

28/1/19

Basic Tools

- Flow chart
- Run chart
- ~~Avg-~~ control chart
- causes & effect diagram (fishbone)
- check sheet
- histogram

~~Pareto analysis~~

80% of the problems may be attributed to 20% of the cause.

Arithmetic mean ( $\bar{X}$ )

It is the average of all the values of the variant in the sample. If  $x_1, x_2, \dots, x_n$  are the 'n' values of the variant 'X' in the sample, then arithmetic mean is given by

$$\bar{X} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{\sum x}{n}$$

If  $x_1$  occurs  $f_1$  times &  $x_2$  occurs  $f_2$  times etc.,  $x_n$  occurs  $f_n$  times will be 'n' observations all together.

$$\frac{f_1x_1 + f_2x_2 + f_3x_3 + \dots + f_nx_n}{f_1 + f_2 + f_3 + \dots + f_n}$$

Median :

When all the observations are arranged in ascending or descending order, then median is the magnitude of the middle case, it has half the observations above it & below it.

$$\text{Median} = M_d - \left( m + \left[ \frac{n/2 - c_{fm}}{P_m} \right] \right) x_i^*$$

where  $l_m$  = lower number of the cell with the median.

$N$  = total number of observations

$c_{fm}$  = cumulative frequency of all cells below  $l_m$

$f_m$  = frequency of median.

$q$  = cell intervals.

### Mode

It is the value that occurs most

frequently, in a frequency histogram, it is the observed

value corresponding to the high point of the graph

for example, the recorded observation 2, 3, 2, 4, 2, 2, 5, 7, 2, 4,

here 2 occurs most of times hence mode is '2'.

### Range

It is a Simplest measure of dispersion in a sum.

### Standard deviation

defined as root mean square the differences b/w observation & median.

$$\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{n}}$$

### Variance

This is the sum of the square of the deviation arithmetic mean divided by the no of observations 'n'  
In other words,

$$\text{Variance} = (S.D)^2$$

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five thermostatic control are tested, to determine the temperature the measured values are  $344^\circ\text{C}$ ,  $338^\circ\text{C}$ ,  $342^\circ\text{C}$ ,  $335^\circ\text{C}$ ,  $336^\circ\text{C}$ . These values constitute to first subgroup for certain control chart. Compute the arithmetic mean, median, range, S.D & Variance of this subgroup.

80.62 202.8 8 12.9 - 82.0

84.51 20.81 262.8 8 82.4 - 83.0

83.7 20.1P 202.5P 8 12.9 - 82.0

84.51 20.1P 230.2P 8 82.4 - 82.0

83.7 20.1P 200.8P 8 12.9 - 80.0

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Imp  
Machine shop produces steinless. the width of 100 pins whether check after machining (data goes) recorded as follows.

width mm	frequency f	X	fX	$\bar{x}$	$f\bar{x}^2$
9.50 - 9.51	6	9.505	57.03	90.35	540.1
9.52 - 9.53	2	9.525	19.05	90.73	38.10
9.54 - 9.55	20	9.545	190.9	91.11	182.21
9.56 - 9.57	32	9.565	306.08	91.49	2927.61
9.58 - 9.59	22	9.585	210.87	91.87	2021.41
9.60 - 9.61	8	9.605	76.84	92.3560	738.08
9.62 - 9.63	6	9.605	57.75	92.6406	555.84
9.64 - 9.65	4	9.645	38.98	93.02	372.12
			= 785.24	= 917.1	= 9160.4

$$\text{Variance} = (SD)^2 \quad JS.D \\ = 9.5481 \times 10^{-4}$$

$$b) \quad Z_1 = \frac{x_1 - \bar{x}}{\sigma}$$

$$Z_2 = \frac{x_2 - \bar{x}}{\sigma}$$

$$Z_1 = \frac{9.52 - 9.571}{0.0309} = -1.65 \\ = 0.0495$$

$$Z_2 = \frac{9.63 - 9.571}{0.0309} = 1.9 \\ = 0.9913$$

a) where find the actual mean, S.D & Variance.

b) what % of the pins manufactured has width

$$9.52 - 9.63.$$

lower limit - upper limit

$$\text{Mean} = \bar{x} = \frac{\sum fx}{n} = \frac{917.1}{100} = 9.171$$

$$\begin{aligned} Z_2 - Z_1 \\ = 0.0495 - 0.9913 \\ = 0.923 \end{aligned}$$

(c)

$$S.D = \sqrt{\frac{\sum f(x - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{9160.4 - (9.571)^2}{100}} = 0.4750$$

$$\begin{aligned} 100/2 \\ = 25 + P + 1 + P + 1 \\ = \sqrt{\frac{9008.796}{100}} \end{aligned}$$

$$= \sqrt{91.604} - 91.6$$

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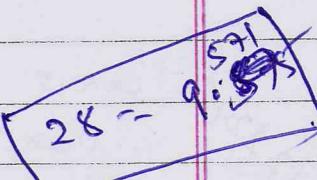
SD. 0.0309

100/2

8/2/19 Median :-

$$l.m + \left[ \frac{N/2 - cf_n}{f_m} \right] x_i$$

$$l.m = 9.565 + \left[ \frac{100/2 - 40}{32} \right]$$



$$9.565(10/32) \times 0.01$$

$$\begin{aligned} &= 9.565 + 0.325 \times 0.01 \\ &= 9.562 + 0.00325 \\ &= 9.56525 \end{aligned}$$

weight (grams)	frequency f	x	$f_x$	$x^2$	$f_x x^2$
3.0 - 3.25	6	3.125	18.75	9.7656	351.5625
3.25 - 3.50	19	3.375	64.125	11.3906	716.421
3.50 - 3.75	35	3.625	126.875	13.1406	459.921
3.75 - 4.00	44	3.875	170.5	15.0156	660.687
4.00 - 4.25	47	4.125	193.875	17.0156	799.734
4.25 - 4.50	29	4.375	126.875	19.1406	555.0724
4.50 - 4.75	15	4.625	69.375	21.3906	320.889
4.75 - 5.00	5	4.875	24.375	23.7656	118.828

$$\text{median} = \bar{x} + \left[ \frac{\frac{n}{2} - cf_n}{f_m} \right] \alpha$$

$$= 3.875 + \left[ \frac{\frac{200}{2} - 60}{44} \right] \times 0.25$$

$$= 3.875 \left[ \frac{100 - 60}{44} \right] \times 0.25$$

$$= 4.1022$$

mean, median, mode, S.D., Variance & range.

$$\text{mean } \bar{x} = \frac{\sum f_x}{n}$$

$$\frac{200 \times 3.875}{200} = 3.875$$

Actual SD

$$\sqrt{\frac{\sum f_x^2}{n}} = \sqrt{3190.448} = 15.784$$

$$= \sqrt{15.95 - 15.78}$$

$$= 0.41$$

$$\text{range} = H - L$$

$$= 5 - 3$$

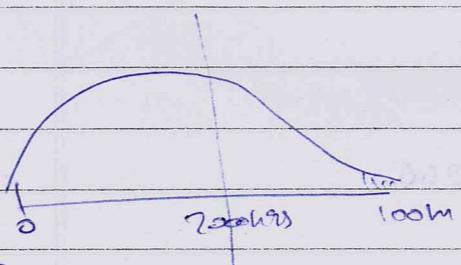
$$= 2$$

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$$\text{mode} = l + \left[ \frac{f_1/f_m}{f_1 + f_2} \right] \alpha$$

$$= 3.875 \left[ \frac{29}{29+44} \right] \times 0.25$$

lowest of next of middle value + frequency of below & above



Problem

a) more above 30000

$$= 4 + 0.0993$$

$$= 4.099$$

$$\bar{x} = 2000$$

$$N = 2000$$

$$\sigma = 400$$

$$Z_1 = \frac{x_1 - \bar{x}}{\sigma}$$

$$= \frac{30000 - 2000}{400} = 2.5 = 0.9938$$

$$= 1 - 0.9938$$

$$\text{median} = l_m + \left[ \frac{n/2 - cf_n}{f_n} \right] x_i$$

$$= 502.75 + \left[ \frac{\frac{250}{2} - 103}{49} \right] \times 0.5$$

$$= 502.75 + \left[ \frac{125 - 103}{49} \right] \times 0.5$$

$$= +502.4\overline{5} \Rightarrow$$

$$\text{Mode} = l + \left[ \frac{f_2 - f_1}{f_1 + f_2} \right] \times p$$

$$= 502.75 \left[ \frac{29}{30+29} \right] \times p$$

$$= \underline{\underline{502.745}}$$

$$\text{Mean} = \frac{\sum fx}{n}$$

$$= \frac{125566.5}{250} = 502.266$$

(D)

$$\sqrt{\frac{\sum x^2 - (\bar{x})^2}{n}}$$

$$= \sqrt{63068245 - 20270399.13}$$

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250  
1.358

## UNIT - 3

### Problem

The control chart  $\bar{X}$  & R are maintained on certain dimension of manufactured part, measured in mm.

The Subgroup Size is 4. The values of  $\bar{X}$  & R are compiled for each subgroup. After 20 subgroups  $\sum \bar{X} = 412.83$ ,  $\sum R = 3.39$ . Compute for value of 3 sigma limit for the  $\bar{X}$  & R charts and estimate value of  $\sigma^1$ . Assumption that the process present statistical control.

### Step 1 :

determine  $\bar{\bar{X}}$  which is equal to  $\frac{\sum \bar{X}}{N}$

$$(\sum \bar{X} = \bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_{20})$$

### Step 2

determine  $\bar{R} = \frac{\sum R}{N}$

$$(\sum R = R_1 + R_2 + \dots + R_{20})$$

(N = is subgroup size used to determine  $\sum \bar{X}$  &  $\sum R$ )

### Step 3 :

determine  $\sigma^1$

(S.D for different no. of nodes.)

$$\sigma^1 = \frac{\bar{R}}{d_2}$$

( $d_2$  = value u from table 'C'

where  $d_2$  is the value obtain for specific subgroup of interest (n).

### Step 4:

determine control chart

$$\text{Date / 120} \quad \text{To determine } UCL_{\bar{X}} = \bar{X} + 3\sigma_{\bar{X}}$$

$$\text{where } 3\sigma_{\bar{X}} = \frac{3\sigma^1}{\sqrt{n}}$$

- lower control limit

$$LCL_{\bar{X}} = \bar{X} - 3\sigma_{\bar{X}}$$

### Step 5 :

determine Rchart

$$UCL_R = D_4 \bar{R}$$

where  $D_4$  can be obtained from table 'D' for specific

Subgroup n.

lower control limit

$$LCL_R = D_3 \bar{R}$$

$D_3$  can be obtain from 'D' for specific subgroup size

$$\Rightarrow \bar{\bar{X}} = \frac{\sum \bar{X}}{N} \\ = \frac{412.83}{20} = 20.6415$$

$$\bar{R} = \frac{\sum R}{N} \\ = \frac{3.39}{20} \\ = 0.169$$

$$\sigma^1 = \frac{\bar{R}}{d_2} \\ = \frac{0.169}{2.059} \\ = 0.0820$$

$$UCL_{\bar{x}} = \bar{x} + 3\sigma_{\bar{x}}$$

$$\sigma_{\bar{x}} = \frac{3\sigma}{\sqrt{n}}$$

$$= \frac{3 \times 0.082}{\sqrt{10}} = 0.10010$$

~~20.6415 +~~

$$= \underline{20.7000} \quad \underline{20.76}$$

$$LCL_{\bar{x}} = \bar{x} - 3\sigma_{\bar{x}}$$

$$= 20.6415 - 0.0615 \times 0.123$$

$$= \underline{20.518}$$

$$UCL_R = D_4 \bar{R}$$

$$= 2.28 \times 0.169$$

$$= 0.3853$$

$\approx$

$$LCL_R = D_3 \bar{R}$$

$$= 0 \times 0.169$$

$\approx$

Control chart

$$UCL_{\bar{x}} = 20.76$$

$$\bar{x} = 20.64$$

$$LCL_{\bar{x}} = 20.518$$

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R chart

$$UCL_R = 0.386$$

$$\bar{R} = 0.169$$

$$LCL_R = 0$$

- 2) In the capability study of a lathe used for turning to a size of  $23.75 \pm 0.1$ . A sample of 6 consecutive pieces were taken each day for 8 days. Data of all these pieces are as below:

1st day	2nd	3rd	4th	5th	6th	7th	8th day
23.77	23.80	23.77	23.79	23.75	23.78	23.76	23.76
23.80	23.78	23.78	23.76	23.78	23.76	23.78	23.79
23.78	23.76	23.77	23.79	23.78	23.73	23.75	23.77
23.70	23.70	23.77	23.74	23.77	23.76	23.76	23.72
23.76	23.81	23.80	23.82	23.76	23.74	23.81	23.78
23.75	23.77	23.74	23.76	23.79	23.78	23.80	23.78
= 23.765	= 23.77	= 23.771	= 23.776	= 23.771	= 23.758	= 23.776	= 23.766

construct the  $\bar{x}$ , R chart & find out process capability for the machine.

$$\bar{x} = \frac{\sum x}{n} = 23.760$$

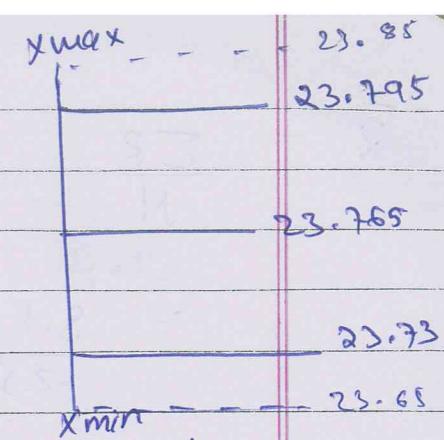
$$\frac{190.12}{8} = 23.765$$

$$UCL_{\bar{x}} = \bar{x} + 3\sigma_{\bar{x}}$$

$$= 23.769 +$$

$$\bar{R} = \frac{\sum R}{N}$$

$$LCL_{\bar{R}} = \frac{23.765}{202.68} - 0.0325 \\ = 23.73$$



$$R_1 = H - L \\ = 23.80 - 23.73 \\ = 0.07$$

$$R_2 = 0.01$$

$$R_3 = 0.08$$

$$R_4 = 0.08$$

$$R_5 = 0.04$$

$$R_6 = 0.05$$

$$R_7 = 0.06$$

$$R_8 = 0.07$$

$$\bar{R} = 0.0675$$

$$UCL_{\bar{x}} = d_4 \bar{R}$$

$$= 2 \times 0.07 \\ = 0.14H$$

$$LCL_{\bar{x}} = D_3 \bar{R}$$

$$= 0 \times 0.07 \\ = 0$$

$$\sigma' = \frac{\bar{R}}{d_2} = \frac{0.0675}{2.534}$$

$$= 0.0266$$

$$UCL_{\bar{x}} = \bar{x} + 3\sigma_{\bar{x}}$$

$$3\sigma_{\bar{x}} = \frac{3\sigma'}{\sqrt{n}} = \frac{3 \times 0.0266}{\sqrt{25}}$$

$$= 0.0325$$

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$$= 23.765 + 0.0325$$

$$= 0.054 + 23.795$$

A subgroup of 5 items each are taken from a manufacturing process at a regular interval. A certain quality character is measured &  $\bar{x}$  &  $\bar{R}$  value are computed. After 25 subgroups it is found that  $\sum \bar{x} = 357.50$  &  $\sum R = 8.80$ . If the specification limits are  $14.10 \pm 0.4$ , & the process is in statistical control, what conclusion can you draw about the ability of the process produced items within specification.

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{N} = \frac{357.50}{25} = 14.3$$

$$\bar{R} = \frac{\sum R}{N}$$

$$= \frac{8.80}{25} = 0.352$$

$$\sigma' = \frac{\bar{R}}{d_2} = \frac{0.352}{2.326} = 0.151$$

$$UCL_{\bar{x}} = \bar{x} + 3\sigma_x$$

$$3\sigma_x = \frac{3\sigma'}{\sqrt{n}}$$

$$= \frac{3 \times 0.15}{\sqrt{5}} = 0.202$$

$$= 1H.3 + 0.202$$

$$\approx 1H.\underline{502}$$

$$LCL_{\bar{x}} = 1H.3 - 0.202$$

$$\approx 1H.\underline{098}$$

$$UCL_{\bar{R}} = D_4 \bar{R}$$

$$= 2.11 \times 0.352$$

$$= 0.742$$

$$LCL_{\bar{R}} = d_3 \bar{R} = 0 \times 0.352$$

$$(x_{\max} - x_{\min})$$

$$x_{\max} = 1H.40 + 0.4 = 1H.8$$

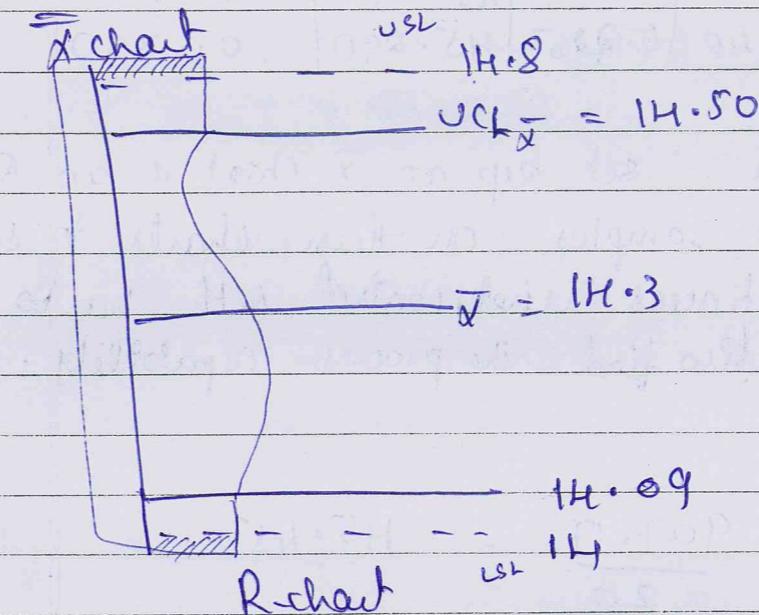
$$x_{\min} = 1H.40 - 0.4 = 1H.$$

$$= 1H.8 - 1H = 0.8$$

$$6\sigma'$$

$$= 6 \times 0.151$$

$$= 0.906$$



$$UCL_{\bar{R}} = 0.742$$

$$0.15)$$

$$LCL_{\bar{R}} = 0$$

The following table shows the averages & ranges of Spindle diameter in millimeters. For 30 subgroups of 5 items each.

$$UCL_{\bar{x}} = \bar{x} + 3\frac{\sigma}{\bar{x}}$$

$$3\sigma = \frac{3\sigma}{\sqrt{n}}$$

$$= \frac{3 \times 0.134}{\sqrt{5}}$$

$$= 0.179$$

$$= 4\bar{x} - 0.179$$

$$= 4\bar{x} - 0.639$$

$$LCL_{\bar{x}} = \bar{x} - 3\frac{\sigma}{\bar{x}}$$

$$= 4\bar{x} - 0.179$$

$$= 4\bar{x} - 2.71$$

$\bar{x}$	$R$	$\bar{x}$	$R$	$\bar{x}$	$R$
HS.020	0.375	HS.600	0.275	HS.260	0.180
HS.950	0.450	HS.020	0.175	HS.650	0.200
HS.480	0.450	HS.320	0.200	HS.680	0.400
HS.320	0.150	HS.560	0.425	HS.480	0.225
HS.280	0.200	HS.140	0.250	HS.380	0.125
HS.820	0.250	HS.620	0.375	HS.680	0.050
HS.580	0.275	HS.800	0.475	HS.260	0.225
HS.400	0.475	HS.500	0.200	HS.640	0.375
HS.660	0.475	HS.780	0.275	HS.390	0.650
HS.680	0.275	HS.640	0.225	HS.290	0.350

For the 1st 20 Samples set up an  $\bar{x}$  chart & an R chart  
plot the next 10 samples on these charts to see  
whether the process continues under control. Both has to  
average and range. Also find the process capability.

$$UCL_R = D_4 \bar{R}$$

$$= 2.11 \times 0.312$$

$$= 0.658$$

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{N} = \frac{909.17}{20} = HS.455$$

$$LCL_R = D_3 \bar{R}$$

$$= 0 \times 0.312$$

$$= 0$$

$$\bar{R} = \frac{\sum R}{N} = \frac{6.25}{20} = 0.3125$$

$\bar{x}$  chart

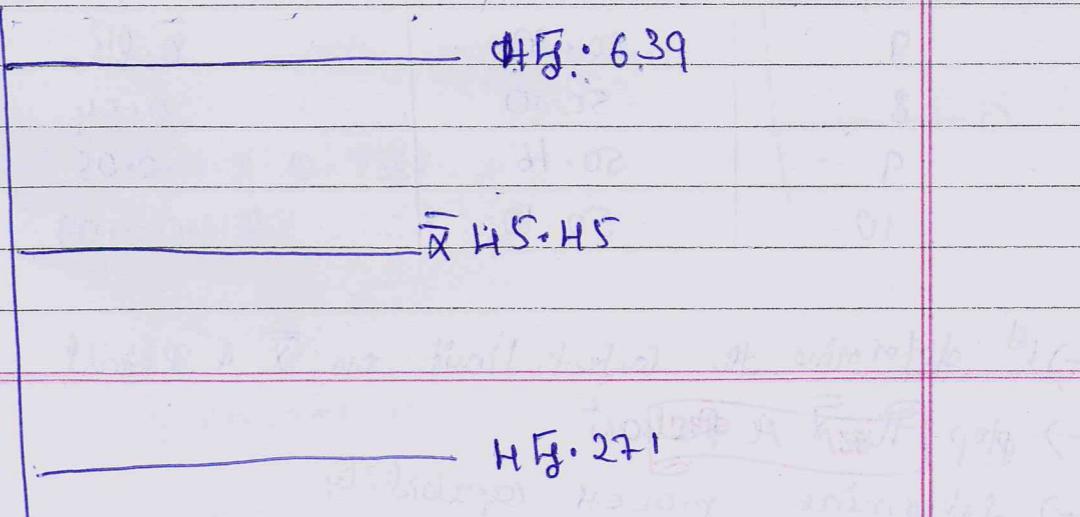
HS.639

$$\sigma = \frac{R}{d_2} = \frac{0.3125}{2.326} = 0.134$$

Process capability

$$6\sigma = 6 \times 0.134$$

$$= 0.8$$



0.658

 $\bar{R} = 0.3125$ 

0

$$\bar{x} = \frac{\sum x}{N} = 50.16$$

$$R = \frac{\sum R}{n} = 0.056$$

$$\sigma^2 = \frac{\bar{R}}{d_2} = \frac{0.056}{2.326} = 0.024$$

25/2/1X

$\Rightarrow$  The  $\bar{x}$  chart this should be established based on the standard values of  $\mu = 60.00$  &  $\sigma = 12$  where  $n=9$

In the manufacturer of precision pins in which the diameter of the pins is the quality characteristic should be controlled, 10 samples of 5 pins each were collected after a lapse of every 30 min periods.  $\bar{x}$  &  $R$  for each sample were calculated & results were recorded as given below.

Sample no.	$\bar{x}$	R
1	50.04	0.07
2	50.24	0.08
3	50.14	0.03
4	50.08	0.05
5	50.28	0.04
6	50.16	0.09
7	50.30	0.04
8	50.10	0.04
9	50.16	0.05
10	50.10	0.07

$$UCL_{\bar{x}} = \bar{x} + 3\sigma_{\bar{x}}$$

$$3\sigma = \frac{3\sigma}{\sqrt{n}}$$

$$= \frac{3 \times 0.024}{\sqrt{5}}$$

50.19

50.16

50.128

$$= 50.16 + 0.032$$

$$= 50.192$$

$$LCL_{\bar{x}} = 50.128$$

0.18

0.056

0

$$UCL_{\bar{R}} = D_4 \bar{R}$$

$$= 2.11 \times 0.056$$

$$= 0.118$$

$$LCL_{\bar{R}} = D_3 \bar{R}$$

$$0 \times 0.056$$

$$= 0$$

$\Rightarrow$  1<sup>st</sup> determine the control limit on  $\bar{x}$  &  $R$  chart

$\Rightarrow$  plot  $\bar{x}$  &  $R$  chart

$\Rightarrow$  determine process capability

control chart for  $\bar{X}$  & R are maintained on a

certain quality char. of a manufacturer product.

Subgroup size is 7 & the values of  $\bar{X}$  & R are computed for each subgroup. After 35 subgroups,  $\sum \bar{X} = 7805$  &

$\sum R = 12,000$ . Compute the control limit for the

above chart. The Specification quality characteristics is

$220 \pm 35$ , what is the process capability, what %

of the component likely to be rejected if so is there any way the situation can be improved.

process capability

$$X_{\max} - X_{\min}$$

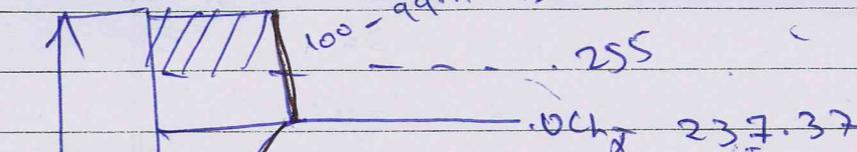
$$= 255 - 185$$

$$= 70$$

$$6^{-1} = 6 \times 12.679$$

$$= 76.074$$

$$= 0.59$$



$$\bar{X} = \frac{\sum \bar{X}}{N} = \frac{7805}{35} = 223$$

$$\bar{R} = \frac{\sum R}{N} = \frac{12000}{35} = 34.28$$

$$\sigma' = \frac{\bar{R}}{d_2} = \frac{34.28}{2.704} = 12.679$$

$$UCL = \bar{X} + 3\sigma$$

$$3\sigma = 3 \times 12.679$$

$$= 223 + 37.77$$

$$= 237.37$$

$$LCL = 208.63$$

$$Z_1 = \frac{185 - 223}{12.679} = -2.997$$

$$= 0.0014 \approx 0.1\%$$

$$Z_2 = \frac{255 - 223}{12.679} = 2.52$$

$$= 0.9941 \approx 99.41\%$$

~~$$UCL_R = D_3 \bar{R}$$~~

$$= 1.92 \times 34.28$$

$$= 65.8126$$

$$\begin{aligned} &= 2.2 \rightarrow 2.1 \\ &= 0.9941 - 0.0014 \\ &= 0.9927 \\ &= \underline{\underline{99.27\%}} \end{aligned}$$

$$LCL = D_3 \bar{R} = 0.08 \times 34.28$$

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$$= 2.74$$

$$\begin{aligned} &0.59 + 0.1\% \\ &= 0.73 \end{aligned}$$

## $C_p$ & $C_{pk}$

A machine is producing product to a specification of  $12.58 \pm 0.05$ . A study of 10 subgroups of size 5 each shows following results.  $\bar{x} = 12.598$ ,  $\bar{R} = 0.055$  process exhibits statistical control. (u)

- determine  $C_p$  &  $C_{pk}$  & comment on the process
- compute % non conformative if any
- suggest possible ways to improve the process.

$$C_p = \frac{USL - LSL}{6\sigma}$$

$$C_{pk} = \min \left( \frac{\bar{x} - LSL}{3\sigma}, \frac{\bar{x} - USL}{3\sigma} \right)$$

$$\sigma = \frac{\bar{R}}{d_2} = \frac{0.055}{2.326} = 0.023$$

$$USL = 12.63$$

$$LSL = 12.53$$

$$= \frac{12.63 - 12.53}{6 \times 0.023} = 0.7062$$

$$= \underline{0.62}$$

$$= \min \left( \frac{12.598 - 12.53}{3 \times 0.0236}, \frac{12.598 - 12.63}{3 \times 0.0236} \right)$$

$$\min \left( \frac{0.068}{0.0708}, \frac{-0.032}{0.0708} \right)$$

$$(0.962, -0.45)$$

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$$Z_1 = \frac{\bar{x}_1 - \bar{x}}{\sigma}$$

$$= \frac{12.53 - 12.598}{0.023} = -2.88$$

$$= 0.0020$$

$$= 0.2\%$$

$$Z_2 = \frac{\bar{x}_2 - \bar{x}}{\sigma} = \frac{12.63 - 12.598}{0.0236}$$

$$= +0.38 \quad \approx 0.9115$$

$$= 91.15\%$$

$$100 - 91.15$$

$$= 8.85\%$$

$$\text{Total rejection} = 8.85\% + 0.2\% \\ = \underline{9.05\%}$$

## RELIABILITY :-

Simp:

A reliability of a device is a probability of a device performing its purpose adequately for the period intended under the given operating condition.

$$R(t) = e^{-ct}$$

where  $R(t)$  is the reliability of the components

$t$  = intended duration

$C$  = failure rate given by  $C = 1/MTTF$

## MTTF :-

Mean Time To Failure

Mean time to failure is the avg life span of the component.

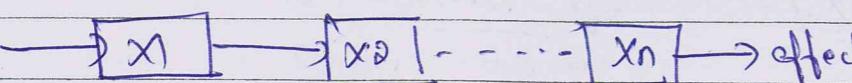
However, MTTF is not equal to actual life span. Therefore MTTF can be less than actual life span or greater than actual life span.

MTBF

mean Time Between failures

In cases of repairable item, if the items can be repaired & put back to service repeatedly, the avg time b/w failure is called as MTBF.

Series System :



cause

Let  $X_1, X_2 \dots X_n$  be the corresponding components assembled in series with their respective failure rates  $\alpha$   $c_1, c_2 \dots c_n$ .

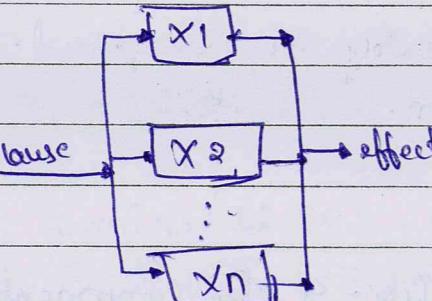
$$R(S) = R(X_1) \cdot R(X_2) \cdots R(X_n)$$

$$= e^{-c_1 t} \cdot e^{-c_2 t} \cdots e^{-c_n t}$$

Let  $t_1, t_2 \dots t_n$  be the respective expected working life of respective component.

$$= e^{-c_1 t_1} \cdot e^{-c_2 t_2} \cdots e^{-c_n t_n}.$$

Parallel system :



Let  $X_1, X_2 \dots X_n$  be the components connected in parallel.

Let the probability of each of the units  $P(X_1), P(X_2) \dots P(X_n)$  respectively. Also let  $R(X_1), R(X_2) \dots R(X_n)$  be the respective reliability & their corresponding failure rates be  $c_1, c_2 \dots c_n$ .

$$P(S) = P(X_1) \cdot P(X_2) \cdots P(X_n)$$

Reliability of the system  $R(S) = 1 - P(S)$

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$$= 1 - [P(X_1) \cdot P(X_2) \cdots P(X_n)]$$

$$= 1 - [(1 - R(X_1)) \cdot (1 - R(X_2)) \cdots (1 - R(X_n))]$$

$$R(S) = 1 - [(1 - e^{-c_1 t}) \cdot (1 - e^{-c_2 t}) \cdots (1 - e^{-c_n t})]$$

Problem

- D) A machine has MTTF of 10,000 hrs. Find the reliability of the system for operating life of  
 a) 100 hrs  
 b) 1000 hrs  
 c) 10000 hrs

Soln :

$$R(100) = e^{-(0.0001 \cdot 100)}$$

$$= 0.99$$

$$c = 1/10000 = 0.0001$$

$$R(1000) = e^{-(0.0001 \cdot 1000)}$$

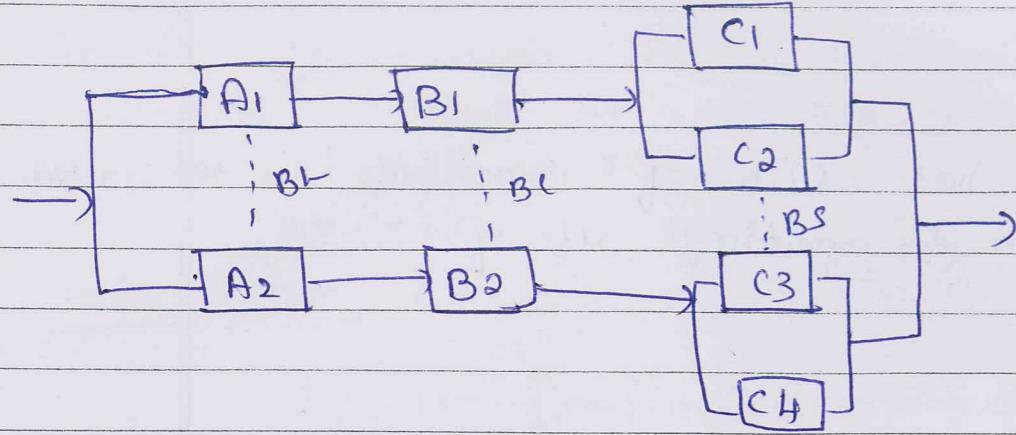
$$= 0.90$$

$$R(10000) = e^{-(0.0001 \cdot 10000)}$$

$$= 0.3678$$

2) There are two sets of brake in bicycle one for front wheel & one for rear. It may be assumed that 1 break is sufficient for safe operation. Each break set consisting a hand operated lever, cable & two break shoes. 1 break shoe is enough for stopping the bicycle. Hence two breaks are in parallel in the each branch. break shoes in parallel & in series the combination with break lever & cable. If MTBF of components are for break lever (A) = 500 hrs, break cable (B) = 100 hrs, break shoes (C) = 20 hrs.

Calculate the reliability of overall system for a 2 hrs down hill run.



$$\begin{aligned}
 R(S) &= 1 - [(1 - 0.967) \cdot (1 - 0.967)] \\
 &= [1 - (0.033) \cdot 0.033] \\
 &= 1 - 0.001089 \\
 &= \underline{\underline{0.9989}}
 \end{aligned}$$

$t = 2\text{hrs}$

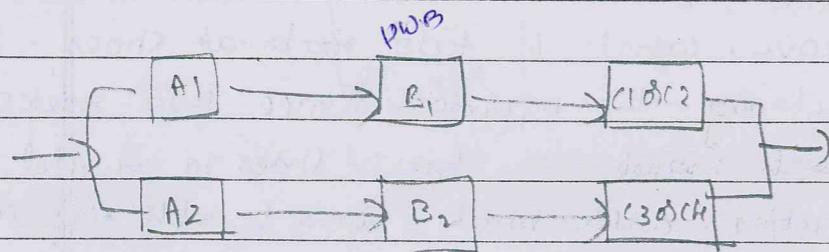
$$R(A) = e^{-ct}$$

$$= e^{-(0.0002 \times 2)} e^{-2/500} = 0.99$$

$$\begin{aligned}
 R(B) &= e^{-ct} = e^{-2t} = e^{-2/100} \\
 &= \underline{\underline{0.98}}
 \end{aligned}$$

$$R(c) = e^{-ct} = e^{-2/20} = \underline{\underline{0.904}}$$

$$\begin{aligned}
 R(C_1 \text{ or } C_2) &= 1 - [(1 - e^{-ct}) \dots (1 - e^{-ct})] \\
 &= 1 - [(1 - 0.904) \cdot (1 - 0.904)] = 0.990,
 \end{aligned}$$



$$R(pwB) = R(A) \cdot R(B) \cdot R(R1 \text{ or } R2) \approx 0.967$$

Consider the braking system of an automobile as shown in below. There are two partly dependent systems. One is the pedal operated (A) & the other is a mechanical system (B). both activate same set of break shoes for safe functions. Activate either both the front wheels breaks (C) or both the rear wheels (D). For safe functions both front & rear will breaks must function otherwise uncontrolling. Calculating the reliability of overall system. If individual reliability are given as follows.

$$R(A_1) = 0.990$$

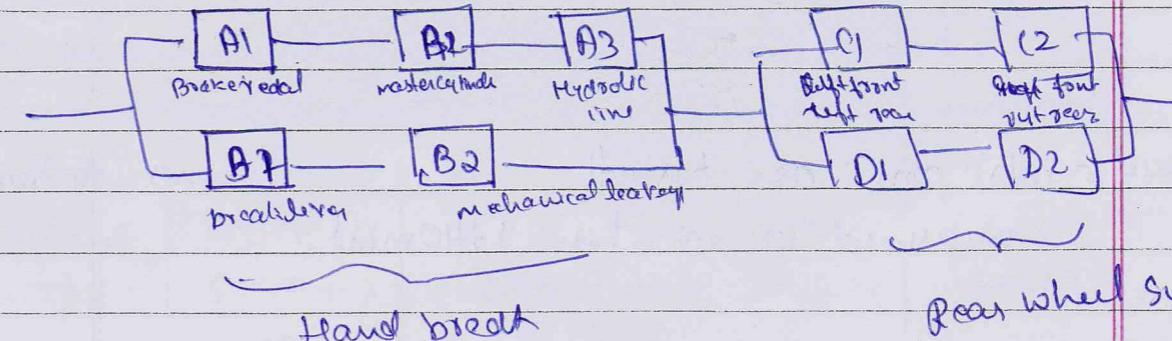
$$R(A_2) = 0.970$$

$$R(A_3) = 0.968$$

$$R(B_1) = 0.94$$

$$R(B_2) = 0.975$$

$$R(C_1) = R(C_2) = R(D_1) = R(D_2) = 0.75$$



$$R(A_1) = e^{-ct}$$

$$\begin{aligned} R(A) &= R(A_1) \cdot R(A_2) \cdot R(A_3) \\ &= 0.9295 \quad = 92.95\% \end{aligned}$$

$$\begin{aligned} R(B) &= R(B_1) \cdot R(B_2) \\ &= 0.9165 \quad = 91.65\% \end{aligned}$$

$$\begin{aligned} R(C_1 \text{ or } C_2) &= R(C_1) \cdot R(C_2) \\ &= 0.75 \times 0.75 \\ &= 0.5625 \quad = 56.25\% \end{aligned}$$

$$R(D_1 \text{ or } D_2) = 0.5625 \quad = 56.25\%$$

$$\begin{aligned} R(A, B) &= 1 - [1 - R(A)] \cdot [1 - R(B)] \\ &= 1 - [1 - 0.9295] \cdot [1 - 0.9165] \\ &= 0.9941 \end{aligned}$$

$$\begin{aligned} R(C, D) &= 1 - [1 - R(C_1 \text{ or } C_2)] \cdot [1 - R(D_1 \text{ or } D_2)] \\ &= 1 - [1 - 0.5625] \cdot [1 - 0.5625] \\ &= 1 - (0.4375 \cdot 0.4375) \\ &= 0.808 \end{aligned}$$

10 ball points pens are tested times to runny measured in minutes 1740 min.

2000 min

1421 min

1857 min

1246 min

16.83 min  
Date / 120  
18.90

16.78 min

Compute the mean life & rate of failure. Under the reliability that a brand new refil will last 20 hrs of writing.

Soln:

mean life : 1720.5

$$= 1/1720.5 = 0.00005812$$

#### UNIT - 4

8/3/19: A plaster cabinet supply company is a fully home subsidiary of an international firm that has major interest in housing industry. The plant located in Boston, Seattle, Miami. A plants produced 3 fabricated housing components that are delivered to other company assembly plants in Chicago, Denmark & Nashville. Demand has grown to the point where company may justified construction of another plant. Immatical problem is determining the location that will minimize production & transportation cost to the existing assembly plants. In order to go over to ~~new~~ material & to service to other potential market, the alternative plant location have been narrowed down to Omaha & Phoenix. Cost, demand & production data on the various alternative are as shown in table (a) & (b) below.

Production Data			Assembly Demand	
Plant	units/month	cost/unit	Plant	units/month
Boston	2000	\$7	Chicago	6000
Seattle	6000	\$7.08	Denmark	5000
Miami	5000	\$6.90	Nashville	6000
Omaha	4000	\$6.90		
Phoenix	4000	\$6.20		

Transportation cost/unit (\$)

13

To	From				
	Boston	Seattle	Miami	Omaha	Phoenix
Chicago	\$5	\$7	\$5	\$4	\$6
Den	\$6	\$4	\$7	\$3	\$4.5
Nash	\$5.50	\$7	\$3	\$5	\$5

	Boston	Sea	name	phonet	almond			
Chi	2000	5	1000	7	5	6	6000	0 0 1 1
Den		6	5000	4	7	45	5000	25
Nas	150	1000	7	5000	3	1000	6000	20 0.5 2
Supply	2000	6000	5000	1000 3000	7000	1700		
	0.5	3	2		0.5			
	0.5	0	2		1			
	0.5	0	0		1			
	0	0	0		1			

## Solving (Penalty method)

	Boston	Sea	Miami	Omaha	Demand	Penalty
Supply	2000	5	7	5	4000	2000
Chi	2000	5	7	5	4000	2000
Den	6	4	7	3	5000	+
Nash	5-50	1000	7	3	5	1000
	2000	6000	5000	4000	= 17000	2 205
penalty	0.5	3	2	1		

production = 24800  $\in$  1000 x 6-20  
(Avant)

$$= 96000 + 24900$$

$$= \underline{\underline{1,00,800}}$$

~~set~~ light briefly from row 10

Select lowest of demand & supply

0.5	0	2	1
0.5	0	0	1
0.5	0	0	0

Total transportation cost considering Omaha

$$\begin{aligned}
 & 2000 \times 5 + 4000 \times 4 + 5000 \times 4 + 4000 \times 7 + 5000 \times 3 \\
 = & 10000 + 16000 + 20000 + 7000 + 15000 \\
 = & \$68000
 \end{aligned}$$

$$\text{production cost} = 4000 \times 6.90 \\ = 27600$$

$$\text{Total production} = \$68000 + 27600$$

transportation is = 95600

Imp      Error

Type I : There no problem with process but we have concluded there is a error in process.

typed : Process is correct but in practice it is  
not correct

## 6 sigma

13/3/19

unit - 4

## ENTERPRENEURSHIP

### Key elements

- Innovation
- Risk taking
- Vision
- Organizing skill
- Operational excellence

### Entrepreneurial process

### Concept of Entrepreneurship

Entrepreneur - Person

enterprise - Object

entrepreneurship - process of action