Assignment 1

Foundations of Audio Signal Processing

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Exercise 1.1

a)

$$(4-3i)(2+2i) = 8+8i-6i+6 = 14+2i$$

b)

$$(3-5i)^{-1} = \frac{1}{3-5i} = \frac{3+5i}{(3-5i)(3+5i)} = \frac{3+5i}{9+25} = \frac{3+5i}{34} = \frac{3}{34} + \frac{5}{34}i$$

c)

$$2e^{\frac{i\pi}{4}} + 2e^{i\pi} = 2(e^{\frac{i\pi}{4}} + e^{i\pi}) = 2((-1)^{\frac{1}{4}} + (-1) = 2(-1)^{\frac{1}{4}} - 2 = \frac{2}{\sqrt{2}} + \frac{2i}{\sqrt{2}} - 2 = \sqrt{2} - 2 + \sqrt{2}i$$

d)

$$6i\left(\frac{1-i}{1+i}\right)^2 = 6i\left(\frac{(1-i)^2}{(1+i)^2}\right) = 6i\left(\frac{1-2i+i^2}{1+2i+i^2}\right) = 6i\left(\frac{1-2i-1}{1+2i-1}\right) = 6i\left(\frac{-2i}{2i}\right) = -6i$$

e)

$$\begin{aligned}\frac{i(5-i)}{(1-i)(5+i)} &= \frac{5i+1}{5+i-5i-i^2} = \frac{5i+1}{6-4i} = \frac{(5i+1)(6+4i)}{(6-4i)(6+4i)} \\ &= \frac{30i-20+6+4i}{36+24i-24i+16} = \frac{-14+34i}{52} = \frac{-7+17i}{26} \\ &= -\frac{7}{26} + \frac{17}{26}i\end{aligned}$$

Exercise 1.2

a)

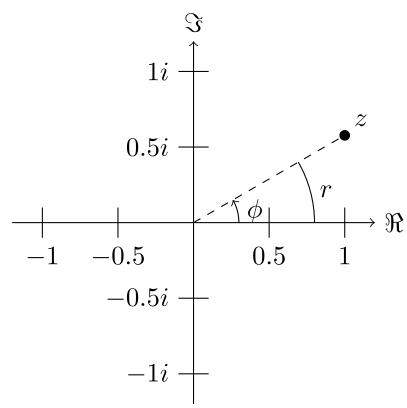
$$1 + i\frac{1}{\sqrt{3}}$$

$$z = a + bi \to |z| = \sqrt{a^2 + b^2} : |z| = \sqrt{1 + \frac{1}{3}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$

$$|z| = \frac{2}{\sqrt{3}} = r : r(\cos\phi + i\sin\phi) = \frac{2}{\sqrt{3}}(\frac{\sqrt{3}}{2} + i\frac{1}{2})$$

$$\phi = \arctan\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$z = \frac{2}{\sqrt{3}}e^{i\frac{\pi}{6}}$$



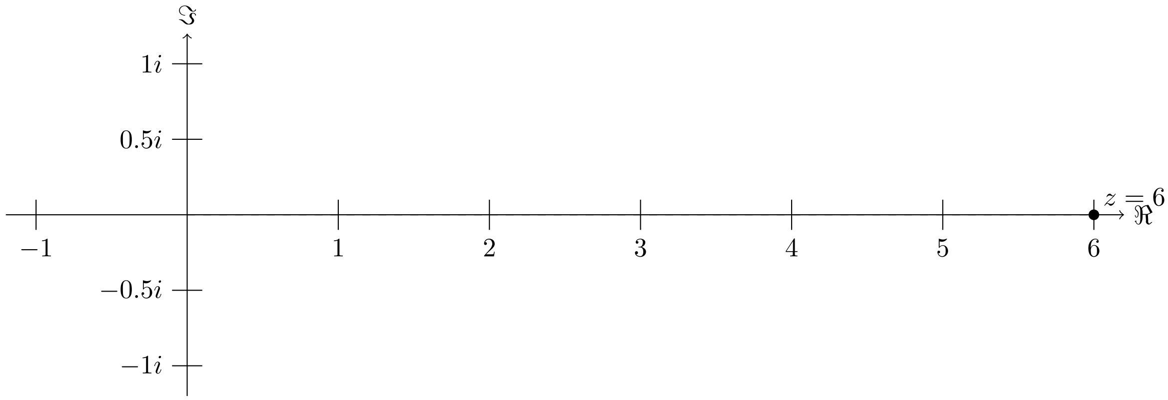
b)

$$6 = a + bi \to |z| = \sqrt{6^2 + 0} = 6$$

$$z = re^{i\phi} = 6(\cos\phi + i\sin\phi) = 6(1 + 0)$$

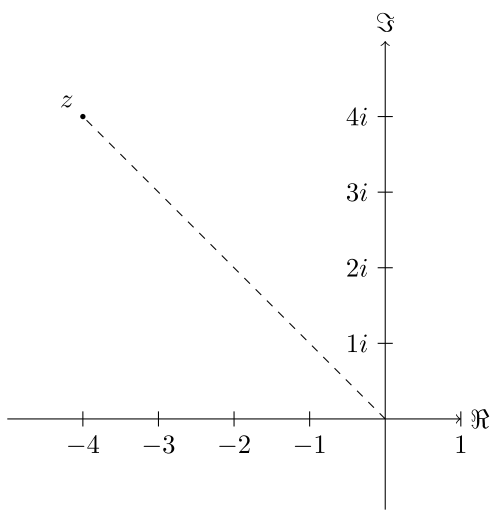
$$\phi = \arctan\left(\frac{0}{1}\right) = 0$$

$$z = 6e^{i0} = 6$$



c)

$$
\begin{aligned}
&-4+4i = a+bi \rightarrow |z| = |r| = \sqrt{(-4)^2+4^2} = 4\sqrt{2} \\
&re^{i\phi} = r(\cos\phi + i\sin\phi) = 4\sqrt{2}(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) \\
&\phi = \arctan\left(\frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}\right) = \frac{3\pi}{4} \\
&z = 4\sqrt{2}e^{i\frac{3\pi}{4}}
\end{aligned}
$$



d)

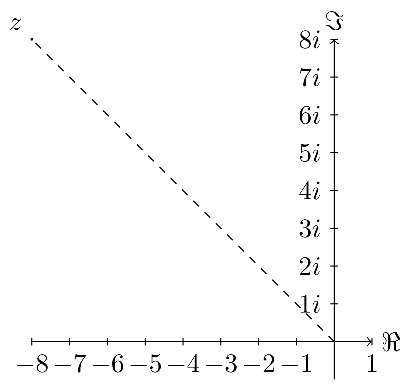
$$(-1+i\sqrt{3})^4 = (a+bi)^4 = z^4 = (re^{i\phi})^4 : r = \sqrt{1+3} = 2$$

$$2^4(-\frac{1}{2}+i\frac{\sqrt{3}}{2})^4 = 2^4((-\frac{1}{2})^2 - (\frac{\sqrt{3}}{2})^2 + i(-\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}))^2 = 2^4(-\frac{1}{2}+i(-\frac{\sqrt{3}}{2}))^2$$

$$= 2^4((-\frac{1}{2})^2 - (-\frac{\sqrt{3}}{2})^2 + i(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4})) = 2^4(-\frac{1}{2}+i\frac{\sqrt{3}}{2})$$

$$\phi = \arctan\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right) = \frac{2\pi}{3}$$

$$(z)^4 = 2^4(e^{i\frac{2\pi}{3}})$$



e)

$$\frac{e^{-i\frac{\pi}{6}}}{1-i\sqrt{3}} = \frac{z}{w}$$

$$w = 1 - i\sqrt{3} \rightarrow |w| = |r| = \sqrt{1+3} = 2$$

$$\rightarrow w = 2\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 2(\cos\varphi + i\sin\varphi)$$

$$\mathrm{Arctg}\left(\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = \mathrm{Arctg}(-\sqrt{3}) = \varphi = -\pi/3$$

$$\rightarrow \frac{z}{w} = \frac{e^{-i\pi/6}}{2e^{-i\pi/3}} = \frac{e^{i\pi/3}}{e^{i\pi/6}} = \frac{e^{i\frac{\pi}{6}}}{2}$$

$$\Rightarrow \frac{1}{2}\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) \Rightarrow \left(\frac{\sqrt{3}}{4}, \frac{1}{4}\right)$$