$$S_0(x) = \begin{cases} \sin(2\pi x), & x \in [-3,3] \\ 0, & x \in (-\infty,-3) \cup (3,+\infty) \end{cases}$$

$$p(x) = \begin{cases} \sin(20\pi x), & x \in [-3,3) \\ 0, & x \in (-\infty,-3] \cup (3,+\infty) \end{cases}$$

$$\Rightarrow S(x) = S_0(x) + p(x)$$

$$= \begin{cases} \sin(2\pi x) + \sin(20\pi x), & x \in [-3,3] \\ 0, & x \in (-\infty,-3) \cup (3,+\infty) \end{cases}$$

(a) $$\int_{-\infty}^{\infty} S_0(y) S_0(y+x)\, dy$$

$$= \int_{-3}^{3} \sin(2\pi y) S_0(y+x)\, dy$$

$$= \int_{\max(-3,-3-x)}^{\min(3,3-x)} \sin(2\pi y) \sin[2\pi(x+y)]\, dy$$

$$= \int_{\max(-3,-3-x)}^{\min(3,3-x)} \frac{1}{2} \cdot \left[\cos(-2\pi x) - \cos[2\pi(x+2y)]\right] dy$$

$$= \begin{cases} \left[\frac{y}{2}\cos 2\pi x - \sin 2\pi(x+2y)\frac{1}{8\pi}\right]\Big|_{-3}^{3-x}, & x \geq 0 \\ \left[\frac{y}{2}\cos 2\pi x - \sin 2\pi(x+2y)\cdot\frac{1}{8\pi}\right]\Big|_{-3-x}^{3}, & x < 0 \end{cases}$$

$$= \begin{cases} \frac{6-x}{2}\cos 2\pi x + \frac{1}{4\pi}\sin 2\pi x, & x \geq 0 \\ \frac{6+x}{2}\cos 2\pi x - \frac{1}{4\pi}\sin 2\pi x, & x < 0 \end{cases}$$

(b) $(s * r)(x) = \int_{-\infty}^{+\infty} s(y)\, r(x-y)\, dy$

$$= \frac{1}{\sqrt{4\pi\sigma^2}} \int_{-3}^{3} \left[\sin(2\pi y) + \sin(20\pi y)\right] e^{-\frac{(x-y)^2}{2\sigma^2}} dy$$