MSDM 5003 Homework 1 Hints

1. (a) You need to compute the PDF for the variable Z using

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(x) f_Y(z-x) \, dx. \tag{1}$$

Although Prof Qian has outlined during lecture how to obtain this by differentiating the CDF of Z

$$F_Z(z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{z-x} f_X(x) f_Y(y) \, dy \, dx, \tag{2}$$

it would be nice if you could reproduce the derivation, filling in details to the level of your satisfaction. When carrying out the integral in Eq. (1), you are advised to first write the exponent into a polynomial in x and then complete the square for the degree 2 and 1 terms. Then, write the remaining terms into a polynomial in z and simplify. You are also reminded of the fact that the integral in Eq. (1) is with respect to x and so z is a constant in that regard.

(b) For the fitting, you are reminded that Prof Qian mentioned during lecture the results of maximising the likelihood function when the model used is the normal distribution. You are **NOT** supposed to construct the likelihood function with your generated samples of Z and maximise it numerically. For the plot, you are advised to visualise the distribution of your generated samples of X, Y and Z using a histogram with uniform interval. Think about how would you choose a suitable number and width of interval. You are also reminded of the fact that the accuracy is affected by the number of samples used. You can have a visual reference as to how well your simulation is doing by plotting the fitted curve and the theoretical curve in (a) on the same plot.