



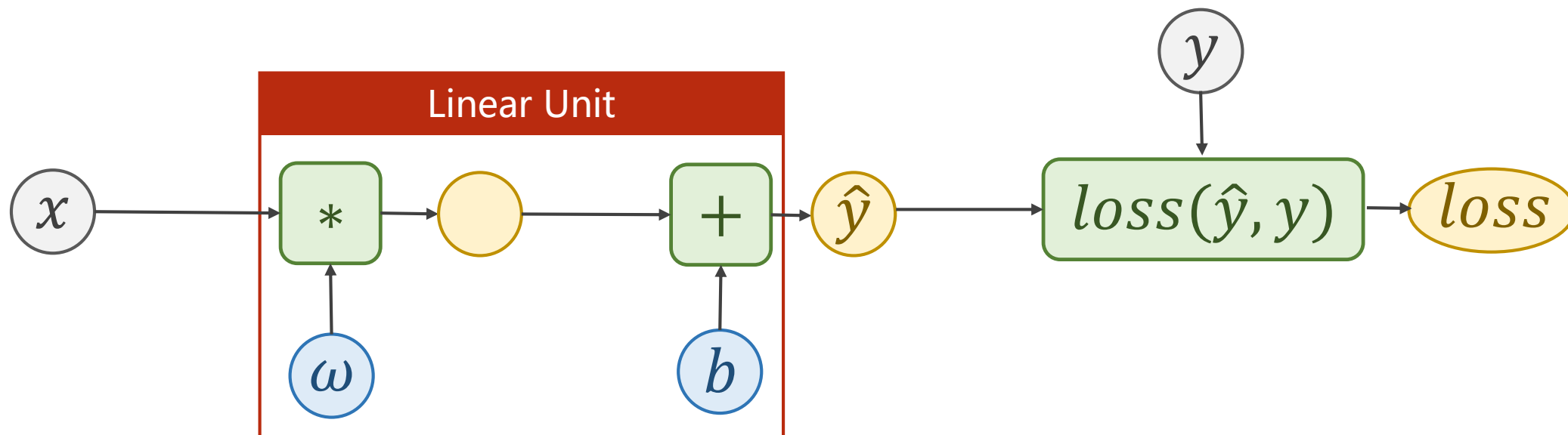
要估计的 $\hat{y}$ 是连续变量——回归 (Regression)

# PyTorch Tutorial

## 06. Logistic Regression

逻辑斯蒂回归虽然叫回归，但是其用logistic函数处理 $\hat{y}$ ，使输出值转换为[0,1]之间的概率

# Revision - Linear Regression



## Affine Model

$$\hat{y} = x * \omega + b$$

## Loss Function

$$loss = (\hat{y} - y)^2 = (x \cdot \omega - y)^2$$

# Revision - Linear Regression

x (hours)	y (points)
1	2
2	4
3	6
4	?

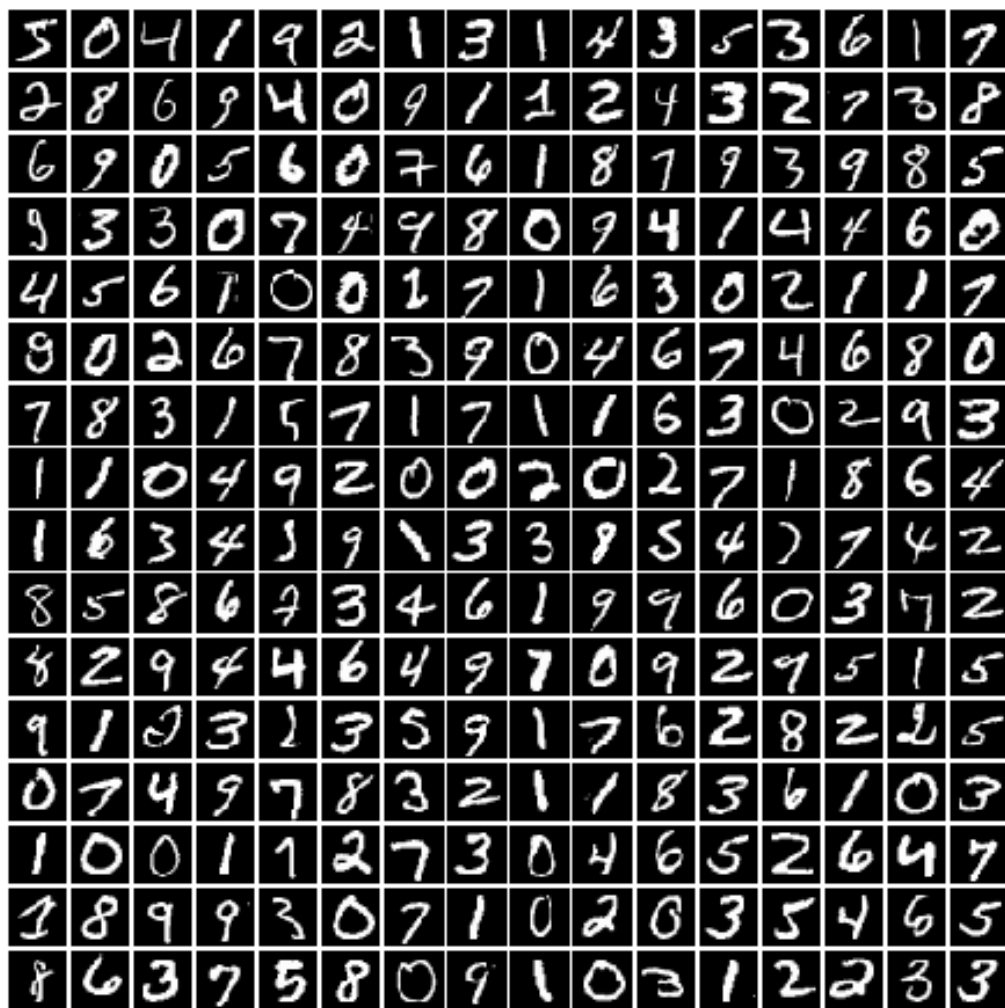
Affine Model

$$\hat{y} = x * \omega + b$$

Loss Function

$$loss = (\hat{y} - y)^2 = (x \cdot \omega - y)^2$$

# Classification – The MNIST Dataset



The database of handwritten digits

- Training set: 60,000 examples,
- Test set: 10,000 examples.
- Classes: 10

分类的结果不应该是一个可以比较大小的标量  
分类算法的结果应该是属于每一个分类的概率

```
import torchvision
train_set = torchvision.datasets.MNIST(root='../dataset/mnist', train=True, download=True)
test_set = torchvision.datasets.MNIST(root='../dataset/mnist', train=False, download=True)
```

# Classification – The CIFAR-10 dataset

- Training set: 50,000 examples,
- Test set: 10,000 examples.
- Classes: 10

```
import torchvision
train_set = torchvision.datasets.CIFAR10(...)
test_set = torchvision.datasets.CIFAR10(...)
```

**airplane**



**automobile**



**bird**



**cat**



**deer**



**dog**



**frog**



**horse**



**ship**



**truck**



# Regression vs Classification

x (hours)	y (points)
1	2
2	4
3	6
4	?

回归 (Regression)



x (hours)	y (pass/fail)
1	0 (fail)
2	0 (fail)
3	1 (pass)
4	?

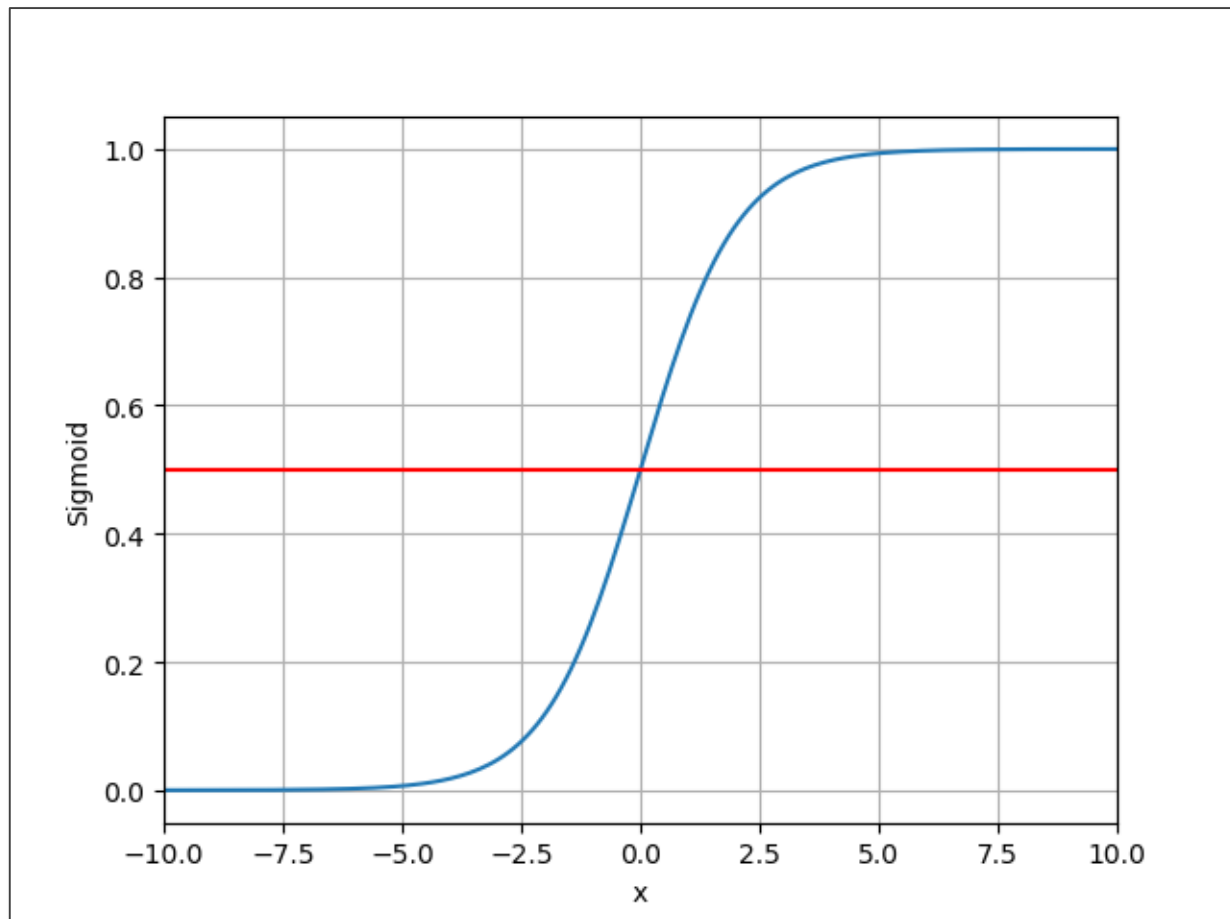
分类 (Classification)

In classification, the output of model is the probability of input belongs to the exact class.

# How to map: $\mathbb{R} \rightarrow [0, 1]$

$y_{\text{hat}} = wx + b$

$y_{\text{hat}}$ 如何转换为 $[0, 1]$ 之间的概率? —— Logistic 函数



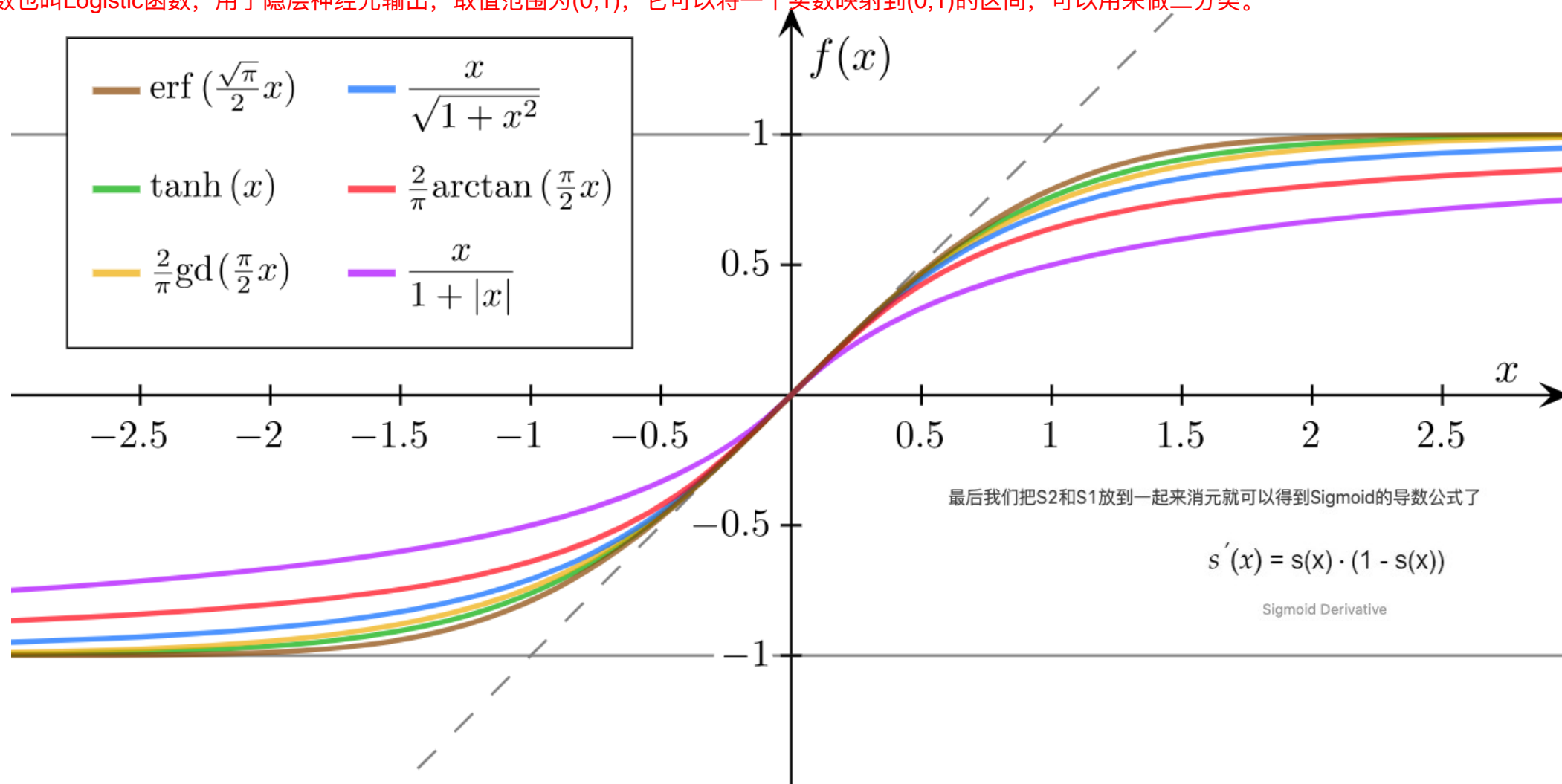
## Logistic Function

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

[https://en.wikipedia.org/wiki/Logistic\\_function](https://en.wikipedia.org/wiki/Logistic_function)

# Sigmoid functions

Sigmoid函数得名因其形状像S字母，Sigmoid函数常被用作神经网络的激活函数，将变量映射到0,1之间。  
sigmoid函数也叫Logistic函数，用于隐层神经元输出，取值范围为(0,1)，它可以将一个实数映射到(0,1)的区间，可以用来做二分类。

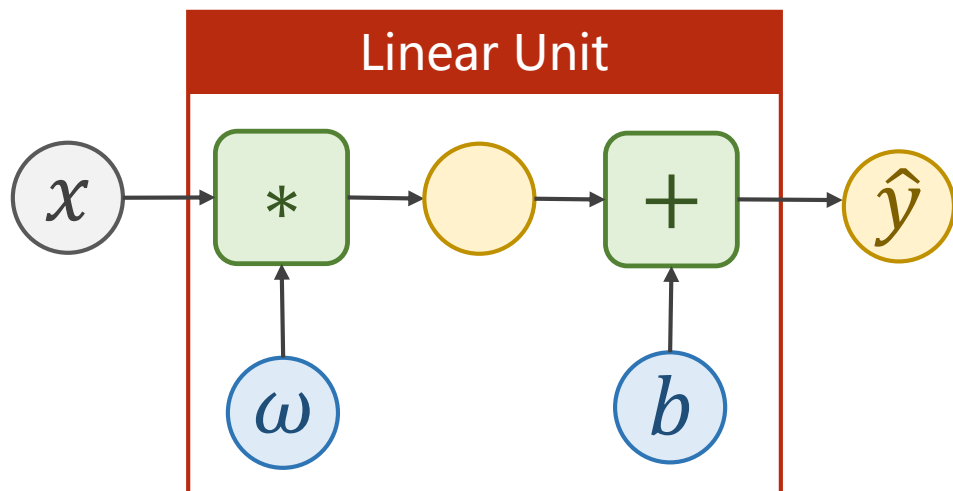




# Logistic Regression Model

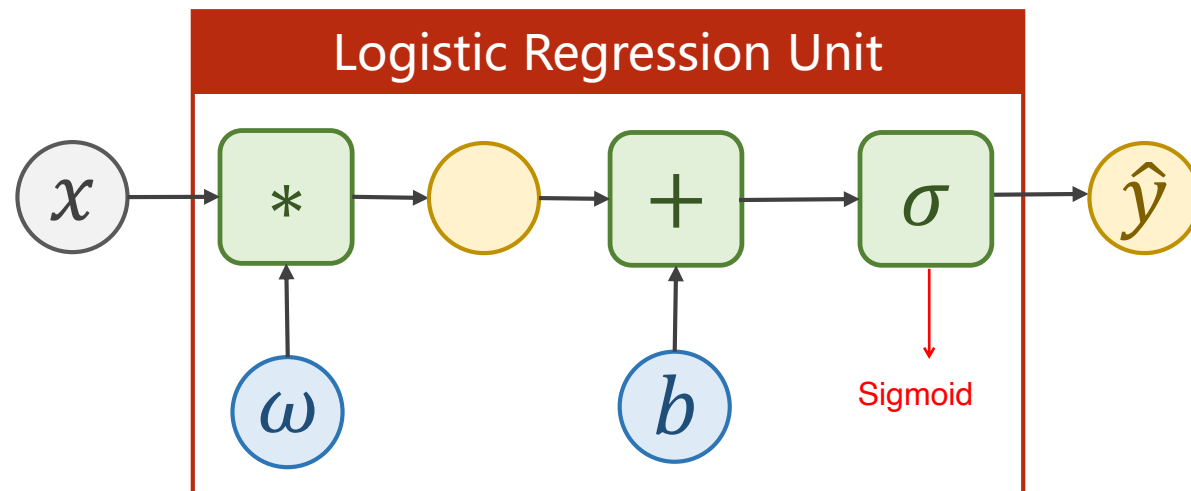
## Affine Model

$$\hat{y} = x * \omega + b$$



## Logistic Regression Model

$$\hat{y} = \sigma(x * \omega + b)$$



Sigmoid函数求导非常简单，运算快。

# Loss function for Binary Classification

Loss Function for Linear Regression

$$loss = (\hat{y} - y)^2 = (x \cdot \omega - y)^2$$



二分类交叉熵 (Binary Cross Entropy)

Loss Function for Binary Classification

$$loss = -(y \log \hat{y} + (1 - y) \log(1 - \hat{y}))$$

损失函数用来衡量不同概率分布之间的差异大小

# Mini-Batch Loss function for Binary Classification

二分类交叉熵 (Binary Cross Entropy)

Loss Function for Binary Classification

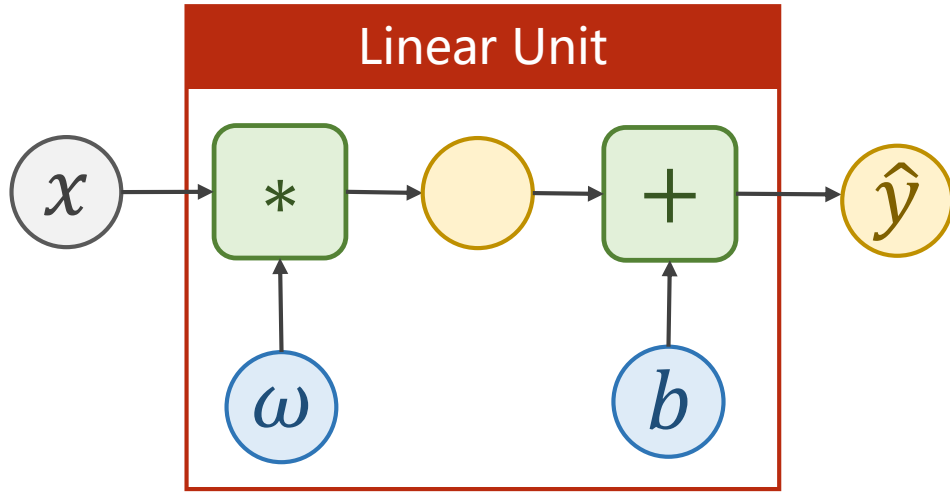
$$loss = -(y \log \hat{y} + (1 - y) \log(1 - \hat{y}))$$

Mini-Batch Loss Function for Binary Classification

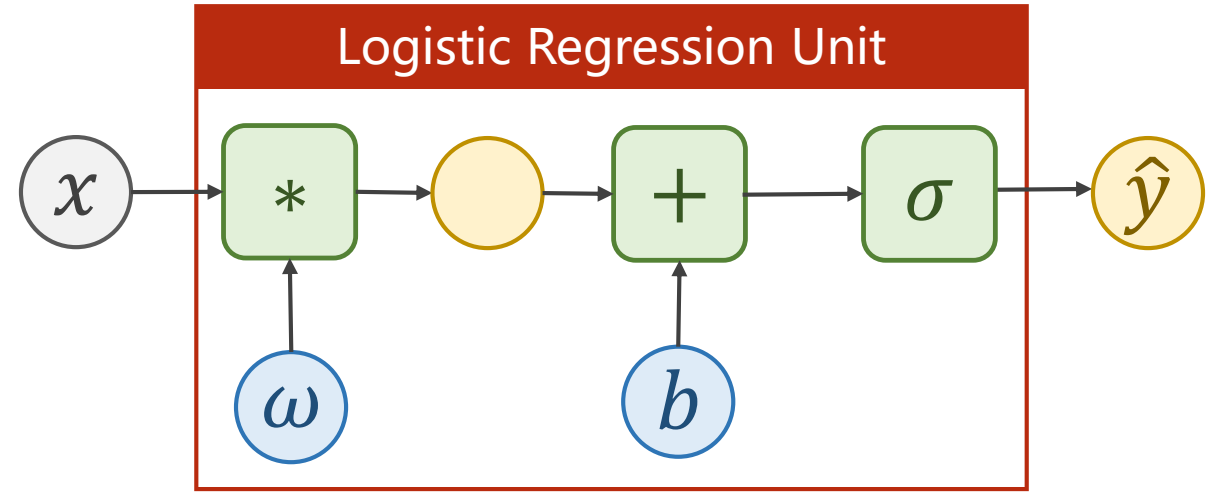
$$loss = -\frac{1}{N} \sum_{n=1}^N y_n \log \hat{y}_n + (1 - y_n) \log(1 - \hat{y}_n)$$

$y$	$\hat{y}$	BCE Loss
1	0.2	<b>1.6094</b>
1	0.8	<b>0.2231</b>
0	0.3	<b>0.3567</b>
0	0.7	<b>1.2040</b>
Mini-Batch Loss		<b>0.8483</b>

# Implementation of Logistic Regression



```
class LinearModel(torch.nn.Module):  
    def __init__(self):  
        super(LinearModel, self).__init__()  
        self.linear = torch.nn.Linear(1, 1)  
  
    def forward(self, x):  
        y_pred = self.linear(x)  
        return y_pred
```



```
import torch.nn.functional as F  
  
class LogisticRegressionModel(torch.nn.Module):  
    def __init__(self):  
        super(LogisticRegressionModel, self).__init__()  
        self.linear = torch.nn.Linear(1, 1)  
  
    def forward(self, x):  
        y_pred = F.sigmoid(self.linear(x))  
        return y_pred
```

# Implementation of Logistic Regression

## Mini-Batch Loss Function for Binary Classification

$$loss = -\frac{1}{N} \sum_{n=1}^N y_n \log \hat{y}_n + (1 - y_n) \log(1 - \hat{y}_n)$$

```
criterion = torch.nn.BCELoss(size_average=False)
```

# Implementation of Logistic Regression

```
x_data = torch.Tensor([[1.0], [2.0], [3.0]])
y_data = torch.Tensor([[0], [0], [1]])
#-----#

class LogisticRegressionModel(torch.nn.Module):
    def __init__(self):
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        self.linear = torch.nn.Linear(1, 1)

    def forward(self, x):
        y_pred = F.sigmoid(self.linear(x))
        return y_pred

model = LogisticRegressionModel()
#-----#

criterion = torch.nn.BCELoss(size_average=False)
optimizer = torch.optim.SGD(model.parameters(), lr=0.01)
#-----#

for epoch in range(1000):
    y_pred = model(x_data)
    loss = criterion(y_pred, y_data)
    print(epoch, loss.item())

    optimizer.zero_grad()
    loss.backward()
    optimizer.step()
```

1

Prepare dataset  
we shall talk about this later

# Implementation of Logistic Regression

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2

Design model using Class  
inherit from nn.Module

# Implementation of Logistic Regression

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3

Construct loss and optimizer  
using PyTorch API



# Implementation of Logistic Regression

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4

Training cycle  
forward, backward, update

# Implementation of Logistic Regression

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1

Prepare dataset  
we shall talk about this later

2

Design model using Class  
inherit from nn.Module

3

Construct loss and optimizer  
using PyTorch API

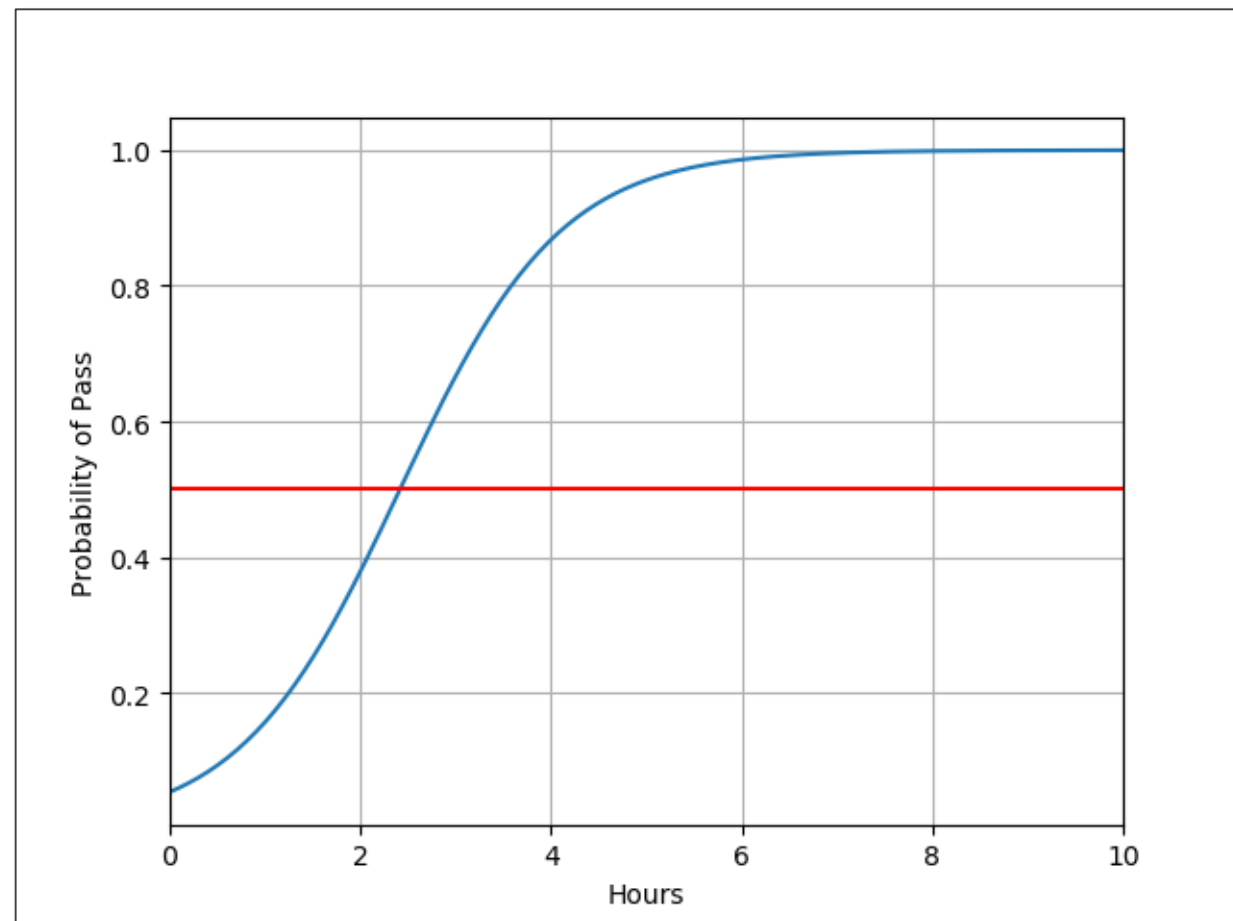
4

Training cycle  
forward, backward, update

# Result of Logistic Regression

```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(0, 10, 200)
x_t = torch.Tensor(x).view((200, 1))
y_t = model(x_t)
y = y_t.data.numpy()
plt.plot(x, y)
plt.plot([0, 10], [0.5, 0.5], c='r')
plt.xlabel('Hours')
plt.ylabel('Probability of Pass')
plt.grid()
plt.show()
```





# PyTorch Tutorial

## 06. Logistic Regression