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MSDM5004

Homework 3 (Part II)

4. Write a code using MATLAB (or some other software) to solve the following boundary value problem of the Poisson equation, using the five-point scheme and the Jacobi iteration method for solving the resulting linear system. Please use a uniform square grid of size 1/2.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2(x^2 + y^2 - 2), \quad -1 \le x \le 1, \ -1 \le y \le 1,$$

The boundary conditions are $u(\pm 1, y) = u(x, \pm 1) = 0$.

**Warning**: Please write your own code for the Jacobi iteration. No credit will be given if you directly use the available function for the Jacobi iteration in MATLAB or some other programming language/software.

$J = 4$

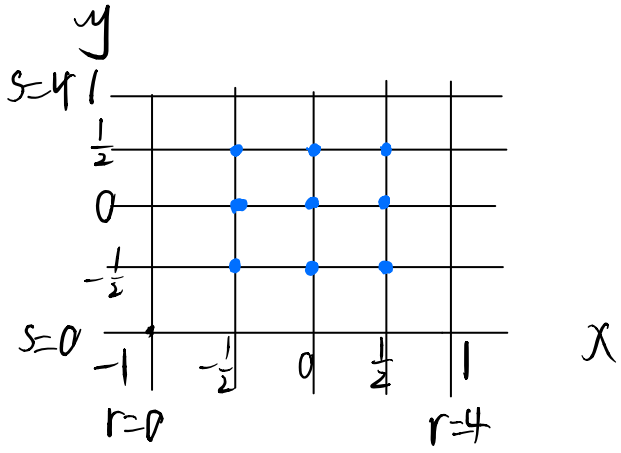
$\Delta x = \Delta y = \frac{1}{2}$

Natural order:

$U_{1,1}, U_{2,1}, U_{3,1},$

$U_{1,2}, U_{2,2}, U_{3,2},$

$U_{1,3}, U_{2,3}, U_{3,3}.$




five-point scheme:

$$\frac{U_{r+1,s} + U_{r-1,s} + U_{r,s+1} + U_{r,s-1} - 4U_{r,s}}{(\Delta x)^2} = f_{r,s}$$

$r = 1, 2, 3, \quad s = 1, 2, 3, \quad u(\pm 1, y) = u(x, \pm 1) = 0$

$$\Rightarrow \begin{bmatrix} -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 \end{bmatrix} \cdot \begin{bmatrix} U_{1,1} \\ U_{2,1} \\ U_{3,1} \\ U_{1,2} \\ U_{2,2} \\ U_{3,2} \\ U_{1,3} \\ U_{2,3} \\ U_{3,3} \end{bmatrix} = (\Delta x)^2 \cdot \begin{bmatrix} f_{1,1} \\ f_{2,1} \\ f_{3,1} \\ \vdots \\ \\ \\ \\ \\ f_{3,3} \end{bmatrix}$$

$$Au = b$$

$$A = D - L - U$$

$$u = D^{-1}(L+U)u + D^{-1}b$$

$$u_i^{(k)} = \frac{1}{a_{ii}}\left[\sum_{\substack{j=1 \\ j\neq i}}^{n} (-a_{ij} u_j^{(k-1)}) + (\Delta x)^2 f_i\right] \text{ for } i = 1, \ldots, 9$$