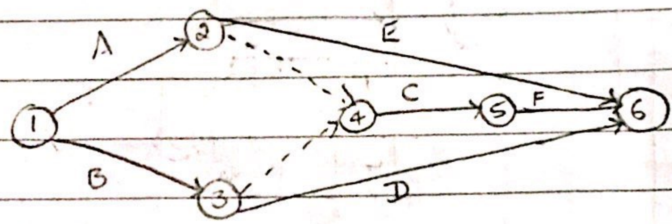
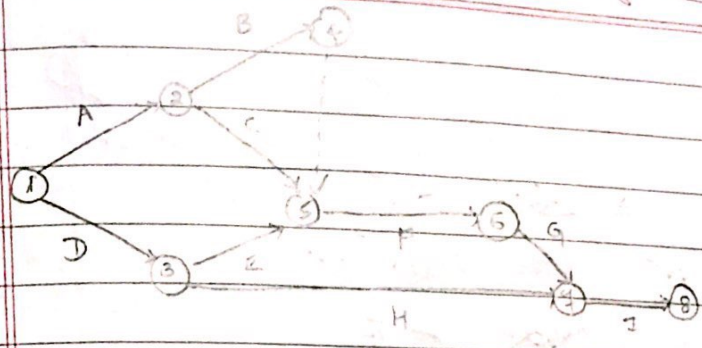
Arrow Network Diagram



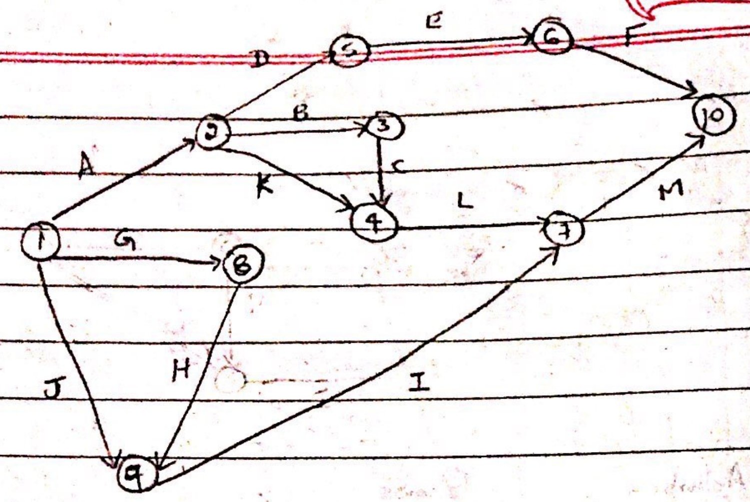
3)

Tasks	Predecressor Task(s)
A	—
B	A
C	A
D	—
E	D
F	B, C, E
G	F
H	D
I	G, H



4)

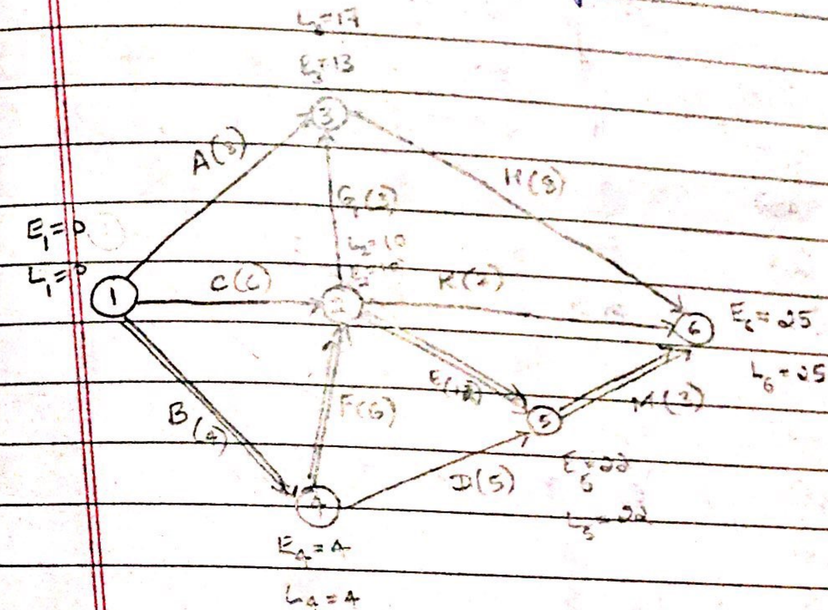
Activities	Follows Activites
A	—
B	A
C	B
D	A
E	D
F	E
G	—
H	G
I	H, J
J	—
K	A
L	C, K
M	I, L



5)

Tasks	Follows Task(s)	Duration (Days)
A	—	8
B	—	4
C	—	6
D	B	5
E	C, F	12
F	B	6
G	C, F	3
H	A, G	9
K	C, F	2
M	D, E	3

Identify the critical path & Length

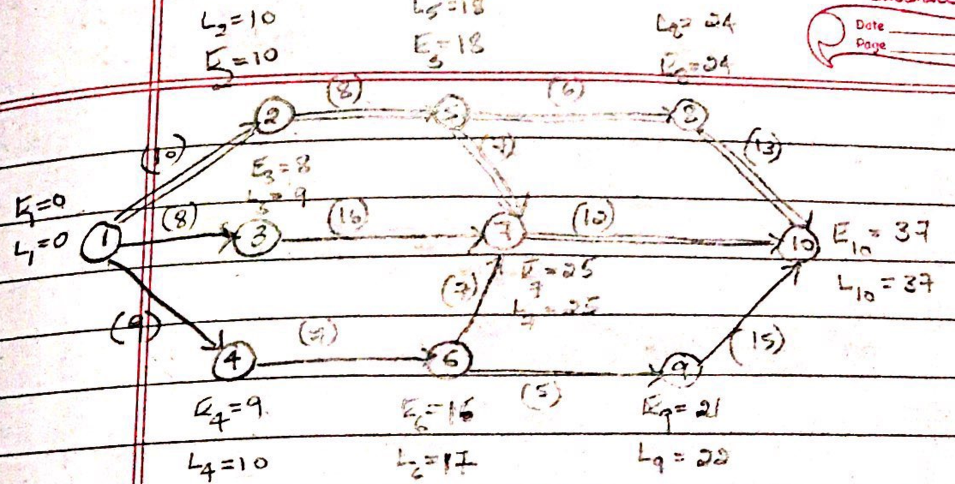


Critical Path :→ 1 → 4 → 2 → 5 → 6

Tasks → B → F → E → M

Length 4+6+12+3 = 25 days

(i,j)	Dij (Days)	
(1,2)	10	10
(1,3)	8	8
(1,4)	9	9
~~(1,5)~~	8	.
(2,5)	8	8
(4,6)	~~6~~ 7	7
(3,7)	7	16
(5,7)	7	7
(6,7)	6	7
(5,8)	5	6
(6,9)	2	5
(7,10)	13	12
(8,10)	5	13
(9,10)		5

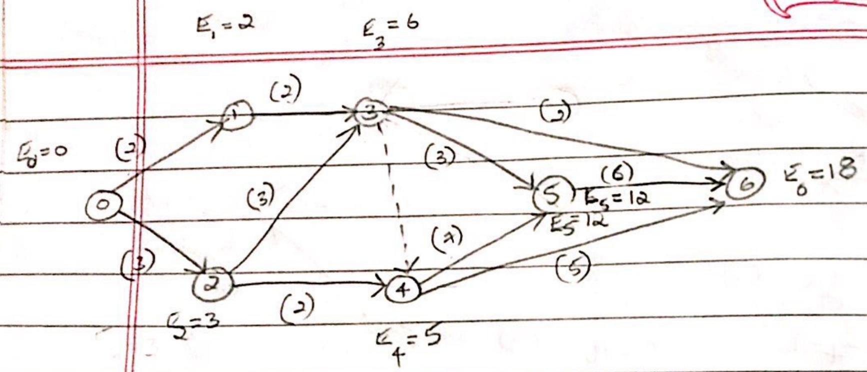


①→②→⑤→⑧→⑩ = 37 days

①→③→⑤→⑦→⑩ = 37 days

7)

(i,j)	(Dij) Months
0-1	2
0-2	3
1-3	2
2-3	3
2-4	2
5-4 (Dummy arrow)	0
3-5	3
3-6	2
4-5	7
4-6	5
5-6	6



Project Evaluation & Review Technique [PERT]

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

where, t_e = Expected time to complete an activity.

t_o = Optimistic time to complete an activity

t_m = Most likely time to complete an activity

t_p = Pessimistic time to complete an activity.

$$\sigma = \left[\frac{t_p - t_o}{6}\right] \quad \text{and} \quad \sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$$

$\therefore$ T_e = Expected project Completion Time.
(Critical path length)

$\sigma_T = \sqrt{\sum \sigma^2}$ of critical path activities

The probability of completing the project by scheduled time:

$$\text{Prob:} \left(Z \leq \frac{T_s - T_e}{\sigma_T} \right)$$

where, T_s = Scheduled time to complete the project

P.T.O.

$t_p \rightarrow$ big value
$t_m \rightarrow$ medium
$t_o \rightarrow$ small

classmate
Date
Page

4

1)

Tasks	Follows Tasks	Time (Days) t_o	t_m	t_p	t_e	σ	σ^2
A	—	4	7	16	8	2	4
B	—	1	5	15	6	2.33	5.44
C	A	6	12	30	14	4	16
D	A	2	8	15	8.17	2.17	4.09
E	C	5	11	17	11	2	4
F	D	3	6	15	7	2	4
G	B	3	9	27	11	4	16
H	E, F	1	4	7	4	1	1
I	G	4	19	28	18	4	16

1) Draw a PERT Network diagram
2) Identify critical path & expected project completion time
3) Find probability that the project is completed in 36 days
4) If you would like to have a guarantee at 99% about the completion of the project, what must be the scheduled time of the project.

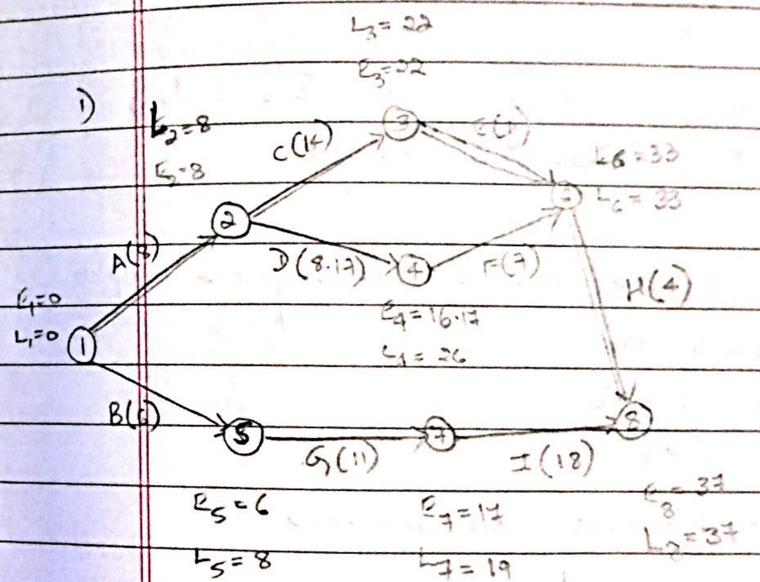
A: $t_e = \frac{t_o + 4t_m + t_p}{6} = \frac{4 + 4(7) + 16}{6} = \frac{4 + 28 + 16}{6} = \frac{48}{6} = 8$

B: $t_e = \frac{t_o + 4t_m + t_p}{6} = \frac{1 + 4(5) + 15}{6} = \frac{1 + 20 + 15}{6} = \frac{36}{6} = 6$

C: $t_e = \frac{t_o + 4t_m + t_p}{6} = \frac{6 + 4(12) + 30}{6} = \frac{6 + 48 + 30}{6} = \frac{84}{6} = 14$

⋮

1)



2) Critical path: ① → ② → ③ → ⑥ → ⑧

Critical path activities: $A \rightarrow C \rightarrow E \rightarrow H$

$\therefore T_e = 8 + 14 + 11 + 4 = 37$ days

$\sigma_T = \sqrt{\Sigma\sigma^2 \text{ of critical path activities}}$

$= \sqrt{4+16+4+1} = \sqrt{25} = 5$

$z = \frac{T_s - T_e}{\sigma_T}$

if $T_s = 36$ days

$z = \frac{36-37}{5} = -1/5 = -0.2$

$\therefore$ Probability of Completing project if T_s is 36 days
$= 0.4207 \approx 42.07\%$

To be guaranteed at 99% that the project is complete the scheduled time:

$z = \frac{T_s - T_e}{\sigma_T}$

at 99% ≈ 0.9960, the $z = 2.33$

$z = \frac{T_s - 37}{\sigma_T}$

$2.33 = \frac{T_s - 37}{5}$

$T_s = 48.65$ days

2) The following activities of a project are completed in series.

Tasks	Time (~~weeks~~ days) t_o	t_m	t_p	t_e	σ	σ^2
A	8	11	14	11	1	1
B	7	10	19	11	2	4
C	10	19	22	18	2	4

Calculate:

(a) t_e

(b) σ & σ^2

(c) T_e and σ_T

(d) Probability of completing the project in

(i) 40 days.

(ii) 46 days.

→

$$1 \xrightarrow{A(11)} 2 \xrightarrow{B(11)} 3 \xrightarrow{C(18)} 4$$

Critical path :- $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$

Critical path activities: $A \rightarrow B \rightarrow C$

$T_e = 11 + 11 + 18$

$= \underline{40 \text{ days}}.$

$$\boxed{t_e = \frac{t_o + 4t_m + t_p}{6}}$$

$$\frac{8+4(11)+14}{6} = \frac{66}{6} = 11 \qquad \frac{7+4(10)+19}{6} = 11 \qquad \frac{10+4(19)+22}{6} = \frac{108}{6} = 18$$

$$\boxed{\sigma = \frac{t_p - t_o}{6} \quad \text{and} \quad \sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2}$$

$$\sigma_T = \sqrt{1+4+4} = \sqrt{9} = 3.$$

(d) $$z = \frac{T_s - T_e}{\sigma_T}$$

i) if $T_s = 40$ days.

$$Z = \frac{40-40}{3} = 0$$

∴ Probability of completing if $T_s = 40$ days $= 0.5000 \approx 50\%$.

ii) if $T_s = 46$ days

$$z = \frac{46-40}{3} = \frac{6}{3} = 2.00$$

∴ Probability of completing if $T_s = 46$ days $= 0.9773 = 97.73\%$

3) Given the details of a project:

Activity	Time (months) a (t_o)	b (t_m)	c (t_p)	t_e	σ	σ^2
1-2	2	6	10			
1-3	4	8	12			
2-3	2	4	6			
2-4	2	3	4			
3-4	0	0	0			
3-5	3	6	9			
4-6	6	10	14			
5-6	1	3	5			

What is the probability of completing the project in 22 m

Time-Cost Trade-off [Crashing]

What is crashing?

The process of reducing the total completion time of the project by reducing the time of completion of one of the critical activities is known as crashing. Crashing is carried out by spending additional money & resources on some of the critical activities.

Steps in crashing [Time-cost optimization Procedure]:

Step 1: (a) Find the normal critical path & identify the critical path activities

(b) Estimate the total cost where total cost = direct cost + indirect cost.

Step 2: Estimate cost-time slope:

$$\text{slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{crash time}} = \frac{C_c - C_n}{T_n - T_c}$$

Step 3: Begin crashing an activity on the critical path with lowest slope to the maximum extent possible & calculate the new direct cost by cumulative adding the cost of crashing to the normal cost.

Step 4: Parallel crashing:

As the critical path duration is reduced by crashing, the other paths will also become critical, i.e. we get parallel critical paths. This means that the project duration can be reduced by simultaneous crashing of activities on the parallel critical paths & only one activity on each path has to be crashed.

Step 5: Decission making:

Continue crashing till the total cost (if the project involves indirect cost) continues to decrease & stop crashing the moment the total cost start increasing & consider the length of critical path where the total cost is at its minimum as final.

(OR)

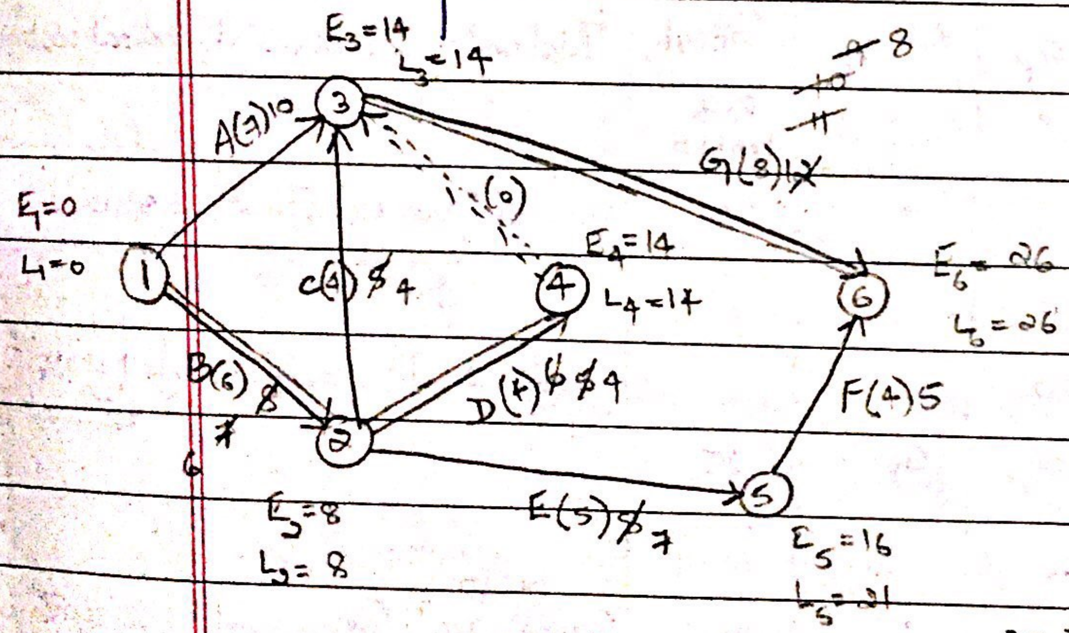
Continue crashing till no further crashing on critical path is possible.

1) The following table shows the details of a project.

Tasks	Follows Task(s)	Normal Time (weeks)	Normal Cost (₹×1000)	Crash Time (weeks)	Crash Cost (₹×1000)
A	—	10	20	7	30
B	—	8	15	6	20
C	B	5	8	4	14
D	B	6	11	4	15
E	B	8	9	5	15
F	E	5	5	4	8
G	A, D, C	12	3	8	4

Indirect cost is ₹400 per day ₹1,000

Find the optimum duration & the associated minimum project cost



critical path using normal time only

~~20~~ ~~19~~ 18

~~24~~

(tracking should start with highest critical path value)

~~B~~ → ~~D~~ → Dummy → ~~G~~ : ~~26~~ ~~25~~ 24 ~~23~~ 22

~~B~~ → ~~C~~ → ~~G~~ : ~~25~~ 24 ~~23~~ ~~22~~ ~~21~~ ~~20~~ ~~19~~ 18

~~B~~ → E → F : ~~21~~ ~~20~~ ~~19~~ 18

A → ~~G~~ : ~~22~~, ~~21~~ ~~20~~ ~~19~~ 18

(For step III cancel dummy)

(All cancellation dummy step 3)

TC = Direct cost + Indirect cost

$= ₹71,000 + [(₹400 \times 7) \times 26] = ₹1,43,800$

Tasks	A	B	C	D	E	F	G
slope	3333.3	2500	6000	2000	2000	3000	250

$$\left.\begin{array}{c}\text{Time-cost slope}\\ (₹)\end{array}\right\} = \frac{C_c - C_n}{T_n - T_c}$$

(for A)

$$\frac{30000 - 20000}{10 - 7} = \frac{10,000}{3} = 3333.3$$

III step

Crash	Options	Slope (₹)	Activity Crashed	Critical Path Length	Total cost = Direct cost + Indirect cost
—	—	—	—	26	71000 + [400×7×26] = ₹1,43,800.
1	B	2500			
	D	2000			[71000 + 250] + [400×7×25] = 1,41,250₹
	G	250 (min)	G	25	
2	B	2500			
	D	2000	G	24	[71000 + 250 (+250)] + [2800×24] = 1,38,700₹
	G	250			
3	B	2500			
	D	2000	G	23	[71500 + 250] + [2800×23] = 1,36,150
	G	250			
4	B	2500			
	D	2000			
	G	250	G	22	[71,750 + 250] + [2800×22] = 1,33,600₹
5	B	2500			
	D	2000	D	21	[72,000 + ~~250~~ 2000] + [2800×21] = 1,32,800₹
6	B	2500	B	20	[74000 + 2500] + [2800×20] = 1,32,500₹
	DCE	10000			
	DCF	11,000			
7	B	2500	B	19	[76500 + 2500] + [2800×19] = 1,32,200₹
	DCE	10000			
	DCF	11,000			
8	DCE	10,000	DCE	18	[79000 + 10,000] + [2800×18] = 1,39,400₹
	DCF	11,000			

Parallel crashing
1) B is common
2) Take 1 activity for each path & group.
3)"

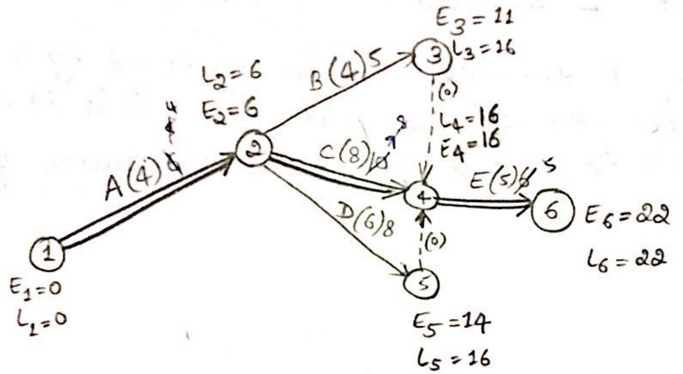
The project duration can be ~~crashed~~ reduced by 7 weeks where critical path length is 19 weeks at which the total cost is at its minimum of ₹1,32,200.

P.T.O.

2) Given the details of a project :

Tasks	Follows Tasks (s)	Time (Days)		Cost (₹x1000)	
		Normal	Crash	Normal	Crash
A	–	6	4	60	90
B	A	5	4	30	80
C	A	10	8	80	160
D	A	8	6	50	100
E	B, C, D	6	5	40	60
				~~[illegible]~~ 260	

What is the critical path length after crashing?



~~A~~ → ~~C~~ → ~~E~~ : 22 21 20 19 18 17

~~A~~ → B → Dummy → ~~E~~ : 17 16 15 14 13 12

~~A~~ → D → Dummy → ~~E~~ : 20 19 18 17 16 15

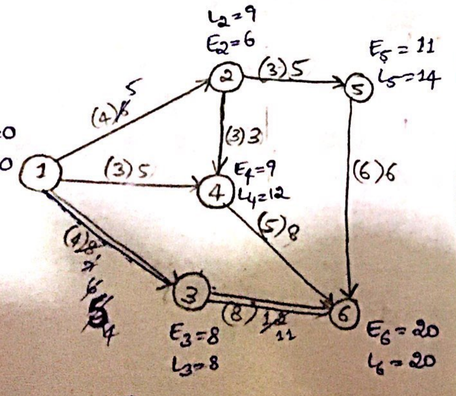
Tasks	A	B	C	D	E
Slope	15	50	40	25	20

Crash	Options	Slope	Task Crashed	Critical path length	Total cost (TC)
—	—	—	—	22	260
1	A ✓ C E	15 ✓ 40 20	A	21	260+15=275
2	A ✓ C E	15 ✓ 40 20	A	20	275+15=290
3	C E ✓	40 20 ✓	E	19	290+20=310
4	C	40	C	18	310+40=350
5	C	40	C	17	350+40=390

∴ Critical path length = 18.

Activity	Time (Months)		Time cost slope (₹/month)
	Normal	Crash	
1 - 2	6	4	80
1 - 3	8	4	90
1 - 4	5	3	30
2 - 4	3	3	–
2 - 5	5	3	40
3 - 6	12	8	200
4 - 6	8	5	50
5 - 6	6	6	–

Overhead cost per month is ₹160.
Cost of completing 8 activities in normal time is ₹6,500. Find out optimum cost & time after crashing.



~~1 → 3 → 6 : 20~~

(1-3) → (3 → 6) : ~~20~~, 19, 18, 17, 16
(1-4) → (4 - 6) : 13
(1-2) → (2-5) → ~~(5-6)~~ : 17
(1-2) → ~~(2-4)~~ → (4-6) : 17

Since normal & crash are the same for (2-4) & (5-6), cancel them in the beginning.

Crash	Options	Slope	Task crashed	Critical path	Total cost = Direct cost + Indirect cost
–	–	–	–	20	6500+[160×20] = 9700
1	1-3 ✓ 3-6	90 ✓ 200	(1-3)	19	[6500+90]+[160×19] = 9630
2	1-3 ✓ 3-6	90 ✓ 200	(1-3)	18	[6500+90]+[160×18] = [illegible]
3	1-3 ✓ 3-6	90 ✓ 200	(1-3)	17	[6500+90]+[160×17] = 9490
4	1-3 ✓ 3-6	90 ✓ 200	(1-3)	16	[6500+90]+[160×16] = 9150
5	3-6	200	(3-6)	15	[6500+200]+[160×15] = 9100
6	3-6	200	(3-6)		
	(1-3), (1-2) (1-3), (2-5), (4-6) (3-6)				

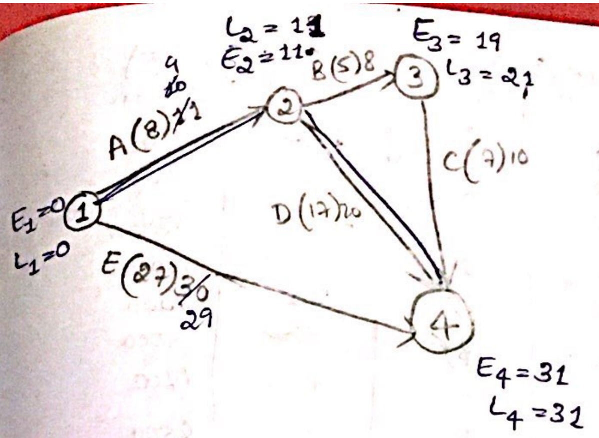
4	(1-3), (1-2)	170	(1-3)(1-2)	16	[6770+170]+[160×16]
	(1-3), (2-5), (4-6)	180			= 9500
	(3-6), (1-2)	280			
	(3-6), (2-5), (4-6)	290			Stop here, coz we got greater value than the previous value

Combination from all 3 critical paths with length 17

The project can be crashed by 3 months. Optimum critical path = 17 months & total cost = ₹9490

4) ₹200 per day charged as penalty for any delay in completion of the project beyond 26 weeks and each of the activities can be accelerated by 3 weeks at an estimated cost of ₹1000 per week reduction. Decide on optimum cost and project the completion time after crashing.

Activities	Follow Activities	Duration (weeks)	
A	—	11	→ 11-3=8
B	A	08	8-3=5
C	B	10	10-3=7
D	A	20	20-3=17
E	—	30	30-3=27



A → D : 31, 30, 29
A → B → C : 29
E : 30, 29

1 → 2 → 4 : 31
1 → 4 : 30
1 → 2 → 3 → 4 : 29

200×7 = 1400

Crash	Options	Slope	Task Crashed	Critical path	TC = Direct cost + Indirect cost
–	–	–	–	31	0 + [1400×5] = 7000 (31-26=5)
1	A D	1000 2000	A	30	1000 + [1400×4] = 6600
2	A, E D, E	2000 2000	A, E	29	[1000+2000]+[1400×3] = 7200

∴ Project can be crashed by one week.

Optimum critical path = 30

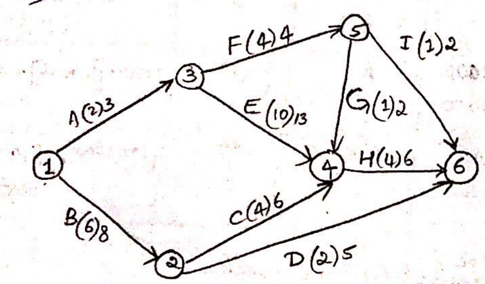
Total cost = ₹6600

5)

Tasks	Follows Tasks	Time (weeks) Normal	Time (weeks) Crash	Cost (in ₹) Normal	Cost (in ₹) Crash
A	—	3	2	8000	9000
B	—	8	6	600	1000
C	B	6	4	10,000	12000
D	B	5	2	4000	10000
E	A	13	10	3,000	9000
F	A	4	4	5,000	5000
G	F	2	1	1,200	1400
H	C,E,G	6	4	3,500	4500
I	F	2	1	700	800

If a deadline of 17 weeks is imposed for the completion of the project, what activities will be crashed? What would be the additional cost of critical path activities after crashing the projects.

→



Depreciation:-

Two types:-

→ Physical.

→ Functional.

Physical depreciation reasons:-

→ with respect to time.

→ accident.

Problems:-

1) Given the details of an article:

First cost = ₹40,000

Life = 8 years.

Salvage value = ₹6000

(a) Calculate annual rate of depreciation.

(b) ~~Depreciation~~ Depreciation fund at the end of 5 years of useful life.

(c) Book value at the end of 5 years of useful life.

Straight-line method:-

$$D_t = \frac{C - S}{n}$$

where, C = First cost or Initial cost

n = Useful economic life.

S = Salvage value or scrap value or residual value.

→ (a) Annual rate of depreciation, $D_t = \frac{C-S}{n} = \frac{₹40,000 - ₹6000}{8} = \underline{\underline{₹4250}}$

$(C-S)$ = Total amount of depreciation during economic life

(b) $DF_t = C - [D_t \times t]$

$= ₹40,000 - [₹4250 \times 5]$

$= \cancel{₹28,250}$ $\underline{\underline{₹18,750}}$

(c) $BV_t = C - DF_t$

$= ₹40,000 - ₹18,750$

$= \underline{\underline{₹21,250}}$

2) A machine was purchased and the ~~tickets~~ details are as below:

Invoice cost = ₹80,000

Ordering cost = ₹2000

Transportation charges = ₹6,000

Installation charges = ₹12,000

Accessories = ₹7,000

$\Big\}$ C = 1,07,000

Salvage value = ₹16,000 → S

Life span = 12 years → n

Estimate the bookvalue after 9 years of useful life.

$D_t = \frac{C-S}{n} = \frac{1{,}07{,}000 - 16{,}000}{12} = \underline{\underline{₹7583.33}}$

$DF_t = C - [D_t \times t]$

$= 1{,}07{,}000 - [7583.33 \times 9]$

$= 1{,}07{,}000 - 68249.97$

$= \underline{\underline{₹38{,}750.03}}$

$\therefore BV_t = C - DF_t$

$= 1{,}07{,}000 - 38{,}750.03$

$= \underline{\underline{₹68{,}249.97}}$

3) A machine was purchased at ₹[illegible] on 1st [illegible] 2009 ~~and~~ with estimated salvage value of ₹[illegible] The machine ~~is~~ will be replaced with a new [illegible] 31st December 2029. What is the annual rate of depreciation and depreciation fund as on [illegible] 2018.

(Incomplete)

Depreciation is done at the beginning of the yr always.

on 15th july 2018.

The machine is replenished with an additional cost of ₹8000 after 12 years of running. What is the new annual rate of depreciation.

From 1st jan 2009 to 31st Dec 2029 (21 yrs, counting 2009 also)

$\therefore D_t = \frac{C-S}{n}$ (Annual Depreciation)

$C = ₹80,000$ $\Rightarrow \frac{80,000 - 6000}{21}$

$S = ₹6000$

$n = 21 yrs$ $= ₹3523.80$

$DF_{2018} = D_t \times t$ (Depreciation Fund)

$= 3523.80 \times 10 yrs$

$= ₹35238.0$ (value of product ~~remaining~~ after 12 yrs)

After 12 yrs of working.

$BV_{2020} = C - DF_t$ (Book value)

$= 80000 - [3523.8 \times 12]$

$= 37,714.4$ ₹

$\therefore C = 37,714.4 + 8000$ { Adding of additional cost after 12 yrs.

$= ₹45,714.4$

$n = (21-21) = 9$ yrs

$S = ₹6000$

$\therefore$ New $D_t = \frac{45,714.4 - 6000}{9}$

$= 4,412.71$ ₹

Sunk - cost

Sunk - cost is ~~dead~~ that part of the company expenditure, cannot be recoverd irrespective of any course of action. and as a result of the replacement decisions. It is also called as historical cost or past cost.

Sunk cost = [Book value at time 't'] - [Realizable market value]

1) A machine ~~must~~ was purchased at ₹80000 with a salvage value of ₹12000 after 8 yrs of useful life. The machine is replaced after 5 yrs of working where-in the dealer of the new machine offers to buy the existing machine at ₹16000. Is the replacement justifiable and what is the sunk cost if replacement happens.

$\therefore D_t = \frac{C-S}{n} = \frac{80000 - 12000}{8} = \frac{68000}{8} = ₹8500$

$BV_5 = C - DF_t$.

$= 80000 - [8500 \times 5] = ₹37,500$

$\therefore$ Sunk cost = BV_5 - Realizable market value

$= 37,500 - 16000$

$= 21500$ ₹

Problem on Crashing:

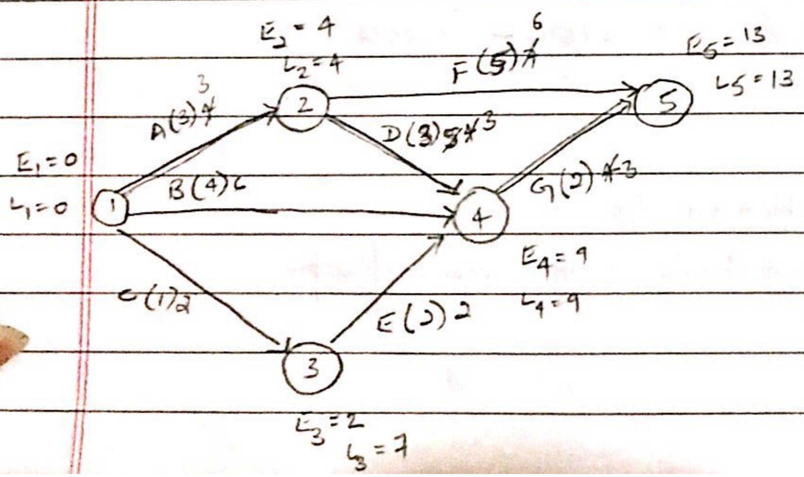
Given below are the details of a project

Tasks	Follows Tasks	Time (Days)		Cost (₹)	
		Normal	Crash	Normal	Crash
A	–	4	3	60	90
B	–	6	4	150	250
C	–	2	1	35	60
D	A	5	3	150	250
E	C	2	2	100	100
F	A	7	5	115	175
G	D, B, E	4	2	100	240

Indirect cost:

Days	15	14	13	12	11	10	9	8	7	6
Cost (₹)	600	500	400	250	175	100	75	50	35	25

Determine the project duration which will result in minimum project duration cost.



A → ~~D~~ → G : ~~13~~ ~~12~~ ~~11~~ ~~10~~ 9

~~A~~ → F : ~~11~~ ~~10~~ 9

B → G : ~~10~~ 9

C → ~~E~~ → G : 08

Task	Slope
A	30
B	50
C	25
D	50
E	–
F	30
G	70

Crash	options	slope	Task crashed	Critical path (Days)	TC = Direct cost + Indirect cost (in ₹)
–	–	–	–	13	₹713 + ₹400 = ₹1113
1	A D	30 ✓ 50	A	12	[₹713 + ₹30] + ₹250 = ₹993
	G	70			
2	D	50 ✓	D	11	[743 + 50] + 175 = ₹968
	G	70			
3	D	50 ✓	D	10	[773 + 50] + 100 = ₹943 ✓
	G	70			
4	G, F	100	GF	9	[843 + 100] + 75 = ₹1018
	only GF can be taken since G is found in both				

After crashing, project duration is 10 days and min cost is 943. Project is crashed for 3 days.