Contents

ECE 8 Robotics Notes 11-17-22

Recall

$$x_{k+1} = a \cdot x_k$$

- where all variable in this equations are real numbers
- The value of a can tell us about the stability or attractiveness of our equilibrium point x_{eq}

$$\begin{cases} a \neq 0 \\ |a| < 1 \end{cases}$$

Recall: For the cruise control

$$v_{k+1} = v_k + \Delta \frac{-bv_k}{m}$$

$$v_{k+1} = v_k - (\Delta \frac{b}{m})v_k$$

Lecture 16

Derivation of Δ Bounds for any m or b

The equilibrium point is : Asymptotically stable In this lecture, we will use the design conditions

$$\begin{cases} a \neq 0 \\ |a| < 1 \end{cases}$$

to assure mathematically that the equilibrium point is stable and attractive. (Asymptotically stable x_{eq}

RECAP from lecture 15:

$$v_{k+1} = v_k + \Delta \frac{-bv_k}{m}$$

$$v_{k+1} = v_k - (\Delta \frac{b}{m})v_k$$

$$v_{k+1} = (1 - \Delta \frac{b}{m})v_k$$

Where we let a be

$$a = 1 - \Delta \frac{b}{m}$$

Apply condition $a \neq 0$

$$a = 1 - \Delta \frac{b}{m} \neq 0$$

$$1 - \Delta \frac{b}{m} \neq 0$$

$$\Delta \frac{b}{m} \neq 1$$

So, given the physics parameters b and m we need to pick a Δ that

$$\Delta \frac{b}{m} \neq 1$$

or equivalently

$$\Delta \neq \frac{m}{b}$$

- Question:
  - why do we let $a = 1 - \Delta \frac{b}{m} \neq 0$
  - Recall the model must be of this form:

    $$x_{k+1} = something \cdot x_k$$

    or

    $$v_{k+1} = something \cdot v_k$$
  - in the cruise control (something $= 1 - \Delta \frac{b}{m}$)
  - let $a = 1 - \Delta \frac{b}{m}$
  - next, we use our required condition $a \neq 0$
  - so then we get again $a = 1 - \Delta \frac{b}{m} \neq 0$
- Question:
  - why does $a \neq 0$
  - examine if our system does have $a = 0$

    $$x_{k+1} = a \cdot x_k$$

    $$x_{k+1} = 0 \cdot x_k$$

    $$x_{k+1} = 0$$
  - all values wil be zero and stay zero, and these systems are not interesting to examine, so we want to ignore them

Apply 2nd condition $|a| < 1$

- take what we know about a

  $$a = 1 - \Delta \frac{b}{m}$$
- apply absolute values

  $$|a| = |1 - \Delta \frac{b}{m}| < 1$$
- this is equivalent to the following, replacing a

  $$-1 < a < 1$$

  $$-1 < 1 - \Delta \frac{b}{m} < 1$$

- given b and m we need to pick Δ range
- We do this by solving for Δ by breaking up the inequality into two equations, the LHS

$$-1 < 1 - \Delta\frac{b}{m}$$
$$\Delta\frac{b}{m} - 1 < 1$$
$$\Delta\frac{b}{m} < 1 + 1$$
$$\Delta\frac{b}{m} < 2$$

$$\Delta < 2\frac{m}{b}$$

  We can divide by m or b if we assume they are positive and non zero. This is the "upper bound" Next we do the other side (RHS),

$$1 - \Delta\frac{b}{m} < 1$$
$$1 < 1 + \Delta\frac{b}{m}$$
$$1 - 1 < \Delta\frac{b}{m}$$
$$0 < \Delta\frac{b}{m}$$

$$0 \cdot \frac{m}{b} < \Delta$$

$$0 < \Delta$$

  This is the lower bound

- So now, our bounds on Delta:

$$0 < \Delta < 2\frac{m}{b}$$

- Now, if we are given any m or b we can plug those values in and pick any Δ in this range to get a stable attractive trajectory.
- BOUNDS: To clarify
  - Upper bound: $2\frac{m}{b}$
    o Δ, our step size cannot be larger than this or equal to this
  - Lower bound: 0
    o Δ, our step size cannot be lower than this (so not negative) or equal to this, so not zero.

Switched to Matlab

- possible quiz subject
  - logical 0 = False
  - logical 1 = True

Example 10

Consider the cruise control problem with proportional control. We are going to plug in u^* as u_k

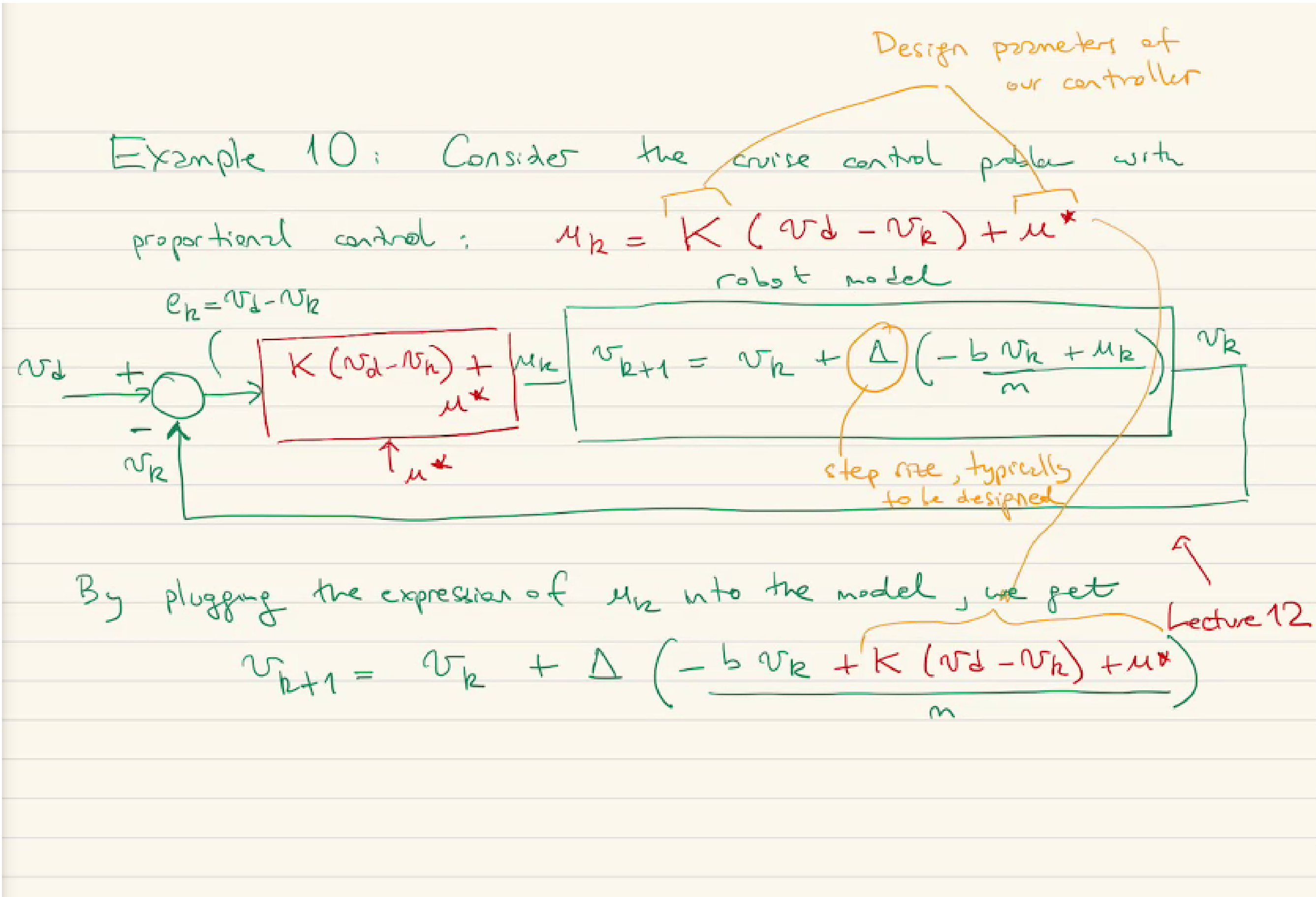

Figure 1: Figure of our cruise control system using friction and proportional control

- recall K and u^* are designed

  We want the form

$$v_{k+1} = something \cdot v_k$$

so we get,

$$\begin{aligned}
v_{k+1} &= v_k + \Delta(\frac{-bv_k + K(v_d - v_k) + \mu^*}{m}) \\
&= v_k - \Delta\frac{bv_k}{m} + \Delta\frac{K(v_d - v_k)}{m} + \Delta\frac{\mu^*}{m} \\
&= v_k - \Delta\frac{bv_k}{m} + \Delta\frac{Kv_d}{m} - \Delta\frac{Kv_k}{m} + \Delta\frac{\mu^*}{m} \\
&= v_k - \Delta\frac{bv_k}{m} - \Delta\frac{Kv_k}{m} + \Delta\frac{Kv_d}{m} + \Delta\frac{\mu^*}{m} \\
&= (1 - \Delta\frac{b}{m} - \Delta\frac{K}{m}) \cdot v_k + \frac{Kv_d}{m} + \Delta\frac{\mu^*}{m}
\end{aligned}$$

so we can let a be,

$$a = 1 - \Delta\frac{b}{m} - \Delta\frac{K}{m}$$

So to guarantee stability and attractively of $v_{eq} = v_d$ we apply the design parameters to that the two conditions are met

$$\begin{cases} a = 1 - \Delta\frac{b}{m} - \Delta\frac{K}{m} \neq 0 \\ |a| < 1 \end{cases}$$

The second condition is the same as:

$$-1 < a < 1 - 1 < 1 - \Delta\frac{b}{m} - \Delta\frac{K}{m} < 1$$

but if we are given Δ we can then solve for the bounds of K.

To do: Suppose that b, m, Δ are given Find the values of K such that conditions 1 and 2 are satisfied.