Calculation procedure for control charts on $R$ & $\bar{X}$ :-

step 1 : calculate the average $\bar{x}$ & Range $R$ each sub group or sample

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$R$ = highest of $x_i$ − lowest of $x_i$.

step 2 : calculate the grand average $\bar{\bar{x}}$ & average Range $\bar{R}$

where, $\bar{\bar{x}} = \frac{\sum \bar{x}}{N}$ & $\bar{R} = \frac{\sum R}{N}$

step 3 :- calculate the $3\sigma$ limits for the control charts.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$\bar{x}$ control chart

$$UCL_{\bar{x}} = \bar{\bar{x}} + 3\sigma_{\bar{x}}$$

$$= \bar{\bar{x}} + 3\frac{\sigma}{\sqrt{n}}$$

$$d_2 = \frac{\bar{R}}{\sigma}$$

Relative range $(W) = \frac{\text{Range}}{\sigma} = \frac{R}{\sigma}$

The Mean of $W = d_2$

we can write $\sigma$ as,

$$\sigma = \frac{\bar{R}}{d_2}$$

$$\therefore UCL_{\bar{x}} = \bar{\bar{x}} + \frac{3\bar{R}}{d_2\sqrt{n}}$$

$\boxed{UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}}$ where $A_2 = \frac{3}{d_2\sqrt{n}}$

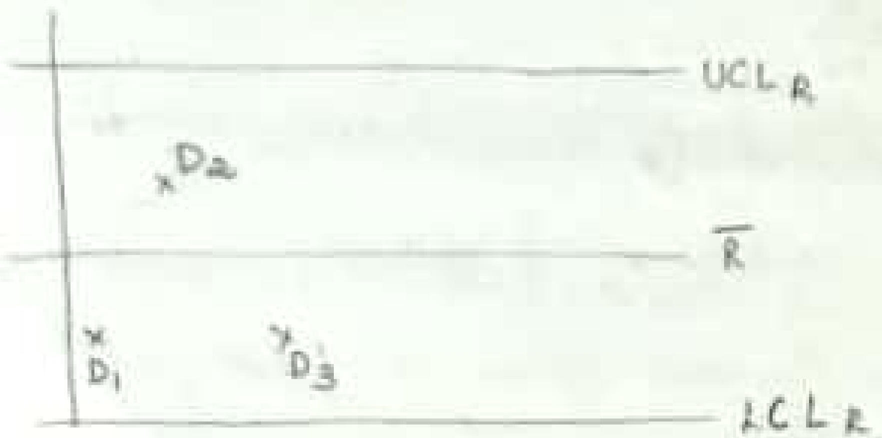
UCL : upper control limit , LCL : lower control limit

$$\boxed{LCL_{\bar{x}} = \bar{\bar{X}} - A_2 R}$$

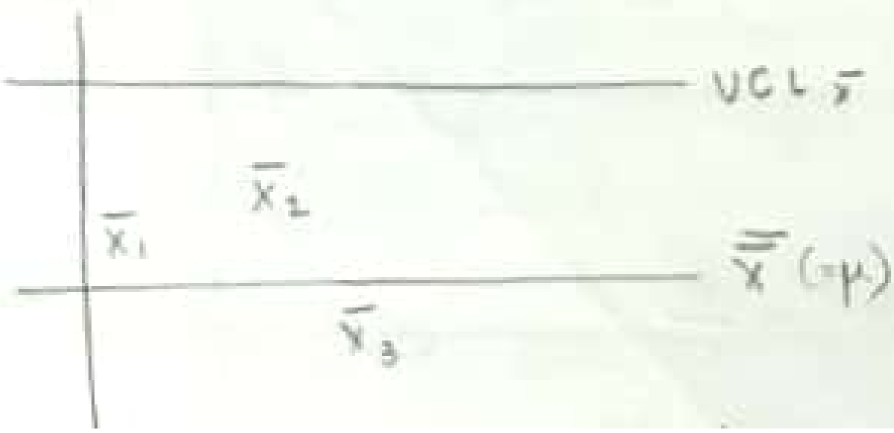
∴ UCL & LCL for control chart on Range (R) can be ge

$$\boxed{\begin{array}{c} UCL_R = D_4 \bar{R} \\ \hline LCL_R = D_3 \bar{R} \end{array}}$$

Control chart for R :




Control chart for $\bar{X}$



If USL − LSL > 6σ process is capable of meeting specification.

... chart is used to find process capability,

problems

1.) In the capability study of the lathe used in turning a shaft to a diameter of 23.75 ± 0.1 mm. A sample of 6 consecutive pieces was taken each day for 8 days. The diameter of these shafts are as shown in the table. Construct $\bar{X}$ & R chart. Find the process capability of the machine.

→

Days

| I | II | III | IV | V | VI | VII | VIII | |
|---|---|---|---|---|---|---|---|---|
| 23.77 | 23.8 | 23.77 | 23.79 | 23.78 | 23.78 | 23.76 | 23.76 | |
| 23.8 | 23.78 | 23.78 | 23.76 | 23.76 | 23.76 | 23.78 | 23.79 | |
| 23.78 | 23.76 | 23.77 | 23.79 | 23.78 | 23.73 | 23.75 | 23.77 | |
| 23.73 | 23.70 | 23.77 | 23.74 | 23.76 | 23.76 | 23.76 | 23.72 | |
| 23.76 | 23.81 | 23.8 | 23.82 | 23.74 | 23.74 | 23.81 | 23.78 | |
| 23.75 | 23.77 | 23.74 | 23.76 | 23.79 | 23.78 | 23.8 | 23.78 | |
| 0.07 | 0.11 | 0.06 | 0.08 | 0.04 | 0.05 | 0.08 | 0.07 | R (H.V − L.V) |
| 23.765 | 23.77 | 23.771 | 23.776 | 23.771 | 23.768 | 23.776 | 23.766 | $\bar{X}$ |

m = 6
N = 8

USL = 23.75 + 0.1 = 23.85 mm

LSL = 23.75 − 0.1 = 23.65 mm

USL − LSL = 0.2 mm

$$\bar{R} = \frac{\Sigma R}{N} = \frac{0.07 + 0.11 + 0.06 + 0.08 + 0.04 + 0.05 + 0.06 + 0.0\ldots}{8}$$

$$\bar{R} = 0.0675$$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{23.765 + 23.77 + \cdots + 23.766}{8}$$

$$\bar{X} = 23.769$$

$UCL_R = D_4\bar{R} = \begin{pmatrix} \text{for } n=6 \\ D_4 = 2 \end{pmatrix} \quad 2 \times 0.06 = 0.135$

$LCL_R = D_3\bar{R} = 0 \times 0.06 = 0$

$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 23.769 + 0.48(0.06) = \underline{\underline{23.8014}}$

$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 23.769 - 0.48(0.06) = \underline{\underline{23.7366}}$

$A_2 = \dfrac{3}{d_2\sqrt{n}} \qquad d_2 = 2.534 \quad (n=6)$

$\underline{\underline{A_2 = 0.4833}}$

$\therefore$ the process is "in control"

$\therefore$ process capability $= 6\sigma = 6\bar{R}/d_2 = \dfrac{6 \times 0.0675}{2.534}$

$\therefore \underline{\underline{6\sigma = 0.1598}}$

Since $6\sigma < USL - LSL$, the process is capable of meeting the specification.

| $\bar{x}$ | R | $\bar{x}$ | R | $\bar{x}$ | R |
|---|---|---|---|---|---|
| 45.02 | 0.375 | 45.6 | 0.275 | 45.26 | 0.15 |
| 44.95 | 0.45 | 45.02 | 0.175 | 45.65 | 0.2 |
| 45.48 | 0.45 | 45.32 | 0.2 | 45.62 | 0.4 |
| 45.82 | 0.15 | 45.56 | 0.425 | 45.48 | 0.225 |
| 45.28 | 0.2 | 45.14 | 0.25 | 45.38 | 0.125 |
| 45.82 | 0.25 | 45.62 | 0.375 | 45.66 | 0.35 |
| 45.58 | 0.275 | 45.8 | 0.475 | 45.46 | 0.225 |
| 45.4 | 0.475 | 45.5 | 0.2 | 45.64 | 0.375 |
| 45.66 | 0.475 | 45.78 | 0.275 | 45.39 | 0.65 |
| 45.68 | 0.275 | 45.64 | 0.225 | 45.29 | 0.35 |

Given :- $n = 5$, $N = 20$

(Consider first 20 & draw the R & $\bar{x}$ control charts)

Also Find the process capability.

$$\bar{\bar{x}} = \frac{\Sigma \bar{x}}{N} = \frac{(45.02 + 44.95 + \cdots\cdots + 45.64)}{20} = 45.5085$$

$$\bar{R} = \frac{\Sigma R}{N} = \frac{0.375 + 0.45 + 0.45 + 0.15 + 0.2 + \cdots\cdots + 0.225}{20}$$

$$= 0.33625$$

for $n = 5$

$D_3 = 0$

$D_4 = 2.11$

$A_2 = 0.58$

Control limits for R

$UCL_R = D_4 \bar{R} = 2.11 \times 0.33625 =$

$LCL_R = D_3 \bar{R} = 0$

Control limits for $\bar{x}$

$UCL_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{R} = 45.5085 + 0.58 \times 0.33625 = 45.7035$

$LCL_{\bar{x}} \; \bar{\bar{x}} - A_2 \bar{R} = 45.5085 - 0.58 \times 0.33625 = 45.3134$

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process capability :-

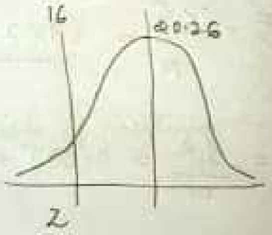
$$6\sigma = \frac{6\bar{R}}{d_2} \quad \Big| \quad d_2 = \frac{\bar{R}}{\sigma} = 2.326 \text{ (from chart)}$$

$$6\sigma = \frac{6 \times 0.33625}{(2.326)} = \underline{0.372901}$$

3. Control charts on $\bar{x}$ & $R$ for sample size of $n=5$ are to be maintained on tensile strength in pounds of yarn. 30 samples were selected and the mean and the range where computed & this yielded $\Sigma\bar{x} = 607.8$ $\Sigma R = 144$

(i) Find the control limits for both $\bar{x}$ & $R$ charts

(ii) if both charts exhibit control of and a lower specification limit is given as 16 pounds. what fraction of yarn would fail to meet the specification




$$\rightarrow \bar{\bar{x}} = \frac{\Sigma\bar{x}}{N} = \frac{607.8}{30} = \boxed{20.26}$$

$$\bar{\bar{R}} = \frac{\Sigma R}{N} = \frac{144}{30} = \underline{\underline{4.8}}$$

$$Z = \frac{x_i - \mu}{\sigma}$$

$$\sigma = \frac{\bar{R}}{d_2} = \frac{4.8}{2.326} = \underline{\underline{2.063}}$$

$$[d_2 = 2.326]$$

$$\therefore z = \frac{16 - 20.26}{2.063}$$

$$= -\underline{\underline{2.067}} = -\underline{\underline{2.07}}$$

Area under normal curve $= 0.0192 = \underline{\underline{1.92\%}}$

process capability = $6\sigma$

process capability Index $C_{pL} = \min\left[\dfrac{\bar{\bar{x}} - LSL}{3\sigma}, \dfrac{USL - \bar{\bar{x}}}{3\sigma}\right]$

process capability ratio (PCR) $= \dfrac{USL - LSL}{6\sigma}$

$20.26 - 16 = 4.26$
$USL = 20.26 + 4.26 = 24.52$
$LSL = 20.26 - 4.26 = 16$

$C_{pL} = \min\left(\dfrac{20.26 - 16}{3\sigma}, \dfrac{24.52 - 20.26}{3\sigma}\right)$

$C_{pL} = \min(0.6881, 0.6881)$ $\therefore C_{pL} = 0.6881$

4.) A machine is producing products to a specification of 12.58 $\pm$ 0.05 mm. A study 10 subgroups of size 5 each shows the following results.

$\bar{\bar{x}} = 12.598$, $\bar{R} = 0.055$

if the process exhibits statistical control (i) determine $C_p$ & $C_{pk}$. (ii) Compute % non conformity if any. (iii) Suggest the possible ways to improve the process

→ (i) $C_p = \dfrac{USL - LSL}{6\sigma}$ $USL = 12.58 + 0.05 = 12.63$
$LSL = 12.58 - 0.05 = 12.53$

$\sigma = \dfrac{\bar{R}}{d_2} = \dfrac{0.055}{2.326}$

$d_2 \Rightarrow n = 5$
$\Rightarrow d_2 = 2.326$

$\sigma = 0.0236$

$C_p = \dfrac{12.63 - 12.53}{6 \times 0.0236} = \dfrac{0.10}{0.1416} = 0.1416$

(ii) $C_{pL} = \min\left[\dfrac{12.598 - 12.53}{3 \times 0.02364}, \dfrac{12.63 - 12.598}{3 \times 0.02364}\right]$

$C_{pL} = \min[0.96, 1.579] \Rightarrow C_{pL} = 0.96$

$$z_1 = \frac{12.58 - 12.598}{0.02364} = -2.87$$

Area under signal curve = $A_1$ = %Scrap = $A_1 \times 100$

$\therefore A_1 = 0.0021$

%Scrap = 0.2%

$$z_2 = \frac{12.63 - 12.598}{0.02364} = 1.35$$

Area under normal curve $A_2 = 0.9115$

% rework = $(1 - A_2) \times 100 = 8.85\%$

% non confirming items = % scrap + % rework

= 0.2 + 8.85 = 9.05%

Reliability:

$R(t) = e^{-ct}$   t : operating time
c : failure rate

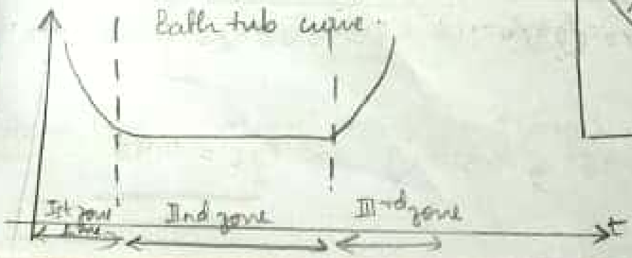
failure rate : No of units that fail per unit time.

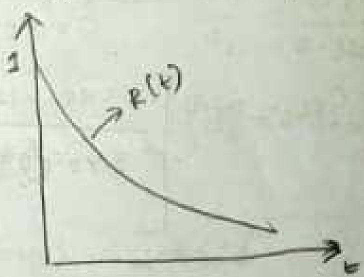
I zone – Early failures (debugging

II zone – chance failure

III zone ⇒ wear out failure.

Bath tub curve.



Mean time to failure (MTTF)    (Reliability is probability of survival)

failure rate = $C = \frac{1}{MTTF}$

MTTF used for, majorly use & throw items

Mean time between failure (MTBF)

Reliability of systems:-

→ Reliability of systems in series 

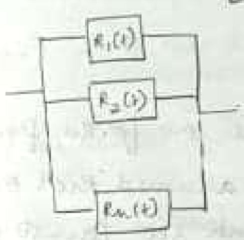
$R(t) = R_1(t) \cdot R_2(t) \cdots R_n(t)$

→ Reliability of systems in parallel

$P(t) = P_1(t) \cdot P_2(t) \cdots P_n(t)$   where $P_i(t)$ prob of failure.

$P(t) = 1 - R(t)$

$\Rightarrow 1 - R(t) = [1 - R_1(t)] \cdot [1 - R_2(t)] \cdots [1 - R_n(t)]$




where $P_i(t)$ = probability of failure.

1) A machine has MTTF of 10,000 hrs, Find the reliability of s/m for operating time of 100hrs, 1000hrs & 2000 hrs.

— MTTF = 10,000 hours.

i) t = 100 hrs.

$R(t) = e^{-ct} = e^{-(1/MTTF)t} = e^{-1/10000 \times 100} = \underline{0.9900}$

(ii) $R(t) = e^{-ct} = e^{-(1/MTTF)t} = e^{-1/10000 \times 1000} = \underline{0.904}$

(iii) $R(t) = e^{-1/10000 \times 2000} =$

⇒ 10 ball point pens were tested for life. The times to run dry measured in minutes were 1740 + 2000 + 1421 + 1857 + 1246 + 1683 + 1890 + 1676 + 1909 + 1783 respectively. Compute mean life / mean life to failure, failure rate. What is the reliability that a brand new refill will last 20 hrs of writing.

→ avg time of refills (write) = $\dfrac{1740+2000+1421+1857+1246+1683+1890+1676+1909+1783}{10}$

$= \underline{\underline{1920.5}}$

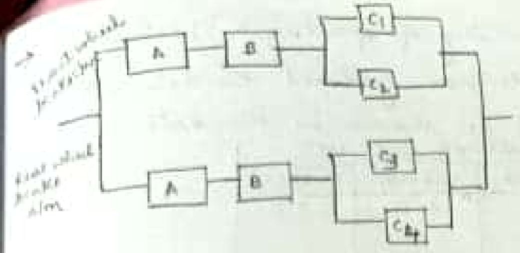
i) $c = \dfrac{1}{MTTF} = \dfrac{1}{1920.5} = \underline{5.812 \times 10^{-4}}$

ii) $R(t) = e^{-ct} = e^{-(5.812 \times 10^{-4} \times 20 \times 60)}$ ↳ in minutes

$R(t) = \underline{0.4978}$

3) There are 2 sets of breaks in a bicycle one for the front wheel & one for the rear wheel. It may be assumed that one break is sufficient for safe operation. Each break set consists of hand operated lever cable & 2 breakershoes. One break shoe is enough for stopping the bicycle. If ~~exupty~~ MTTF of various components of brake levers (A) : 500 hrs, Brake cable (B) - 100 hrs. Brake shoes (C) : 20 hrs.

Calculate the reliability of overall system for a 2 hr downhill run (t = 2hrs).




$C_1 = C_2 = C_3 = C_4 = C$

Reliability of brake shoes $= 1 - \left[(1 - R(C_1))(1 - R(C_2))\right]$ $\quad C = \frac{1}{MTTF}$

$$= 1 - \left(\left(1 - e^{-(1/20 \times 2)}\right)\left(1 - e^{-1/20 \times 2}\right)\right)$$

$$R(C) = \underline{0.9909}$$

Reliability of front wheel brake subsystem =

$R_{sub\,s/m} = R(A) \cdot R(B) \cdot R(C)$

$$= e^{-(1/500 \times 2)}\, e^{-(1/100 \times 2)} \times 0.9909$$

$$= 0.9960 \times 0.98019 \times 0.9909$$

$R(sub\,s/m) = \underline{\underline{0.9673}} = 0.967$

$\therefore$ the reliability of the s/m $= 1 - \left[\underbrace{(1 - 0.967)(1 - 0.967)}_{\text{2 sub s/m are in parallel}}\right]$

$$= \underline{\underline{0.9989}}$$

1) A businessman thinking of opening a factory in one of the three places. He has gathered the data on fixed cost of variable cost as shown in the data.

| | Fixed cost | Variable per unit cost: Material | Labour | Overhead |
|---|---|---|---|---|
| A | 200000 | 0.2 | 0.4 | 0.4 |
| D | 180,000 | 0.25 | 0.75 | 0.75 |
| N | 170,000 | 1 | 1 | 1 |

over what range of anual volume is each location advantanges

→ For A

$TC_A = TFC + TVC$

$= 200000 + (0.2 + 0.4 + 0.4)Q$

$TC_A = 200000 + Q$ — ①

IIIly. FOR D

$TC_D = TFC + TVC$

$= 180,000 + (0.25 + 0.75 + 0.75)Q$

$= 180,000 + 1.75Q \rightarrow$ ②

IIIly FOR N

$TC_N = 170,000 + (1 + 1 + 1)Q$

$TC_N = 170,000 + 3Q \rightarrow$ ③.

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$TC_A = TC_D$

$200000 + Q = 180000 + 1.75Q$

$20000 = 0.75Q$

$\underline{Q = 26\,667}$ units

$TC_D = TC_N$

$180000 + 1.75Q = 170000 + 3Q$

$10000 = 1.25Q$

$\underline{Q = 8000 \text{ units}}$

For production of 0 - 8000 units city N is preferred

For production of 8000 - 26,667 units city D is preferred.

For production ~~of~~ beyond 26,667 units city N is preferred.

If the product is sold at ₹ 200/- which alternative is preferred if sales of 10,000 units is ~~expected~~ expected

$$\text{profit} = TR - TC$$

$$= RQ - TFC + VC \cdot Q$$

$$\text{profit}(A) = 200 \times 10{,}000 - (200{,}000 + 10{,}000 \times 1) = 17{,}90{,}00$$

$$\text{profit}(D) = 200 \times 10{,}000 - (180{,}000 + 10{,}000 \times 1.75) = 18{,}02$$

$$\text{profit}(N) = 200 \times 10000 - (170{,}000 + 10{,}000 \times 3) =$$

∴ choose D.