HW2 - Dror Ettlinger-Levy

1. Iterative techniques are more efficient for large systems, uses less memory because they don't use storing the full matrix.
Iterative ~~teq~~ techniques also works well with matrices with alot of zeros.

2.
$$\begin{cases} 17x_1 - 2x_2 - 3x_3 = 500 \\ -5x_1 + 21x_2 - 2x_3 = 200 \\ -5x_1 - 5x_2 + 22x_3 = 30 \end{cases}$$

$$\begin{cases} x_1 = \dfrac{500 + 2x_2 + 3x_3}{17} \\ x_2 = \dfrac{200 + 5x_1 + 2x_3}{21} \\ x_3 = \dfrac{30 + 5x_1 + 5x_2}{22} \end{cases}$$

Iterations

	0	1
x_1	0	$x_1^{(1)} = \dfrac{500 + 2\cdot 0 + 3\cdot 0}{17} = 29.41$
x_2	0	$x_2^{(1)} = \dfrac{200 + 5\cdot 0 + 2\cdot 0}{21} = 9.52$
x_3	0	$x_3^{(1)} = \dfrac{30 + 5\cdot 0 + 5\cdot 0}{22} = 1.36$

iteration 2

$$x_1^{(2)} = \frac{1}{17}\left(500 + 2\cdot 9.52 + 3\cdot 1.36\right) = 30.77$$

$$x_2^{(2)} = \frac{1}{21}\left(200 + 5\cdot 29.41 + 2\cdot 1.36\right) = 16.65$$

$$x_3^{(2)} = \frac{1}{22}\left(30 + 5\cdot 29.41 + 5\cdot 9.52\right) = 10.21$$

iterations

$$x_1^{(3)} = \frac{1}{17}(500 + 2\cdot 16.85 + 3\cdot 10.211) = \boxed{33.17}$$

$$x_2^{(3)} = \frac{1}{21}(200 + 5\cdot 30.77 + 2\cdot 10.211 = \boxed{17.82}$$

$$x_3^{(3)} = \frac{1}{22}(30 + 5\cdot 30.77 + 5\cdot 16.65) = \boxed{12.14}$$

the condition to convert is if the system is dominated by it main variable → the absolute value of a number in front of the variable must be bigger then the sum of the absolute values in the row.

like:

$|17| > |-2|$ and

$|21| > |-5|$

③ $\begin{cases} -5x_1 + 12x_3 = 80 \\ 4x_1 - x_2 - x_3 = -2 \\ 6x_1 + 8x_2 = 45 \end{cases}$

$$x_1 = \frac{-12x_3 + 80}{-5}$$

$$x_2 = \frac{-2 + x_3 - 4x_1}{-1}$$

$x_3 =$ ~~45~~ directly from ~~x~~ x_1

⊛ → $x_3 = \dfrac{80 + 5x_1}{12}$

iteration 0 → $x_1 = 0$ $x_2 = 0$ $x_3 = 0$

(3) <u>iteration 1</u>

$$x_1^{(1)} = \frac{12\cdot 0 + 80}{-5} = -16$$

$$x_2^{(1)} = \frac{-2+0 - 4\cdot(-16)}{-1} = -62$$

$$x_3^{(1)} = \frac{80+5\cdot(-16)}{12} = 0$$

<u>iteration 2</u>

$$x_1^{(2)} = \frac{-12\cdot 0 + 80}{-5} = -16$$

$$x_2^{(2)} = \frac{-2+0 - 4\cdot(-16)}{-1} = -62$$

$$x_3^{(2)} = \frac{80+5\cdot(-16)}{12} = 0$$

also for iteration 3 ...

differences: jacobi updates variables simultaneously
Gauss-Seidel updates variables immediately

Advantages: Jacobi easy for parallel
Gauss-Seidel converges faster

Disadvantages: Jacobi slower
Gauss-Seidel can be less stable compared to Jacobi (like in the exercise)

(4) (a) SOR (successive over relaxtion) is like Gauss-Seidel method for solving linear system equations, but faster.

it adds a relaxation factor w to adjust the solution for each step

(b) it is advises to use for when gauss-Seidel is working but slowly.

(c) it is NOT always better, if w is picked wrong so it can be unstable and slow.