4. (c)

$$I = \int_0^{\infty} \frac{x^{\frac{1}{2}}}{1+e^x} dx$$

$$w(x) = \frac{1}{1+e^x}$$

$$s = \int_0^x w(t) dt$$

$$= \int_0^x \frac{1}{e^t+1} dt$$

$$= \int_0^x \left(1 - \frac{e^t}{e^t+1}\right) dt$$

$$= x - \int_0^x \frac{de^t}{e^t+1}$$

$$= x - \ln(e^x+1) + \ln 2$$

$$= \ln \frac{2}{1+e^{-x}}$$

$$= F(x)$$

$x \in [0, \infty) \Rightarrow$ N uniform deviate in $s \in [0, \ln 2)$

$$x = \ln \frac{1}{2e^{-s}-1}$$

$$\frac{f}{w} = \sqrt{x} = \sqrt{\ln \frac{1}{2e^{-s}-1}}$$

(d) $P(x\to y) = R(x\to y) A(x\to y)$ for $y \neq x$

$$R(x\to y) = \begin{cases} 1 , & |y-x| \le \eta \\ 0 , & \text{otherwise} \end{cases}$$

$$A(x\to y) = \min\left[1, \frac{W(y)}{W(x)}\right]$$

$$\int_0^\infty \frac{1}{1+e^x} dx = \left[\ln \frac{e^x}{1+e^x}\right]_0^\infty = \ln 2$$

$$\Rightarrow W(x) = \frac{1}{1+e^x} / \ln 2$$

$$f(x) = \ln 2 \cdot \sqrt{x}$$

$$M_1 = \frac{1}{N} \sum_{i=0}^{N-1} f(x_i)$$

$$M_2 = \frac{1}{N} \sum_{i=0}^{N-1} f^2(x_i)$$

$$I \approx M_1$$

$$\text{error} \approx \sqrt{\frac{M_2 - M_1^2}{N-1}}$$