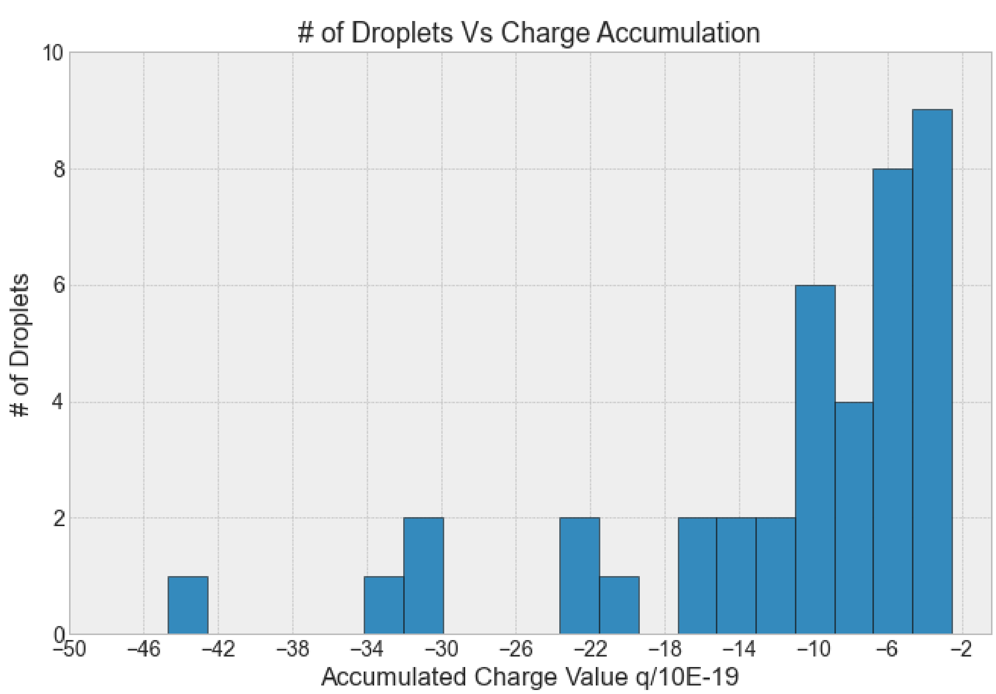
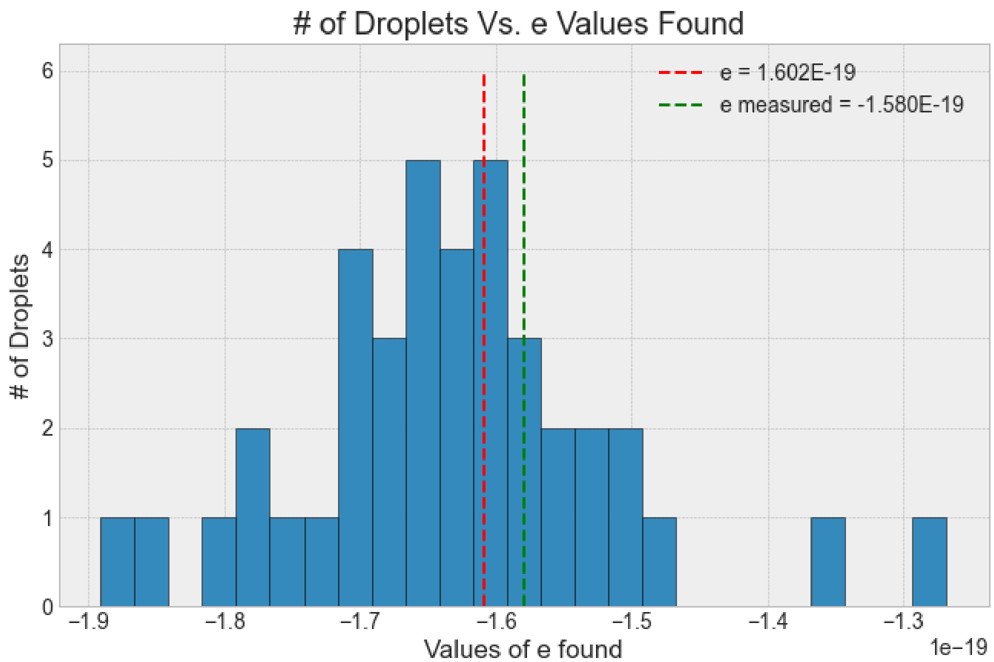


Figure 11: Histogram of e values from Table (7)



Unfortunately, interpretation is also lost on me. At first glance I assume this Z Score and p-value mean we are in agreement of the null hypothesis i.e. in agreement with the value e. But, the Z Score in comparison to the tail values makes me believe it lies inside the rejection zone and should indicate a rejection of the null.

With more time these values could be better understood but for now this is where we conclude our analysis. If preformed again we would address all issued outlined in our Table of errors, the section titled *Additional Error*, and prefect the final analysis. As it stands this is the conclusion of the report.

5 Additional Images

Figure 12: Apparatus Platform (PASCO)

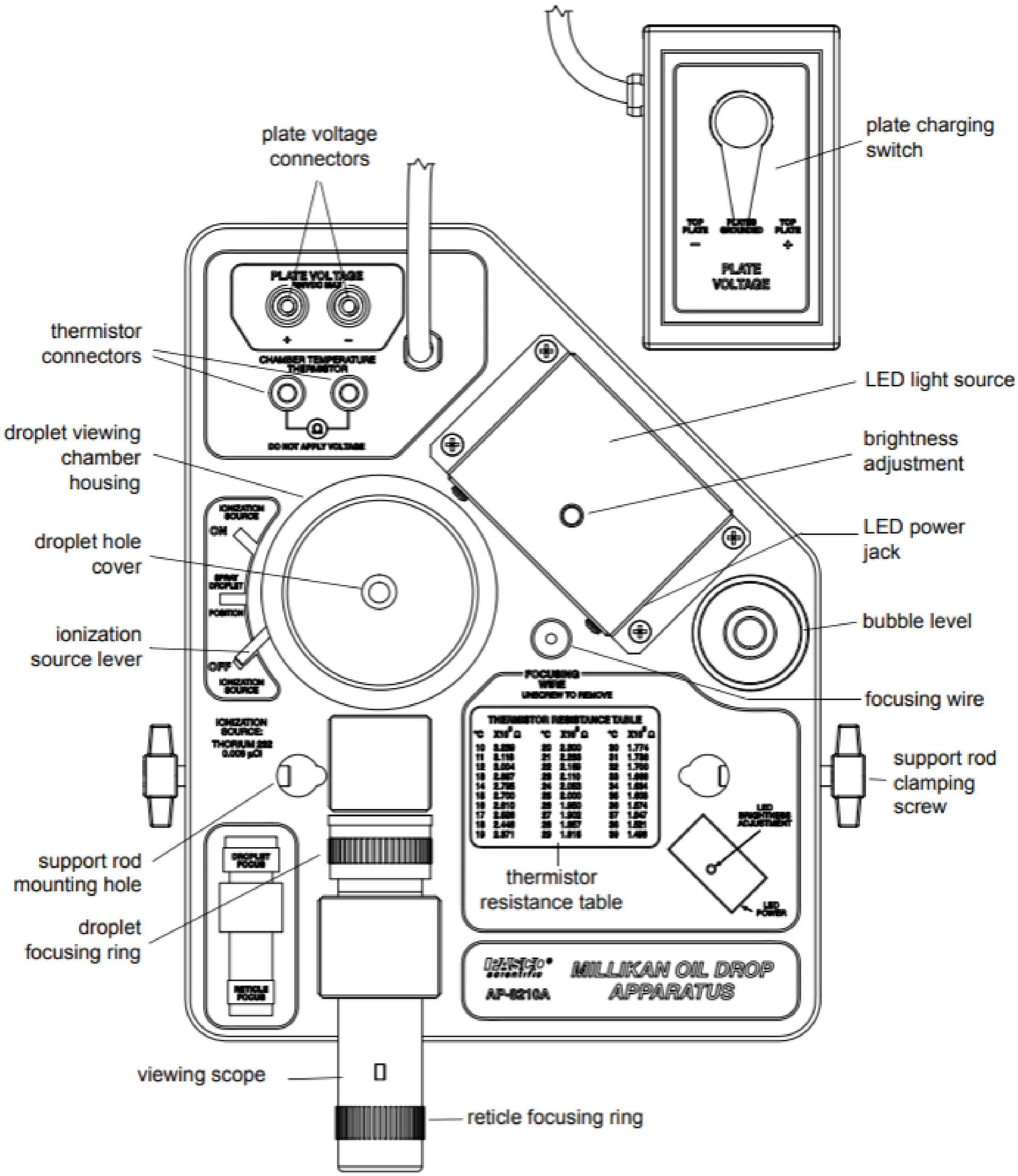


Figure 13: Thermistor Resistance Table

THERMISTOR RESISTANCE TABLE

°C	X10^6 Ω	°C	X10^6 Ω	°C	X10^6 Ω
10	3.239	20	2.300	30	1.774
11	3.118	21	2.233	31	1.736
12	3.004	22	2.169	32	1.700
13	2.897	23	2.110	33	1.666
14	2.795	24	2.053	34	1.634
15	2.700	25	2.000	35	1.603
16	2.610	26	1.950	36	1.574
17	2.526	27	1.902	37	1.547
18	2.446	28	1.857	38	1.521
19	2.371	29	1.815	39	1.496

References

[1] Bishop, I. et al.(2019). Robert A. Millikan and the Oil Drop Experiment. *Phys.Teach.* 57, 442: http://doi.org/10.119/1.5126819

[2] Jones.(1995). The Millikan oildrop experiment:Making it worthwhile. *American Association of Physics Teachers*63, 970; doi:10.1119/1.18001

[3] Ken-A-Vision.(2013). *Software Instruction Manual*.Applied Vision™ 4.

[4] Nadia Robotti (1995): J.J Thomson at the cavendish laboratory: The history of an electric charge measurement, *Annals of Science*, 52:3, 265-284.

[5] Niaz et al.(2021). Reconstruction of the History and the Photoelectric Effect and its Implications for General Physics Textbooks. *Sci Ed.* 94:903-931

[6] PASCO.Instruction Manual and Experiment Guide for the PASCO scientific Model AP-8210 *Millikan Oil Drop Apparatus*

[7] U.C Santa Cruz.(2022)*Advanced Physics Laboratory*

[7] Unknown. http://chemed.chem.purdue.edu/genchem/history/millikan.html