Contents

ECE 8 Robotics 11-22-22

- Today is the last zoom instruction
- review lecture is on Friday during office hours and it will be Hybrid
- The next lectures are in the lab
- Friday,Sat,Sun of this next week there are office hours to complete lab 4
- OH 1hr per day (see canvas for times):
  - Tomorrow
  - Friday
  - Saturday
  - Sunday
- Similar set up for the final

Lecture 17

- Question from Lecture 16
  - Find a range for k
  - Suppose that b, m, Δ are given Find the values of K such that conditions 1 and 2 are satisfied.

$$? \quad < K < \quad ?$$

  - what are the upper and lower bounds
- Our previous equation was:

$$-1 < a < 1$$

$$-1 < 1 - \Delta\frac{b}{m} - \Delta\frac{K}{m} < 1$$

Solve for K

- First we solve for arbitrary K by leaving b, m, Δ as variables
- At the end of lecture 16 we posed the question asking for the appropriate range of K when b, m, Δ are given. To find the range of K we proceed from example 9:
- Bound on the Left:

$$\begin{aligned}
-1 &< 1 - \Delta\frac{b}{m} - \Delta\frac{K}{m} \\
\Delta\frac{K}{m} - 1 &< 1 - \Delta\frac{b}{m} \\
\Delta\frac{K}{m} &< 1 - \Delta\frac{b}{m} + 1 \\
\Delta\frac{K}{m} &< 2 - \Delta\frac{b}{m} \\
K &< \frac{m}{\Delta}(2 - \Delta\frac{b}{m})
\end{aligned}$$

- Bound on the Right:

$$
\begin{aligned}
1 - \Delta\frac{b}{m} - \Delta\frac{K}{m} &< 1 \\
1 - \Delta\frac{b}{m} &< 1 + \Delta\frac{K}{m} \\
1 - \Delta\frac{b}{m} - 1 &< \Delta\frac{K}{m} \\
-\Delta\frac{b}{m} &< \Delta\frac{K}{m} \\
-\frac{\Delta b}{m} \cdot \frac{m}{\Delta} &< K \\
-b &< K
\end{aligned}
$$

- Full Bounds:

$$
-b < K < \frac{m}{\Delta}(2 - \Delta\frac{b}{m})
$$

In class Exercise

- Given the discrete time system:

$$
x_{k+1} = Qx_k + bu_k \tag{1}
$$

- Controlled by the proportional controller:

$$
u_k = K(x_d - x_k) + u_* \tag{2}
$$

- Where x_d is the desired reference value for x, K is the proportional gain, and u_* is a controller parameter inducing the desired equilibrium point.

---

1. Design u_* so that the equilibrium point $x_{eq} = x_d$

- Answer: let $x_eq = x_k = x_d$ in the previous equations (1)(2)

$$
\begin{aligned}
x_d &= ax_d + bK(x_d - x_d) + u_* \\
x_d &= ax_d + u_* \\
u_* &= x_d\frac{(1-a)}{b}
\end{aligned}
$$

Work:

- if this is what we want, then solving for u_k

$$
\begin{aligned}
f(x_{eq}) = ax_{eq} + bu_k &= x_{eq} \\
x_{eq}(a-1) + bu_k &= 0 \\
x_{eq}(a-1) &= -bu_k \\
x_{eq} &= \frac{-bu_k}{(a-1)}
\end{aligned}
$$

$$
\begin{aligned}
x_{eq} &= \frac{-bu_k}{a-1} \\
&\leftrightarrow \\
u_k &= -x_{eq}\frac{(a-1)}{b} \\
u_k &= x_{eq}\frac{(1-a)}{b}
\end{aligned}
$$

- next we plug in $ uk = K(xd - xk) + u_$ and solve for u_*

$$
\begin{aligned}
u_k &= K(x_d - x_k) + u_* \\
u_k &= K(x_d - x_k) + u_* \\
u_k &= K(x_{eq} - x_{eq}) + u_* \\
u_k &= u_*
\end{aligned}
$$

- therefor if $x_{eq} = x_d$ we get

$$
u_* = u_k = x_d\frac{(1-a)}{b} \quad b \neq 0 \tag{3}
$$

---

2. Design K so that the equilibrium point $x_{eq} = x_d$ is stable and attractive, were a, b, x_d are given

- To find K we need the system in the correct form

$$
x_{k+1} = Qx_k + bu_k
$$

- Answer: $K \neq \frac{a}{b}$ and

$$
\frac{a-1}{b} < K < \frac{a+1}{b}
$$

Work:

$$
\begin{aligned}
x_{k+1} &= Qx_k + bu_k \\
&= ax_k + b(K(x_d - x_k) + u_*) \\
&= ax_k + bKx_d - bKx_k + bu_* \\
&= (a - bK)x_k + bKx_d + bu_*
\end{aligned}
$$

- Let $a' = (a - bK)$
- apply conditions
- Condition 1: $a' \neq 0$

$$
\begin{aligned}
a' &\neq 0 \\
&\implies \\
a - bK &\neq 0 \\
-bK &\neq -a \\
K &\neq \frac{a}{b}
\end{aligned}
$$

- Condition 2: $-1 < a' < 1$

$$
\begin{aligned}
-1 < a' < 1 \\
-1 < a - bK < 1
\end{aligned}
$$

Other control strategies (FYI)

Proportional control is just one type of control strategy for robotics. There are many others!

- We used: $u_k = K(x_d - x_k) + u_*$
- e.g.: Proportional - Derivative Control (PD control)
  - In PD control we add an extra term that incorporates the derivative of the error signa

$$u_k = K_p(x_d - x_k) + u_* + "K_d(\dot{x_d} - \dot{x_k})"$$

  - where K_p is proportional gain
  - where K_d is derivative gain
  - NOTE: this is discrete time so derivative is not always possible, so we implement the following approximation
    - o Given a function of time $g(t)$ its derivative is

$$\frac{dg(t)}{dt} = \lim_{h \to 0} \frac{g(t+h) - g(t)}{h} \approx \frac{\xi_k - \xi_{k-1}}{\Delta} = \text{first order approximation of } \frac{dg(t)}{dt}$$

  - more info on this google: `NUMERICAL RECIPES`

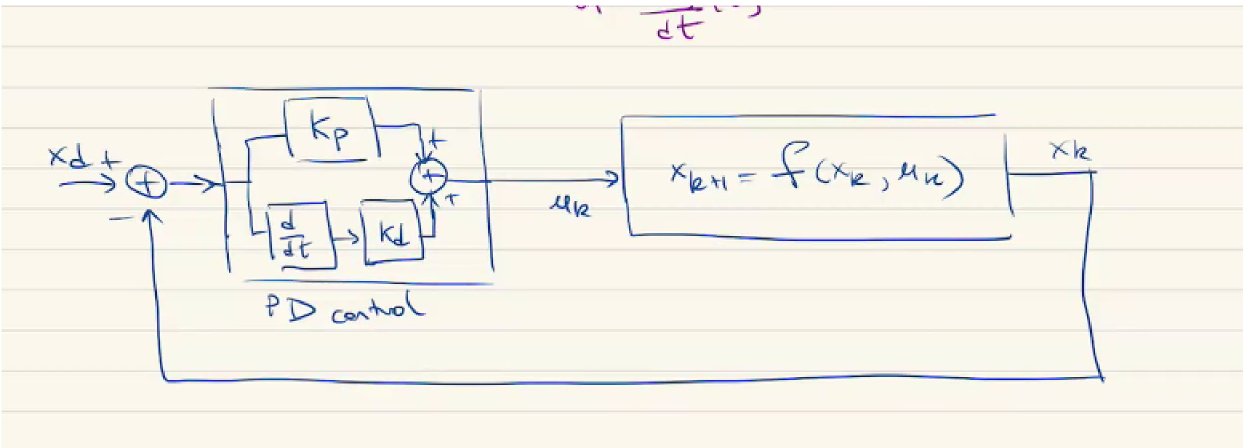

Figure 1: Figure showing the proportional and derivative control diagram

- Proportional Integral Derivative Control
  - K_p: proportional gain
  - K_d: derivative gain
  - K_I: integral gain

End of class

- Making sure programs work for lab
- Lecture 13 has all the notes needed for this lab
- Lecture 13 has code for the circle movement