# Задание 3

1. Функция логистической регрессии:

$$Q(w) = \frac{1}{n} \sum_{i=1}^{n} \ln\left(1 + e^{-y_i \langle w, x_i \rangle}\right)$$

$$Q_r(w) = Q(w) + \frac{\lambda}{2} ||w|| = Q(w) + \frac{\lambda}{2} \sum_{j=1}^{m} w_j^2$$

2. Градиент

$$\frac{\partial Q_r}{\partial w_j} = \frac{1}{n} \sum_{i=1}^{n} -\frac{e^{-y_i \langle w, x_i \rangle} \cdot y_i \cdot x_{ij}}{1 + e^{-y_i \langle w, x_i \rangle}} + \lambda w_j = -\frac{1}{n} \sum_{i=1}^{n} -\frac{y_i \cdot x_{ij}}{1 + e^{y_i \langle w, x_i \rangle}} + \lambda w_j$$

$$\nabla Q = -\frac{1}{n} A_{m,n} b_n + \lambda w_m$$

$$A_{j,i} = y_i \cdot x_{ij}$$

$$b_i = \frac{1}{1 + e^{y_i \langle w, x_i \rangle}}$$

3. Гессиан

$$\frac{\partial^2 Q_r}{\partial w_j \partial w_k} = \frac{\partial}{\partial w_k} \frac{\partial Q_r}{\partial w_j} = \frac{1}{n} \sum_{i=1}^{n} \frac{e^{y_i \langle w, x_i \rangle} \cdot y_i^2 \cdot x_{ij} \cdot x_{ik}}{(1 + e^{y_i \langle w, x_i \rangle})^2} + \lambda \frac{\partial w_j}{\partial w_k}$$

$$\left( \frac{\partial^2 Q_r}{\partial^2 w} \right)_{m,m} = \frac{1}{n} C_{m,n} \cdot D_{n,m} + \lambda I_m$$

$$C_{j,i} = \frac{x_{ij}}{1 + e^{y_i \langle w, x_i \rangle}}$$

$$D_{i,j} = \frac{e^{y_i \langle w, x_i \rangle} \cdot x_{ik}}{1 + e^{y_i \langle w, x_i \rangle}} = \frac{x_{ik}}{1 + e^{-y_i \langle w, x_i \rangle}}$$