Date

Operations Management & Entrepreneurship

Measure of central tendency
mean, median, mode

Measure of dispersion
range, standard deviation

→ Statistical Quality Control – M. Mahajan (for II or I)

Unit IV, Unit V → Management in Entrepreneurship by Poornima Charantimath

~~Facility~~

Facility location decisions
Break even analysis &
Transportation method
} Operation Management Joseph G. Monks
Facility / Plant Location Chapter

Date

Modeling process quality

The concept of variation

No two items are perfectly identical, the variation ~~not~~ may be small, expressed in microns or very large, but the variations exists in all the parts manufactured, using different processes.

Quality improvement involves reduction of variability in processes and products, usage of statistically methods to control and improve quality has led to the concept of statistically quality control or statistically process control.

Numerical summary of data can be classified as measure of central tendency and measure of dispersion.

Measures of central tendency

1) Mean – Suppose $x_1, x_2, x_3, ----x_n$ are the obj observations,

$$\text{Mean} = \text{Average} = \bar{X} = \frac{x_1+x_2+x_3+\_\_\_\_+x_n}{n}$$

$$= \sum_{i=1}^{n} \frac{x_i}{n}$$

% of items below & that will not meet the specification

$0.2514 + 0.0038$
$= 0.2552$
$= 25.52\%$

(ii) USL = 2.835, LSL = 2.825
σ = ~~2.823~~ ~~2.827~~ 0.003
$\bar{x}$ = 2.83 (given question mein)

$$z_1 = \frac{2.825 - 2.83}{0.003} = -1.67$$

Area under curve = 0.0475

~~The %~~

~~1 - 0.0475 = 0.525~~
~~52.50%~~

The percentage of items ~~that~~ will meet the specification

$$z_2 = \frac{2.835 - 2.83}{0.003} = 1.667$$

Area under curve 0.9525

$1 - 0.9525 = 0.0475$

The percentage of items that will meet the specification

$0.0475 + 0.0475 = 0.095$
or (9.50%)

Quality Control Tools

1) Histogram
2) Flowchart
3) Scatter diagram
4) Check sheets
5) Cause and effect diagram
6) Pareto diagram
7) Statistical quality control / statistical process control chart

(~~Statistical process/qu~~)

If x_1 occurs f_1 times, x_2 occurs f_2 times
x_n occurs f_n times

$$\text{Mean} = \bar{X} = \frac{x_1 f_1 + x_2 f_n + \cdots + x_n f_n}{f_1 + f_2 + \cdots + f_n}$$

$$\bar{X} = \sum_{i=1}^{n} \frac{f_i x_i}{n}$$

2) Median - When all the observations are arranged in ascending or descending, median is magnitude of middle case

$$M_d = \left(\frac{n+1}{2}\right)^{th} \text{ value if } n \text{ is odd}$$

M_d = Average of $\frac{n}{2}$ th value & $\left(\frac{n}{2}+1\right)$ value if n is even

When data is grouped into frequency distribution

$$M_d = L_m + \left[\frac{\frac{n}{2} - C_{fm}}{f_m}\right] \cdot i$$

where,

L_m → lower number boundary of ~~⟶~~ with the median
n → total number of observations

C_{fm} → cumulative frequency of all cells below L_m
f_m → frequency of median cell
i → cell interval

3) Mode - Mode is the value that occurs most frequently in a frequency histogram or frequency polygon.
It is an observed value corresponding to the highest point in the frequency histogram.

$$\text{Mode} = L + \left[\frac{f_2}{f_1 + f_2}\right] i$$

where,

f_1 → frequency of class immediately before the modal class
f_2 → frequency of class immediately after the modal class
L → lower ~~number~~ boundary of modal class
i → cell interval

Date

Measures of dispersion

1) Range (R) - It ~~(may be a)~~ is the differe[nce] between the largest observed value a[nd] smallest observed value.

2) Standard deviation (σ) - It is defined a[s] root mean square of the differences betwe[en] the observations and the mean.

If $x_1, x_2, x_3, \ldots x_n$ are the number of observations,

$$\sigma = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n}}$$

When classified data with frequency distribution is given

$$\sigma = \sqrt{\frac{\sum fd^2}{n} - \left(\frac{\sum fd}{n}\right)^2}$$

$$d = x_i - k$$

$$\text{Mean} = \bar{x} = k + \frac{\sum fd}{n}$$

{ mos[t] conv[enient] for d[...] pe[...]

Variance $= \sigma^2$

What makes you happy? #

Date

Find the mean, median, mode and standard deviation for the following data

~~Weight~~

Weight (g)	Frequency (f)	x	$d = (x_i - k)$	fd	fd^2
3 - 3.25	6	3.125	-0.75	-4.5	3.375 ~~20.25~~
3.25 - 3.5	19	3.375	-0.5	-9.5	4.75
3.5 - 3.75	35	3.625	-0.25	-8.75	2.1875
3.75 - 4	44	3.875 (k)	0	0	0
4 - 4.25	47	4.125	0.25	11.75	2.9375
4.25 - 4.5	29	4.375	0.5	14.5	7.25
4.5 - 4.75	15	4.625	0.75	11.25	8.4375
4.75 - 5	5	4.875	1	5	5
	$\sum f = 200$			$\sum fd = 19.75$	$\sum fd^2 = 33.9375$

$$\text{Mean} = k + \frac{\sum fd}{n}$$

$$(\cancel{= 3.875 + 2.46875 = 6.34375})$$

$$= 3.875 + 0.09875 = \underline{3.97375}$$

$$\text{Median} = L_m + \frac{\left[\frac{n}{2} - C_{fm}\right]}{f_m} \cdot i$$

$$= 3.75 + \frac{\left[\frac{200}{2} - (6 + 19 + 35)\right]}{f_m} \times 0.25$$

$$= \cancel{4.659090909} = \underline{3.97}$$

What makes you happy? #[...]CollegeDays # #

$$\text{Mode} = L + \left[\frac{f_2}{f_1+f_2}\right] i$$

$$= 4 + \left(\frac{29}{44+29}\right) 0.25 \quad = \cancel{4.0999}$$

$\cancel{4.0397}$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = \sqrt{\frac{33.93}{200} - \left(19.\right.}$$

$$= 0.399$$

Concept of Universe or population

In order to construct a frequency distributie we normally take small proportion of sa to represent entire collection of items. From the measurement of the characteri of the sample, we draw conclusions abo collection of items ~~is~~ known as univ or population.

~~Cent~~

~~Central~~

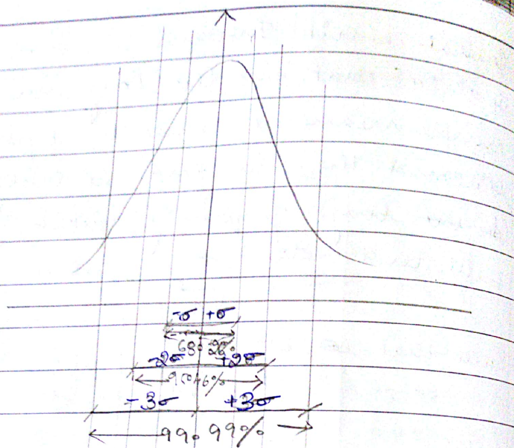
Central Limit Theorem

It states that in the long run the average of sample means $\bar{x}$ tends to population mean μ, and the standard deviation of the (~~In the long~~) frequency distribution of $\bar{x}$ ~~no~~ values will tend to, $\frac{\sigma}{\sqrt{n}}$, where n is the sample size.

Such that ~~a~~. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

In the frequency distribution if the number of observations are increased considerably, then the number of cells will increase and the width of the cell will become smaller and smaller. The series of steps that constitutes the top line of the histogram will then approach a smooth curve. The height of the curve at any point is ~~po~~ proportional to the frequency at that point, and the (~~area~~ area under it between any two limits is proportional to the frequency of occurrences within these limits such a curve is called as frequency curve.

The frequency curve that is of most importance to the statistical quality control is normal distribution curve.



Considering the entire area under standard normal curve (with mean = 0 and standard deviation = 1), the proportion of area less than any given value is tabulated in standard normal distribution table.

This table can made use for finding the proportion or ratio percentage of distribution lying between any two values. If it is desired to calculate or the expected proportion of observations that will be less than or equal to a specified value 'x' then the standard deviate

$$z = \frac{x_i - \bar{x}}{\sigma}$$ is calculated

the required proportion is directly read from the standard normal distribution table.

1) A machine shop produces steel pins. The width of 100 pins was checked after machining and the data was recorded as shown in the table.

(a) Find the mean, median, mode, ~~and~~ standard deviation and variance.

(b) What % of pins manufactured would have width between 9.52mm and 9.63mm

difference 0.01/2 = 0.005

mid value of this

width (in mm)	f		x	d	fd	fd^2
9.5 - 9.51	6	9.495 - 9.515	9.505	-0.06	-0.36	0.0216
9.52 - 9.53	2	9.515 - 9.535	9.525	-0.04	-0.08	0.00032
9.54 - 9.55	20	9.535 - 9.555	9.545	-0.02	-0.4	0.208
9.56 - 9.57	32	9.555 - 9.575	9.565	0	0	0
9.58 - 9.59	22	9.575 - 9.595	9.585	0.02	0.44	0.0088
9.6 - 9.61	8	9.595 - 9.615	9.605	0.04	0.32	0.0128
9.62 - 9.63	6	9.615 - 9.635	9.625	0.06	0.36	0.0216
9.64 - 9.65	4	9.635 - 9.655	9.645	0.08	0.32	0.0256
	$\Sigma f = 100$				$\Sigma fd = 0.6$	$\Sigma fd^2 = 0.1016$

$K = 9.565$

$$\boxed{\text{Mean} = K + \frac{\Sigma fd}{\Sigma f} = 9.565 + \frac{0.6}{100} = 9.571}$$

Date

$$\text{Median} = L_m + \frac{\left[\frac{N}{2} - cf_m\right]}{f_m} i$$

$$= 9.555 + \frac{\left[100/2 - (6+2+20)\right]}{32} \times 0.0$$

$$\boxed{\text{Median} = 9.56875}$$

$$\text{Mode} = L + \left[\frac{f_2}{f_1+f_2}\right] i$$

$$\boxed{\text{Mode} = 9.555 + \left[\frac{22}{20+22}\right] 0.02 = 9.565}$$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\left(\frac{0.1016}{100}\right) - \left(\frac{0.6}{100}\right)^2}$$

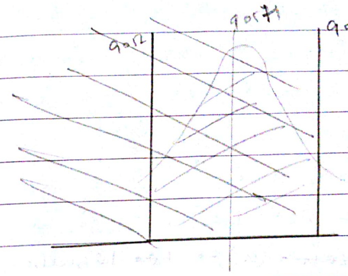
$$= \sqrt{0.001016 - 0.006}$$

$$\boxed{\sigma = 0.03130}$$

$$\boxed{\sigma^2 = 0.00098}$$

Date

(b) The percentage / proportion of items between 9.52mm and 9.63mm



$$z_1 = \frac{x_i - \bar{x}}{\sigma} = \frac{9.52 - 9.571}{0.0313}$$

$$= -\cancel{0}1.6293$$

$$= -1.63$$

∴ Area under normal distribution curve $= 0.0516$

$$z_2 = \frac{9.63 - 9.571}{0.031} = 1.8849$$

$$= \cancel{0.9696}$$

$$= 0.9699$$

The proportion of pins within 9.52 mm and 9.63mm

$0.9699 - 0.0516 = 0.9183$

$= 91.83\%$

Q2) Assuming that the life in hours of an electric bulbs is a random variable for normal distribution with a mean of and std deviation of 400 hrs. Find the expected no. of bulbs from a random of 2000 bulbs having life

(a) more than 3000hrs

(b) b/w 2600 & 2800 hrs

(Answers : ~~Mean~~ (a) → ~~12~~ 12 bulbs, ~~0.6~~

(b) 0.062 %

0.9938 (1 - 0.99...)

(b) 0.0381 %

$z = 1.5$ & 2

Q3) In a normal distribution 31 ~~30~~ % of the items under ~~(45 hours)~~ 45 Ω and 8 % are over 64. Find the mean and std deviation of the distribution.

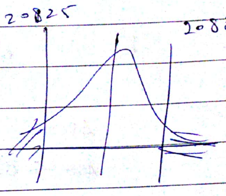
Q4) Dishes stamped out of sheet metal are found have a weight variation conformed into normal distribution with mean weight of 2.827 kg and std deviation of 0.0083 kg and specification weight for these dishes are 2.083 ± 0.0005 kg. Determine:

1) the % of dishes which are not expected to meet the specifications.

2) the minimum % of dishes ~~wil~~ which will not need the specification if the mean of the process can be adjusted to the specification mean

→ upper specification limit $= 2.833 + 0.005 = 2.835$ kg

lower specification limit $= 2.83 - 0.005 = 2.825$ kg



~~For~~ For 31 %

$1 - 0.031 = 0.49$ (area under mid curve)

$$z_1 = \frac{x_i - \bar{x}}{\sigma}$$

$$-0.49 = \frac{45 - \bar{x}}{\sigma}$$

$$(45 - \bar{x}) = -0.49\sigma$$

$1 - 0.08 = 0.92$

$z_2 = 1.41$

$$1.41 = \frac{64 - \bar{x}}{\sigma} \Rightarrow 1.41\sigma = 64 - \bar{x}$$

Answers

Q2) $\bar{x} = 2000$ & $\sigma = 400$

a) $z = \frac{x_i - \bar{x}}{\sigma} = \frac{3000 - 2000}{400} = \frac{1000}{400} = 2.5$

Area under normal distribution curve = 0.9

More than 3000 = $1 - 0.9938 =$ ~~6~~ 0.00

Number = $0.0062 \times 2000 = 12$

b) $z_1 = \frac{2600 - 2000}{400} = 1.5$

Area = 0.9332

$z_2 = \frac{2800 - 2000}{400} = 2$

Area = 0.9773

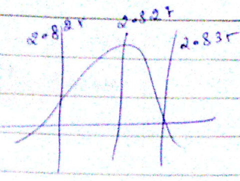
Area between = $0.9773 - 0.9332 = 0.$

number = $0.0441 \times 2000 \simeq 88.2$

Q4) ~~$x = 2.827$~~

$\bar{x} = 2.827$ kg

$\sigma = 0.003$ kg



specification 2.83 ± 0.005

USL = $2.83 + 0.005 = 2.835$ kg

LSL = $2.83 - 0.005 = 2.825$ kg

(i) $z = \frac{x_i - \bar{x}}{\sigma}$

$z_1 = \frac{2.825 - 2.827}{0.003} = -0.6667 = -0.67$

Area under curve = 0.2514

= 25.14%

$z_2 = \frac{2.835 - 2.827}{0.003} = 2.667 = 2.67$

Area under curve = 0.9962

99.62%

(Below 2.825 or above 2.835)

% of items that will be above 2.835 is $1 - 0.9962 = 0.0038$

0.38%