1. For the following languages, construct a DFA accepting the language and prove that your construction is correct.
   (a) $L = \{w \in \{0,1\}^* \mid w \text{ starts with } 01\}$.
   (b) $L = \{w \in \{0,1\}^* \mid w \text{ ends with } 01\}$.
   (c) $L = \{w \in \{0,1\}^* \mid w \text{ contains } 01 \text{ as a substring}\}$.
   (d) $L = \{w \in \{0,1\}^* \mid \text{every block of four symbols in } w \text{ contains at least two 0s}\}$.
   (e) $L = \{w \in \{0,1\}^* \mid w \text{ contains an even number of 0s or contains exactly two 1s}\}$.
   (f) $L = \{0,1\}^* \setminus \{\epsilon\}$.

   For these languages, argue that your construction is correct. First, argue with normal English sentences. Now, try to formalize this by writing the correct induction hypothesis like we did in class.

2. Let Σ be a finite alphabet and let $x, y \in \Sigma^*$ be any two strings. Show that $xy = yx$ iff there exists a string $z \in \Sigma^*$ such that $x^2y^2 = z^2$.

3. Let $\Sigma = \{0,1\}^*$ and $L \subseteq \Sigma^*$ be the set strings w such that the number of 0s in w is not equal to the number of 1s in w. Can you describe the language L^2? Is it regular?