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Research paper

Node similarity measuring in complex networks with relative entropy

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A B S T R A C T

Measuring the similarity of nodes in complex network has been significant research in the analysis of complex characteristic. Several existing methods have been proposed to address this problem, but most of them have their own limitations and shortcomings. So a novel method based on relative entropy is proposed to solve the problems above. The proposed entropy combines the fractal dimension of the whole network and the local dimension of each node on the basis of Tsallis entropy. When the fractal dimension equals to 1, the relative entropy would degenerate to classic form based on Shannon entropy. In addition, relevance matrix and similarity matrix are used to show the difference of structure and the similarity of each pair of nodes. The ranking results show the similarity degree of each node. In order to show the effectiveness of this method, four real-world complex networks are applied to measure the similarity of nodes. After comparing four existing methods, the results demonstrate the superiority of this method by employing susceptible-infected (SI) model and the ratio of mutual similar nodes.

1. Introduction

Complex network has attracted extensive attention in many fields [1] since the discovery of small-world and scale-free properties of networks, including messages spreading [2,3], multi-fractal geometry [4,5], and nonlinear rating projection [6]. In recent years, more efforts have been made to reveal other properties of complex network [7], such as the fractal [8,9] and self-similarity properties of complex network [10–12]. The discovery of these properties has enabled the complex network to solve practical problems in the real world, like decay synchronization [13], information spreading [14], detecting early-warning signals [15], and solving varying preferential dynamics [16]. Because most of the previous research are based on the structure characteristics of complex network, quantifying the structure characteristics has become the focus of complex network. Particularly, node similarity measurement has important theoretical and practical significance, which gradually aroused researchers' interest. For instance, the similarity of different nodes is an important indicator to identify the influential nodes [17–19], assess the relevance of individual characteristics [20], predict potential links [21,22], measure the influence on the wring patterns [23], and detect overlapping community [24].

In general, a selected node has several different similar nodes in the network. Different similarity measuring methods would obtain different similar ranking orders because of the different consideration of network information. Several methods have been proposed to measure the similarity of nodes, and they can be divided into three different categories, which are

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local similarity indices, global similarity indices, and quasi-local indices. These indices contain some classical methods, like Common Neighbors (CN), Katz Index [25], Local Path Index (LP) [26], Local Random Walk [27], and many other measures. These indices have been wildly used in various studies due to its board applicability, such as the information mining in biological network [28], the analysis of social network [29], the process of recommending to users [30], and the classification problem in partially labeled networks [31]. However, these existing methods have their own limitations and shortcomings. For example, CN only considers the neighbor nodes around the selected node which is a narrow horizon. Katz Index is based on the network structure, but the computational complexity is much higher than other methods with the increase of the number of nodes. Recently, some new similarity measuring methods are proposed. For example, Mheich et al. [32] proposed an algorithm called SimiNet to measure similarity when nodes are defined as a priori within a three coordinate system. Wu et al. [33] proposed an effective similarity index based on spatial-temporal position drift model. Zhang et al. [34] proposed a method based on Shannon entropy which focuses on the degree of nodes.

Since entropy can quantify the uncertainty of information [35] of complex network, it has been wildly applied in network mining, such as influential nodes identifying [36,37], topological vulnerability evaluating [38], game theory [39,40], evidence theory [41–43], and dimension presentation [44,45]. Most of the network structure characteristics can be transformed into probability sets. Therefore, the entropy approach gradually becomes an effective measure to explore the properties of complex network.

In this paper, a novel node similarity measure is proposed based on relative entropy. This proposed method combines the local dimension of each node and fractal dimension of complex network, which concentrates on the local structure and whole structure respectively. Unlike previous methods, this proposed method considers both local structure and whole structure rather than only one aspect of complex network. This measure is based on Tsallis entropy [46], so it would degenerate to Shannon entropy based on local dimension when the fractal dimension of complex network equals to 1. In order to show the effectiveness of this method, four real-world complex networks and four different classes of comparison methods are applied in this paper. Furthermore, the ratio of mutual similar nodes and SI model [47] are used to evaluate the performance of this method.

The organization of the rest of this paper is as follows. Section 2 presents the detail model of relative entropy to measure the similar node of each node. Some classical similarity measures are given in Section 3. Meanwhile, numerical experiments are performed to illustrate the effectiveness of the proposed method. Conclusion is conducted in Section 4.

2. Relative entropy based on Tsallis entropy

In this section, a novel relative entropy (RE) is proposed based on Tsallis entropy to measure the similarity of nodes. This proposed method can combine two properties which can consider more details in the network and give a different result which overcomes the shortcomings of previous methods. The flow chart of this proposed method is shown in Fig. 1.

2.1. Fractal dimension

First of all, the fractal dimension of complex network should be obtained, because it is a significant property to describe the uncertain information of network, and it can reveal the structure, fractal, and self-similarity properties. There are

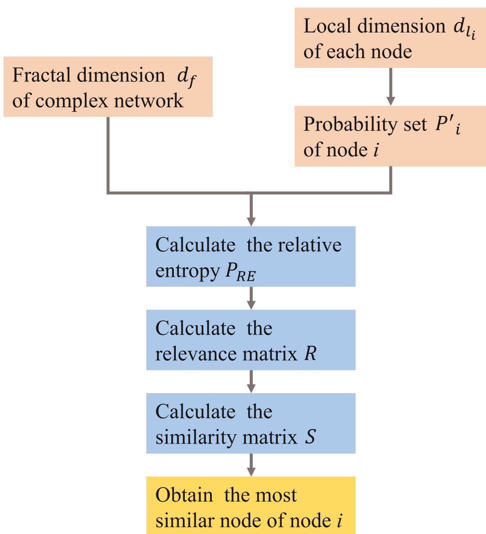

Fig. 1. The flow chart of this proposed method.

several methods to obtain the fractal dimension of complex network, and the box-covering algorithm for fractal dimension of weighted networks (BCANw) [48] is applied in this paper.

For a given complex network G(V, E), V is the set of nodes and E is the set of edges. The shortest distance d_{ij} between node i and node j is shown as follows,

$$d_{ij} = x_{ik_1} + x_{k_1k_2} + \cdots x_{k_mj} \tag{1}$$

where k_1, k_2, $\cdots$, k_m are the IDs of node on the shortest way, $x_{k_1k_2}$ is the edge between node k_1 and node k_2.

When the shortest distance between any two nodes is less than the box size, they would be put into the same box, and the whole network would be covered by boxes in the end. The fractal dimension of network can be obtained as follows,

$$d_f = -\lim_{s\to 0} \frac{\ln N(s)}{\ln s} \tag{2}$$

where $N(s)$ is the number of boxes when the box size equals s. The box size s is increasing from the minimum value of shortest distance to the size that one box can cover the whole network. The fractal dimension can be obtained by the slope of fitted line between $\ln N(s)$ and $\ln s$.

2.2. Local dimension and probability set

The local dimension of node is also an important index to reveal the local structure of complex network which can find the similarity between different nodes. The local dimension is based on a central node to describe the local structure. Therefore, the dimension varies according to the central nodes. When a central node i is chosen, the radius of box gradually grows from the shortest distance from the central node to the largest distance in the network. For each central node, the whole network would be covered by one box in the end. The local dimension of node i is shown as follows,

$$d_{l_i} = r\frac{n_i(r)}{N_i(r)} \tag{3}$$

where $n_i(r)$ is the number of node whose shortest distance from central node i equals to radius r, and $N_i(r)$ is the number of node whose shortest distance from central node i is less than radius r (include r). The local dimension of node i would be calculated by the slope between $N_i(r)$ and r on a double logarithmic scale.

The local network of node i contains the central node i and the nodes directly connected to the central node i, so it can be shown as $G_i(V, L)$, where V is the set of nodes in the local network and L is the set of local dimensions of nodes in set V. The degree of node i is defined as d_i, and the largest value of node degree can be obtained by $D = \max_{i\in V} d_i$, and the scale of probability set can be obtained as follows,

$$m = D + 1 \tag{4}$$

Because the biggest local network contains $|D|$ neighbor nodes and central node itself, Eq. (4) is suitable for all nodes in the network. The probability set of node i is shown as follows,

$$P_i = [p_i(1), p_i(2), \cdots, p_i(k), \cdots, p_i(m)] \tag{5}$$

When the degree of central node d_i equals to D, the whole element of probability set is obtained based on the local dimension. But when the degree of central node d_i is smaller than D, some elements of probability set would equal to zero. The detail definition of $p_i(k)$ is shown as follows,

$$p_i(k) = \begin{cases} \frac{d_{l_k}}{\sum_{k=1}^{m} d_{l_k}} & k \le d_i + 1 \\ 0 & k > d_i + 1 \end{cases} \tag{6}$$

where d_{l_k} is the local dimension of node k, d_i is the degree of central node i.

2.3. Relative entropy

The proposed relative entropy based on Tsallis entropy is applied to measure the similarity of nodes in this subsection. The traditional relative entropy (Kullback-Leibler divergence) [49] in information theory is to measure the difference between two different probability distributions, and it is shown as follows,

$$D_{KL}(P||Q) = \sum_{i=1}^{n} P(i) \ln \frac{P(i)}{Q(i)} \tag{7}$$

where P and Q are two different probability distributions, but have the same number of components.

We modify the traditional relative entropy and obtain a novel relative entropy based on Tsallis entropy to measure the similarity between different nodes, and it is shown as follows,

$$P_{RE}(P'(i)\,\|\,P'(j)) = k\sum_{k=1}^{m'} \frac{\left(\frac{p_i'(k)}{p_j'(k)}\right)^{d_f} - \left(\frac{p_i'(k)}{p_j'(k)}\right)}{1-d_f} \tag{8}$$

where k equals to constant 1, d_f is the fractal dimension of complex network. Because the element order in each probability distribution would affect the value of relative entropy and the similarity result. The order in probability set needs adjustment in advance, $P'(i)$ and $P'(j)$ are the decreasing order of $P(i)$ and $P(j)$ in Eq. (5). m' can be obtained by

$$m' = \min(k_i, k_j) + 1 \tag{9}$$

The purpose of m' is to avoid $\frac{p_i'(k)}{p_j'(k)}$ being 0 or positive infinity, which is beneficial for the calculation. When the fractal dimension d_f equals to 1, the relative entropy based on Tsallis entropy would degenerate to traditional relative entropy based on local dimension.

2.4. Similarity measuring

Finally, the similarity degree of each pair of nodes can be obtained. The value of relative entropy between two nodes shows the difference between their structure. Based on the relative entropy, a relevance matrix R of the whole network is shown as follows,

$$R = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & r_{nn} \end{pmatrix} \tag{10}$$

where n is the number of nodes in the network. Because the relative entropy is not symmetry, some changes are needed to accommodate the symmetry of the relevance matrix R. The value of the element can be obtained as follows,

$$r_{ij} = P_{RE}(P'(i)\,\|\,P'(j)) + P_{RE}(P'(j)\,\|\,P'(i)) \tag{11}$$

Based on this, the relevance matrix would be symmetry matrix ($r_{ij} = r_{ji}$). The relevance matrix can show the relationship between any two nodes structure in the network. This matrix is based on local dimension of node and fractal dimension of network, which considers more information in the network. The bigger value of the element of relevance matrix means there are more differences in their structure. So a similarity matrix is calculated to show the similarity between any two nodes, and it is shown as follows,

$$S = \begin{pmatrix} s_{11} & \cdots & s_{1n} \\ \vdots & \ddots & \vdots \\ s_{n1} & \cdots & s_{nn} \end{pmatrix} \tag{12}$$

where s_{ij} is the similarity of each pair of nodes, and it is obtained as follows,

$$s_{ij} = 1 - \frac{r_{ij}}{\max\limits_{i,j\in V}(r_{ij})} \tag{13}$$

The element in similarity matrix shows the similarity degree of any two nodes in the network. When two nodes are more similar, the difference between them is less, so r_{ij} is closer to $\max\limits_{i,j\in V}(r_{ij})$ and s_{ij} is closer to 0.

3. Experiments and results

3.1. Data

In this section, four real-world complex networks are applied to show the relationship between CN, Katz, LP, LRE, RE indices.

These four real-world complex networks are Jazz musicians network (Jazz) [50], USAir lines network (USAir) [51], Zacharys Karate Club network (Karate) [52], and political blogs network (Blog) [53]. The topological properties of these four real-world complex networks are shown in Table 1. $\langle k\rangle$ and $k_{\max}$ are the average and maximum value of degree, $\langle d\rangle$ and $d_{\max}$ are the average and maximum value of shortest distance.

Table 1
The topological properties of different networks.

Network	Nodes	Edges	$\langle k\rangle$	$k_{\max}$	$\langle d\rangle$	$d_{\max}$
Karate	34	78	4.5882	17	2.3374	5
USAir	332	2126	12.8072	139	2.7299	6
Jazz	198	5484	27.6970	100	2.2238	6
Blog	1222	19,021	27.352	351	2.7353	8

3.2. Comparison methods

Some existing methods are introduced as the comparison method in this paper. There are four different kinds of methods which focus on different aspects. They are Common Neighbors (local similarity index), Katz Index (global similarity index), Local Path Index (quasi-local index), Local Relative Entropy (entropy index).

Common Neighbors (CN) is a local similarity index. For any two nodes, if there are more common nodes between them, they would be more similar. It can be obtained as follows,

$$S_{xy}{}^{CN} = |\Gamma(x) \cap \Gamma(y)| = (A^2)_{xy} \tag{14}$$

where $\Gamma(x)$ and $\Gamma(y)$ are the sets of neighbor node of x and y respectively, and $|X|$ is the cardinality of the set X. $(A^2)_{xy}$ demonstrates the number of paths which connect node x and y with length 2, and A is the adjacency matrix of the complex network. The element of adjacency matrix A_{xy} shows whether two nodes have a connection, $A_{xy} = 1$ demonstrates there is a connection between node x and y and $A_{xy} = 0$ is the opposite situation.

Katz index [25] is a different method from CN, and it is a global similarity index. This index focuses on the global structure, which sums over the different length of paths between two nodes and gives shorter paths more weight. The detail of this index is shown as follows,

$$S_{xy}{}^{Katz} = \sum_{l=1}^{\infty} \beta^l \left| path_{xy}^{<l>} \right| = \beta A_{xy} + \beta^2 (A^2)_{xy} + \beta^3 (A^3)_{xy} + \cdots \tag{15}$$

where $path_{xy}^{<l>}$ is the set of paths which connects node x and node y with length l, and β is the weight parameter depending on the path length. The parameter β must be smaller than reciprocal of the largest eigenvalue of the adjacency matrix which can make Eq. (15) convergence.

Local Path Index (LP) [26] is a quasi-local index. This index is proposed for a tradeoff between computational complexity and accuracy which is based on local path but better than CN. It is shown as follows,

$$S_{xy}{}^{LP} = (A^2)_{xy} + \varepsilon (A^3)_{xy} \tag{16}$$

where ε is a free parameter. When $\varepsilon = 0$, this index would degenerate to CN. $(A^3)_{xy}$ is the number of paths which connected node x and y with length 3.

Local Relative Entropy (LRE) [34] is a method based on entropy approach. This method is based on the degree of each node and uses Shannon entropy to measure the similarity of pair of nodes. The detail of this method is shown as follows,

$$D_{KL}(P(i)||P(j)) = \sum_{k=1}^{m} p(i,k) \ln \frac{p(i,k)}{p(j,k)} \tag{17}$$

where m is the smaller one in degree of node i and node j, $p(i, k)$ is shown as follows,

$$p(i,k) = \begin{cases} \frac{D(k)}{\sum\limits_{k=1}^{m} D(k)} & k \le Degree(i) + 1 \\ 0 & k > Degree(i) + 1 \end{cases} \tag{18}$$

where $D(k)$ is the degree of node k.

3.3. The ratio of mutual similar nodes

Firstly, the ratio of mutual similar nodes of complex network is applied to show the efficiency of this proposed method. In general, if node A′s most similar node is node B, the most similar node of B would have large probability to be node A. So if the above logic is true, node A and node B would be mutual similar node. According to the above argument, the criterion for judging such methods is that the more mutual similar nodes are found in the network, the more effective the method is. The ratio of mutual similar node P(s) is defined as follows,

$$\mathrm{P}(s) = \frac{N_s}{N} \tag{19}$$

where N_s is the number of mutual similar nodes, N is the total number of nodes in complex network. The details of P(s) in different networks and methods are shown in Table 2. $Katz_1$ represents that the Katz index only considers the edges which

Table 2
The ratio of mutual similar node in different networks.

P(S)	CN	$Katz_1$	$Katz_2$	$Katz_3$	LP	LRE	RE
Karate	0.2941	0.1176	0.1765	0.1765	0.1765	0.5471	0.5294
USAir	0.0783	0.0843	0.0602	0.0542	0.3735	0.5542	0.5663
Jazz	0.1111	0.0505	0.1111	0.0505	0.1111	0.5364	0.5354
Blog	0.0180	0.2357	0.0327	0.0131	0.0327	0.4566	0.4681

connect node x and y directly, $Katz_2$ represents that the Katz index considers the connections between node x and y equal to 1 and 2, and $Katz_3$ represents that the Katz index considers the connections between node x and y equal to 1, 2, and 3. Observed from Table 2, the value of P(s) obtained by CN is relatively low in these methods, and the P(s) obtained by Katz does not have a regular change rule when the connection length between two nodes is changed. For example, the P(s) is decreasing in Blog, and it is increasing in Karate, and it is growing first and then decreasing in Jazz. In addition, the value of P(s) obtained by LP is relatively large in previous three methods. At last, the value of P(s) obtained by LRE and RE is the largest in all of these methods. In USAir and Blog, P(s) obtained by RE is larger than LRE, and the opposite situation happens

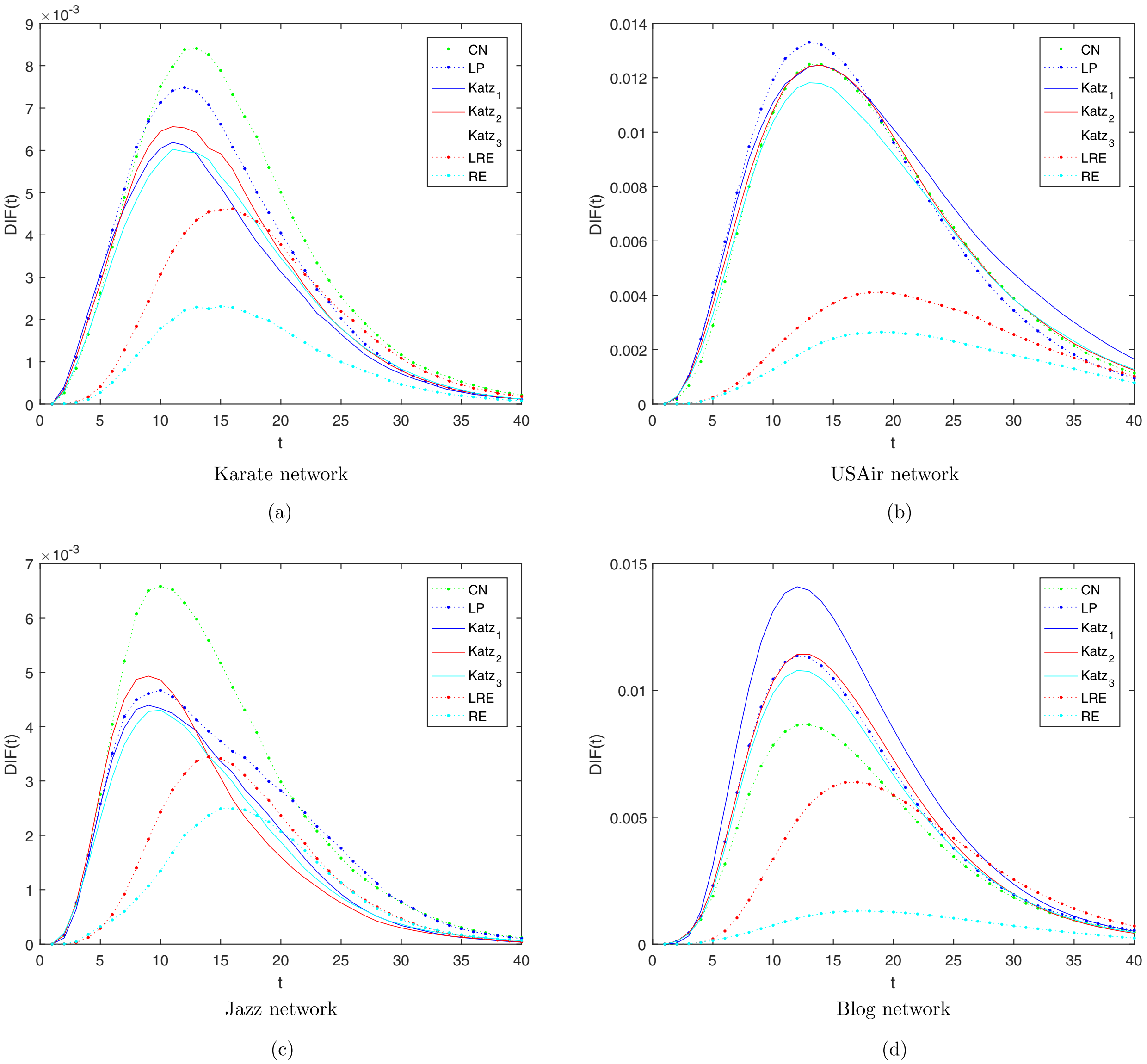

Fig. 2. The variance of affected ability difference $DIF(t)$ in different time t.

in Karate and Jazz, but the difference between these two methods is small. Based on the above results, RE and LRE are more reasonable than the previous methods, and the difference between these two methods is small.

3.4. SI model

The standard of susceptible-infected model is used to evaluate the performance of this proposed method. When two nodes are similar to each other, their influential abilities would be the same. So the epidemic spreading model is a useful tool to compare different methods. The detail steps are shown as follows,

1. Setting the key parameters of SI model firstly, like the spreading rate $\lambda = 0.125$, the affecting time $T = 40$, and repetition time $R = 100$.
2. Setting every node and their most similar node as initial nodes, and affecting other nodes in the network respectively.
3. The number of infected nodes affected by node i in time t is denoted as $f_i(t)$, and the number of infected nodes affected by the most similar node of node i in time t is denoted as $f_i'(t)$.
4. The affected ability difference between node i and its most similar node in time t can be obtained as follows,

$$d_i(t) = \left| \frac{f_i(t) - f_i'(t)}{N} \right| \tag{20}$$

   where N is the number of nodes in the network.
5. The variance of all of nodes in the network is obtained as $DIF(t) = \mathrm{var}(d_i(t))$.

The variance of affected ability difference $DIF(t)$ in different time t is shown in Fig. 2. The horizontal axis is the infect time t, and the vertical axis is the variance of affected ability difference $DIF(t)$. It can be found that the variance of affected ability difference $DIF(t)$ first grows from 0 until it reaches the peak when the time is $20 - 30$ which is different from the method, and then it falls to 0 after 40 times when all of the network would be affected after that time. Besides, the variance of affected ability difference $DIF(t)$ obtained by RE is lower than other methods during most of time. It means the similar nodes obtained by RE is more reasonable, and the difference between two nodes obtained by RE is smaller than the nodes obtained by other methods. The biggest value of affected ability difference $DIF(t)$ can be used as a quantification to show the difference between the selected node and its most similar node, and it can be obtained as follows,

$$D_{\max} = \max(DIF(t)) \tag{21}$$

The selected node and its most similar node would have similar affect abilities to the whole network when $D_{\max}$ is close to 0. The $D_{\max}$ of karate, USAir, Jazz, Blog network are 0.0023, 0.0026, 0.0025, 0.0013 respectively, and all of them are smaller than the values obtained by other methods. Because a method with small $D_{\max}$ would be more reasonable, RE would obtain more reasonable results than previous methods, and it is more effective to measure the node similarity between different nodes in complex network.

4. Conclusion

The node similarity measuring gradually becomes a focus in the network theory. In this paper, a novel relative entropy based on Tsallis entropy is proposed in order to overcome the shortcomings and limitations of existing methods. Combining fractal dimension and local dimension, this proposed entropy takes both the local structure property and whole structure property into consideration. Because the relative entropy is asymmetric between two nodes, it needs a summation operation. When two nodes are more similar, the relative entropy would be bigger and the element in similarity matrix would be closer to 0. Different from previous methods, this proposed method considers both the local structure property and whole structure property, and it would degenerate to Shannon entropy based on local dimension when the fractal dimension of complex network equals to 1. To evaluate the performance of this proposed method, four real-world complex networks and SI model are applied. Through the ratio of mutual similar nodes, the rationality of this method is demonstrated. Furthermore, by employing the variance of affected ability difference $DIF(t)$, the similarity degree between two nodes are evaluated. Compared with four existing methods (CN, Katz, LP, LRE), this measure has a more rational and effective result.

However, there are still some potential problems to be solved to improve the effectiveness of this method. One key point is that how to determine the most similar node of selected node when two nodes have the same value of s_{ij} to selected node. In future studies, the relative entropy can be extended to other forms, or the probability set can be obtained from other features in order to get a better result. Therefore, the framework of this method can be significantly improved to measure the similarity of nodes.

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