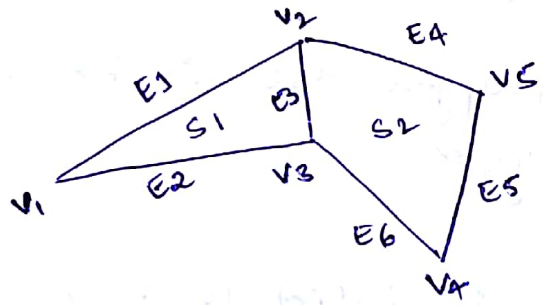
(1) Polylist = sort polygons based on highest zcordinate
clip polygon = first polygon (polylist);
(2) WA_subdivide (clippolygon, polylist);

unit III

# Three-Dimensional Object Representation

## Representation of polygon surfaces

- usually in computer graphics many objects are represented in terms of combination of the polygon surfaces.
- The reason for this is the polygon surfaces are easy to render on the screen since the polygon surface uses linear equation of the form $Ax + By + Cz + D = 0$ which is easy to process in terms of mathematical operations.




- To represent a polygon there are 3 geometric tables

1) vertex table
2) Edge table
3) polygon surface table.

vertex table : An vertex table for a polygon consists of the collection of vertices as index and within each index it holds the (x, y, z) coordinate values corresponding to a particular vertex.

| vertex table | | | |
|---|---|---|---|
| V1 | $x_1$ | $y_1$ | $z_1$ |
| V2 | $x_2$ | $y_2$ | $z_2$ |
| V3 | $x_3$ | $y_3$ | $z_3$ |
| V4 | $x_4$ | $y_4$ | $z_4$ |
| V5 | $x_5$ | $y_5$ | $z_5$ |

Edgetable : An edge table is basically as pointer to the vertex table which points to the vertices of that defines a particular edge.
Like vertex table edge table maintains information pertaining to each edge in a seperate index value.

| Edgetable | |
|---|---|
| E1 | V1 , V2 |
| E2 | V1 , V3 |
| E3 | V2, V3 |
| E4 | V2 , V5 |
| E5 | V5 , V4 |
| E6 | V3 , V4 |

polygon surface table: This geometric table will define the edges corresponding to a particular polygon surface. The table consists of n number of indices and each index in the table will hold the information pertaining to a particular surface in terms of the edges defining it.

- This table is basically a pointer pointing to the edge table

| Polygonsurfacetable | |
|---|---|
| S1 | E1 , E2 , E3. |
| S2 | E3 , E4 , E5 , E6 |

# Plane equation for the polygon

- A plane equation is represented as $Ax + By + Cz + D = 0$ ①
- Use three vertices in clockwise or anticlockwise direction. V1 V2 V3

$$V1 = Ax_1 + By_1 + Cz_1 + D = 0 \Rightarrow (A/D)x_1 + (B/D)y_1 + (C/D)z_1 = -1$$

$$V2 = Ax_2 + By_2 + Cz_2 + D = 0 \Rightarrow (A/D)x_2 + (B/D)y_2 + (C/D)z_2 = -1$$

$$V3 = Ax_3 + By_3 + Cz_3 + D = 0 \Rightarrow (A/D)x_3 + (B/D)y_3 + (C/D)z_3 = -1$$

According to Cramer's Rule

$$A = \begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix} \quad B = \begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{vmatrix} \quad C = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$D = -\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$A = y_1(z_2 - z_3) - y_2(z_1 - z_3) + y_3(z_1 - z_2) \qquad x \rightarrow y \rightarrow z$$

$$B = z_1(x_2 - x_3) - z_2(x_1 - x_3) + z_3(x_1 - x_2)$$

$$C = x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)$$

$$D = x_1(y_2z_3 - y_3z_2) - y_1(x_2z_3 - x_3z_2) + z_1(x_2y_3 - y_2x_3)$$

for any plane if $A = 1$, $B = 0$, $C = 0$ and $D = -1$ then the plane is infront of viewplane else it is backside.

For any point $(x, y, z)$ if $Ax + By + Cz + D < 0$, then point is inside the plane

For any point $(x, y, z)$ if $Ax + By + Cz + D = 0$, then point is on the edge of the plane

For any point $(x, y, z)$ if $Ax + By + Cz + D > 0$, then point is outside the plane.

## Spline Representation

- Spline came into existence in drafting technology
- Spline in drafting technology is a flexible strip that is used to produce a curve shape through a designated set of points.
- Spline in computer graphics basically represents a composite curve that is being generated by fitting a polynomial expression to each section of the curve that satisfies specific continuity conditions.


## Types of splines

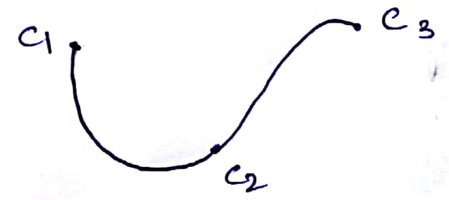
- Based on the way the spline intersects with the control points there are 2 types of splines.

1: Interpolation spline
2. Approximation spline

## Interpolation spline

In interpolation spline a curve will intersect all the designated control points

eg:

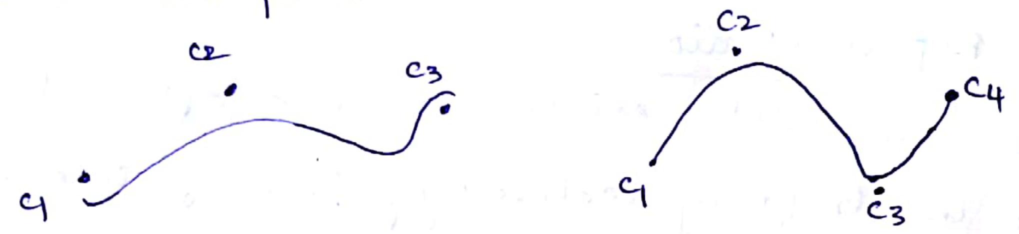


- Here a polynomial equation is fitted such a way that it ensures the intersection with each control point.

2. Approximation spline

In approximation spline a curve is generated such a way that it need not necessarily intersect each control point.

eg:

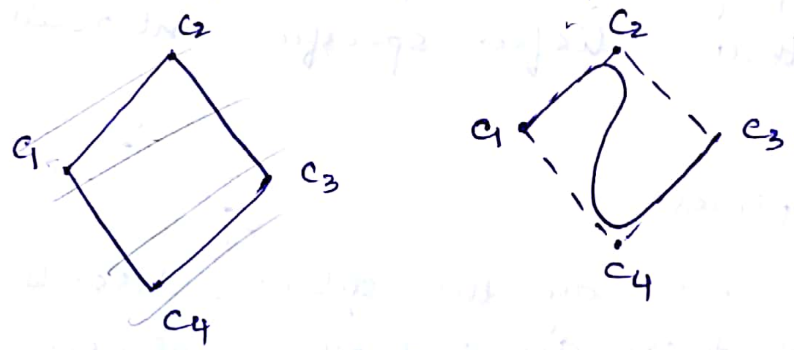



- For more finer images we normally use this approach.

## Convex hull

It is a convex polygon formed out of the designated set of control points.

eg:




If we have to control the shape of the curve locally we can use this approach.

## Continuity Conditions

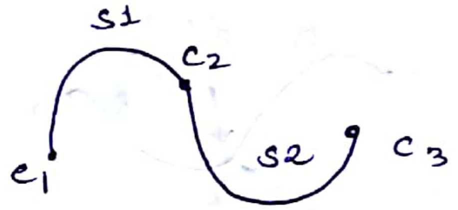
- condition that ensures the smoother transition of curve from one section to another section.

There are two ~~points~~ types.

1) parametric continuity
2) Geometric continuity.

parametric continuity : Here the parametric polynomial section is used between the curve to ensure the smoothness of the curve. ~~(where the par~~

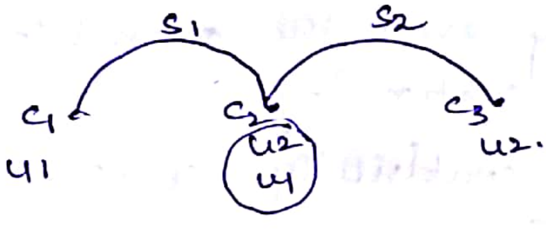
A parametric equation is fixed to each coordinate value at the point which joins two sections in the form $x = x(u)$ , $y = y(u)$ and $z = z(u)$




3 types of parametric continuity.

1. zero order continuity → $C^0$ continuity
2. first order continuity → $C^1$ continuity
3. second order continuity → $C^2$ " "

1) zero order continuity also called as $C^0$ continuity & in this continuity the u2 value for section 1 is equal to u1 value of section 2

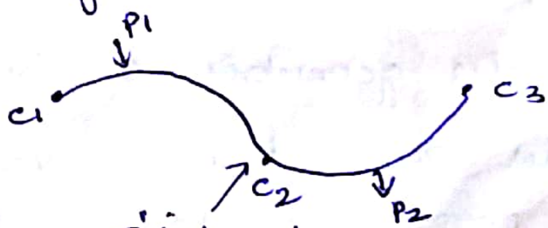



2) First order parametric continuity

In first order parametric continuity the first order parametric derivative for the two sections of the curve is equal at their joining point.

$P_1(u) \Rightarrow P_1'(u)$ :

$P_2(u) \Rightarrow P_2'(u)$

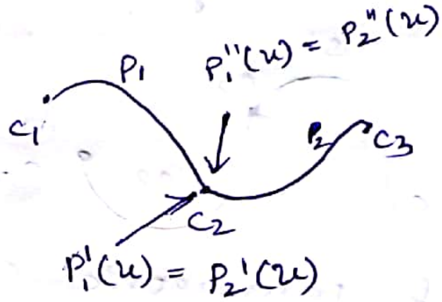



$P_1'(u) = P_2'(u)$

This also called as $C^1$ continuity

3) Second order parametric continuity : In second order parametric continuity the first order parametric derivative as well as second order parametric derivative for the 2 sections of the curve has to be same at the joining points.

It is also called as $c'$ continuity




Geometric continuity : Geometric continuity is an alternate method for the parametric continuity where the geometric continuity doesnt impose the parametric derivatives to be ~~th~~ necessarily the same but they impose the condition of the parametric derivatives to be proportional.

Like parametric continuity even the geometric continuity has the three order of continuity

1) Zero order geometric continuity : It is described as $G^0$ continuity and is very similar to $c^0$ continuity that requires the parametric section of the ~~ding~~ point joining 2 sections of the curve to be same. ie the parametric value of $u_2$ for the $1^{st}$ curve is equal to the parametric value $u_1$ of the $2^{nd}$ curve.

2) First order geometric continuity : Here it requires the $1^{st}$ order parametric derivatives of one curve section to be proportional to the first order parametric derivative of the another curve section.

eg: If $P_1'(u)$ is the $1^{st}$ order parametric derivative of

$P_1'(u)$ and $P_2'(u)$ is the first order parametric derivate of second curve section $(P_2(u))$ then if we get a result in which $P_2'(u) = \frac{1}{P_1'(u)}$. then the type of continuity is called as first order geometric continuity
It is also described as $G'$ continuity.

3) Second order geometric continuity : Here it requires both the first order parametric derivative as well as second order parametric derivative to be proportional to eachother.
It is also called as $G^2$ continuity.

## Bezier curves and surfaces

A Bezier's spline is one of the method for producing the curved surfaces called as Bezier curves
Bezier curve was orginally invented by Piere Bezier for the design of the body of Renault Automobiles
Bezier splines is basically one of the easiest splines to be generated and hence is used widely in different areas of computer graphics.

- Properties of Bezier curve

1) A bezier curve always intersects with the first and last control points
2) A bezier curve with n+1 control points has a Polynomial section of degree n
3) Shape of the bezier curve will always lie inside the convex hull.
4) The slope of the bezier curve at the first

section will always be towards the line joining the first two control sections (points) and the slope of the Bezier curve at the last section will always be towards the line joining the last 2 control points

disadvantage 5) Even though the Bezier curve lies within the convex hull the curve couldnot be controlled or modified locally at a specified control point due to the property of its polynomial section

Polynomial section for the Bezier curve

$$\boxed{P(u) = \sum_{k=0}^{n} p_k \cdot BEZ_{k,n}(u)} \quad 0 \le u \le 1$$

$P_k$ is any coordinate value on to which a section of Bezier curve needs to be fitted

$$P_k = \{x_k, y_k, z_k\}$$

$BEZ_{k,n}(u)$ is called as Blending function.

$$BEZ_{k,n}(u) = C(n,k).u^k.(1-u)^{n-k}$$

$$C(n,k) = \frac{n!}{k!(n-k)!}$$

6marks *** imp

n is degree

Derivation of cubic Bezier Spline

- cubic Bezier spline is a Bezier spline having the polynomial section with degree = 3. (n)

no of control points = 3+1 = 4.

Blending functions = 4

$$P(u) = \sum_{k=0}^{n} P_k \; BEZ_{k,3}(u) \qquad 0 \le u \le 1$$

$$BEZ_{(0,3)}(u) = C(3,0)\, u^0 (1-u)^3$$

$$= \frac{3!}{0!(3-0)!} \cdot 1 (1-u)^3 = \underline{\underline{(1-u)^3}}$$

$$\therefore\; BEZ_{(0,3)} = (1-u)^3.$$

$$BEZ_{1,3}(u) = C(3,1)\, u^1 (1-u)^2$$

$$= \frac{3!}{1!(2)!} u(1-u)^2 = \underline{\underline{3u(1-u)^2}}$$

$$BEZ_{2,3}(u) = C(3,2)\, u^2 (1-u)$$

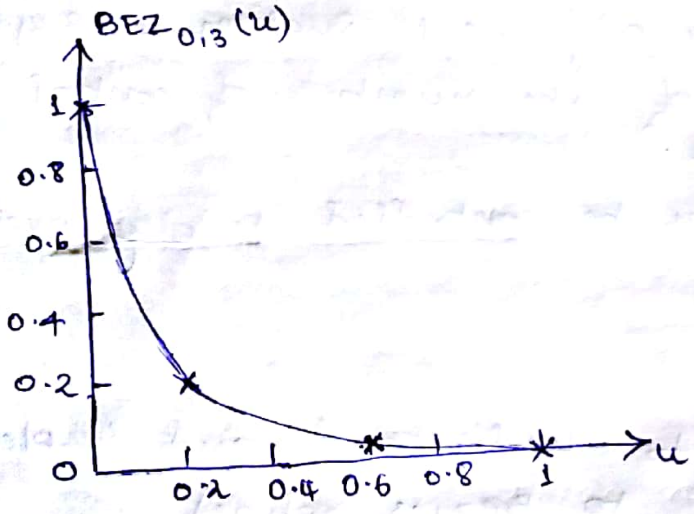
$$= \frac{3!}{2!(3-2)!} u^2 (1-u) = \underline{\underline{3u^2(1-u)}}$$

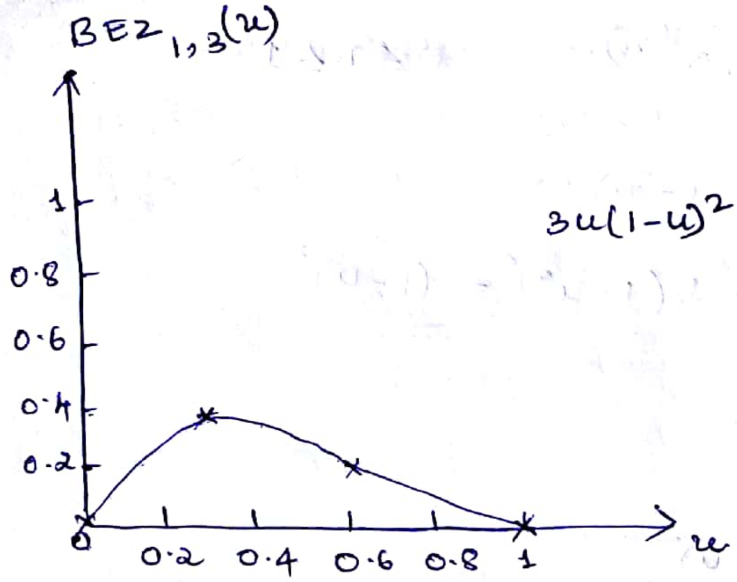
$$BEZ_{(3,3)}(u) = C(3,3)\, u^3 (1-u)^0$$

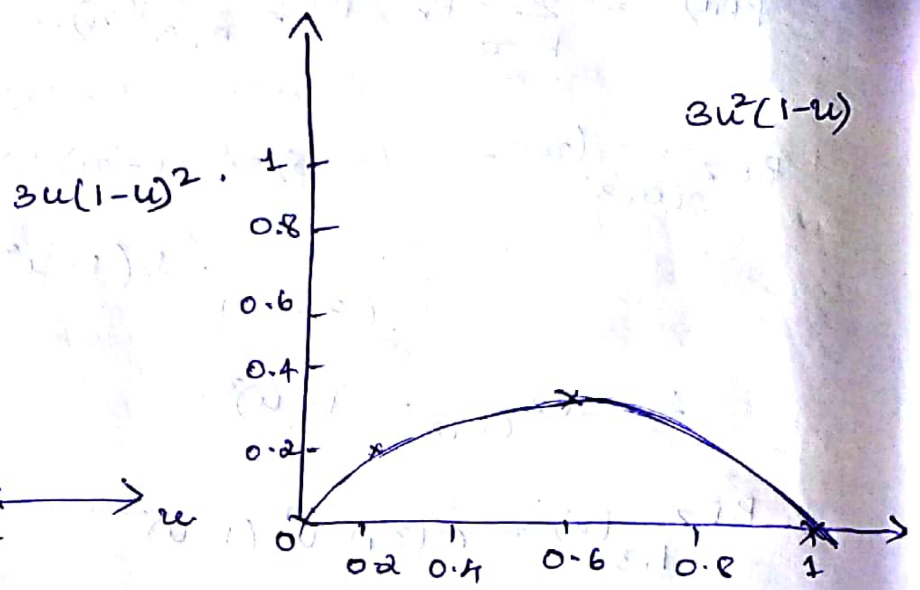
$$= \underline{\underline{u^3}}$$

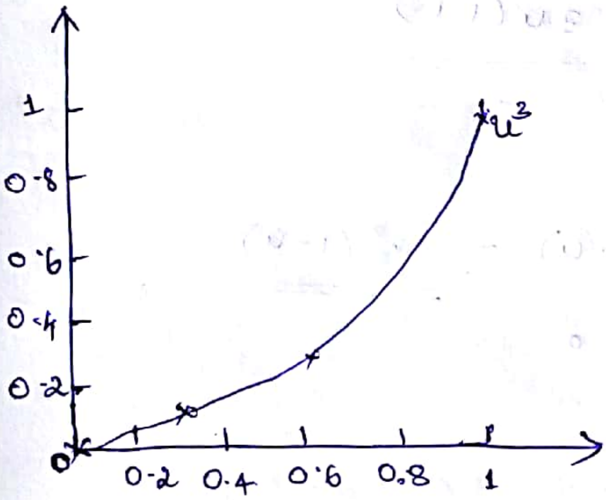
$0 \le u \le 1$

$u = 0/3,\; 1/3,\; 2/3,\; 3/3 \Rightarrow 0,\; 0.33,\; 0.66,\; 1$






## B Spline Curve

### Advantages of B spline curve over Bezier Curve

1. The degree of the polynomial equation in B spline curve is independent of the number of control points with some condition.
2. B spline curve can be controlled locally over the control point.

### Drawback

It is difficult to understand and implement (when compared to Bezier spline)

## Polynomial Equation for B-spline curve

$$P(u) = \sum_{k=0}^{n} P_k B_{k,d}(u) \quad u_{min} \le u \le u_{max}$$

$$2 \le d \le n+1$$

where $B_{k,d}(u)$ is blending function for the Bspline curve

$d$ is the degree of the polynomial.

$n+1$ is the number of control points.

Each of the Blending function has $d$ subintervals that is defined by the value of knot vector $u$

$$B_{k,d}(u) = \frac{u - u_k}{u_{k+d-1} - u_k} B_{k,d-1}(u) + \frac{u_{k+d} - u}{u_{k+d} - u_{k+1}} B_{k+1,d-1}(u)$$

$$B_{k,1}(u) = \begin{cases} 1 & u_k \le u \le u_{k+1} \\ 0 & \text{otherwise.} \end{cases}$$

## construction of knot vector

A knot vector is basically an array that is constructed for selecting the range of $u$ values that ranges from minimum to maximum value.

Each value in the knot vector should be greater or equal ~~than~~ to its previous value and less than & equal to the next value

If there are $n+1$ control points for a curve and if the degree of the polynomial chosen is $d$ which is $2 \le d \le n+1$, then the knot vector will

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have n+d+1 values inside it.

1.) <u>Uniform periodic B spline</u>

In this type of knot vector there will be an uniform distribution of values between any 2 elements of u.

Eg: $n=3$, $d=3$

no of values $= n+d+1 = 7$

Knot vector $= \{u_0, u_1, u_2, u_3, u_4, u_5, u_6\}$

$= \{-1.0, -0.5, 0, 0.5, 1.0, 1.5, 2.0\}$.

or

$= \{1, 2, 3, 4, 5, 6, 7\}$

2) <u>Uniform open B spline</u>

$$u_j = \begin{cases} 0 & \text{, for first } d \text{ lines} \\ j-d+1 & \text{for the } j\text{th position where } d \le j \le n \\ n-d+2 & j > n \end{cases}$$

$n=3$ $d=3$

no of value $= 7$

Knot vector $= \{u_0, u_1, u_2, u_3, u_4, u_5, u_6\}$

$= \{0, 0, 0, 1, 2, 2, 2\}$

3) <u>Non uniform B spline</u>

$n=3$, $d=3$

no of values $= 7$

Knot vector $= \{u_0, u_1, u_2, u_3, u_4, u_5, u_6\}$

$= \{2, 5, 6, 6, 7, 7, 10\}$.

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Design of Quadratic uniform B-spline curve

$\downarrow$

4 control points. $(n+1=4)$

$n=3,\ d=3$

no of elements $= n+d+1 = 3+3+1 = 7$

Knot vector $= \{u_0, u_1, u_2, u_3, u_4, u_5, u_6\} \rightarrow$ uniform periodic form.

$= \{0, 1, 2, 3, 4, 5, 6\}$

$$P(u) = \sum_{k=0}^{n} P_k B_{k,d}(u)$$

$$P(u) = \sum_{k=0}^{3} P_k B_{k,3}(u)$$

$$B_{k,d}(u) = \frac{u-u_k}{u_{k+d-1}-u_k} B_{k,d-1}(u) + \frac{u_{k+d}-u}{u_{k+d}-u_{k+1}} B_{k+1,d-1}(u)$$

$$B_{k,1}(u) = \begin{cases} 1, & u_k \le u \le u_{k+1} \\ 0, & \text{otherwise} \end{cases}$$

$$B_{0,3}(u) = \frac{u-u_0}{u_2-u_0} B_{0,2}(u) + \frac{u_3-u}{u_3-u_1} B_{1,2}(u)$$

( $u_0\ u_1 \ldots$ values are taken from knot vector)

$$= \underbrace{\frac{u}{2}\left\{\frac{u-u_0}{u_1-u_0} B_{0,1}(u) + \frac{u_2-u}{u_2-u_1} B_{1,1}(u)\right\}}_{\text{expansion of } B_{0,2}(u)}$$

$$+ \frac{3-u}{2}\left\{\frac{u-u_1}{u_2-u_1} B_{1,1}(u) + \frac{u_3-u}{u_3-u_2} B_{2,1}(u)\right\}$$

$$= \frac{u}{2}\left\{u \cdot \underbrace{B_{0,1}(u)}_{0} + (2-u) B_{1,1}(u)\right\} + \frac{3-u}{2}\left\{(u-1) B_{1,1}(u) + (3-u) B_{2,1}(u)\right\}$$

$\because k=0$

all $B_{k,1}$ where $k=0$ is set to 1

rest are set to 0.

choosing intervals for $B_{0,3}(u)$

$u_k \le u \le u_{k+1}$

$0 \le u \le 1$

$1 \le u \le 2$.

$2 \le u \le 3$

$k = 3$

$\therefore$ 3 intervals

For, $0 \le u \le 1$, $k = 0$.

$$\therefore B_{0,3}(u) = \frac{u}{2}\left[u \cdot 1 + (2-u)\, 0\right] + \frac{3-u}{2}\left[(u-1)\, 0 + (3-u)\, 0\right]$$

$$= \frac{u}{2}(u) = u^2/2$$

For $1 \le u \le 2$, $k = 1$ $u_k = 1$

$$\therefore B_{0,3}(u) = \frac{u}{2}\left[u \cdot 0 + (2-u)\, 1\right] + \frac{3-u}{2}\left[(u-1)\, 1 + (3-u)\, 0\right]$$

$$= \frac{u}{2}\left[2-u\right] + \frac{3-u}{2}\left[u-1\right]$$

$$= \frac{u}{2}(2-u) + \frac{(3-u)(u-1)}{2}$$

$$= \frac{2u - u^2 + 3u - u^2 + u - 3}{2}$$

$$= \frac{6u - 2u^2 - 3}{2}$$

For $2 \le u \le 3$ $k = 2$ $\because u_k = 2$.

$$B_{0,3}(u) = \frac{u}{2}\left[u \cdot 0 + (2-u)\, 0\right] + \frac{3-u}{2}\left[(u-1)\, 0 + (3-u)\, 1\right]$$

$$= \frac{3-u}{2}(3-u) = \frac{(3-u)^2}{2}$$

$$\therefore\ B_{0,3}(u) = \begin{cases} \dfrac{u^2}{2} & 0 \le u \le 1 \\ \dfrac{u(2-u) + (3-u)(u-1)}{2} & 1 \le u \le 2 \\ \dfrac{(3-u)^2}{2} & 2 \le u \le 3 \end{cases}$$

This technique only works if we use uniform periodic form for knot vector

$$B_{1,3}(u) = \begin{cases} \dfrac{(u-1)^2}{2} & 1 \le u \le 2 \\ \dfrac{(u-1)(3-u) + (4-u)(u-2)}{2} & 2 \le u \le 3 \\ \dfrac{(4-u)^2}{2} & 3 \le u \le 4 \end{cases}$$

$$B_{2,3}(u) = \begin{cases} \dfrac{(u-2)^2}{2} & 2 \le u \le 3 \\ \dfrac{(u-2)(4-u) + (5-u)(u-3)}{2} & 3 \le u \le 4 \\ \dfrac{(5-u)^2}{2} & 4 \le u \le 5 \end{cases}$$

$$B_{3,3}(u) = \begin{cases} \dfrac{(u-3)^2}{2} & 3 \le u \le 4 \\ \dfrac{(u-3)(5-u) + (6-u)(u-4)}{2} & 4 \le u \le 5 \\ \dfrac{(6-u)^2}{2} & 5 \le u \le 6. \end{cases}$$

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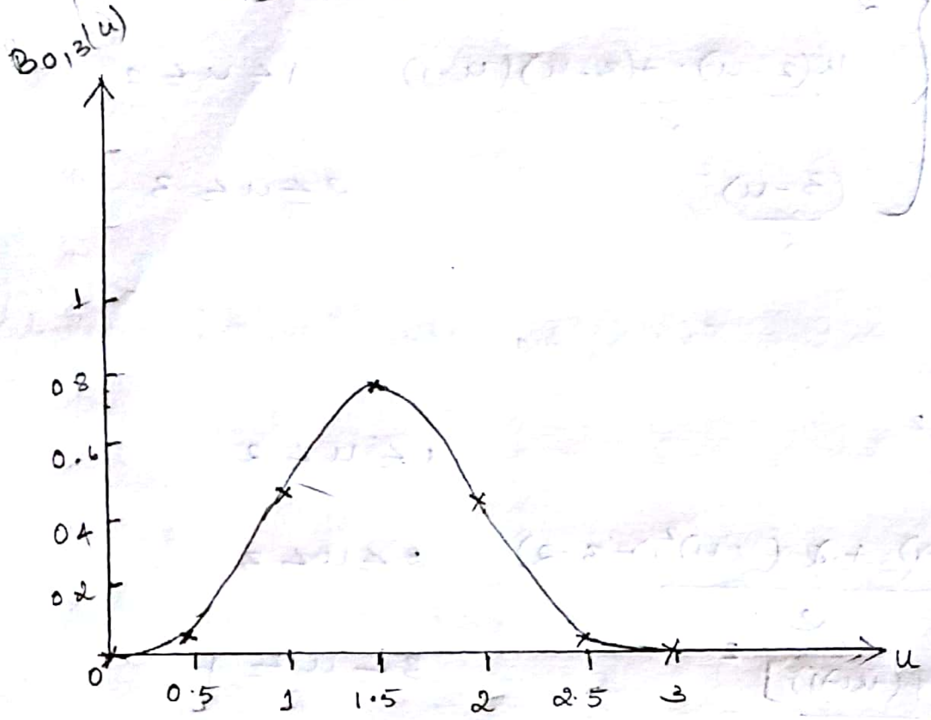
graph for $B_{0,3}(u)$




Plot for other 3

**Properties of B spline curve**

1) Polynomial curve of B-spline has a degree $d-1$ and a continuity of $C^{d-2}$ over a range of u
2) A polynomial curve with $n+1$ control points has a blending functions of $n+1$ starting from $k=0$ and ending with $k=10$
3) Each blending function is defined over d number of subintervals with each subinterval having starting value equivalent to k and ending value equivalent to $k+1$, chosen from knot vector U
4) The total number of elements for the knot vector is equal to $n+d+1$ values which is arranged in ~~equal~~ the higher order such that the previous value is less than or equals to the next value.
5) Each section of curve is controlled by d number of control points.

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6) Any one control point can effect only the 4 sections to which it belongs