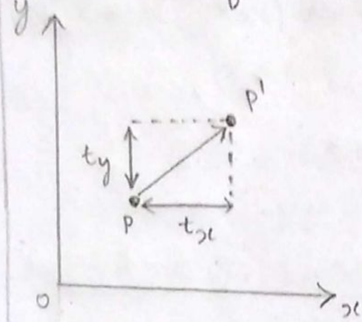
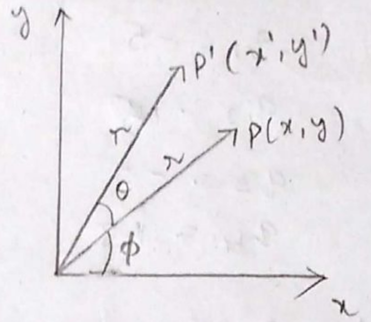
Geometric Transformations.

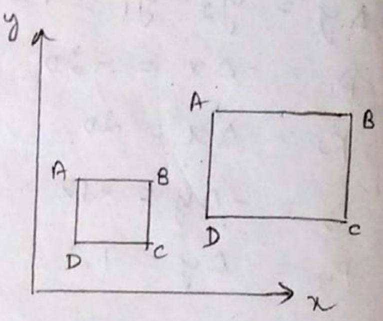
2D Transformation techniques :



Translation



Rotation



Scaling

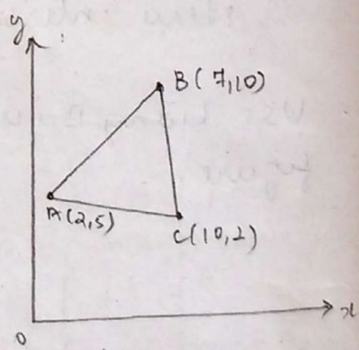
~~Transformation~~ Translation is a process of changing position of an object in a straight line path from one co-ordinate location to another.

Consider a point $P(x, y)$ where x and y are the initial co-ordinates. To identify the new position of point P' with the co-ordinates x' and y', where t_x and t_y are known as translation vectors or shift vectors.

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \qquad P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\boxed{P' = P + T}$$

1. Translate the polygon with co-ordinates A(2,5), B(7,10), C(10,2) by 3 units in x-direction & 4 units in y-direction.



→ $t_x = 3$, $t_y = 4$

$A' = A + T$

$$= \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$ i.e., (5, 9)

$B' = B + T$

$$= \begin{bmatrix} 7 \\ 10 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$ i.e., (10, 14)

$C' = C + T$

$$= \begin{bmatrix} 10 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 13 \\ 6 \end{bmatrix}$$ i.e., (13, 6)

Rotation.

- Let us consider the rotation of an object about the origin (0,0). Where R is a constant distance from point P from the origin. ϕ is the original angular position of the point from the origin and θ is the rotation angle. The transformed co-ordinates of P' can be represented in terms of ϕ and θ:

$$x' = r\cos(\phi+\theta) = r\cos\theta\cos\phi - r\sin\theta\sin\phi \quad — ①$$

$$y' = r\sin(\phi+\theta) = r\sin\phi\cos\theta + r\cos\phi\sin\theta \quad — ②$$

The original co-ordinates of point P are,

$$x = r\cos\phi$$

$$y = r\sin\phi$$

Sub; the values of x and y in ① & ②

$$x' = x\cos\theta - y\sin\theta.$$

$$y' = x\sin\theta + y\cos\theta.$$

The above eq^ns. can be represented in the form of matrices.

$$[x' \ y'] = [x \cdot y]\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$\Rightarrow \boxed{P' = P.R}$$

where R is the rotation matrix or rotation vector and

$$R = \begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$$ (For anticlockwise rotation)

2. ~~A point (4,3)~~ i.e., $\theta = -ve$

For clockwise rotation, $R = \begin{bmatrix}\cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta)\end{bmatrix}$

$$R = \begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$$

2. A point (4,3) is rotated counterclockwise ~~when~~ by an angle of 45°. Find the rotation matrix and the resultant point.

Scanned by CamScanner

→ $(x, y) = (4, 3)$

$\theta = 45°$ [Anti clockwise rotation]

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$P' = P \cdot R$ To identify resultant point.

$$= \begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 7/\sqrt{2} \end{bmatrix}$$

3. A point (8, 6) is rotated clockwise by an angle of 60°. Find the rotation matrix and the resultant point.

→ $(x, y) = (8, 6)$

$\theta = 60°$

$$R = \begin{bmatrix} \cos 60° & \sin 60 \\ -\sin 60 & \cos 60 \end{bmatrix} = \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}$$

$P' = P \cdot R$

$$= \begin{bmatrix} 8 & 6 \end{bmatrix} \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 9.196 & -3.928 \end{bmatrix}$$

Scaling.

Initial Point, $P = [x \;\; y]$ & $P' = [x' \;\; y']$

$\Rightarrow \boxed{x' = x \cdot S_x + y \cdot S_y}$

$$S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

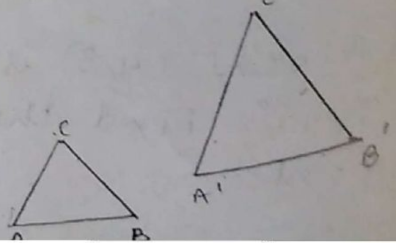
Q. Scale the polygon with co-ordinates A(2, 5), B(7, 10) C(10, 2) by 2 units in x-direction and 2 units in y-direction.

→ $A' = A \cdot S$

$$= [2 \;\; 5] \cdot \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = [4 \;\; 10]$$

$B' = B \cdot S$

$$= [7, 10] \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = [14 \;\; 20]$$



$c' = c \cdot S$

$= [10, 2] \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = [10\ 4]$

can be used only when the object is at (0,0)

Homogeneous co-ordinates and matrix represention of 2D transformations.

Homogeneous co-ordinates for translation:

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

$P' = PT$

$$[x'\ y'\ 1] = [x\ y\ 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix} = [x + t_x \quad y + t_y \quad 1]$$

=

Homogeneous co-ordinates for rotation:

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore P' = PR$

$$[x'\ y'\ 1] = [x\ y\ 1] \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [x\cos\theta - y\sin\theta \quad x\sin\theta + y\cos\theta \quad 1]$$

Homogeneous co-ordinates for scaling:

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$P' = PS$

$$[x'\ y'\ 1] = [x\ y\ 1] \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = [xS_x \quad yS_y \quad 1]$$

1. If a 3x3 co-ordinate transformation matrix for each of the foll. translations:
   i) Shift the image to the right 3 units
   ii) Shift the image up 2 units

iii) Move the image down half unit and right 1 unit.
iv) Move the image down 2/3 units and left 4 units.

→ i) $t_x = 3 \quad t_y = 0$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

ii) $t_x = 0 \quad t_y = 2$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

~~$P' = P \cdot T =$~~

iii) $t_y = -1/2, \; t_x = 1$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1/2 & 1 \end{bmatrix}$$

iv) $t_x = -4 \quad t_y = \frac{-2}{3}$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & \frac{-2}{3} & 1 \end{pmatrix}$$

2. Find the transformation matrix that transforms a given square ABCD to half its size, with center still remaining at the same position. The co-ordinates of square are A(1, 1), B(3, 1), C(3, 3), D(1, 3) and centered at (2, 2). Also find the resultant co-ordinates of the square.

→ Move the square to (0, 0). Scale it. And move it back to the same position.

$$T^{-1} S T = T^{-1} \cdot S \cdot T$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1/2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P' = P \cdot \begin{bmatrix} 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \\ 1 & 1 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \cdot [T^{-1} S T] = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 3 & 3 & 1 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & 3/2 & 1 \\ 5/2 & 3/2 & 1 \\ 5/2 & 5/2 & 1 \\ 3/2 & 5/2 & 1 \end{bmatrix}$$

3. Find the transformation of the Δle $A(1,0)$, $B(0,1)$, $C(1,1)$ by i) Rotating 45° above the origin & then translating 1 unit in x & y direction.

ii) Translating 1 unit in x & y direction & rotating 45° about the origin.

$\rightarrow$ $\theta = 45^\circ \Rightarrow R$

$t_x = 1,\ t_y = 1 \Rightarrow T$

i) Rotation and translation i.e.,

$$R.T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Now find new vertices $A'B'C'$.

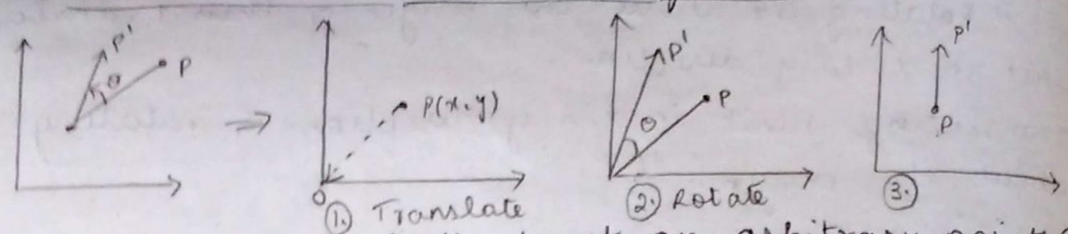
$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} \cdot [R\,T] = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1+1/\sqrt{2} & 1+1/\sqrt{2} & 1 \\ 1-1/\sqrt{2} & 1+1/\sqrt{2} & 1 \\ 1 & \sqrt{2}+1 & 1 \end{bmatrix}$$

ii) $$T \cdot R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & \sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & \sqrt{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} [T\,R] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & \sqrt{2} & 1 \end{bmatrix}.$$

Rotation about an arbitrary point.



To rotate an object about an arbitrary point (x_p, y_p) we have to carry out 3 steps:

① Translate the point (x_p, y_p) to the origin.

② Rotate it about the origin.

③ Translate the center of rotation back to where it belongs.

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_p & -y_p & 1 \end{bmatrix} \quad R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

∴ the overall transformation matrix for counter clockwise axis for an angle θ about the point (x_p, y_p) is given as,

$$T_1 \cdot R \cdot T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -t_x & -t_y & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -t_x\cos\theta + t_y\sin\theta & -t_x\sin\theta - t_y\cos\theta & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -t_x\cos\theta + t_y\sin\theta + t_x & -t_x\sin\theta - t_y\cos\theta + t_y & 1 \end{bmatrix}$$

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1. Perform a counter clockwise $45°$ rotation of a Δle $A(2,3)$, $B(5,5)$, $C(4,3)$ about the point $(1,1)$.

⇒ $\theta = 45°$ $t_x = 1$, $t_y = 1$.

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\cancel{T_1 \cdot R \cdot T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}}$$

$$T_1 R T_2 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -t_x\cos\theta + t_y\sin\theta + t_x & -t_x\sin\theta - t_y\cos\theta + t_y & 1 \end{bmatrix} \qquad \theta = 45.$$

$$-1\times\frac{1}{\sqrt{2}} + 1\times\frac{1}{\sqrt{2}} + 1 \qquad -1\times\frac{1}{\sqrt{2}} - 1\times\frac{1}{\sqrt{2}} + 1$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 1-\sqrt{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} [T_1 \; R \; T_2]$$

$$= \begin{bmatrix} 2 & 3 & 1 \\ 5 & 5 & 1 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 1-\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} 1-1/\sqrt{2} & 1+3/\sqrt{2} & 1 \\ 1 & 8/\sqrt{2}+1 & 1 \\ 1+1/\sqrt{2} & \frac{5}{\sqrt{2}}+1 & 1 \end{bmatrix}$$

2. Perform a clockwise 45° rotation of Δle A(3, 4), B(7, 7) C(5, 3) about point (1, 1).

→ $\theta = -45^\circ$. $t_x = 1$, $t_y = 1$.

$$T_1 R T_2 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -t_x\cos\theta + t_y\sin\theta + t_x & -t_x\sin\theta - t_y\cos\theta + t_y & 1 \end{bmatrix}$$

$$-1\times\frac{1}{\sqrt{2}} + 1\times-\frac{1}{\sqrt{2}} + 1 \qquad -1\times-\frac{1}{\sqrt{2}} - 1\times\frac{1}{\sqrt{2}} + 1$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1-\sqrt{2} & 1 & 1 \end{bmatrix}$$

$\frac{7}{\sqrt{2}}$

$\frac{7}{\sqrt{2}} + \frac{7}{\sqrt{2}} + 1 - \sqrt{2}$

$\frac{14}{\sqrt{2}} + 1 - \sqrt{2}$

$\frac{12}{\sqrt{2}} + 1$

$\frac{3\sqrt{2}}{\sqrt{2}} + 1 = 4$

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} [T_1 \; R \; T_2]$$

$$= \begin{bmatrix} 3 & 4 & 1 \\ 7 & 7 & 1 \\ 5 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1-\sqrt{2} & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{\sqrt{2}}+1 & 1+\frac{1}{\sqrt{2}} & 1 \\ & & \\ & & \end{bmatrix}$$

3D Transformations.

Translation

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

$$P' = P \cdot T$$

$$[x' \;\; y' \;\; z' \;\; 1] = [x \;\; y \;\; z \;\; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

$$P' = [x+t_x \quad y+t_y \quad z+t_z \quad 1]$$

Scaling

$$S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

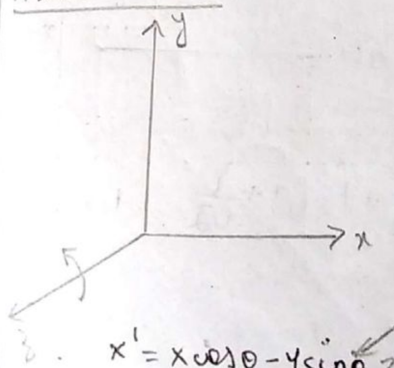
$$P' = P \cdot S$$

$$[x' \;\; y' \;\; z' \;\; 1] = [x \;\; y \;\; z \;\; 1] \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P' = [x \cdot S_x \quad y \cdot S_y \quad z \cdot S_z \quad 1]$$

Rotation:

About Z axis.

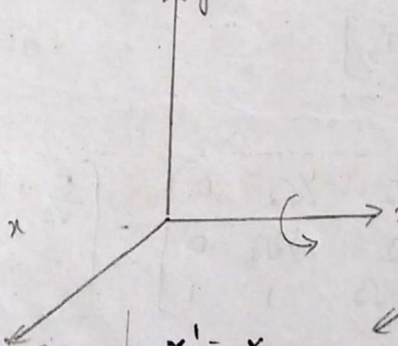


$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$z' = z$

About X axis

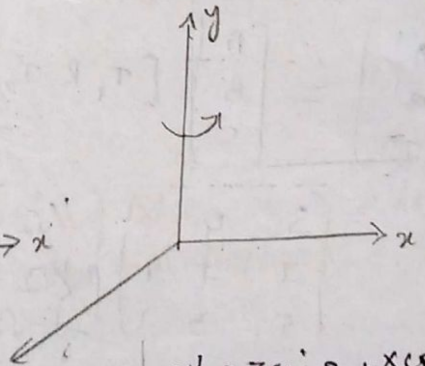


$x' = x$

$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$.

About Y axis



$x' = z\sin\theta + x\cos\theta$

$y' = y$

$z' = z\cos\theta - x\sin\theta$

For Z axis:

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For X axis:

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & +\sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For Y-axis:

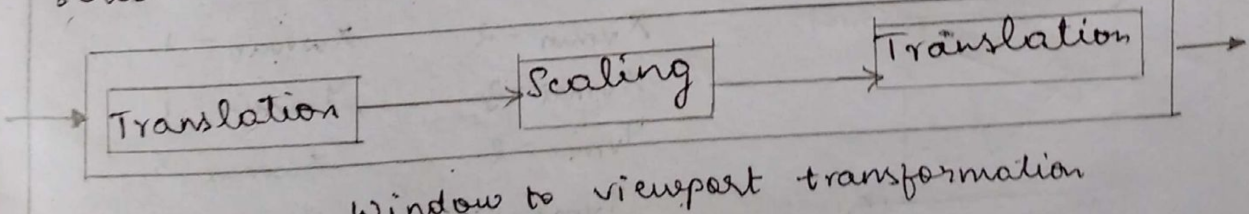
$$R = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & \cancel{\cos\theta}\ 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Window to viewport Transformation (view transformat^n)

- Steps: world co-ordinate system is infinite and device display area is finite.
- To perform viewing transformation, we select a finite world ~~transformation~~ coordinate area for displaying ~~window~~ within the area.
- An area on a device to which a window is mapped is called a viewport.

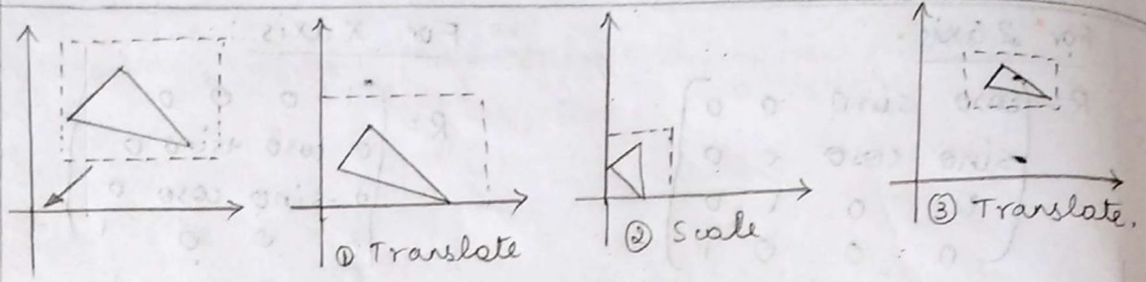
Steps.

1. The object together with its window is translated until the lower left corner of the window is at the origin.
2. Object & window are scaled until the window has a dimension of viewport.
3. Translate the viewport to its correct position on the screen.



Window to viewport transformation

Window co-ordinates : $X_{wmin}, X_{wmax}, Y_{wmin}, Y_{wmax}$



$W = T \cdot S \cdot T^{-1}$

The transformation matrix for indivisual

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -X_{wmin} & -Y_{wmin} & 1 \end{bmatrix} \quad S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ X_{vmin} & Y_{vmin} & 1 \end{bmatrix}$$

$$S_x = (X_{vmax} - X_{vmin}) / (X_{wmax} - X_{wmin})$$

$$S_y = (Y_{vmax} - Y_{vmin}) / (Y_{wmax} - Y_{wmin})$$

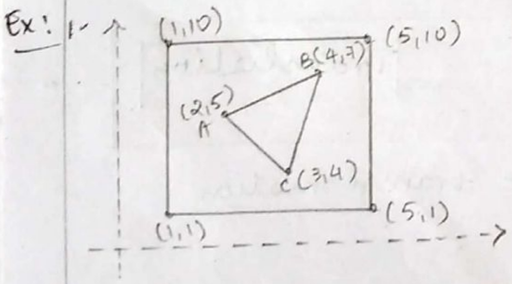
$$W = T \cdot S \cdot T^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -X_{wmin} & -Y_{wmin} & 1 \end{bmatrix} \cdot \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ X_{vmin} & Y_{vmin} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ S_x \cdot X_{wmin} & S_y \cdot Y_{wmin} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ X_{vmin} & Y_{vmin} & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ -S_x \cdot X_{wmin} + X_{vmin} & -S_y \cdot Y_{wmin} + Y_{vmin} & 1 \end{bmatrix}$$

Ex:



$X_{vmin} = 2$ $\quad X_{wmin} = 1$

$X_{vmax} = 3$ $\quad Y_{wmin} = 1$

$Y_{vmin} = 2$ $\quad X_{wmax} = 5$

$Y_{vmax} = 4$ $\quad Y_{wmax} = 10$

Find the window to viewport transformation matrix and Find the primitives of ABC after mapping.

$S_x = (3-2)/(5-1) = 1/4$

$S_y = (4-2)/(10-1) = 2/9$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \quad S = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 2/9 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad T^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 2/9 & 0 \\ -\frac{1}{4}\times 1 + 2 & -\frac{2}{9}\times 1 + 2 & 1 \end{bmatrix} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 2/9 & 0 \\ \frac{7}{4} & \frac{16}{9} & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.2 & 0 \\ 1.75 & 1.7 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} * W$$

$$= \begin{bmatrix} 2 & 5 & 1 \\ 4 & 7 & 1 \\ 3 & 4 & 1 \end{bmatrix} * \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.2 & 0 \\ 1.75 & 1.7 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.25 & 2.7 & 1 \\ 2.75 & 3.1 & 1 \\ 2.5 & 2.5 & 1 \end{bmatrix}$$

These values should be b/w (2, 2) & (3, 4)

2. Find normalization transformation window to viewport with window lower left corner at (1, 1) and upper right corner at (3, 5) to a viewport with lower left corner at (0, 0) and upper right corner at (1/2, 1/2).

→ $X_{vmin} = 0$ $\quad X_{wmin} = 1$

$Y_{vmin} = 0$ $\quad Y_{wmin} = 1$

$X_{vmax} = 1/2$ $\quad X_{wmax} = 3$

$Y_{vmax} = 1/2$ $\quad Y_{wmax} = 5$

$S_x = (1/2 - 0) / (\cancel{1/2 - 0}\ 3-1) = \frac{1}{2}/2$

$S_y = (\cancel{1} - \cancel{0}) / (5-1) = 2/4 = 1/2$.

$$w = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/8 & 0 \\ -1/4 & -1/8 & 1 \end{bmatrix}$$

3. Find the normalization transformation window to viewport with lower left corner at (2, 2) and upper-right corner at (9,10) onto a view port with L.L corner at (6, 3) & U.R corner at (9, 5).

→ Sol: $X_{wmin} = 2$ $\quad X_{vmin} = 6$

$X_{wmax} = 9$ $\quad X_{vmax} = 9$

$Y_{wmax} = 10$ $\quad Y_{vmin} = 3$

$Y_{wmin} = 2$ $\quad Y_{vmax} = 5$

$S_x = \frac{3}{7}$ $\qquad S_y = \frac{2}{8} = \frac{1}{4}$

$$W = \begin{bmatrix} 3/7 & 0 & 0 \\ 0 & 1/4 & 0 \\ -\frac{3}{7}\times 2+6 & -\frac{1}{4}\times 2+3 & 1 \end{bmatrix} = \begin{bmatrix} 3/7 & 0 & 0 \\ 0 & 1/4 & 0 \\ \frac{36}{7} & \frac{10}{4} & 1 \end{bmatrix}$$

Find the transformed object with vertices A(4, 5), B(5, 4), C(8, 4), D(8, 7), E(5, 7).

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \\ E' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix} * W$$

$5\times3 \quad 3\times3$

$$= \begin{bmatrix} 4 & 5 & 1 \\ 5 & 4 & 1 \\ 8 & 4 & 1 \\ 8 & 7 & 1 \\ 5 & 7 & 1 \end{bmatrix} \begin{bmatrix} 3/7 & 0 & 0 \\ 0 & 1/4 & 0 \\ \frac{36}{7} & \frac{5}{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{48}{7} & \frac{15}{4} & 1 \\ \frac{51}{7} & \frac{7}{2} & 1 \\ \frac{60}{7} & 7/2 & 1 \\ \frac{60}{7} & \frac{17}{4} & 1 \\ 51/7 & 17/4 & 1 \end{bmatrix}$$

$4\times\frac{3}{7} + \frac{36}{7} =$

$\frac{5}{4} + \frac{5}{2}$

$\frac{5+10}{4}$

$\frac{15}{7} + \frac{36}{7} =$

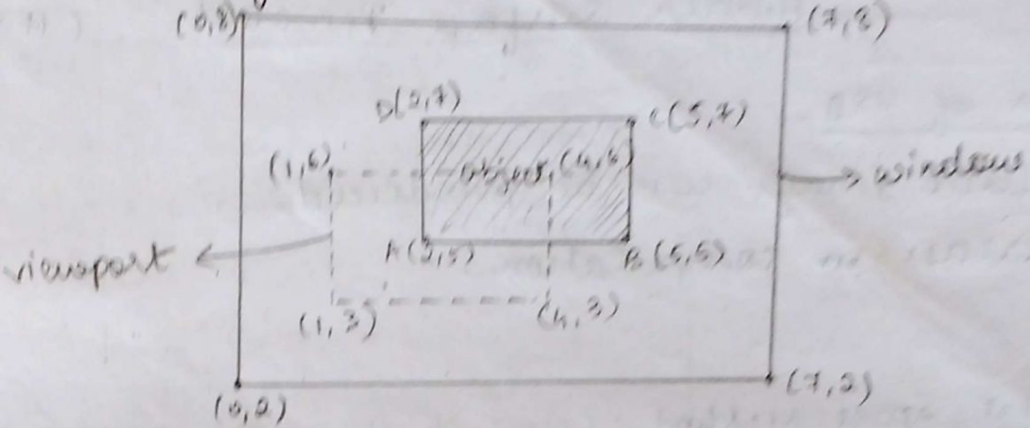
$1 + \frac{5}{2}$

$\frac{7}{4} + \frac{10}{4}$

$\frac{24}{7} + \frac{36}{7}$

$\frac{15}{7} + \frac{36}{7}$

Apply window to viewport transformation to the given figure to identify the new values of ABCD.



$X_{wmin} = 0$ $\quad X_{vmin} = 1$

$X_{wmax} = 7$ $\quad X_{vmax} = 4$

$Y_{wmin} = 2$ $\quad Y_{vmin} = 3$

$Y_{wmax} = 8$ $\quad Y_{vmax} = 6$

$S_x = \frac{3}{7}$

$S_y = \frac{3}{6} = \frac{1}{2}$.

$$W = \begin{bmatrix} \frac{3}{7} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} 2 & 5 & 1 \\ 5 & 5 & 1 \\ 5 & 7 & 1 \\ 2 & 7 & 1 \end{bmatrix} \begin{bmatrix} 3/7 & 0 & 0 \\ 0 & 1/2 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 13/7 & 9/2 & 1 \\ 22/7 & 9/2 & 1 \\ 22/7 & 11/2 & 1 \\ 13/7 & 11/2 & 1 \end{bmatrix}.$$