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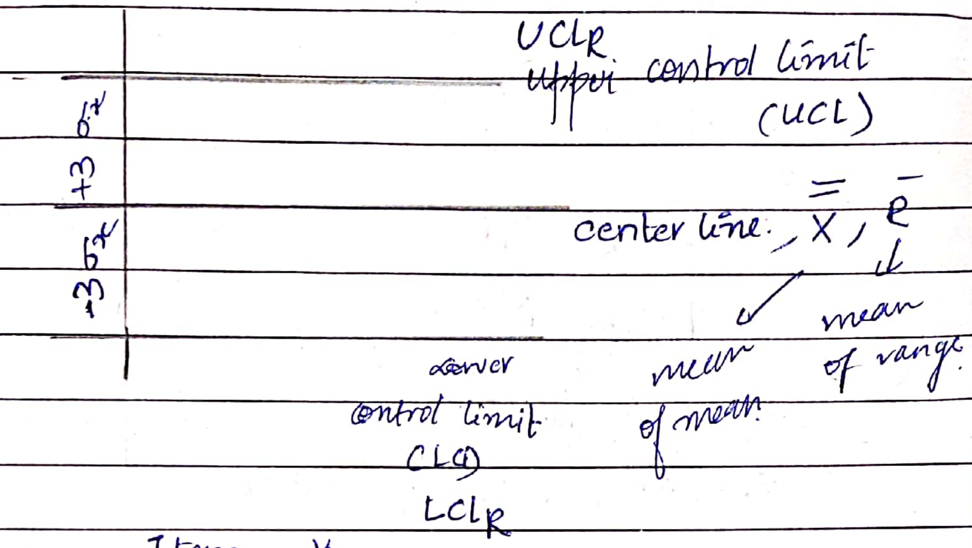
SPC / SQC charts. — eg - $\bar{X}$, R, σ

control charts. — Variables / Attributes

Assignable causes of Variations - "out of control"

chance cause of variations. - "In control"



Hour		Items	$\bar{X}$
1	—	10	$\bar{X}_1$
2	—	10	$\bar{X}_2$
⋮			
8	. . . .	.	$\bar{X}_8$

sample size $n = 10$

No. of samples $N = 8$

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_8}{(N = 8)}$$

$$UCL_{\bar{x}} = \bar{\bar{X}} + 3\sigma_{\bar{x}}$$

$$LCL_{\bar{x}} = \bar{\bar{X}} - 3\sigma_{\bar{x}}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + \frac{3\sigma}{\sqrt{n}}$$

$$d_2 = \frac{\bar{R}}{\sigma}$$

$$= \bar{\bar{X}} + \frac{3\bar{R}}{d_2\sqrt{n}}$$

$$\sigma = \bar{R}/d_2$$

$$\left(A_2 = \frac{3}{d_2\sqrt{n}}\right)$$

$$UCL_{\bar{x}} = \bar{\bar{X}} + A_2 \bar{R}$$

$$UCL_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{R}$$

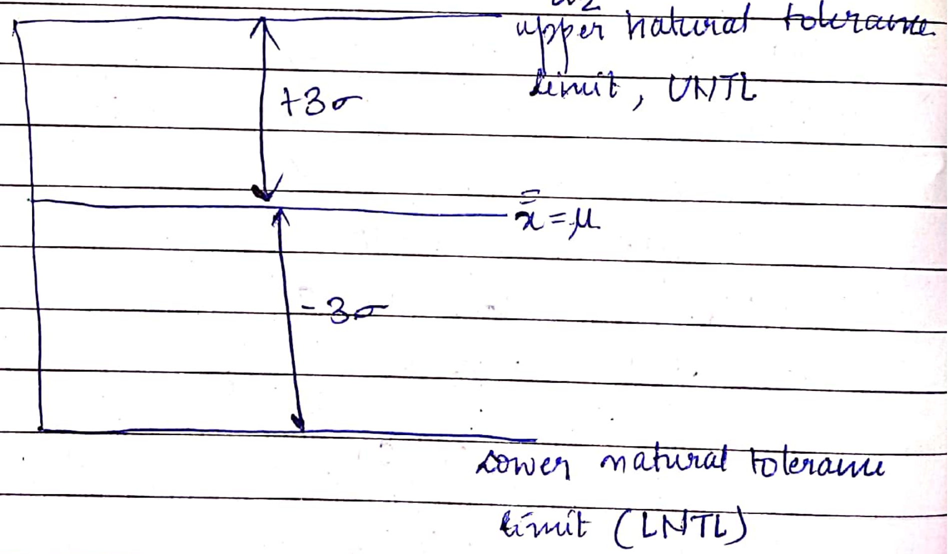
<u>R chart</u>

$$UCL_R = D_4 \bar{R}$$

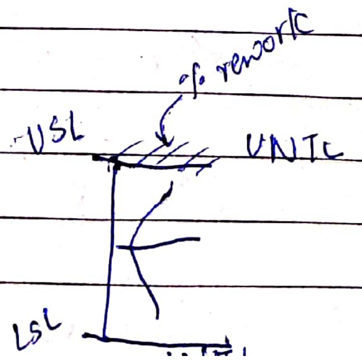
$$LCL_R = D_3 \bar{R}$$

if any point is above or below UCL or LCL, then it is out of control.

Process capability $= 6\sigma = \frac{6\bar{R}}{d_2}$



if $6\sigma < USL - LSL$, process is capable of meeting specification upper control limit $UCL_{\bar{x}}$ UCL_R



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% scrap + % rework + % rejects = % non-confirming.

Better to have rework than scrap.

~~For Robert~~

Type I error -

if you take control points ~~ins~~ closer to center line, then more points falls outside, and ~~contro~~ processes are rejected even if they are in control → false alarm.

Type II →

far from center line — saying it's ~~out of~~ in control but actually out of control.

Q) Capability study of a lathe used in turning a shaft to a diameter of 23.75 ± 0.1mm, A sample of 6 consecutive pieces of were taken each day for 8 day construct $\bar{x}$ & R chart. The diameter of the shaft inspected.

	I day	II day	III day	IV day	V day	VI day	VII day	VIII day
	23.77	23.8	23.77	23.79	23.75	23.78	23.76	23.76
	23.8	23.78	23.78	23.76	23.78	23.76	23.78	23.79
	23.78	23.76	23.77	23.79	23.78	23.73	23.75	23.77
	23.73	23.70	23.77	23.74	23.77	23.76	23.76	23.72
	23.76	23.81	23.80	23.82	23.76	23.74	23.81	23.78
	23.75	23.77	23.74	23.76	23.79	23.78	23.8	23.78
$\bar{X}$	23.765	23.77	23.776	23.7767	23.7717	23.75	23.7767	23.7667
R	0.07	0.11	0.06	0.08	0.04	0.05	0.06	0.07

plot

$\bar{\bar{X}} =$

$\bar{R} =$

✓ difference b/w largest & smallest value.

Sample size $n = 6$

no. of samples = 8.

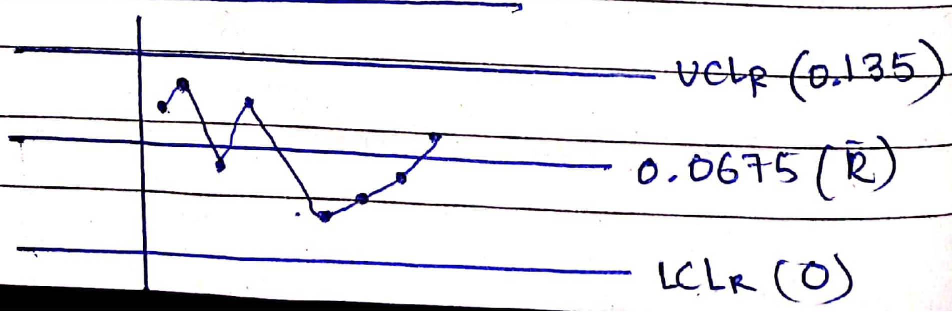
control limits for R

$UCL_R = D_4 \bar{R} = 2.00 \times 0.0675 = 0.135$

$LCL_R = D_3 \bar{R} = 0 \times 0.0675$

check in table, $n = 6$

control chart for R.



plot R values

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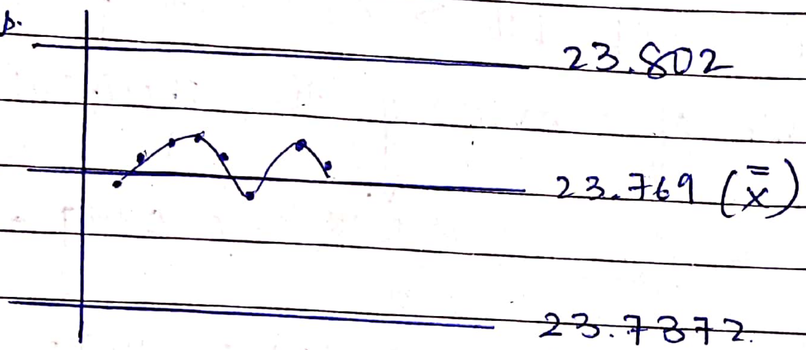
$UCL_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R} = 23.7696 + 0.48 \times 0.0675$

$= \underline{\underline{23.802}}$

$LCL_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R} = 23.7696 - 0.48 \times 0.0675$

$= \underline{\underline{23.7372}}$

plot $\bar{X}$ values.



$\bar{\bar{X}} = \frac{\sum \bar{X}}{N} = \frac{190.1568}{8} = 23.7698$

$\bar{R} = \frac{\sum R}{N} = \frac{0.54}{8} = \underline{0.0675}$

The process is in control. (since it is not above UCL & below LCL)

process capability $= 6\sigma$

$= \frac{6\bar{R}}{d_2} = \frac{6 \times 0.0675}{2.534}$

check in first page for $n=6$.

$= \underline{\underline{0.1598}}$

if the question is asked is process capable of meeting specification then,

upper specification limit (USL)

$= 23.75 + 0.1 = \underline{\underline{23.85 \text{ mm}}}$

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Lower specification limit (LSL):

$23.75 - 0.1 = \underline{23.65}$

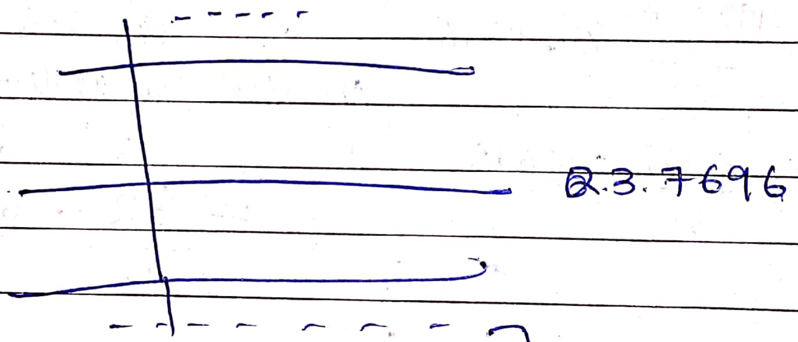
$USL - LSL = 0.2$ mm.

Since $USL - LSL > 6\sigma$, the process is capable of meeting the specification.

$3\sigma = \frac{6\sigma}{2} = \frac{0.1598}{2} = 0.0799 \rightarrow$

can add this to $23.7696 + 0.0799 = 23.8495$

if we plot it.



and if we subtract it we get it here.

$UNTL = 23.7696 + 0.0799$.

$LNTL = 23.7696 - 0.0799$.

some problems you need to ~~at~~ do this

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Q) The following data were obtained for 10 days to initiate $\bar{X}$ & R control chart for quality characteristic for manufactured ~~the~~ product that had required substantial amount of rework. The subgroup size was 5 & 2 subgroups were taken per day. i) Determine the trial control limits of $\bar{X}$ & R chart.

ii) State whether the process is in control.

iii) The specifications of the quality characteristic under study is given as 171 ± 11. If a product falls below the lower specification limit it must be scrap & if it falls above upper specification limit it ~~m~~ must be rework which is desired to hold the scrap to a minimum without causing excessive rework. What would suggest as the value of process mean ~~be~~ if the mean can be changed by a simple machine adjustment.

iv) What would you recommend as new limits for control chart.

$171 \pm 11 = \cancel{0}\, U$

sample no.	$\bar{X}$	R	sample no.	$\bar{X}$	R
1	177.6	$\boxed{23}$	11	179.8	9
2	176.6	8	12	176.4	8
3	178.4	$\boxed{22}$	13	178.4	7
4	176.6	12	14	178.2	4
5	177	7	15	180.6	6
6	179.4	8	16	179.6	6
7	178.6	15	17	177.8	10
8	179.6	6	18	178.4	9
9	178.8	7	19	181.6	7
10	178.2	12	20	177.6	10

$USL = 171 + 11 = 182$, $LSL = 171 - 11 = 160$.

subgroup

$USL - LSL = 182 - 160 = 22$, $n = 5$ $N = 20$

$$\bar{\bar{X}} = \frac{\sum \bar{x}}{N} = \frac{177.6 + 176.6 + \ldots \ldots + 181.6 + 177.6}{20}$$

$$= 17$$

$$\bar{R} = \frac{\sum R}{N} = \frac{23 + 8 + 22 + 12 + \ldots \, 7 + 10}{20}$$

$$= 9.8$$

see for $n=5$

$$UCL_R = D_4 \bar{R} = 2.11 \times 9.8 = 20.678$$

$$LCL_R = D_3 \cdot \bar{R} = 0$$

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23 & 22 is more than 20.678, so the process is not in control.,
so omit those 2 values ~~from~~ & divide by 18.

$$\bar{R} = \frac{(23+8+22+\dots+7+10)-(23+22)}{18}$$

$= 8.388$

$UCL_R = D_4\bar{R} = 2.11 \times 8.388 = 17.698.$

$LCL_R = D_3 . \bar{R} = 0$

no point is above 17.698, so it is in control.

$\bar{\bar{X}}$

leaving sample no. 1 & 3. find $\bar{\bar{X}}$

$$\bar{\bar{X}} = \frac{176.6 + 176.6 + \dots + 177.6}{18}$$

$= 178.51$

$UCL_X = \bar{\bar{X}} + A_2\bar{R}$

$= 178.51 + 0.58 \times 8.388$

$= 183.~~06504~~37$

$LCL_X = \bar{\bar{X}} - A_2\bar{R} = 178.51 - 0.58 \times 8.388$

$= 173.64$

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The process is in control, (becoz all points are in limit)

ⓒ $6\sigma = \dfrac{6\bar{R}}{d_2} = \dfrac{6 \times 8.388}{2.326} = \underline{\underline{21.63}}$

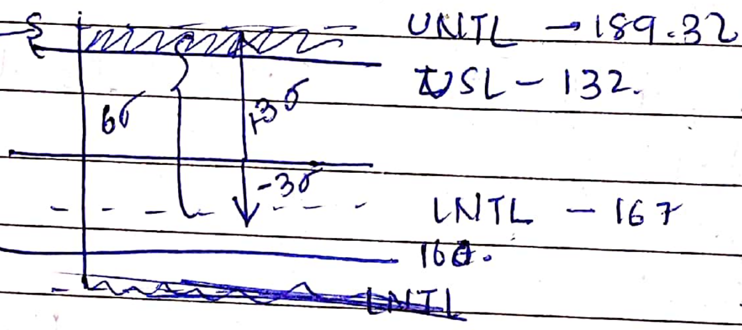
since, $6\sigma <$ USL−LSL. the process is capable of meeting specifications.

$3\sigma = 10.815$

$UNTL = 178.51 + 10.815 = 189.32$

$LNTL = 178.51 - 10.815 = 167.695$

this should be reduced to USL, so the full line will come down, so the mean value also comes down.



when UNTL = USL, then the % rework = 0.

In such a case $\bar{\bar{X}}$ will can be changed to $= 182 - 3\sigma = 182 - 10.815$

$= \underline{\underline{171.18}}$

If we further subtract it with 3σ again, it will be 160 something

161 > 160 ← so No scrap, no rework.

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The new $\bar{X}$ control limits

$UCL_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R} = 171.18 + 0.58 \times 8.388 = 176.045$

$LCL_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R} = 171.18 - 0.58 \times 8.388 = 166.31$

Q) A machine producing a product with specification of 12.58 ± 0.05mm. A study of 10 subgroups of size 5 each was carried out and the following results were obtained $\bar{\bar{x}} = 12.598$ & $\bar{R} = 0.55$

i) Determine Cp & Cpk ii) compute % non confirming if any. iii) Suggest the possible ways to improve the process.

12.58 ± 0.05mm

$N = 10$

$n = 5$

$\bar{\bar{X}} = 12.598$

$\bar{R} = 0.055$

$USL = 12.58 + 0.05 = 12.63mm$

$LSL = 12.58 - 0.05 = 12.53mm$

$\sigma = \frac{\bar{R}}{d_2} = \frac{0.055}{2.326}$

$= 0.0236$

for $n = 5$

$d_2 = 2.326$

i) Cp & Cpk

$$Cpk = \min \left[\frac{\bar{\bar{X}} - LSL}{3\sigma}, \frac{USL - \bar{\bar{X}}}{3\sigma} \right]$$ → process capability index

$= \min [0.96, 0.45]$

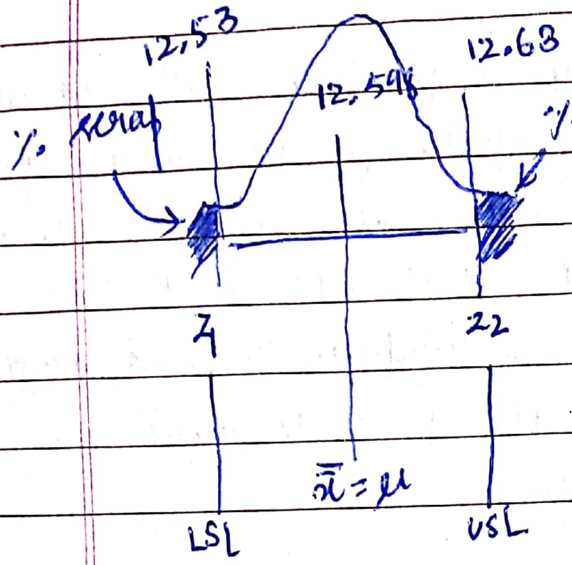
$= 0.45$ (Mean $\bar{\bar{X}}$ is closed to USL)

∴ Higher rework is expected.

Cp process capability ratio :

$$= \frac{USL - LSL}{6\sigma} = \frac{12.63 - 12.53}{6 \times 0.0236} = \underline{0.706}$$

ii) % rejects (non-conforming) = % rework + % scrap



% rework =

$$Z_1 = \frac{X_i - \mu}{\sigma} = \frac{12.53 - 12.598}{0.0236}$$

$$= -2.88$$

Area under normal curve $= A_1 = 0.002$ (0.2%)

% rework :

$$Z_2 = \frac{X_i - \mu}{\sigma} = \frac{12.63 - 12.598}{0.0236} = 1.35$$

Area under normal curve = 0.9115

$A_2 = 1 -$ Area under normal curve

$= 1 - 0.9115 = 0.0885 = \underline{8.85\%}$

% non conforming = % scrap + % rework

$= 0.2 + 8.85\%$

$= 9.05\%$

specification limits > Natural tolerance limits

control limits maybe < "

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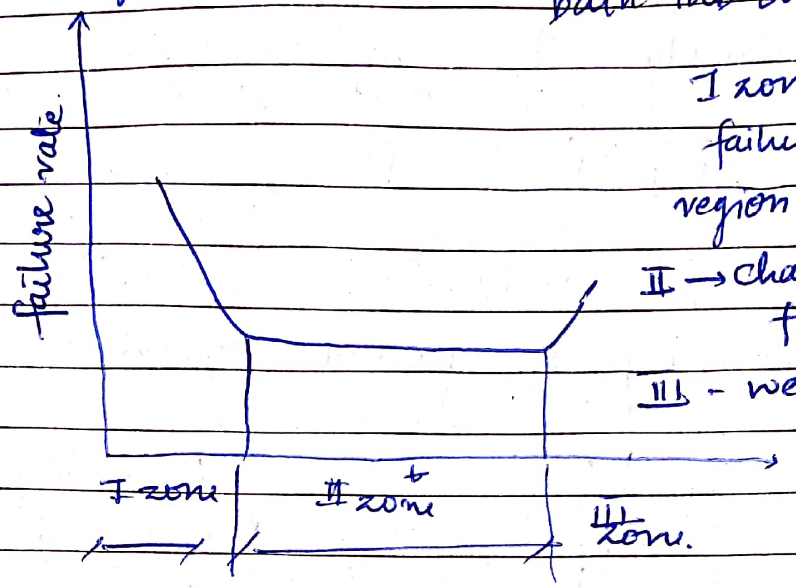
Reliability

bath-tub curve



I zone - Early failures debugging region

II → chance failures

III - wear out

$R(t) = e^{-ct}$

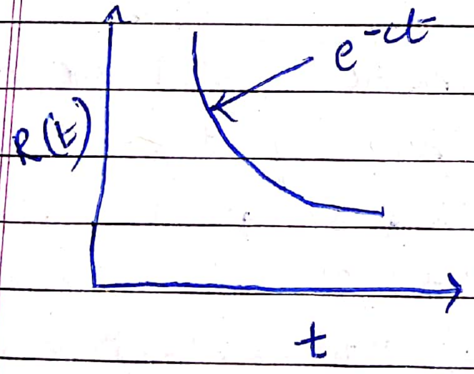
reliability of a component.

c, λ, z = failure rate

Mean time to failure (MTTF) / Mean time between failures (MTBF)

failure rate $c = \dfrac{1}{MTTF} = \dfrac{1}{MTBF}$

$R(t) = e^{-ct}$

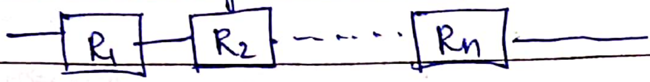


t → operating time.

Reliability of systems:

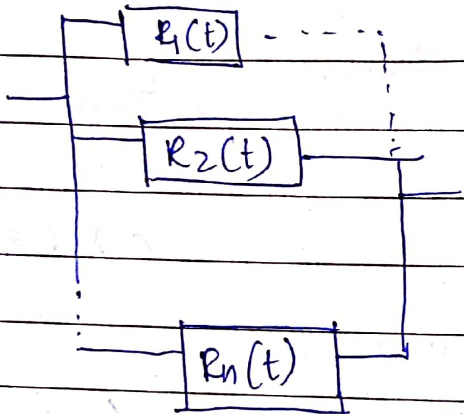
1) Reliability of series systems.

If anyone component fails, everything fails.



$$R(S) = R_1(t) \cdot R_2(t) \ldots\ldots R_n(t)$$

2) Reliability of parallel systems.



All subsystem should fail for the system to fail.

Probability of failure of subsystem.

Probability of failure $\checkmark$ $P(S) = P_1(t) \cdot P_2(t) \ldots\ldots P_K(t)$.

$t \rightarrow$ operating time.

$$1 - R(S) = [\{1 - R_1(t)\}\{1 - R_2(t)\} \ldots (1 - R_n(t))]$$

$$\text{hence } R(S) = 1 - [\{1 - R_1(t)\}\{1 - R_2(t)\} \ldots \{1 - R_n(t)\}]$$

Q) 10 ball point pens were tested for life. The times of their running dry measured in minutes were. 1246, 1683, 1890, 1676, 1909, 1783, 1740, 2000, 1421, 1857. Compute mean life and rate of failure. What is the reliability that a brand new refil will last 20 hrs of writing

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A: MTTF (Mean life) $= \dfrac{1246 + 1683 + 1890 + 1676 + 1909 + 1783 + 1740 + 2000 + 1421 + 1857}{10}$

$= 1720.5$ min

failure rate $= c = \dfrac{1}{MTTF} = 5.812 \times 10^{-4}$ failure/min

$R(t) = e^{-ct} = e^{-\frac{1}{MTTF} \times t} = e^{\left(-\frac{1}{1720.5} \times 20 \times 60\right)}$

$= \underline{\underline{0.4978}}$

Q) A machine has MTTF of ~~of~~ 10000 hrs find the reliability of the system for operating life of i) 100 hours ii) 1000 hrs. iii) 10,000 hrs

Reliability of machine $R(t) = e^{-ct}$

$= e^{-\left(\frac{1}{MTTF}\right)t}$

$R(100) = e^{-\left(\frac{1}{1000}\right)100}$

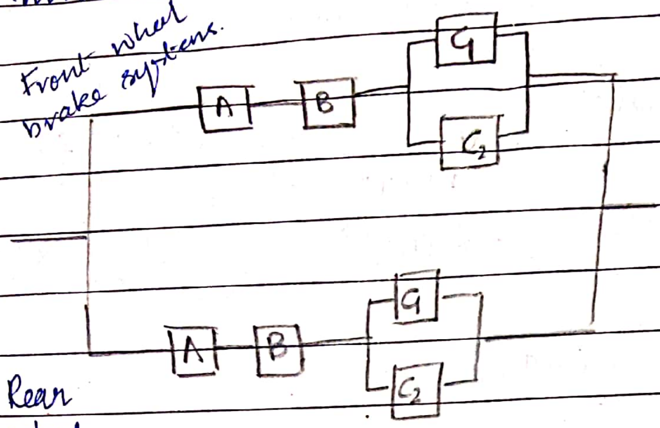
2) $R(1000) = e^{-\frac{1}{10,000} \times 1000} = \underline{0.9048}$

3) $R(10000) = e^{-1/10000\,(10000)} = \underline{0.3678}$

Q) There are 2 sets of brakes in a bicycle, one for the front wheel and one for the gear. It may be assumed that 1 breake is sufficient for safe operation. Each breake set consists of a handoperated lever, a cable and 2 brakeshoes one brakeshoe is enough for stopping the bicycle, if MTBF of various components are brake lever (n) = 5200 hrs

Task-II : Reliability & Control Charts.

Brake cables (B) = 100hrs, Brake shoes (C) = 20hr.
calculate reliability of overall system, for a 2 hour downhill run.



The reliability of brake shoes (C_1 or C_2) =

$$1-\left[(1-R_1(C_1))(1-R(C_2))\right]$$

$$= 1-\left[(1-e^{-C_1t})(1-e^{-C_2t})\right]$$

$$= 1-\left[(1-e^{-\frac{1}{20}\times 2})(1-e^{-\frac{1}{20}\times 2})\right]$$

$$= \underline{\underline{0.9909}}$$

The reliability of front/rear wheel brake system $\Big\}$ = $R(A).R(B).R(C_1 \text{ or } C_2)$

$$= \left(e^{-\frac{1}{500}\times 2}\right)\left(e^{-\frac{1}{100}\times 2}\right)\times 0.9909$$

$$= \underline{\underline{0.9679}}$$

Reliability of system = $1-\left[(1-0.\cancel{9909}\,9674)(1-0.9674)\right]$

$$= \underline{\underline{0.9989}}$$

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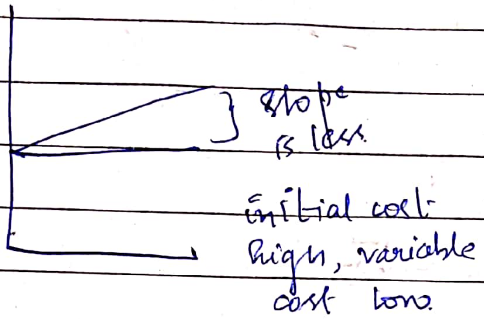
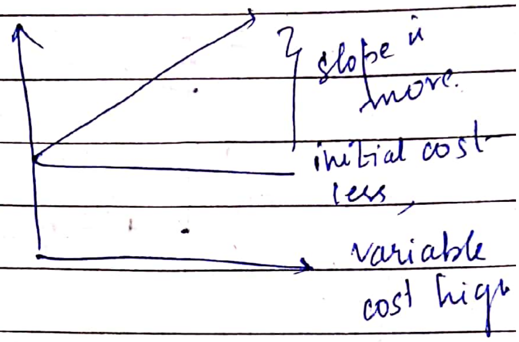
Break even Analysis.

$TR - TC = Profit$

$P.Q - [FC + VC \times Q] = Profit.$

$TC = FC + VC.Q$





Q) Businessman is thinking of opening a factory in one of the 3 places A, D & N. He has gathered the data on fixed cost & variable cost, mentioned on the table, ~~over~~ what range of annual volume does each location have competitive advantage.

	fixed cost/year	Material	Labour.	Overhead.
A	200,000	0.2	0.4	0.4
D	180,000	0.25	0.75	0.75
N	170,000	1	1	1

$TC = FC + VC \times Q$

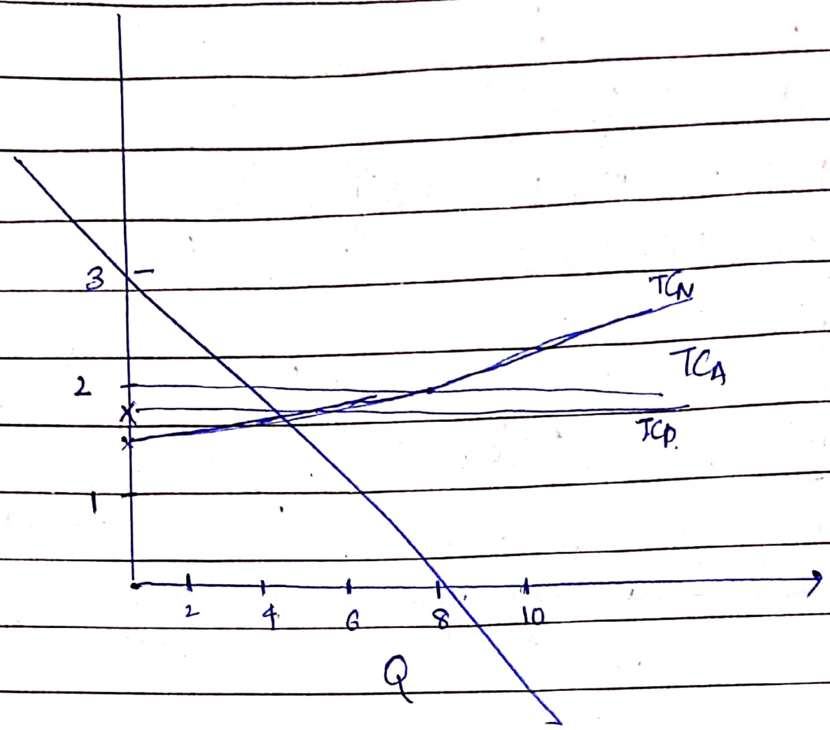
$TC_A = 200,000 + (0.2 + 0.4 + 0.4)Q.$

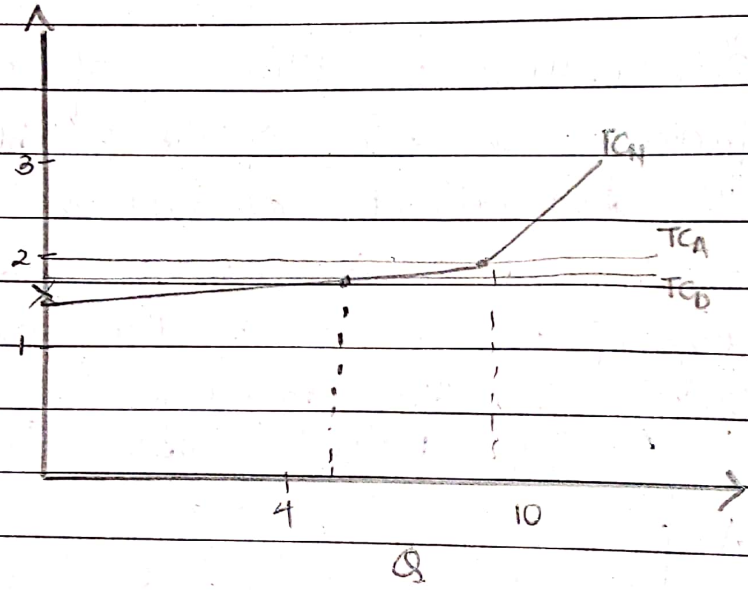
$TC_A = 200,000 + 1Q$ —— (1).

$TC_B = 180,000 + (0.25 + 0.75 + 0.75)Q$

$= 180,000 + 1.75Q$ —— (2)

$TC_N = 170,000 + 3Q \longrightarrow$ (3)





Substitute 10,000 in each of the equation and find TC for each.

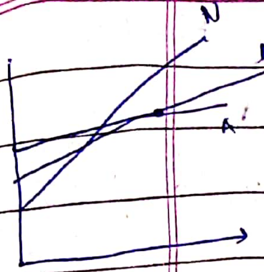
If extend TC_D and TC_A, they will meet.

$$TC_A = TC_D$$

$$200\,000 + Q = 180{,}000 + 1.75Q$$

$$20{,}000 = 0.75Q$$

$$\boxed{Q = 26667 \text{ units}}$$



$TC_D = TC_N$

$180{,}000 + 1.75Q = 170{,}000 + 3Q$

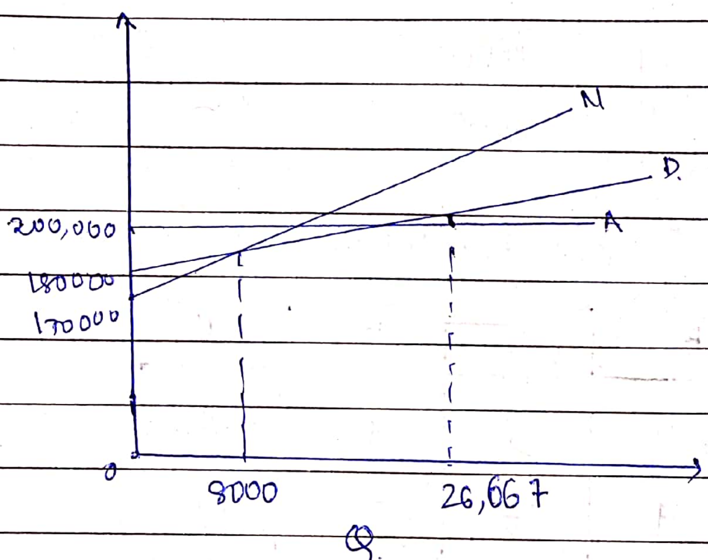
$10000 = 1.25Q$

$Q = 8000$ units.

If the production volume is 0 – 8000 units then Location N is preferred.

If the production volume is 8000 – 26,667 units then location D is preferred.

If production volume is > 26667 units then Location A is preferred.



NOTE: If they ask which is best for 10000 units, don't draw graph, Just substitute Q in ①, ② & ③ and find the least TC, and that is chosen.

Q) A company has clients at A, B, C & supplies goods to 3 market places A1, B1 & C1, because of the growing demand the company wants to establish another plant at D or E, from the cost demand & production data given below, determine where the plant should be located.

Plant	Units/month	Cost/unit (Rs)	~~Market~~
A	2000	7	13,000
B	6000	7.08	
C	5000	6.9	
D	4000	6.9	
E	4000	6.2	

Market	Units/month
A_1	6000
B_1	5000
C_1	6000
	17,000

Transportation cost.

	A	B	C	D	E
A_1	5	7	5	4	6
B_1	6	4	7	3	4.5
C_1	5.5	7	3	5	5

The demand is 17,000, but they produce only 13K, so we need new plant; D or E and find location depending on the cost.

Taking location D first:

~~Infinite~~ Initial feasible solution.

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	A_1	B_1	C_1	Supply	Penalty
A	2000 $\boxed{5}$	$\boxed{6}$	$\boxed{5.5}$	2000 0	0.5
B	$\boxed{7}$	5000 $\boxed{4}$	$\boxed{7}$	~~7000~~ 6000	~~3~~ 0
C	$\boxed{5}$	$\boxed{7}$	5000 $\boxed{3}$	5000 0	2
D	4000 $\boxed{4}$	$\boxed{3}$	$\boxed{5}$	4000 0	1
Demand	~~6000~~ ~~2000~~	~~5000~~ 0	~~6000~~ 1000	17000 17000.	
Penalty	~~1~~ 2	~~1~~	~~2~~ 0.5 (5.5 − 5)		

} difference b/w minimum.

row or column ~~as~~ which has highest penalty is 3.

next, highest penalty now is 2, ~~then go for~~ if there is tie go for least cost.

select highest penalty row or column, check the least cost cell and choose that

Transportation cost:

$2000\times5 + 5000\times4 + 1000\times7 + 5000\times3 + 4000\times4$

=

production cost

$4000 \times 6.9 =$

Total cost = Transportation cost + production cost

= ₹95600

	A_1	B_1	C_1	Supply	Penalty
A	2000 $\boxed{5}$	$\boxed{6}$	$\boxed{6.5}$	~~2000~~ 0	0.5
B	1000 $\boxed{7}$	5000 $\boxed{4}$	$\boxed{7}$	~~6000~~ ~~1000~~ 0	~~3~~ 0
C	$\boxed{5}$	$\boxed{7}$	5000 $\boxed{3}$	5000	2
E	3000 $\boxed{6}$	$\boxed{4.5}$	1000 $\boxed{5}$	~~4000~~ 3000	~~0.5~~ 1
	~~6000~~ ~~4000~~ 1000	~~5000~~ 0	~~6000~~ 1000	~~17000~~ 14000	~~4000~~ ~~1000~~
Penalty	0 0 1	0.5	~~2~~ 0.5 2		

bcoz 4.5 was removed so the value changes.

Transportation cost:

$2000 \times 5 + 1000 \times 7 + 5000 \times 4 + 5000 \times 3 + 3000 \times 6 = \cancel{75{,}000}$

$+ 1000 \times 5 = 80{,}000$

Production cost at E

$4000 \times 6.2 = 24{,}800$

Total cost = Transportation cost + production cost

= ₹ 1,04,800

Location D is preferred ∵ of least total cost