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(a) theory part:

$$\gamma \frac{dx}{dt} = -\frac{d}{dx}U(x) + \xi$$

$$\langle \xi(t_1)\xi(t_2)\rangle = 2\delta(t_1 - t_2)$$

$$\langle \xi \rangle = 0$$

$$U(x) = 0 \Rightarrow \frac{dx}{dt} = \xi$$

$$\Rightarrow dx = \xi\, dt$$

$$\Rightarrow x(t) - x(0) = \int_0^t \xi\, dt$$

$$\Rightarrow \langle [x(t) - x(0)]^2 \rangle$$

$$= \langle (\int_0^t \xi\, dt)^2 \rangle$$

$$= \langle \int_0^t \xi_1 dt_1 \cdot \int_0^t \xi_2 dt_2 \rangle$$

$$= \langle \int_0^t \int_0^t \xi(t_1)\cdot\xi(t_2)\, dt_2 dt_1 \rangle$$

$$= \int_0^t \int_0^t \langle \xi(t_1)\xi(t_2)\rangle dt_2 dt_1$$

$$= \int_0^t \int_0^t 2\delta(t_1 - t_2) dt_2\, dt_1$$

$$= 2t$$

According to Fokker-Planck equation

$$\frac{\partial}{\partial t} p(x,t) = -\frac{\partial}{\partial x}[A(x)\cdot p(x,t)] + \frac{1}{2}\cdot\frac{\partial^2}{\partial x^2}[B(x)\, p(x,t)]$$

assume $x(0) = 0 \Rightarrow \langle x^2(t)\rangle = 2t$

$$A(x) = a(x_0, t)/t = \frac{\langle x(t)\rangle - x(0)}{t} = 0$$

$$B(x) = b(x_0, t)/t = \frac{\langle x^2(t)\rangle - x^2(0)}{t} = 2$$

$$\Rightarrow \frac{\partial}{\partial t} p(x,t) = \frac{\partial^2}{\partial x^2} p(x,t) = D\frac{\partial^2}{\partial x^2} p(x,t)$$

$$\Rightarrow D = 1$$

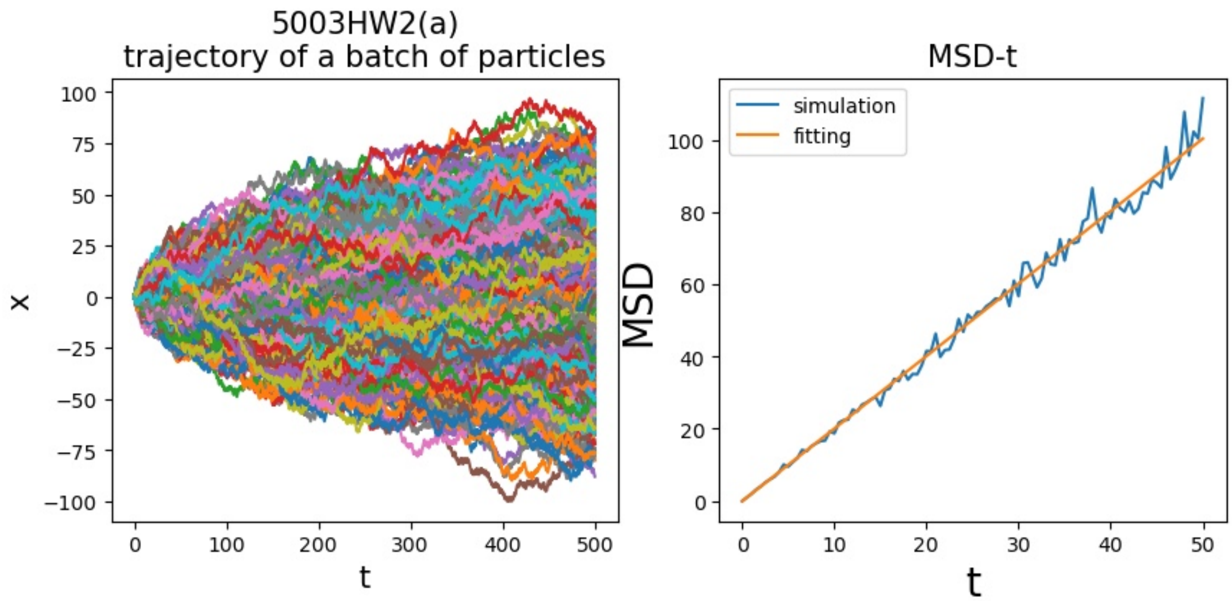
$$MSD = \langle x^2 \rangle = N\Delta^2 = 2t \Rightarrow t = N\Delta^2/2.$$

Simulation part

fitting

D = 0.9979343047151394

MSD = 2Dt



(b) $U(x) = x^2/2$

$$\Rightarrow \frac{dx}{dt} = -x + \xi$$

$$\frac{x(t+\Delta) - x(t)}{\Delta t} = -x(t) + \xi(t)$$

$$\Rightarrow x(t+\Delta) - x(t) = -\Delta t \cdot x(t) + \xi(t)\Delta t.$$