Physica A 526 (2019) 121054

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Measuring the complexity of complex network by Tsallis entropy

Tao Wen, Wen Jiang *

School of Electronics and Information, Northwestern Polytechnical University, Xi'an, shaanxi 710072, China

HIGHLIGHTS

- A novel structure entropy is proposed based on Tsallis entropy.
- This proposed method focuses on both the global structure and the local structure.
- This proposed method can be used to quantify the complexity of networks.
- Some experiments are applied to show the effectiveness of the proposed method.

ARTICLE INFO

Article history:
Received 24 December 2018
Received in revised form 4 April 2019
Available online 2 May 2019

Keywords:
Complex network
Structure entropy
Tsallis entropy
Fractal dimension

ABSTRACT

Measuring the complexity degree of complex network has been an important issue of network theory. A number of complexity measures like structure entropy have been proposed to address this problem. However, these existing structure entropies are based on Shannon entropy which only focuses on global structure or local structure. To break the limitation of existing method, a novel structure entropy which is based on Tsallis entropy is introduced in this paper. This proposed measure combines the fractal dimension and local dimension which are both the significant property of network structure, and it would degenerate to the Shannon entropy based on the local dimension when fractal dimension equals to 1. This method is based on the dimension of network which is a different approach to measure the complexity degree compared with other methods. In order to show the performance of this proposed method, a series of complex networks which are grown from the simple nearest-neighbor coupled network and five real-world networks have been applied in this paper. With comparing with several existing methods, the results show that this proposed method performs well.

1. Introduction

The complex network theory can be used in many fields [1,2] of the real world, so it has attracted many researchers' interest, including brain networks [3], statistical physics [4–6], social networks [7,8], affiliation networks [9]. A lot of new studies have been launched in recent years, like combining with visible graph [10–14], identifying influential nodes [15–18], time series [19], link prediction [20,21], and game theory [22]. Based on these, complex network produced many practical models and achieved a lot of practical results [23,24], like decision making [25,26], influence maximization [27,28], and strategies analysis [29–32]. Meanwhile, the nonlinear science in complex network has also made great progress. For instance, many dimensions which can reveal the property of networks have been put forward, like the

* Corresponding author.
E-mail address: jiangwen@nwpu.edu.cn (W. Jiang).

https://doi.org/10.1016/j.physa.2019.121054
0378-4371/© 2019 Published by Elsevier B.V.

fractal dimension [33,34], information dimension [35–37], local dimension [38], Tsallis dimension [39,40], Hausdorff dimensions [41,42], and so on. In addition, many box-covering algorithms [43] which are beneficial to the solution of dimension are being followed up at the same time, like the box-covering algorithm for complex network [44]. Because the system of real-world are often presented in the form of weighted complex network, the proposal of this algorithm is beneficial to the application of dimension [45] in the real world, such as evaluating the topological vulnerability [46,47], assigning equilibrium traffic [48], efficient spreading strategies [49], H-index of node [50], statistical vaccination [51,52]. In particular, the study on measuring the complexity degree of complex network is of significant theoretical and practical significance, such as brain network [53], earthquake [54,55].

Therefore, many methods have been designed to measure the complexity degree of complex network during the past years [56,57]. For example, Rajaram et al. [58] proposed a complexity measure which is called case-based entropy C_C and it is based on the Shannon–Wiener entropy measure H, and this method can measure and identify the distribution of the diversity of complexity between different real-world systems. Then, Hornero et al. [59] put forward a global measure method of graph complexity which is called Shannon graph complexity, the available range for this method includes weighted graphs and binary graphs. And Barriales [60] elaborated some network characterizing features and entropy-based complexity measures based on Random Network Model, and provided some views about the characterization and generation of random network. Because of the defects in previous methods, Deng et al. [61] gave a method based on structure entropy which focuses on the degree and betweenness distribution to quantify the structure complexity of complex network. Based on these methods mentioned above, it can be found that the entropy gradually becomes the focus of the complexity research, especially the structure entropy.

Because structure entropy can reveal the properties of complex network, it has been applied in many fields, such as brain system [62,63]. For instance, Xiao et al. [64] proposed a structure entropy based on automorphism partition which can quantify the structural heterogeneity of network more precisely than degree-based entropies. Meanwhile, Stanley et al. [65] compared different structure entropies to measure the ability of different index to characterize network heterogeneity, and their studies gave an expansive view of the structural complexity of network. Gao et al. [66] used network structure entropy to evaluate the survivability of military heterogeneous networks, which considers both the nodes and the edges. Lin et al. [67] also studied a recurrence network for earthquake magnitude time series based on structure entropy and dynamical evolution, which gave an approach to characterize main shocks quantitatively. In addition, Fu et al. [68] proposed a structure entropy based on betweenness importance and analyzed the survivability of public transit network. So Structure entropy has been applied in several fields in the real-world system.

In this paper, a novel structure entropy is proposed based on Tsallis entropy. This proposed method combines the fractal dimension and local dimension which are two significant dimensions in complex network to measure the structure complexity degree of complex network. The fractal dimension focuses on the whole structure and properties of network itself, like the fractal and self-similarity property, and the local dimension reveals the local structure and property of center node. Based on this foundation, this proposed structure entropy considers not only the local structure but also the whole structure, which modifies the previous method which can focus on only one of the structure property of network. Because this method is based on Tsallis entropy, it would degenerate to Shannon entropy when fractal dimension equals to 1, and the Shannon entropy is an important solution to measure the structure complexity of complex network [69] before. In order to evaluate the reliability and validity of this method, a series of complex networks which are grown from the simple nearest-neighbor coupled network and five real-world networks have been applied in this paper. With comparing with several previous methods, the results show that this proposed method performs well.

The remainder of this paper is organized as follows. Some brief overview of Tsallis entropy, structure entropy, and dimensions are given in Section 2. In Section 3, this structure entropy based on Tsallis entropy is introduced. A series of complex networks which are grown from the simple nearest-neighbor coupled network and five real-world complex networks are used to show the performance of this method in Section 4. Some conclusions are given in Section 5.

2. Preliminaries

2.1. Fractal dimension of complex network

Fractal dimension can reveal the fractal and self-similarity properties which are important for the uncertain information measuring of complex network. There are lots of methods to obtain the fractal dimension of complex network. In this section, the box-covering algorithm for fractal dimension of weighted networks (BCANw) which is introduced by Wei et al. [44] is used to obtain the fractal dimension. When two nodes' shortest distance d_{ij} is less than the box size s, they can be put into the same box. The fractal dimension describes the relation between the number of boxes needed to cover the whole network and the size of box. It is defined as follows,

Definition 2.1 (*Fractal Dimension [33]*)**.** The fractal dimension of weighted complex networks is obtained as follows,

$$d_{fra} = -\lim_{s\to 0} \frac{\ln N(s)}{\ln s} \tag{1}$$

where $N(s)$ is the number of boxes needed to cover the whole networks when the box size equals to s.

The initial box size is generally set as the minimum value of shortest distance of network, and grows on different rules. The fractal dimension d_{fra} can be obtained by the slope of the fitting line between $\ln N(s)$ and $\ln s$ eventually.

2.2. Local dimension of complex network

To explore the properties of each node in the complex network, local dimension has been proposed by Silva et al. [38] lately. It has been found that not only the real-world networks but also the small-world networks follow a power law distribution. Different center nodes with different radius r have different scaling properties. It means that the number of nodes $N_i(r)$ within the radius r and the radius r have a power law relationship, and it is shown as follows,

$$N_i(r) \sim r^d \tag{2}$$

where d is the dimension of network. The constant d shows that the whole network can be embedded in a d-dimensional space.

When considering the locality of local dimension, the local dimension could be different because of the radius r and the center node i. It can be rewritten as follows,

$$N_i(r) = r^{d_{i\_loc}} \tag{3}$$

$$d_{i\_loc} = \frac{d}{d \log r} \log N_i(r) \tag{4}$$

where d in Eq. (4) is the symbol of derivative. So the local dimension of node i can be obtained by the slope of the double logarithmic scale fitting curves. The radius r expands from 1 to the maximum of shortest distance from center node i, so the radius r is discontinuous. The derivative can be stilled applied because of the discrete nature [38,70] of complex network, and it is shown as follows,

$$d_{i\_loc} = \frac{r}{N_i(r)} \frac{d}{dr} N_i(r) \tag{5}$$

Compared with Eq. (4), Eq. (5) takes a derivative to eliminate *log* and it is more intuitive. This form can be easier to obtain the final form of local dimension.

Definition 2.2 (*Local Dimension [38]*)**.** The local dimension is obtained as follows,

$$d_{i\_loc} \approx r \frac{n_i(r)}{N_i(r)} \tag{6}$$

where $n_i(r)$ is the number of nodes whose shortest distance from center node i equal to radius r. $N_i(r)$ is the number of nodes within the radius r(the distance can be less or equal to the radius r).

2.3. Tsallis nonextensive statistical mechanics

The entropy is defined for thermodynamics [71] by Clausius. Boltzmann–Gibbs entropy is obtained based on a finite discrete set of probabilities, and it is shown as follows,

$$S_{BG} = -k \sum_{i=1}^{N} p_i \log p_i \tag{7}$$

where k is a conventional constant which is the Boltzmann universal constant for thermodynamics, and it is seen as unity for information theory.

Tsallis entropy was proposed by Tsallis [72] in 1988. It is a more general form and it is shown as follows,

$$S_q = -k \sum_{i=1}^{N} p_i \ln_q p_i \tag{8}$$

and the q-logarithmic function of Eq. (8) is shown as follows,

$$\ln_q p_i = \frac{{p_i}^{1-q} - 1}{1-q} (p_i > 0; q \in R; \ln_1 p_i = \ln p_i) \tag{9}$$

Definition 2.3 (*Tsallis Entropy [72]*)**.** Based on Eq. (9), Eq. (8) can be rewritten as follows,

$$S_q = k \frac{1 - \sum_{i=1}^{N} {p_i}^q}{q-1} \tag{10}$$

where N is the total number of subsystems. The nonextensive parameter q in nonextensive statistical mechanics is used to nonextensive additivity of the system. When q equals to 1, the nonextensive additivity degenerated to the classic additivity which is called Shannon entropy.

2.4. Existing structure entropy

To measure the structure complexity of complex network, a lot of measures have been put forward, but most are based on Shannon dimension. In this section, three frequently used methods are detailed introduced.

Definition 2.4 (*Degree Structure Entropy [73]*)**.** The degree structure entropy is wildly used in the real-world networks' structure complexity measuring, and it is based on the Shannon entropy and the degree distribution, which is detailed shows as below,

$$E_{\deg} = -\sum_{i=1}^{N} p_i \log(p_i) \tag{11}$$

where N is the total number of the nodes, and p_i is the weighted degree distribution [74] of node i which is shows as follow,

$$p_i = \frac{S(i)}{\sum_{i=1}^{N} S(i)} \tag{12}$$

where $S(i)$ is the strength of node i which is defined as the summation of the weights of the edges associated with node i, namely

$$S(i) = \sum_{j=1}^{N} w_{ij} \tag{13}$$

where w_{ij} is the weight of edge connected with node i and node j.

Definition 2.5 (*Betweenness Structure Entropy [75]*)**.** Because the degree structure entropy focuses on the local information, the betweenness structure entropy is proposed recently. It is based on the Shannon entropy and the betweenness distribution, which can be shown as follows,

$$E_{bet} = -\sum_{i=1}^{N} {p'}_i \log({p'}_i) \tag{14}$$

where N represents the total number of nodes, and ${p'}_i$ represents the betweenness distribution [74] of node i which can be detailed shown as follows,

$${p'}_i = \frac{\sum_{s\neq t, s\neq i, t\neq i} L_{st}(i)}{\sum_{s\neq t} L_{st}} \tag{15}$$

where L_{st} is the total number of shortest paths between node s and node t, $L_{st}(i)$ is the number of shortest paths between node s and node t which pass through node i.

Definition 2.6 (*A Combination of Degree and Betweenness [61]*)**.** The degree distribution focuses on the local information of the center node, and the betweenness distribution focuses on the whole network's information. So a method which combines degree distribution and betweenness distribution has been proposed recently [61], and it is shown as follows,

$$T_{db} = \sum_{i=1}^{N} \frac{{p_i}^{q_i} - p_i}{1 - q_i} \tag{16}$$

where N is the total number of nodes in the network, p_i represents the degree distribution of node i which is shown as Eq. (12), q_i is based on the betweenness distribution which can be obtained as follows,

$$q_i = 1 + ({p'}_{\max} - {p'}_i) \tag{17}$$

where ${p'}_i$ can be obtained by Eq. (15), and ${p'}_{\max}$ is the maximum betweenness distribution ${p'}_i$ among all the nodes. This makes the index q_i bigger than 1 which shows the influence of subnetwork on the whole network.

3. Structure entropy based on Tsallis entropy

Lots of complexity measures have been studied in this field, but most of them have their own limitations and shortcomings, which only consider whole structure or local property in their methods. In this section, the structure entropy is proposed based on the fractal dimension and local dimension. Because fractal dimension can reveal the whole structure property which is based on the whole structure of network, and local dimension focuses on the local structure property which is different from each other because of the difference of center node, this proposed method can pay attention to both whole structure and local structure. Compared with previous methods like Ref. [61], this method is not straightforward, because this method needs a series of box-covering process to calculate fractal dimension and local dimension. Even this calculating process is not straightforward, it can bring more information of network which is beneficial for complexity measuring, and it is time-saving compared with betweenness centrality.

Definition 3.1 (*Proposed Method*)**.** This structure entropy is based on the nonextensive statistical mechanics and is defined as follow,

$$T_{lf} = k\sum_{i=1}^{N}\frac{{p_i}^{d_{fra}} - p_i}{1 - d_{fra}} \tag{18}$$

where T_{lf} represents this proposed structure entropy of complex network, k equals to constant 1 in this method, p_i is related to the local dimension d_{i_loc} of node i which can be obtained by Eq. (6), and it is detailed shown as follows,

$$p_i = \frac{d_{i\_loc}}{\sum_{i=1}^{N} d_{i\_loc}} \tag{19}$$

This makes each node's p_i is smaller than 1 and related to the local structure.

The fractal dimension of complex network is based on the whole structure of network, so it is a global characteristic to describe the properties of network. The fractal dimension is different for different network which is related to the complexity degree of complex network. Using it to replace the constant parameter q in Tsallis entropy (Eq. (10)) is more reasonable to describe the nonextensive additivity of the network. In this method, p_i is smaller than 1, d_{fra} is larger than 1 in general networks, and the entropic index d_{fra} shows the influence of subsystem to the whole network. When $d_{fra} = 1$, this structure entropy would degenerate to the Shannon entropy based on the local dimension. This proposed structure entropy' property obeys the classic Tsallis entropy.

Entropy is a useful tool for dealing with uncertainty. When the probability distribution is uniform, the system would be unknown and with maximum uncertainty, and the entropy for this system would be maximum. So a network would be more complex with higher entropy because of the greater uncertainty in this network. Otherwise, when a network becomes more order based on certain generation rules, there is less uncertainty in the network and the entropy would decline. So the entropy is also a useful tool to measure the complexity degree of network.

4. Experimental study

4.1. Data

In order to show this proposed method's performance, a series of complex networks which are grown from the simple nearest-neighbor coupled network are applied in this paper. Then, five real world networks are also used in this paper. These real world networks contain karate network, dolphins network, jazz network, USAir network, and email network, which are from (http://vlado.fmf.uni-lj.si/pub/networks/data/).

4.2. Nearest-neighbor coupled network

In this section, a series of complex networks which are grown from the simple nearest-neighbor coupled network are used, and like a scale-free network model (BA model) [1]. The connection edge between any two nodes would change with the development of times T, and the structure complexity of the network would also change. The node with larger value of degree has bigger probability to connect to other nodes. Finally, the network would become more orderly, and more edges are connected with node with larger value of degree. The development progress of this network is shown in Fig. 1, and it is detailed introduced as follows.

Step 1: Set the times of this progress T and the number of nodes N in this network. The simple nearest-neighbor coupled network with N nodes is configured as the initial network. The degree of nodes in the initial network equals to 2.

Step 2: In each time, every nodes have a certain probability to connect other nodes, and the way is random. The probability of connecting to other nodes is based on the value of degree, the nodes with large value of degree have the large probability to connect to other nodes. The connect probability of node i is defined as follows,

$$p_{i\_c} = \frac{D(i)}{\sum_{i=1}^{N} D(i)}$$

where $D(i)$ is the degree of node i.

Step 3: Repeat the step 2 with the development of times T.

Then, five networks are established based on this rule, and these networks have 100 nodes, 300 nodes, 500 nodes, 700 nodes, and 1000 nodes respectively. The proposed structure entropy of these networks in different time T is shown in Table 1. The four kinds of structure entropy of these networks are shown form Figs. 2 to 5.

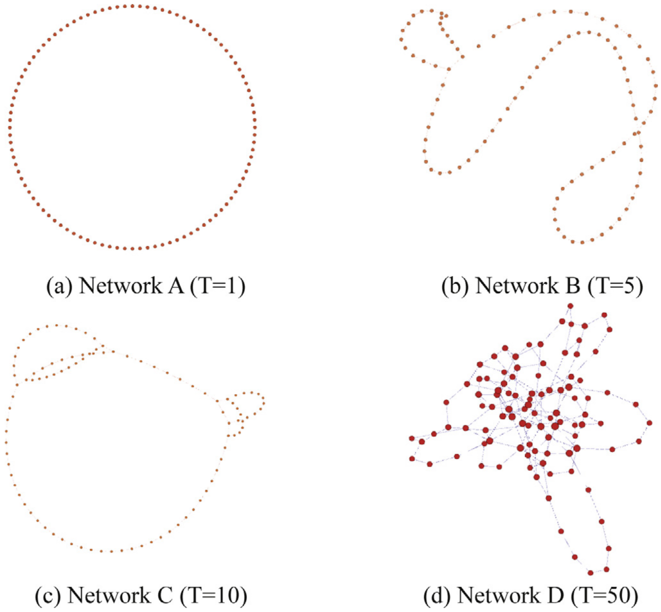

Fig. 1. A series of networks. The network A is a simple nearest-neighbor coupled network, and the degree of each node is equal to 2. The network B, C, D is the network whose growing progress is 5, 10, 50 respectively. With the increasing of growing progress T, the network is more orderly.

Table 1
The proposed structure entropy T_{lf} for networks with different nodes in different times T.

Structure entropy	Nodes number				
	100	300	500	700	1000
$T_{lf}(T=1)$	5.5241	7.1281	7.8925	8.4403	8.9846
$T_{lf}(T=5)$	3.3257	4.2466	4.0258	4.2353	4.5846
$T_{lf}(T=10)$	2.6333	3.1573	3.1566	3.2959	3.3691
$T_{lf}(T=15)$	2.3191	2.7220	2.6401	2.7546	2.8310
$T_{lf}(T=20)$	2.0028	2.3117	2.3703	2.4214	2.5160
$T_{lf}(T=25)$	1.8466	2.0390	2.1687	2.1766	2.2788
$T_{lf}(T=30)$	1.7491	1.9183	2.0329	2.0181	2.0853
$T_{lf}(T=35)$	1.6432	1.7898	1.9643	1.9461	2.0132
$T_{lf}(T=40)$	1.5747	1.6985	1.8244	1.8740	1.8919
$T_{lf}(T=45)$	1.5217	1.6208	1.7201	1.7786	1.8127
$T_{lf}(T=50)$	1.4949	1.5390	1.6280	1.7075	1.7251

From the above results, some conclusions can be obtained as follows.

1. The decline rate of degree structure entropy in these networks is close to a constant, and there is also an exception, like 100 nodes network in Fig. 2. The variation of complexity degree with T can be easily obtained by this method.

2. The changes rate of the betweenness structure entropy E_{bet} and the combination of degree and betweenness structure entropy T_{db} are declined with the development of time T. The main change is shown in the first 30 steps, and the change in the last 20 steps are smaller than the change in the first 30 steps. Because the decline rate is slow and the value of E_{bet} and T_{db} change very small in the last 20 steps, it is not beneficial to obtain the change of complexity degree of complex network.

3. The change of T_{lf} is close to change of E_{bet} and T_{db}, but the change progress is more clear than the previous method. Especially, the change in the last 20 steps can be observed.

4. T_{lf} is easier changed with the number of edges, while the other three structure entropies are more likely to change with the number of nodes.

With the time T going, the network would be more orderly and the complexity degree would decrease, and the structure entropy declines at the same time. So T_{lf} can reveal the complexity changes of network in different time T. From the results in Table 1, the T_{lf} in different series complex network with different number of nodes are monotonous decreasing. From the results from Figs. 2 to 5, it can be found that the previous structure entropies are each roughly equal in the last 20 steps, and it is difficult to judge the complexity degree. But the change of value of this method is obvious, and it is monotonically decreasing in the last 20 steps, which is beneficial to judge the complexity degree. In addition, because betweenness centrality is very time-consuming, this proposed method can obtain the complexity degree of complex network more quickly than the previous method.

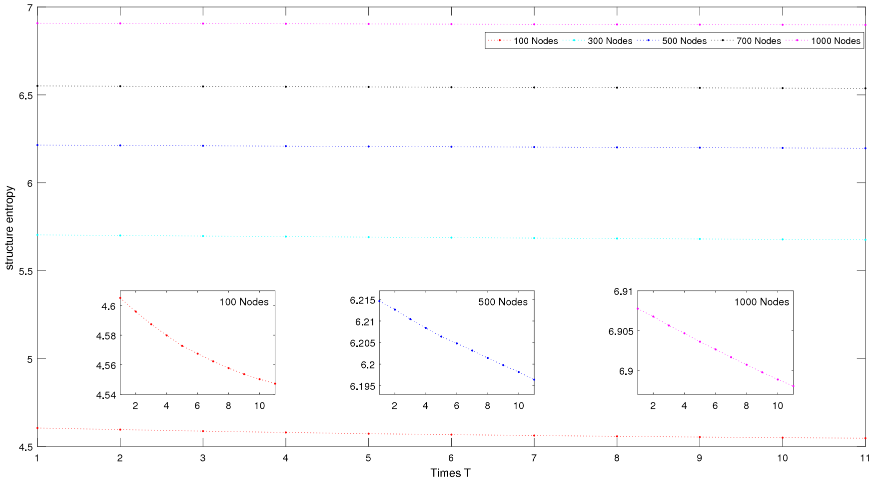

Fig. 2. E_{deg} for five networks.

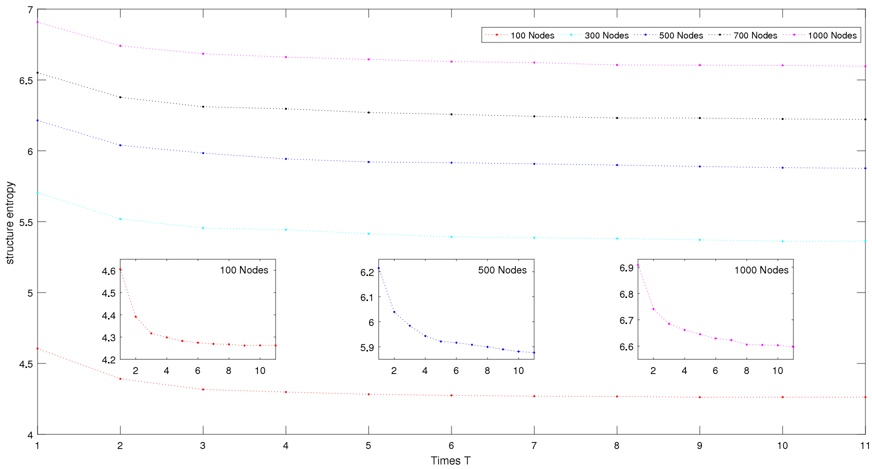

Fig. 3. E_{bet} for five networks.

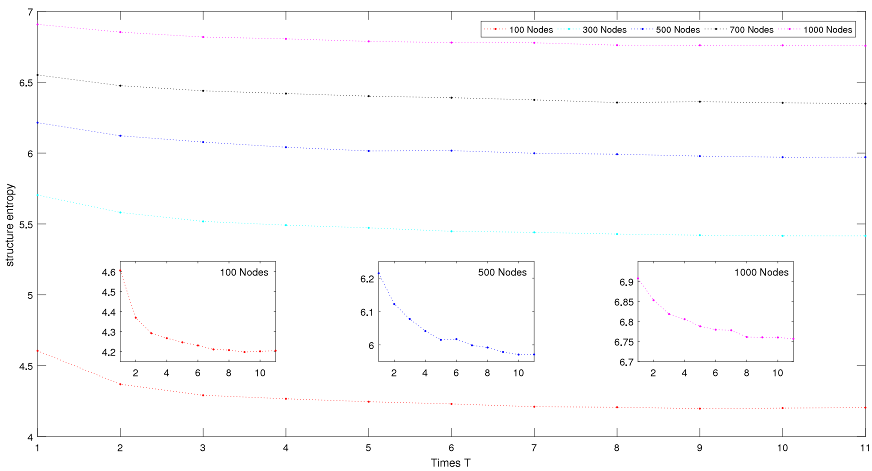

Fig. 4. T_{db} for five networks.

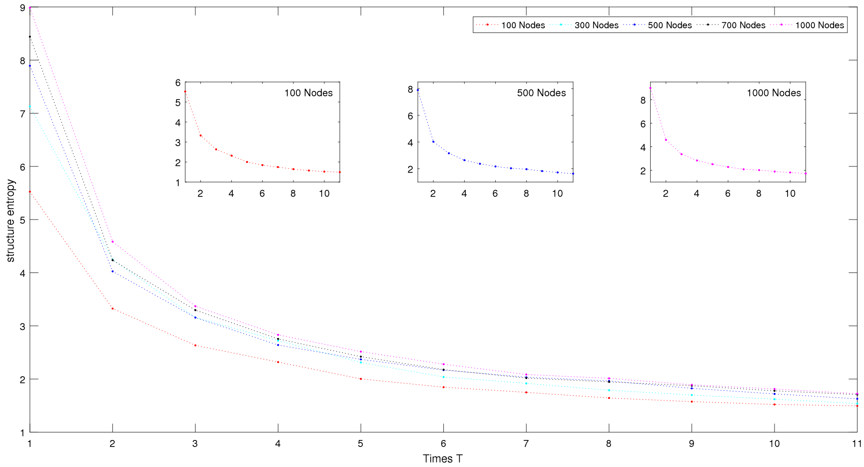

Fig. 5. T_{lf} for five networks.

Table 2
The structure entropy of five real world networks.

Network	Nodes	Edges	E_{deg}	E_{bet}	T_{db}	T_{bc}	T_{cc}	T_{dc}	T_{lf}
karate	34	78	3.2608	1.8678	2.0027	1.7178	2.7514	3.0670	0.5240
dolphins	62	159	3.9513	3.3688	3.3781	3.1265	3.2975	3.7296	0.5274
jazz	198	5 484	5.0849	3.6946	3.5471	3.5785	3.2425	5.0114	0.9659
USAir	332	2 126	5.0250	3.0702	3.4177	2.5465	3.1350	4.7086	0.6390
email	1133	10 902	6.6310	5.8492	6.3184	5.5692	5.0156	6.4383	1.2780

4.3. Real-world complex network

To find the difference between these structure entropy, five real-world complex networks are used in this section, includes karate network, dolphins network, jazz network, USAir network, and email network. To show the advantage of this proposed method, some other centralities are introduced in this section as comparative experiments. The p_i in Eq. (18) is related to local dimension. Then, we use three other methods as comparative experiments which still combine with fractal dimension based on Tsallis dimension. When only the p_i in Eq. (18) is replaced by betweenness centrality, closeness centrality, and degree centrality, three other entropies are obtained and they are shown as T_{bc}, T_{cc}, T_{dc}. The p_i in T_{bc}, T_{cc}, T_{dc} are shown in Eq. (15), Eq. (12), and Eq. (20) respectively.

$$\begin{aligned} C(i) &= \frac{1}{\sum_{j=1}^{N} d_{ij}} \\ p(i) &= \frac{C(i)}{\sum_{i=1}^{N} C(i)} \end{aligned} \tag{20}$$

Some properties and different structure entropies of these five networks are shown in Table 2. From the result in Table 2, it can be found the detail results between these entropies are different, like the result in E_{bet} (dolphins and USAir), T_{bc} (dolphins and USAir), and T_{cc} (dolphins and jazz). In addition, T_{lf} result is more scattered which is beneficial for analyzing complexity degree. T_{lf} concerns both whole structure and local structure, and it gets a similar result with previous methods, because it considers more information in the network than each previous methods. It is also time-saving compared with entropies which consider betweenness centrality.

To explore the relationship between these different measures, Pearson product-moment moment correlation coefficient r_{xy} between two result lists is applied in this paper, which is obtained as follows,

$$\mathrm{r}_{xy} = \frac{\sum (X - \bar{\mathrm{X}})(Y - \bar{Y})}{(\sqrt{\sum_{i=1}^{n} (X_i - \bar{\mathrm{X}})^2})(\sqrt{\sum_{i=1}^{n} (Y_i - \bar{Y})^2})} \tag{21}$$

where X and Y are two different lists which are the experimental object, $\bar{\mathrm{X}}$ and $\bar{\mathrm{Y}}$ are the mean of X and Y respectively, and n is the capacity of the lists. When r_{xy} is close to 1, it demonstrates perfect positive correlation, and -1 illustrates perfect negative correlation. In addition, $\mathrm{r}_{xy} = 0$ illustrates there is no relationship between two lists. The r_{xy} between every two results are shown in Table 3.

Table 3
The r_{xy} between different results.

r_{xy}	$E_{\deg}$	E_{bet}	T_{db}	T_{bc}	T_{cc}	T_{dc}	T_{lf}
$E_{\deg}$	1	0.9269	0.9371	0.8984	0.8738	0.9977	0.9154
E_{bet}	0.9269	1	0.9899	0.9938	0.9646	0.9316	0.9009
T_{db}	0.9371	0.9899	1	0.9730	0.9820	0.9340	0.8836
T_{bc}	0.8984	0.9938	0.9730	1	0.9574	0.9100	0.9160
T_{cc}	0.8738	0.9646	0.9820	0.9574	1	0.8722	0.8662
T_{dc}	0.9977	0.9316	0.9340	0.9100	0.8722	1	0.9368
T_{lf}	0.9154	0.9009	0.8836	0.9160	0.8662	0.9368	1

From Tables 2 and 3, T_{lf} can obtain a similar result with $E_{\deg}$ and T_{dc}, but has some differences with comparative methods T_{bc} and T_{cc}. Different structure entropies have different considerations, and would get different results. When one method combines two kinds of information of network, the result would be similar with other methods which only consider the subset of information in this method. For example, T_{db} takes degree centrality and betweenness centrality of network into consideration, $E_{\deg}$ considers degree centrality and E_{bet} considers betweenness centrality. So T_{db} would be similar with E_{bet} and $E_{\deg}$, and T_{db} is the 1st and 2nd similar method of E_{bet} and $E_{\deg}$ respectively from Table 3.

5. Conclusion

In this paper, a novel complexity degree measure is proposed based on Tsallis entropy. To overcome the shortcoming of previous method, this proposed method is based on Tsallis entropy which combines the fractal dimension and local dimension, which are both the significant properties of complex network. In addition, the fractal dimension can reveals the whole structure property of complex network, and the local dimension reveal the structure property based on the center node. This proposed structure entropy can degenerate to Shannon entropy based on local dimension when fractal dimension equals to 1. To show the performance of this structure entropy, a series of complex networks which are grown from the simple nearest-neighbor coupled network and five real-world complex networks are applied in this paper, and the results show the good performance of this method.

Acknowledgments

The authors greatly appreciate the reviews' suggestions and the editor's encouragement. The work is partially supported by National Natural Science Foundation of China (Program No. 61671384, 61703338), Natural Science Basic Research Plan in Shaanxi Province of China (Program No. 2018JQ6085), Peak Experience Plan in Northwestern Polytechnical University.

References

[1] R. Albert, H. Jeong, A.L. Barabasi, Internet - diameter of the world-wide web, Nature 401 (6749) (1999) 130–131.
[2] D.J. Watts, S.H. Strogatz, Collective dynamics of 'small-world' networks, Nature 393 (6684) (1998) 440–442.
[3] L.K. Gallos, M. Sigman, H.A. Makse, The conundrum of functional brain networks: small-world efficiency or fractal modularity, Front. Physiol. 3 (2012) 9.
[4] K. Chochlaki, F. Vallianatos, G. Michas, Global regionalized seismicity in view of non-extensive statistical physics, Physica A 493 (2018) 276–285.
[5] C. Tsallis, L.J.L. Cirto, Thermodynamics is more powerful than the role to it reserved by Boltzmann-gibbs statistical mechanics, Eur. Phys. J. Spec. Top. 223 (11) (2014) 2161–2175.
[6] F. Vallianatos, A non extensive view of electrical resistivity spatial distribution estimated using inverted transient electromagnetic responses in a karstified formation (keritis basin, crete, Greece), Physica A 505 (2018) 171–178.
[7] K.A. Dawson, J.O. Indekeu, H.E. Stanley, C. Tsallis, Communities detection as a tool to assess a reform of the Italian interlocking directorship network, Physica A 466 (2017) 91–104.
[8] L. Lue, Y.-C. Zhang, C.H. Yeung, T. Zhou, Leaders in social networks, the delicious Case, Plos One 6 (6) (2011).
[9] V. Galvao, J.G.V. Miranda, R.F.S. Andrade, J.S. Andrade, L.K. Gallos, H.A. Makse, Modularity map of the network of human cell differentiation, Proc. Natl. Acad. Sci. USA 107 (13) (2010) 5750–5755.
[10] Z. He, W. Jiang, An evidential dynamical model to predict the interference effect of categorization on decision making, Knowl.-Based Syst. 150 (2018) 139–149.
[11] P. Xu, R. Zhang, Y. Deng, A novel visibility graph transformation of time series into weighted networks, Chaos Solitons Fractals 117 (2018) 201–208.
[12] W. Jiang, C. Huang, X. Deng, A new probability transformation method based on a correlation coefficient of belief functions, Int. J. Intell. Syst. 34 (6) (2019) 1337–1347.
[13] R. Zhang, B. Ashuri, Y. Deng, A novel method for forecasting time series based on fuzzy logic and visibility graph, Adv. Data Analy. Classif. 11 (4) (2017) 759–783.
[14] X. Deng, D. Han, J. Dezert, Y. Deng, Y. Shyr, Evidence combination from an evolutionary game theory perspective, IEEE Trans. Cybern. 46 (9) (2016) 2070–2082.
[15] T. Wen, W. Jiang, Identifying influential nodes based on fuzzy local dimension in complex networks, Chaos Solitons Fractals 119 (2019) 332–342.
[16] L. Fei, Q. Zhang, Y. Deng, Identifying influential nodes in complex networks based on the inverse-square law, Physica A 512 (2018) 1044–1059.
[17] Y. Wang, S. Wang, Y. Deng, A modified efficiency centrality to identify influential nodes in weighted networks, Pramana 92 (4) (2019) 68.
[18] M. Li, Q. Zhang, Y. Deng, Evidential identification of influential nodes in network of networks, Chaos Solitons Fractals 117 (2018) 283–296.

[19] G. Michas, F. Vallianatos, Stochastic modeling of nonstationary earthquake time series with long-term clustering effects, Phys. Rev. E 98 (4) (2018).
[20] J.H. Liu, Y.X. Zhu, T. Zhou, Improving personalized link prediction by hybrid diffusion, Physica A 447 (2016) 199–207.
[21] L.Y. Lu, T. Zhou, Link prediction in complex networks: A survey, Physica A 390 (6) (2011) 1150–1170.
[22] L. Pan, Y. Deng, A new belief entropy to measure uncertainty of basic probability assignments based on belief function and plausibility function, Entropy 20 (11) (2018) 842.
[23] L.K. Gallos, N.H. Fefferman, Simple and efficient self-healing strategy for damaged complex networks, Phys. Rev. E 92 (5) (2015).
[24] Y. Yu, G. Xiao, J. Zhou, Y. Wang, Z. Wang, J. Kurths, H.J. Schellnhuber, System crash as dynamics of complex networks, Proc. Natl. Acad. Sci. 113 (42) (2016) 11726–11731.
[25] X. Deng, W. Jiang, D number theory based game-theoretic framework in adversarial decision making under a fuzzy environment, Internat. J. Approx. Reason. 106 (2019) 194–213.
[26] Z. Huang, L. Yang, W. Jiang, Uncertainty measurement with belief entropy on the interference effect in the quantum-like Bayesian Networks, Appl. Math. Comput. 347 (2019) 417–428.
[27] F. Morone, H.A. Makse, Influence maximization in complex networks through optimal percolation, Nature 524 (7563) (2015) 65–U122.
[28] X. Teng, S. Pei, F. Morone, H.A. Makse, Collective influence of multiple spreaders evaluated by tracing real information flow in large-scale social networks, Sci. Rep. 6 (2016) 11.
[29] L. Yin, Y. Deng, Toward uncertainty of weighted networks: An entropy-based model, Physica A 508 (2018) 176–186.
[30] Y. Han, Y. Deng, A novel matrix game with payoffs of maxitive belief structure, Int. J. Intell. Syst. 34 (4) (2019) 690–706.
[31] W. Jiang, A correlation coefficient for belief functions, Internat. J. Approx. Reason. 103 (2018) 94–106.
[32] Y. Han, Y. Deng, Z. Cao, C.-T. Lin, An interval-valued pythagorean prioritized operator based game theoretical framework with its applications in multicriteria group decision making, Neural Comput. Appl. (2019) http://dx.doi.org/10.1007/s00521-019-04014-1.
[33] C.M. Song, L.K. Gallos, S. Havlin, H.A. Makse, How to calculate the fractal dimension of a complex network: the box covering algorithm, J. Statist. Mech.-Theory Exp. (2007) 16.
[34] E. Rosenberg, Minimal box size for fractal dimension estimation, Community Ecology 17 (1) (2016) 24–27.
[35] E. Rosenberg, Maximal entropy coverings and the information dimension of a complex network, Phys. Lett. A 381 (6) (2017) 574–580.
[36] S.Y. Duan, T. Wen, W. Jiang, A new information dimension of complex network based on r'enyi entropy, Physica A 516 (2019) 529–542.
[37] T. Wen, W. Jiang, An information dimension of weighted complex networks, Physica A 501 (2018) 388–399.
[38] F.N. Silva, L.D.F. Costa, Local dimension of complex networks, Comput. Sci. (2013).
[39] M. Gell-Mann, C. Tsallis, Nonextensive Entropy - Interdisciplinary Applications, p. 440, 2004.
[40] B. Wei, Y. Deng, A cluster-growing dimension of complex networks: From the view of node closeness centrality, Physica A (2019) http://dx.doi.org/10.1016/j.physa.2019.01.125.
[41] E. Rosenberg, Non-monotonicity of the generalized dimensions of a complex network, Phys. Lett. A 381 (28) (2017) 2222–2229.
[42] E. Rosenberg, Generalized hausdorff dimensions of a complex network, Physica A 511 (2018) 1–17.
[43] E. Rosenberg, Minimal partition coverings and generalized dimensions of a complex network, Phys. Lett. A 381 (19) (2017) 1659–1664.
[44] D.J. Wei, Q. Liu, H.X. Zhang, Y. Hu, Y. Deng, S. Mahadevan, Box-covering algorithm for fractal dimension of weighted networks, Sci. Rep. 3 (2013) 8.
[45] X. Deng, W. Jiang, Z. Wang, Zero-sum polymatrix games with link uncertainty: A dempster-shafer theory solution, Appl. Math. Comput. 340 (2019) 101–112.
[46] H. Yang, Y. Deng, J. Jones, Network division method based on cellular growth and physarum-inspired network adaptation, Int. J. Unconvent. Comput. 13 (6) (2018) 477–491.
[47] T. Wen, M.X. Song, W. Jiang, Evaluating topological vulnerability based on Fuzzy fractal dimension, Int. J. Fuzzy Syst. 20 (6) (2018) 1956–1967.
[48] Z. He, W. Jiang, An evidential Markov decision making model, Inform. Sci. 467 (2018) 357–372.
[49] L.K. Gallos, F.Q. Potiguar, J.S. Andrade, H.A. Makse, IMDB Network revisited: Unveiling fractal and modular properties from a typical small-world network, Plos One 8 (6) (2013) 8.
[50] L. Lu, T. Zhou, Q.M. Zhang, H.E. Stanley, The h-index of a network node and its relation to degree and coreness, Nature Commun. 7 (2016) 7.
[51] Z. Wang, C.T. Bauch, S. Bhattacharyya, A. d'Onofrio, P. Manfredi, M. Perc, N. Perra, M. Salathé, D. Zhao, Statistical physics of vaccination, Phys. Rep. 664 (2016) 1–113.
[52] S. Boccaletti, J. Buldu, R. Criado, J. Flores, V. Latora, J. Pello, M. Romance, Multiscale vulnerability of complex networks, Chaos 17 (4) (2007) 4.
[53] E. Bonmati, A. Bardera, M. Feixas, I. Boada, Novel brain complexity measures based on information theory, Entropy 20 (7) (2018) 24.
[54] K. Chochlaki, G. Michas, F. Vallianatos, Complexity of the yellowstone park volcanic field seismicity in terms of tsallis entropy, Entropy 20 (10) (2018).
[55] F. Vallianatos, G. Chatzopoulos, A complexity view into the physics of the accelerating seismic release hypothesis: Theoretical principles, Entropy 20 (10) (2018).
[56] S. Murayama, H. Kinugawa, I.T. Tokuda, H. Gotoda, Characterization and detection of thermoacoustic combustion oscillations based on statistical complexity and complex-network theory, Phys. Rev. E 97 (2) (2018) 8.
[57] H. Zenil, N.A. Kiani, J. Tegner, A review of graph and network complexity from an algorithmic information perspective, Entropy 20 (8) (2018) 15.
[58] R. Rajaram, B. Castellani, An entropy based measure for comparing distributions of complexity, Physica A 453 (2016) 35–43.
[59] J. Gomez-Pilar, J. Poza, A. Bachiller, C. Gomez, P. Nunez, A. Lubeiro, V. Molina, R. Hornero, Quantification of graph complexity based on the edge weight distribution balance: Application to brain networks, Int. J. Neural Syst. 28 (1) (2018) 19.
[60] P.J. Zufiria, I. Barriales-Valbuena, Entropy characterization of random network models, Entropy 19 (7) (2017) 13.
[61] Q. Zhang, M. Li, Y. Deng, A new structure entropy of complex networks based on nonextensive statistical mechanics, Internat. J. Modern Phys. C 27 (10) (2016) 440–455.
[62] D.J.J. Wang, K. Jann, C. Fan, Y. Qiao, Y.F. Zang, H.B. Lu, Y.H. Yang, Neurophysiological basis of multi-scale entropy of brain complexity and its relationship with functional connectivity, Front. Neurosci. 12 (2018) 14.
[63] W. Deng, Y. Deng, Entropic methodology for entanglement measures, Physica A 512 (2018) 693–697.
[64] Y.H. Xiao, W.T. Wu, H. Wang, M. Xiong, W. Wang, Symmetry-based structure entropy of complex networks, Physica A 387 (11) (2008) 2611–2619.
[65] M. Cai, Y. Cui, H.E. Stanley, Analysis and evaluation of the entropy indices of a static network structure, Sci. Rep. 7 (2017).
[66] X. Gao, K.Q. Li, B. Chen, Invulnerability measure of a military heterogeneous network based on network structure entropy, Ieee Access 6 (2018) 6700–6708.
[67] M. Lin, X.X. Fan, G. Wang, G. Zhao, Network structure entropy and its dynamical evolution for recurrence networks from earthquake magnitude time series, Eur. Phys. J. B 89 (5) (2016) 7.
[68] B.B. Fu, L. Zhang, S.B. Li, Y.X. Li, Survivability of public transit network based on network structure entropy, Internat. J. Modern Phys. C 26 (9) (2015) 11.

[69] C.E. Shannon, A mathematical THEORY OF communication, Bell Syst. Tech. J. 27 (4) (1948) 623–656.
[70] E. Ben-Naim, H. Frauenfelder, Z. Toroczkai, Complex Networks, Springer Berlin Heidelberg, 2004, p. xviii,520.
[71] J.W. Gibbs, Elementary principles in statistical mechanics, developed with especial reference to the rational foundation of thermodynamics. library of american civilization. c, Dover Publ. Inc New York 14 (1) (1902) A55–A59.
[72] C. Tsallis, Possible generalization of Boltzmann-gibbs statistics, J. Statist. Phys. 52 (1) (1988) 479–487.
[73] B. Wang, H. Tang, C. Guo, Z. Xiu, Entropy optimization of scale-free networks robustness to random failures, Physica A 363 (2) (2006) 591–596.
[74] L. Lu, D. Chen, X.-L. Ren, Q.-M. Zhang, Y.-C. Zhang, T. Zhou, Vital nodes identification in complex networks, Phys. Rep.-Rev. Sect. Phys. Lett. 650 (2016) 1–63.
[75] Q. Zhang, M. Li, Y. Deng, A betweenness structure entropy of complex networks, arXiv, 2014.