Dantzig's Simplex Algorithm

How to Write Fast Numerical Code

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Linear Programming

Optimising a Linear Program in standard form:

```
Maximize
2x - 3y + z

Subject To
 x   +   y + z <= 10
4x   - 3y + z <=  3
2x   +   y - z <= 6
```

Restrictions

- all coefficients positive (simplicity)
- all coefficients $\leq 10^6$ (stability)

Steps

- Tableau form $$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 10 \\ 4 & -3 & 1 & 0 & 1 & 0 & 0 & 3 \\ 2 & 1 & -1 & 0 & 0 & 1 & 0 & 6 \\ 2 & -3 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$
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- Termination, worst runtime $O(e^m)$, but often $O(m)$

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Comparison

- GLPK (GNU Linear Programming Kit), solid standard solver
- CPPLEX, mathematical OO-implementation
- Gurobi (CPLEX), fastest (multithreaded) solver available
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Properties

- Tableau: $(m+1) \times (m+n+2)$
  (requires full access each iteration)
  Memory reads: $m(m+n) + 2m + n$
  (all capacity misses for bigger problems)
  Flops: $m(m+n) + m$
- Computational intensity $I = \frac{m^2+mn+2m+n}{8(m^2+n)} \sim \frac{1}{4}$

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- ssa
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Performance

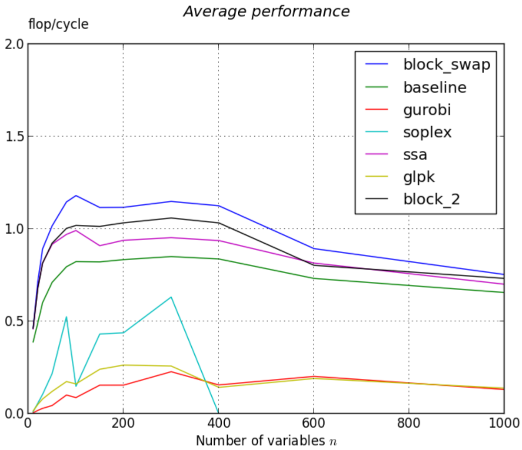

Figure : performance comparison

Questions?

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