MSDM5004

Homework 1 (Part II)

Remarks:

(1) For all problems, write down the formulas and the calculate the results by calculators and **do not** compute it by MATLAB or other software, unless it is specified in the problem that you are required to write a code.

(2) When you are required to write a code, you can use MATLAB or any other programming language.

2. Solve the nonlinear system

$$f_1(x_1, x_2) = 4x_1 + 6x_1^2 + 4x_1^3 - 2x_2 - 2$$
$$f_2(x_1, x_2) = -2x_1 + 2x_2 + 2$$

Write down the iteration algorithm of Newton's method, then perform 2 iterations with the starting point $\mathbf{x}^{(0)} = (0.5, -0.4)^T$. (Use the inverse formula for a 2×2 matrix.)

3. Write a code using MATLAB (or other programming language) to solve the following system using Newton's method

$$f_1(x_1, x_2) = 1 + \frac{1}{4}x_1^2 - x_2^2 + e^{\frac{x_1}{2}} \cos x_2 = 0$$
$$f_2(x_1, x_2) = x_1 x_2 + e^{\frac{x_1}{2}} \sin x_2 = 0$$

Use starting values $x_1^{(0)} = -2$ and $x_2^{(0)} = 4$. Perform 5 iterations.

4. (1) Find the Lagrange interpolating polynomial for these data:

x	-2	0	1
$f(x)$	1	2	0

(2) Find approximation of $f(-1)$ using the interpolating polynomial.

5. Find the least squares polynomial of degree 1 for the data in the table, and compute the error E.

x_i	1.0	1.1	1.3	1.5	1.9	2.1
y_i	1.77	1.89	2.14	2.38	2.87	3.11

Ans: 2. $X^{(k)} = X^{(k-1)} - J(X^{(k-1)})^{-1} F(X^{(k-1)})$

$$J(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(x) & \frac{\partial f_1}{\partial x_2}(x) \\ \frac{\partial f_2}{\partial x_1}(x) & \frac{\partial f_2}{\partial x_2}(x) \end{bmatrix}$$

$$= \begin{bmatrix} 4+12x_1+12x_1^2 & -2 \\ -2 & 2 \end{bmatrix}$$

$k=1$: $X^{(0)} = (0.5, -0.4)^T$

$$J(x^{(0)})^{-1} = \begin{bmatrix} 13 & -2 \\ -2 & 2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \frac{1}{11} & \frac{1}{11} \\ \frac{1}{11} & \frac{13}{22} \end{bmatrix}$$

$$F(x^{(0)}) = (2.8, 0.2)^T$$

$$\Rightarrow X^{(1)} \approx \begin{bmatrix} 0.5 \\ -0.4 \end{bmatrix} - \begin{bmatrix} \frac{1}{11} & \frac{1}{11} \\ \frac{1}{11} & \frac{13}{22} \end{bmatrix} \cdot \begin{bmatrix} 2.8 \\ 0.2 \end{bmatrix}$$

$$\approx (0.2273, -0.7727)^T$$

$k=2$:

$$X^{(2)} = X^{(1)} - J(X^{(1)})^{-1} \cdot F(X^{(1)})$$

$$= (0.0755, -0.9245)^T$$

4. (1) $L_0(x) = \dfrac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \dfrac{1}{6}x(x-1)$

$L_1(x) = \dfrac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = -\dfrac{1}{2}(x+2)(x-1)$

$L_2(x) = \dfrac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \dfrac{1}{3}(x+2)x$

$\Rightarrow p(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2)$

$= \dfrac{1}{6}x(x-1) - (x-1)(x+2)$

(2) $f(x) = p(x) = \dfrac{1}{6}x(x-1) - (x-1)(x+2)$

$f(-1) = \dfrac{1}{3} + 2 = \dfrac{7}{3}$

5. f: $x_i m + c = y_i$.

$X_* = \begin{bmatrix} m \\ c \end{bmatrix}$ $A = \begin{bmatrix} 1.0 & 1 \\ 1.1 & 1 \\ 1.3 & 1 \\ 1.5 & 1 \\ 1.9 & 1 \\ 2.1 & 1 \end{bmatrix}$. $b = \begin{bmatrix} 1.77 \\ 1.89 \\ 2.14 \\ 2.38 \\ 2.87 \\ 3.11 \end{bmatrix}$

$X^* = (A^TA)^{-1}A^Tb = \begin{bmatrix} 1.2198 \\ 0.5509 \end{bmatrix}$

$E = \|AX^* - b\|^2 = 2.7194 \times 10^{-5}$