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(a) theory part:

$\gamma \frac{dx}{dt} = -\frac{d}{dx}U(x) + \xi$

$\langle \xi(t_1)\xi(t_2)\rangle = 2\delta(t_1 - t_2)$

$\langle \xi \rangle = 0$

$U(x) = 0 \Rightarrow \frac{dx}{dt} = \xi$

$\Rightarrow dx = \xi dt$

$\Rightarrow x(t) - x(0) = \int_0^t \xi dt$

$\Rightarrow \langle [x(t) - x(0)]^2 \rangle$

$= \langle (\int_0^t \xi dt)^2 \rangle$

$= \langle \int_0^t \xi_1 dt_1 \cdot \int_0^t \xi_2 dt_2 \rangle$

$= \langle \int_0^t \int_0^t \xi(t_1) \cdot \xi(t_2) dt_2 dt_1 \rangle$

$= \int_0^t \int_0^t \langle \xi(t_1)\xi(t_2)\rangle dt_2 dt_1$

$= \int_0^t \int_0^t 2\delta(t_1 - t_2) dt_2 dt_1$

$= 2t$

According to Fokker-Planck equation

$\frac{\partial}{\partial t} p(x,t) = -\frac{\partial}{\partial x}[A(x) \cdot p(x,t)] + \frac{1}{2} \cdot \frac{\partial^2}{\partial x^2}[B(x) p(x,t)]$

assume $x(0) = 0 \Rightarrow \langle x^2(t) \rangle = 2t$

$A(x) = a(x_0, t)/t = \frac{\langle x(t)\rangle - x(0)}{t} = 0$

$B(x) = b(x_0, t)/t = \frac{\langle x^2(t)\rangle - x^2(0)}{t} = 2$

$\Rightarrow \frac{\partial}{\partial t} p(x,t) = \frac{\partial^2}{\partial x^2} p(x,t) = D \frac{\partial^2}{\partial x^2} p(x,t)$

$\Rightarrow D = 1$

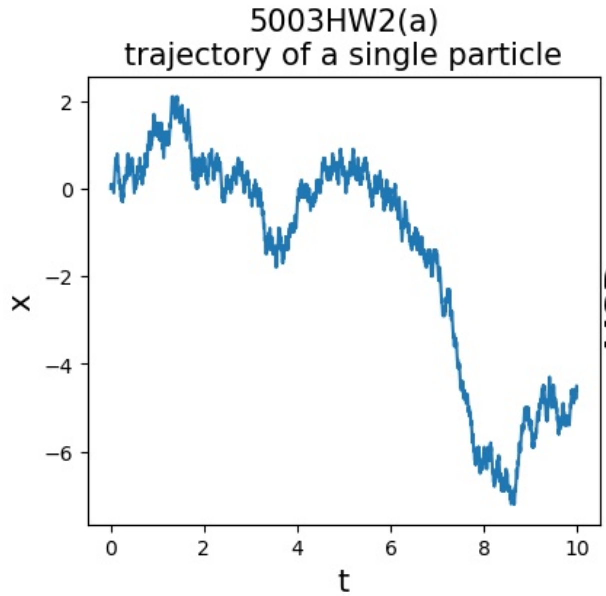
$MSD = \langle x^2 \rangle = N\Delta^2 = 2t \Rightarrow t = N\Delta^2/2$.

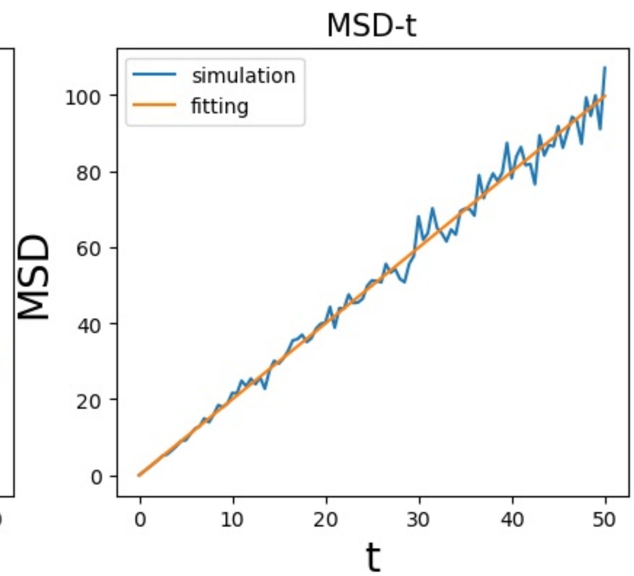
Simulation part

fitting

D = 0.9979343047151394

MSD = 2Dt





(b) $U(x) = x^2/2$

$$\Rightarrow \frac{dx}{dt} = -x + \xi$$

$$\frac{x(t+\Delta) - x(t)}{\Delta t} = -x(t) + \xi(t)$$

$$\Rightarrow x(t+\Delta) - x(t) = -\Delta t \cdot x(t) + \xi(t) \Delta t.$$