1. Consider the problem of solving the equation $f(x) = 0$, where

$$f(x) = \frac{e}{2}e^x + \frac{2^{-x}}{4} + \cos(x+1) - 3.$$

    (1) Write down the iteration algorithm of Newton's method, then perform 4 iterations with the starting point $x_0 = 0.5$. (Write down the formulas and the calculate the results by calculators. **Do not** compute it by MATLAB or other software if you are not asked to do so.)

    (2) Write codes using MATLAB to solve this equation using (i) Newton's method and (ii) the secant method.

$$\text{Ans. (1)} \quad f'(x) = \frac{e}{2}e^x - \frac{\ln 2}{4} \cdot 2^{-x} - \sin(x+1)$$

$$X_{n+1} = X_n - \frac{f(x_n)}{f'(x_n)} = X_n - \frac{\frac{e}{2}e^{x_n} + \frac{2^{-x_n}}{4} + \cos(x_n+1) - 3}{\frac{e}{2}e^{x_n} - \frac{\ln 2}{4} \cdot 2^{-x_n} - \sin(x_n+1)}$$

$$n = 0,$$

$$X_1 = X_0 - \frac{\frac{e}{2}e^{x_0} + \frac{1}{4} \cdot 2^{-x_0} + \cos(x_0+1) - 3}{\frac{e}{2}e^{x_0} - \frac{\ln 2}{4} \cdot 2^{-x_0} - \sin(x_0+1)}$$

$$= 0.5 - \frac{\frac{1}{2}e^{1.5} + \frac{1}{4\sqrt{2}} + \cos(1.5) - 3}{\frac{1}{2}e^{1.5} - \frac{\ln 2}{4\sqrt{2}} - \sin 1.5}$$

$$\approx 0.9564897211$$

$$n = 1,$$

$$X_2 = X_1 - \frac{\frac{e}{2}e^{x_1} + \frac{1}{4} \cdot 2^{-x_1} + \cos(x_1+1) - 3}{\frac{e}{2}e^{x_1} - \frac{\ln 2}{4} \cdot 2^{-x_1} - \sin(x_1+1)}$$

$$\approx 0.8415330610$$

$n=2,$
$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \approx 0.8295060132$$

$n=3,$
$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \approx 0.8293836145$$