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Vital spreaders identification in complex networks with multi-local dimension[☆]

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ABSTRACT

The important nodes identification has been an interesting problem in this issue. Several centrality methods have been proposed to solve this problem, but most previous methods have their own limitations. To address this problem more effectively, multi-local dimension (MLD) which is based on the fractal property is proposed to identify the vital spreaders in this paper. This proposed method considers the information contained in the box and q plays a weighting coefficient for this partition information. MLD would have different expressions with different values of q, and it would degenerate to local information dimension and variant of local dimension when $q = 1$ when $q = 0$ respectively, both of which have been effective identification methods for influential nodes. Thus, MLD would be a more general method which can degenerate to some existing centrality methods. In addition, different from classical methods, the node with low MLD would be more important in the network. Some real-world and theoretical complex networks and comparison methods are applied in this paper to show the effectiveness and reasonableness of this proposed method. The experiment results show the superiority of this proposed method.

1. Introduction

The complex network has become a useful approach in recent research, because it is inextricably correlated with various research issues. For example, the Cyber–Physical Systems (CPS) can be transformed into complex networks to study the system operation [1], optimization [2,3], and reliability [4,5] issues. The traffic network can also use complex networks to study traffic congestion [6], path planning [7], intelligent transportation [8], et al. Therefore, the study of the basic property of complex networks has become more important [9,10], like the fractal property [11] and self-similarity property [12] of complex networks. These properties have been used in various fields in the network. Currently, lots of relevant studies have been carried out to study the significant properties of the network, like measuring the similarity between nodes to find the same user in different apps [13,14]; predicting the potential links in networks to find possible relationships in social software [15,16]; exploring the game theory in networks to find the role of evolutionary game in human progress [17–20]; measuring the vulnerability of networks to guide the reconstruction of networks [21–23]. In particular, only a part of nodes plays an important role in most network properties, i.e. a small number of individuals has a great influence on society [24–26]. In network, the influence ability of each node means the speed of propagation caused by this node. Thus, finding the influential nodes in networks not only has significant theoretical significance but also practical significance. These nodes would have a more important influence on the function and structure of networks [27,28].

Lots of centrality methods have been proposed to identify these nodes with huge influence in the complex network [29], the number of vital nodes is very small, but the impact would be indeed much larger than the other nodes. The classical centrality methods contain Betweenness Centrality [30], Closeness Centrality [31], Degree Centrality [32], PageRank [33], and lots of other methods [34]. In addition, some algorithms have been wildly used in various aspects of society, like ranking relevant website [35], detecting threat and managing disaster [36,37], designing searching algorithm [38,39], affecting synchronization of interconnected network [40,41] and so on [42–45]. However, these existing centralities have their own limitations. For instance, Betweenness Centrality has a high computational complexity, and lots of nodes' value would be 0 which cannot identify their importance; Closeness Centrality cannot be applied in the

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network with disconnected components; Degree Centrality considers the neighbor nodes' influence but ignores the influence all over the network.

Recently, some novel centralities have been proposed in this field to address this problem. For example, Tang et al. [46] presented a probabilistic greedy-based local search strategy to enhance the exploitation operation of discrete bat algorithm which can maximize the spread of influence. Shi et al. [47] considered influence maximization from an online–offline interactive setting and proposed the location-driven influence maximization problem. Deng et al. [48] identified the vital nodes by inverse-square law in the complex network. Zhou et al. [49] modified the gravity model to detect the influential nodes in the complex network which achieve a good performance. There still are lots of methods used in this field, such as TOPSIS [50], evidence theory [51,52], entropy-based method [53], nodes' relationship [54,55], evidential network [56,57], optimal percolation theory [58,59], and so on [60–63].

The fractal property and self-similarity property in networks can not only show the network's feature [64,65], but also reveal the nodes' properties [66]. Recently, Pu et al. [67] modified the local dimension in the network to identify the influential nodes. Then, Bian et al. [68] measured the information dimension of nodes to rank the influence of node which is a new research perspective. After that, Jiang et al. [69] proposed the fuzzy local dimension to identify the influential nodes. Thus, the fractal and self-similarity properties have been proved to be significant for nodes' importance identification. Although the local dimension can describe the local structure properties around the central nodes, it is inadequate to use a single local dimension to describe the fractal property of nodes in the complex network. To describe the spatial heterogeneity of fractal objects systematically, the multi-local dimension is introduced in this paper. By adjusting the weighting coefficient q, MLD can measure the fractal property around the central node from different scales, and there are different representations about the MLD. In particular, MLD can degenerate to the local information dimension [70] and variant of local dimension [67] when $q = 1$ and $q = 0$ respectively. The different scales of MLD can give sufficient consideration to the fractal property of local structure.

In this paper, a novel centrality method is proposed based on the multi-local dimension which is from the view of the fractal property. This proposed method considers the structure around the central node by the box. The box radius would increase from 1 to the maximum value of the shortest distance from the central node. The information in each box is represented by the number of nodes in the box. Then, a weighting coefficient q is used to deal with the information. The different chosen values of q would consider the information in different scale which can cause different representations of the multi-local dimension. Finally, the multi-local dimension of nodes can be obtained by the slope of linear regression. Thus, this proposed method is a more general method to identify the vital nodes because of the existence of coefficient q. Some real-world and theoretical complex networks have been used in this paper, the effectiveness and reasonableness of this proposed method are demonstrated in comparison with some existing centrality methods. Observing from the experiment results, the superiority of this proposed method and the relationship between this proposed method and other comparison methods can be obtained.

The organization of the rest of this paper is as follows. This proposed multi-local dimension is defined in Section 2 to identify the vital spreaders in the network. Meanwhile, some real-world complex networks and existing comparison methods are performed to illustrate the reasonableness and effectiveness of the proposed method in Section 3. The conclusion is conducted in Section 4.

2. The proposed vital spreaders identification method

In this section, a novel method is proposed based on the *multi-local dimension* (MLD) to identify the influential spreaders in the complex network. This proposed method can consider the information in boxes with different scale q. When q has different values, different expressions of MLD would be given to identify influential nodes. In addition, this proposed method would degenerate to local information dimension and variant of local dimension when $q = 1$ and $q = 0$ respectively. The flow chart of MLD is shown in Fig. 1.

2.1. The structure of the complex network

In a given *complex network* $G(N, E)$, N is the set of nodes and E are the set of edges in the network, $|N|$ and $|E|$ is the number of nodes and edges respectively in the network. Firstly, the *adjacency matrix* A is given to describe the topological structure of the complex network. The element a_{ij} in the adjacency matrix A shows the connection edge between node i and node j. $a_{ij} = 1$ represents there is an edge between node i and j, and $a_{ij} = 0$ is the opposite. Then, the *shortest distance* between any two nodes can be obtained by the adjacency matrix A (the known information) through *Dijkstra algorithm* [71], and the definition of the shortest distance ω_{ij} between node i and node j can be shown below,

$$\omega_{ij} = \min_{h} \left(a_{ih_1} + a_{h_1h_2} + \cdots + a_{h_mj}\right) \tag{1}$$

where $a_{h_1h_2}$ is one element of A which can show network's connection, $h_1, h_2, \ldots, h_m$ are the IDs of different nodes. The shortest distance matrix can be constructed by the known shortest distance between any two nodes, and it is denoted as W. The element ω_{ij} represents the shortest distance between node i and j, and the shortest distance matrix W would be a symmetric matrix. The *maximum value of the shortest distance* from node i is denoted as ξ_i and defined as follows,

$$\xi_i = \max_{j\in N, j\neq i} \left(\omega_{ij}\right) \tag{2}$$

where ξ_i would vary from the chosen of central node i, which can show the scale of the locality of central node i.

2.2. The local dimension of complex network

After getting the relevant basic characteristics of complex networks, the local dimension which is the basis of this proposed method is introduced in this section. The *local dimension* (LD) is modified from the fractal dimension and firstly proposed to accurately measure the local property of each node, i.e. the change of dimensionality among vertices in the network [72]. Then, Pu et al. [67] modified the local dimension to identify the vital nodes in the complex network. In this method, the volume scaling property has been considered in different topological scale. In general, the number of nodes $B_i(r)$ within a given radius r (including r) for any node i follows a power law which is shown as follows,

$$B_i(r) \sim r^{LD_i} \tag{3}$$

where LD_i is the local dimension of node i. Thus, the local dimension of any node can be obtained by the slope of double logarithmic curves which is detailed shown below,

$$LD_i = \frac{d}{d \ln r} \ln B_i(r) \tag{4}$$

where d is the symbol of the derivative. Due to the discrete property [73] of complex networks, Eq. (4) can be rewritten as follows,

$$\begin{aligned} LD_i &= \tfrac{r}{B_i(r)} \tfrac{d}{dr} \ln B_i(r) \\ LD_i &= \tfrac{r}{B_i(r)} b_i(r) \end{aligned} \tag{5}$$

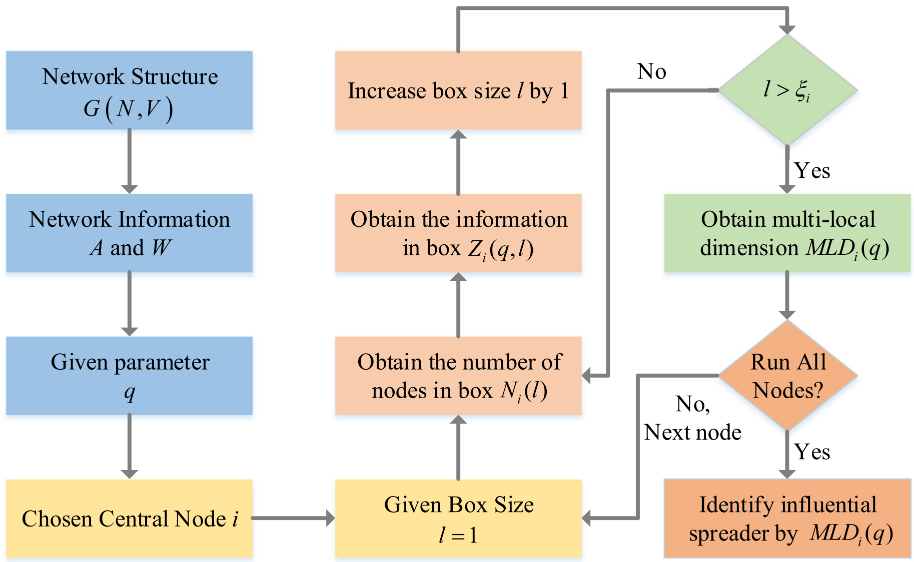


Fig. 1. The flow chart of this proposed method.

where r is the radius of the box, $b_i(r)$ represents the number of nodes whose shortest distance from central node i equal to r, and $B_i(r)$ represents the number of nodes whose shortest distance from central node i is less than or equal to r. The radius r whose central node is node i would increase from 1 to ξ_i, and the local dimension LD_i of node i would be the slope of double logarithmic curves.

2.3. This proposed multi-local dimension of complex network

Lots of methods have been proposed to identify the vital spreaders in the network, and these methods concentrate on different parts of information in the network. In this subsection, the multi-local dimension is proposed to identify the vital nodes in complex networks based on the local structure of each node. To illustrate this method more detail, the network in Fig. 2 is taken as an example to show the process of this proposed method.

Firstly, the network work structure can be shown in Fig. 2, including the adjacency matrix A and the shortest distance matrix W. Then, let node i be the central node, and $q=0$, $q=1$, $q=2$ are selected as three example cases in this subsection. The reason why choose these three values is that 0 and 1 are two special situations of MLD, and 2 represents other general situations.

In this proposed method, there is a box covering the network with node i as the central node. The radius of the box l would increase from 1 to ξ_i, and the entire network would be covered by this box when $l=\xi_i$ eventually. The box with different radii would cover different number of nodes, which is determined by the shortest distance from the central node to other nodes. These nodes in the example network with different distances from central node are shown in Fig. 2. The information $\mu_i(l)$ in this box with chosen central node i and box radius l is related with the number of nodes in this box, and it is defined as follows,

$$\mu_i(l)=\frac{N_i(l)}{|N|} \tag{6}$$

where $N_i(l)$ is the number of nodes covered by this box, i.e. the shortest distance from these nodes to the central node i is less than or equal to the radius of the box l, and $|N|$ is the number of nodes in the network. In this example network, $N_i=\{4,11,25\}$ and $\mu_i=\{0.16,0.44,1\}$ for every values of q.

Table 1
The results of $Z_i(q,l)$ with different values of q and radii l for central node i.

$Z_i(q,l)$	$l=1$	$l=2$	$l=3$
$Z_i(0,l)$	6.2500	2.2727	1
$Z_i(1,l)$	−0.4230	−0.5211	0
$Z_i(2,l)$	0.0256	0.1936	1

For this given information $\mu_i(l)$, the partition consideration of the box $Z_i(q,l)$ is defined as follows,

$$Z_i(q,l)=\begin{cases}[\mu_i(l)]^q & q\neq 0 \,\&\, q\neq 1\\ 1/\mu_i(l) & q=0\\ \mu_i(l)\ln\mu_i(l) & q=1\end{cases} \tag{7}$$

where q is the real number ($q\in R$) which can be changed. In addition, q plays a weighting coefficient for Eq. (7). The setting of $Z_i(0,l)$ is to avoid $q-1$ less than 0 and cause MLD to be less than 0. Moreover, $Z_i(1,l)$ obeys the expression of Shannon entropy which is similar with the multi-fractal dimension. Observing from the definition above, the partition consideration of the box would have the following property: $Z_i(q,l)\geq 0$. $q=0$ and $q=1$ are two special cases in MLD, which would be discussed later. The results of $Z_i(q,l)$ for different values of q are shown in Table 1.

Then, based on the partition consideration $Z_i(q,l)$, the multi-local dimension $MLD_i(q)$ of node i is defined as follows,

$$MLD_i(q)=\begin{cases}\frac{\tau_i(q,l)}{q-1}, & q\neq 1\\ \lim_{l\to 0}\frac{Z_i(1,l)}{\ln l}, & q=1\end{cases} \tag{8}$$

because the denominator $q-1$ would equal to 0 when $q=1$, MLD would have different expression in this situation ($q=1$). The value of $Z_i(1,l)$ when $q=1$ can be obtained by Eq. (7), where the partition consideration follows the expression of Shannon entropy. When $q\neq 1$, the numerator $\tau_i(q,l)$ would be defined as follows,

$$\tau_i(q,l)=\lim_{l\to 0}\frac{\ln Z_i(q,l)}{\ln l} \tag{9}$$

where $Z_i(q,l)$ has been given in Eq. (7), and l is the radius of the box.

The multi-local dimension of nodes can be obtained by the slope of the linear regression of related variables. More detail, when $q\neq 1$, the numerical estimation of MLD would be obtained

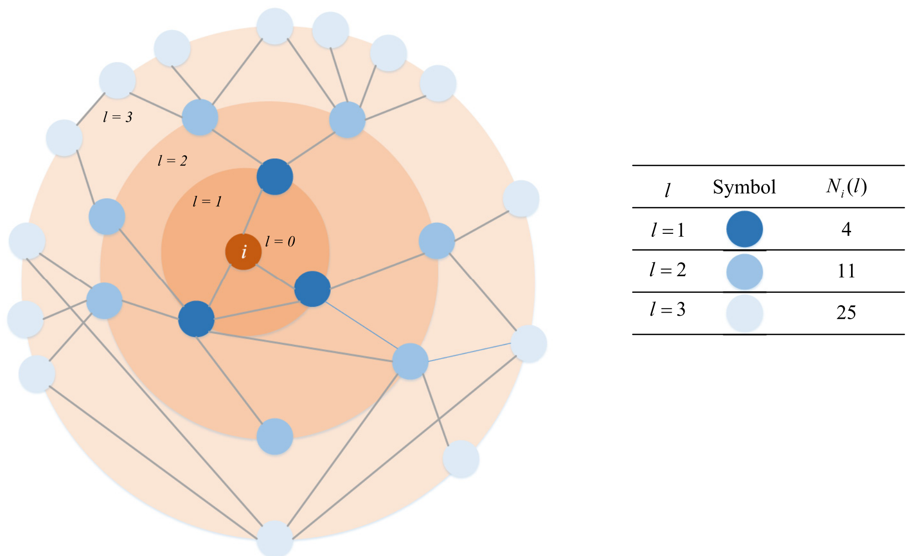


Fig. 2. The example network with different nodes' distance from central node *i*. These nodes with different colors mean the different shortest distance from central node.

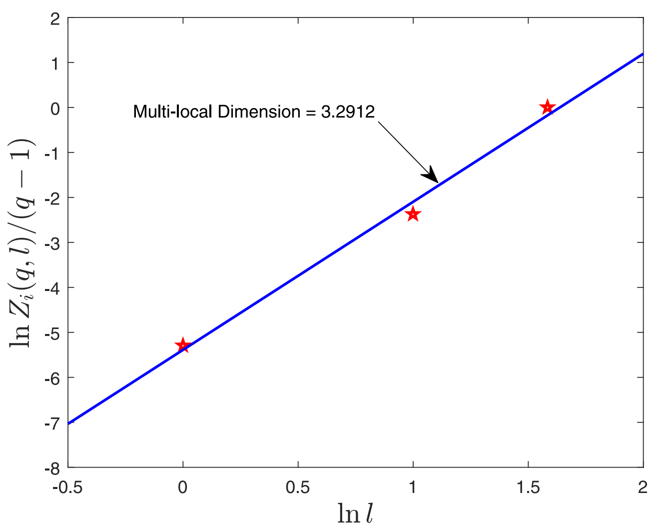


Fig. 3. The linear regression of the multi-local dimension of node i when $q = 2$.

by the slope of the linear regression of $\ln Z_i(q, l)/(q-1)$ against $\ln l$, and when $q = 1$, MLD would be obtained by the slope of the linear regression of $Z_i(1, l)$ against $\ln l$. The example of linear regression for node i when $q = 2$ is shown in Fig. 3, and it is $MLD_i(2) = 3.2912$. The multi-local dimensions of node i when $q = 0, q = 1, q = 2$ are 1.6456, 0.2276, 3.2912 respectively. Lastly, the MLD of the remaining nodes can also be obtained according to the same process.

In this proposed method, the scale of locality for each node is different, which is decided by the maximum value of the shortest distance ξ_i from central node i. The box radius l would change from 1 to ξ_i for each node. Similar to the multi-fractal dimension, the multi-local dimension can degenerate to other dimensions with different values of q, and it is detailed shown below:

- When $q = 1$, MLD would degenerate to local information dimension [70].
- When $q = 0$, MLD would degenerate to variant of local dimension [67].

Both of these two methods have been applied to identify the influential spreaders in the complex network. Thus, this proposed method MLD is a more general method. In addition, MLD can consider the information of networks in different scales, which can give a more effective result.

2.4. Vital spreaders identification

When the multi-local dimension MLD_i is obtained, the importance of spreaders can be ranked by the value of the multi-local dimension. Different from previous methods, the spreader would be more important with smaller MLD. The details can be shown in Section 3.

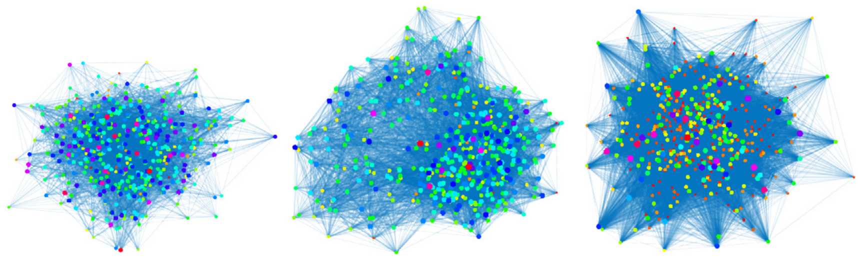
3. Experimental study

In this section, six different scale real-world complex networks, three theoretical complex networks, and three comparison methods are used in this section to show the reasonableness and effectiveness of this proposed method. Four kinds of experiments are utilized in this section, including giving top-10 nodes lists, obtaining the individuation of each nodes' rank results, measuring the infectious ability of initial nodes, describing the relationship between different methods and infectious ability obtained by SI model.

3.1. Data

There are six different scale real-world complex networks and three theoretical complex networks used in this section to show the effectiveness and reasonableness of this proposed method, and they are:

(1) The Zacharys Karate network: This network demonstrates the relationship between many individuals in one USA university karate club;
(2) The Jazz musicians network: This network shows the collaborations between different jazz musicians;
(3) The USA airline network: This network represents the airlines between the big city airports in the USA;
(4) The Political blogs network: This network demonstrates the blogs' connection in two camps in the USA.



(a) Random network (b) Small-World network (c) Scale-free network

Fig. 4. The structure of three theoretical networks. The color and size of nodes mean the degree of nodes.

Table 2
The topological properties of real-world and theoretical complex networks.

Network	$\|N\|$	$\|E\|$	$\langle k\rangle$	$k_{\max}$	$\langle\omega\rangle$	$\omega_{\max}$
Karate	34	78	4.5882	17	2.4082	5
Jazz	198	2742	27.6970	100	2.2350	6
USAir	332	2126	12.8072	139	2.7381	6
Political blogs	1222	16714	27.3552	351	2.7375	8
Email	1133	5451	9.6222	71	3.6060	8
Power	4941	6594	2.6691	19	18.9892	46
Random network	500	5000	20.0000	29	2.3933	3
Small-world network	500	10000	40.0000	51	2.0676	3
Scale-free network	500	18440	73.7600	234	1.8527	3

(5) The Email network: This network shows the email exchanges among members of the University Rovira i Virgili.
(6) The Power network: This network shows the topology of the Western States Power Grid of the USA.
(7) Scale-free network: The degree distribution of this network follows a power law, and this network has 500 nodes and 40 seed nodes in this section.
(8) Small-world network: This network shows the small-world property, and the number of nodes is 500 and the average degree is 40 in this section.
(9) Random network: This network is generated randomly, and there are 500 nodes and the average degree is 20 in this section.

These real-world networks can download from http://vlado.fmf.uni-lj.si/pub/networks/data/ and http://www-personal.umich.edu/~mejn/netdata/. The structure of three theoretical networks is shown in Fig. 4. The detailed structural information of these networks isare] shown in Table 2. $|N|$ and $|E|$ are the number of nodes and edges in the network respectively. $\langle k\rangle$ and $k_{\max}$ are the average value and maximum value of the degree of nodes in the entire network. $\langle\omega\rangle$ and $\omega_{\max}$ represent the average value and maximum value of the shortest distance in the network.

3.2. Existing centrality methods

Before the experiment begins, let us introduce some existing centrality methods to identify the influential nodes as comparison methods in this section. Because MLD would degenerate to local information dimension and variant of local dimension, these two methods would not be used as comparison methods in this section.

Definition 3.1. Betweenness centrality (BC) [30]. The betweenness centrality of node i is expressed as $C_B(i)$, and it is defined as follows,

$$C_B(i) = \sum_{s,t\neq i} \frac{g_{st}(i)}{g_{st}} \tag{10}$$

where $g_{st}(i)$ means the shortest path between node s and node t which go through node i, and g_{st} means the shortest path between node s and node t. Node s and node t would traverse all nodes in the network. Thus, BC highlights the intermediary role of the selected node.

Definition 3.2. Closeness centrality (CC) [31]. The closeness centrality of node i is expressed as $C_C(i)$, and it is defined as follows,

$$C_C(i) = \left(\sum_{j\in N} \omega_{ij}\right)^{-1} \tag{11}$$

where ω_{ij} is the shortest distance between node i and node j which belongs to the shortest distance matrix W, and node j would traverse all nodes in the network. Thus, CC highlights that the selected node can quickly reach any node in the network.

Definition 3.3. Degree centrality (DC) [32]. The degree centrality of node i is expressed as $C_D(i)$, and it is defined as follows,

$$C_D(i) = \sum_{j\in N} a_{ij} \tag{12}$$

where a_{ij} is the element in adjacency matrix A, and node j would traverse all nodes in the network. When there is an edge between node i and node j, a_{ij} would equal to 1, and $a_{ij} = 0$ represents the opposite situation. In fact, the degree centrality of node i represents the number of edges connected with node i. Thus, DC highlights the number of neighbor nodes around the selected node in the network.

3.3. Experiment I: Top-10 nodes

In this experiment, the top-10 nodes lists are obtained by different methods to show the difference and correlation between these methods, and these lists are shown in Table 3. Because these methods consider different parts of information in the network, their rank lists may be different from the others. When two methods' top-10 nodes lists are similar, their consideration information would be similar. In addition, the same nodes between MLD and other methods can bring more credibility to this proposed method. These nodes which only appear in MLD result would have a significant improvement to the propagation process.

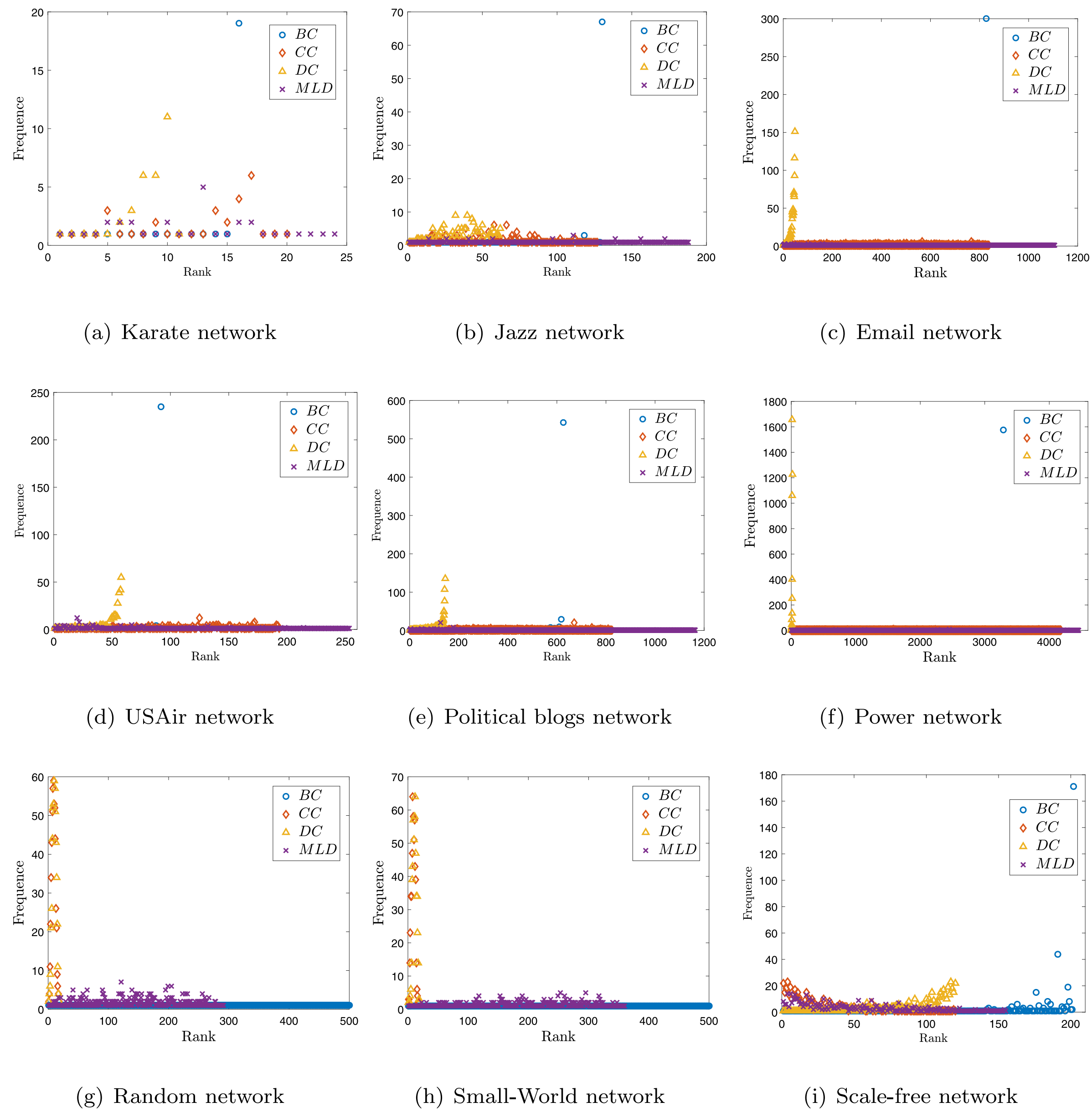
(1) Observing the result in Karate network from Table 3, the most similar lists to MLD is BC, and there are eight same nodes between BC and MLD. The number of same nodes between CC, DC, and MLD is five and six nodes respectively, which is relatively low compared to the results between BC and MLD. The result means that the most similar method to MLD is BC, which is different from the later experiments.

Table 3
The top-10 nodes ranked by different centrality methods in six real-world and three theoretical complex networks.

Rank	Karate network				Jazz network				Email network			
	BC	CC	DC	MLD	BC	CC	DC	MLD	BC	CC	DC	MLD
1	1	1	34	34	136	136	136	60	23	333	105	105
2	3	3	1	1	60	60	60	136	105	23	333	333
3	34	34	33	33	153	168	132	132	333	105	42	23
4	33	32	3	24	5	70	168	83	76	42	23	42
5	32	33	2	3	149	83	70	168	42	41	16	16
6	6	14	32	2	189	132	108	99	578	76	41	434
7	2	9	4	30	167	194	99	108	135	233	196	41
8	28	20	24	6	96	122	158	158	41	52	233	14
9	24	2	14	7	115	174	83	194	52	135	76	468
10	9	4	9	28	83	158	7	7	355	378	21	299
Rank	USAir network				Political blogs network				Power network			
	BC	CC	DC	MLD	BC	CC	DC	MLD	BC	CC	DC	MLD
1	118	118	118	118	12	28	12	12	4164	1308	2553	3312
2	8	261	261	261	304	12	28	28	4219	2594	4458	2554
3	261	67	255	152	94	16	304	304	2543	2605	4345	1166
4	47	255	182	230	28	14	14	14	2528	1131	3468	4458
5	201	201	152	255	145	36	16	16	69	2606	831	2553
6	67	182	230	182	6	67	94	94	108	1243	3895	1170
7	313	47	166	112	16	94	6	6	70	1476	2585	2434
8	13	248	67	147	300	35	67	67	1308	2557	2575	4345
9	182	166	112	166	163	145	35	35	2606	2528	2542	3351
10	152	112	201	293	35	304	145	36	4120	2532	2382	2575
Rank	Random network				Small-world network				Scale-free network			
	BC	CC	DC	MLD	BC	CC	DC	MLD	BC	CC	DC	MLD
1	9	327	345	316	202	401	453	386	42	500	45	45
2	7	56	316	345	12	339	386	453	41	499	56	42
3	26	396	409	393	31	326	430	98	47	498	42	56
4	25	108	393	409	75	490	98	430	5	497	47	47
5	18	62	347	214	38	482	202	75	43	496	46	46
6	2	37	214	347	64	119	75	202	16	495	43	43
7	63	487	370	370	43	421	406	137	45	493	41	41
8	16	486	270	172	15	417	312	201	6	492	50	50
9	30	466	219	270	210	393	201	312	4	491	51	51
10	23	297	172	219	104	320	137	406	2	490	48	48

(2) In Jazz network, the result between BC and MLD is the most dissimilar, and there are only three same nodes between these two methods which is the lowest same number of all results. Compared to CC with this proposed method, there are seven same top-10 nodes. In addition, the top-10 nodes lists are almost the same using MLD and DC, and it is nine same nodes in the top-10 nodes lists.
(3) Observing from the result in Email network, DC is the most similar list with MLD and there are five same nodes. In addition, the other two comparison methods only have three same nodes with MLD. This result shows that DC the most similar method with MLD, but the number of similar nodes is relatively smaller than other networks.
(4) Similar to Jazz network's result, the number of the same top-10 nodes between DC and MLD in USAir network is the highest in three comparison methods, and it is eight same nodes. There are six same nodes between this proposed method and CC in these top-10 nodes lists. The number of same top-10 nodes between BC and MLD is also the lowest in three comparison methods, and there are only four same nodes between two methods, which means there is difference between BC and MLD. The most influential node identified by three comparison methods and MLD is the same, and it is node 118, which means the accuracy of this proposed method.
(5) Observing the Political blogs network's result from Table 3, all comparison methods have many same nodes in top-10 lists. CC and DC both have nine same top-10 nodes with MLD, and this only one different node is the ninth and tenth node respectively. The lowest number of same nodes is between BC and MLD, and it is seven, which is bigger than the results in other networks. The top-2 nodes are the same in CC, DC, and MLD. From the result in this network, it can be found the similarity between this proposed method and other comparison methods is high.
(6) In Power network, the results show that these three comparison methods are totally different from MLD. The detail infectious ability difference of these methods would be shown in the following experiments.
(7) In these three theoretical networks, the results are similar, so we analyze them together. CC differs from MLD in all three theoretical networks, that is, no same nodes between CC and MLD. In contrast, DC and MLD are identical in the three networks, but the order is not exactly the same. Lastly, BC shows different performances in three networks, and there are zero, two, and five same nodes in Random, Small-world, Scale-free network respectively. The results in three theoretical networks mean DC is the most similar method with MLD, and CC is the most dissimilar method.

In conclusion, observing from the number of the same top-10 nodes, this proposed method has close performance with DC, and it is far from the other two methods. The effectiveness and superiority would be demonstrated in the following sections. Because this proposed method MLD can degenerate to local information dimension and variant of local dimension, these two methods would not be contained in the following experiments.



(a) Karate network (b) Jazz network (c) Email network

(d) USAir network (e) Political blogs network (f) Power network

(g) Random network (h) Small-World network (i) Scale-free network

Fig. 5. The frequency of nodes in each rank obtained by different methods in complex networks. Fewer nodes with the same score and more rankings mean the effectiveness of this method. It can be found MLD is the most effective method in these four real-world networks, and BC is the most effective method in three theoretical networks.

3.4. Experiment II: Individuation

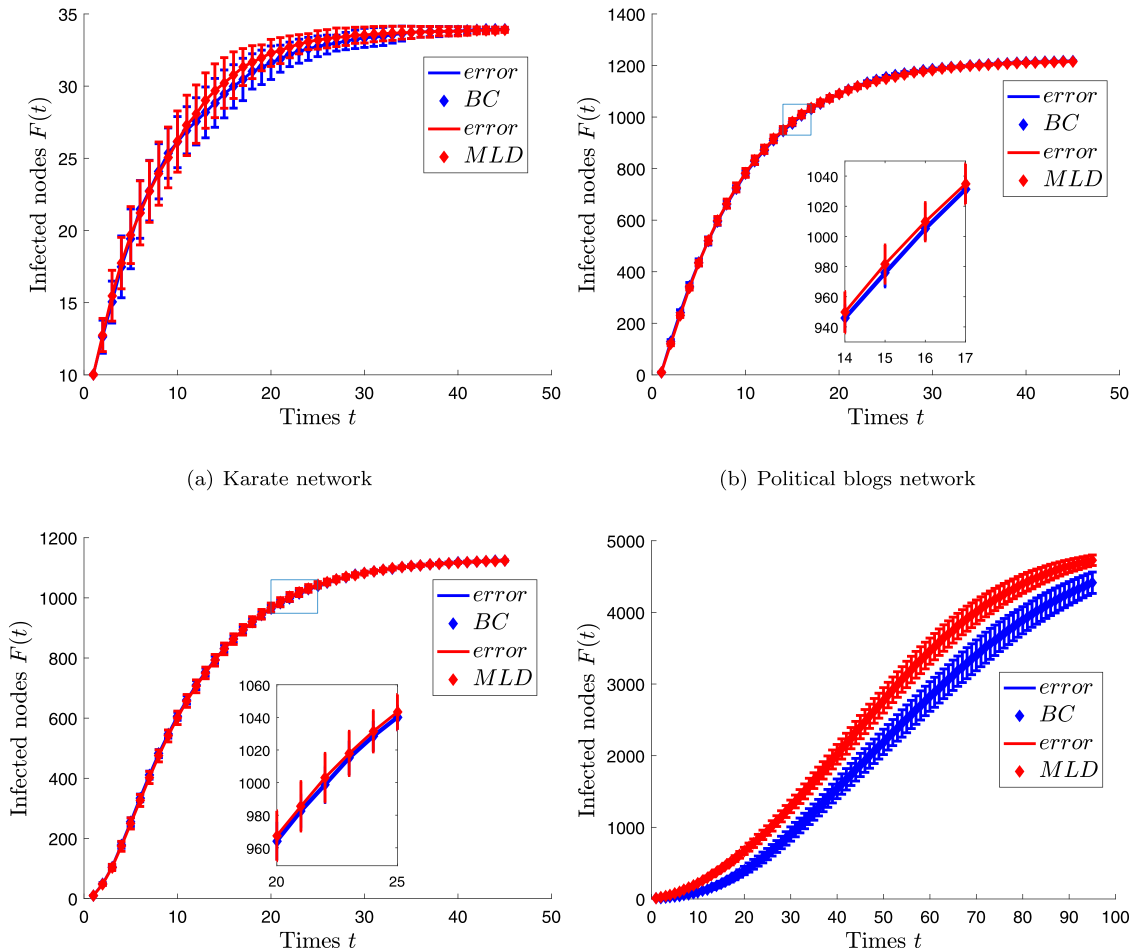
Then, different methods' capability to identify influential nodes are explored in this section. The importance of these nodes with the same score (frequency) cannot be distinguished correctly, but it is a common situation in this field. Thus, a more useful method should be found to give nodes as individual values as possible. If one method can give all these nodes with a unique score, this method can give a reasonable importance rank list to avoid ambiguous rank results. So the individual of each method can be considered an effective indicator to show the quantity of different methods. The higher the individual of one method is, the more effective this method is.

Definition 3.4. The individuation of one method is defined as follows,

$$\gamma(\cdot) = \frac{N_S(\cdot)}{|N|} \tag{13}$$

where $N_S(\cdot)$ is the number of nodes with a unique score, $|N|$ is the number of nodes in the entire network. $\gamma(\cdot)$ is the individuation of one method.

The frequency of nodes in each rank obtained by different methods is shown in Fig. 5. In these real-world complex networks, it can be found that MLD has the least number of nodes in the same rank, and there are more ranks in this proposed method. In contrast, the other three comparison methods have more nodes with the same rank. In these real-world networks, DC has the least rank which mean there are lots of nodes with the same



(a) Karate network

(b) Political blogs network

(c) Email network

(d) Power network

Fig. 6. The number of infectious nodes $F(t)$ with different initial nodes obtained by BC and MLD in four real-world complex networks. The details of these figures are enlarged to facilitate observation. The high $F(t)$ in each time t means the strong infectious ability of these initial nodes.

ranks. Most nodes have the same rank in the previous period, resulting in the inability to distinguish lots of nodes' infectious capabilities. The frequency of nodes in most of the top ranks is relatively low in BC, but the last few ranks have a very high frequency (almost half node), which means BC cannot identify these nodes with low C_B. CC can give a relatively reasonable rank, because most of the frequency in each rank is low and there are relatively more ranks. However, compared with CC, MLD is more effective to identify the influential nodes. That is because the frequency of nodes in each rank is the least in these four methods, and there are the most ranks (almost one node has one unique rank) in these networks.

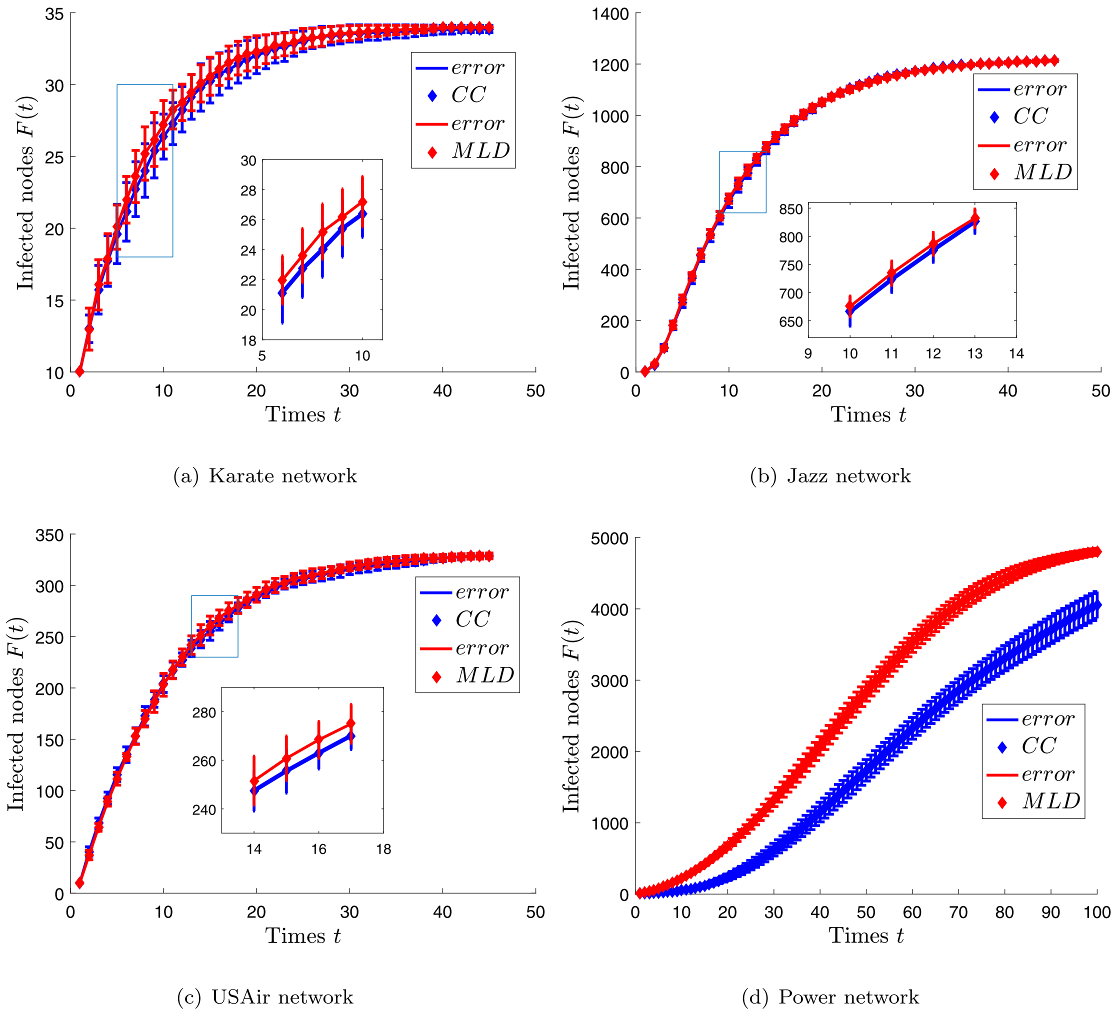
In these theoretical networks, the situation is different from the actual network. BC has most ranks which means BC is more effective than other methods, and all nodes have unique ranks in Random and Small-world network ($\gamma(BC) = 1$). Only the last few ranks have a very high frequency with BC in Scale-free network, and about half of nodes cannot be identified effectively by this method. MLD is the second effective method in these four methods, and some nodes in the last ranks have a relatively high frequency which means poorly infectious nodes cannot be sorted well. BC and CC are similar, and most nodes have the same rank in the Random and Small-world network. In Scale-free network, BC and CC perform better, and these methods can give relatively

Table 4
The individuation $\gamma(\cdot)$ of different methods in complex networks.

Network	$\gamma(BC)$	$\gamma(CC)$	$\gamma(DC)$	$\gamma(MLD)$
Karate	0.4705	0.5882	0.3235	**0.7058**
Jazz	0.6565	0.6414	0.3131	**0.9494**
USAir	0.2771	0.5813	0.1746	**0.7620**
Political blogs	0.5114	0.6743	0.1178	**0.9525**
Email	0.7308	0.7405	0.0424	**0.9762**
Power	0.6650	0.8463	0.0032	**0.9016**
Random network	**1.000**	0.0360	0.0360	0.5820
Small-world network	**1.000**	0.0380	0.0380	0.7200
Scale-free network	**0.404**	0.2400	0.2400	0.3100

reasonable ranks with low frequency. In theoretical networks, BC is the most effective method which has the least nodes in each rank. However, the difference of BC value is small which would be shown in Experiment IV.

The individuation $\gamma(\cdot)$ of different methods in these complex networks are shown in Table 4, where the highest $\gamma(\cdot)$ is bold. It can be found that MLD has the highest individuation $\gamma(\cdot)$ in six real-world complex networks, and DC has the lowest individuation $\gamma(\cdot)$. In three theoretical networks, BC has the highest $\gamma(\cdot)$, and equals 1 in Random and Small-world network. MLD is the second effective method and it is far better than the other



(a) Karate network (b) Jazz network

(c) USAir network (d) Power network

Fig. 7. The number of infectious nodes $F(t)$ with different initial nodes obtained by CC and MLD in four real-world complex networks. The details of these figures are enlarged to facilitate observation. The high $F(t)$ in each time t means the strong infectious ability of these initial nodes.

two methods. The difference of BC value is small with the highest individuation, and it would be introduced detailed in Experiment IV. These results mean that this proposed method is an effective method to identify the influential nodes in the complex network.

3.5. Experiment III: SI model

In this section, Susceptible–Infected (SI) model [24] is applied to show the effectiveness and reasonableness of this proposed method. The details of SI model are introduced below.

Step 1 For the entire network, all nodes are classified into two states, and they are *susceptible states* and *infected states*.

Step 2 At the beginning, the top-10 nodes obtained by centrality methods (shown in Table 3) are set as the initial infected state, and the other nodes are set as the susceptible state.

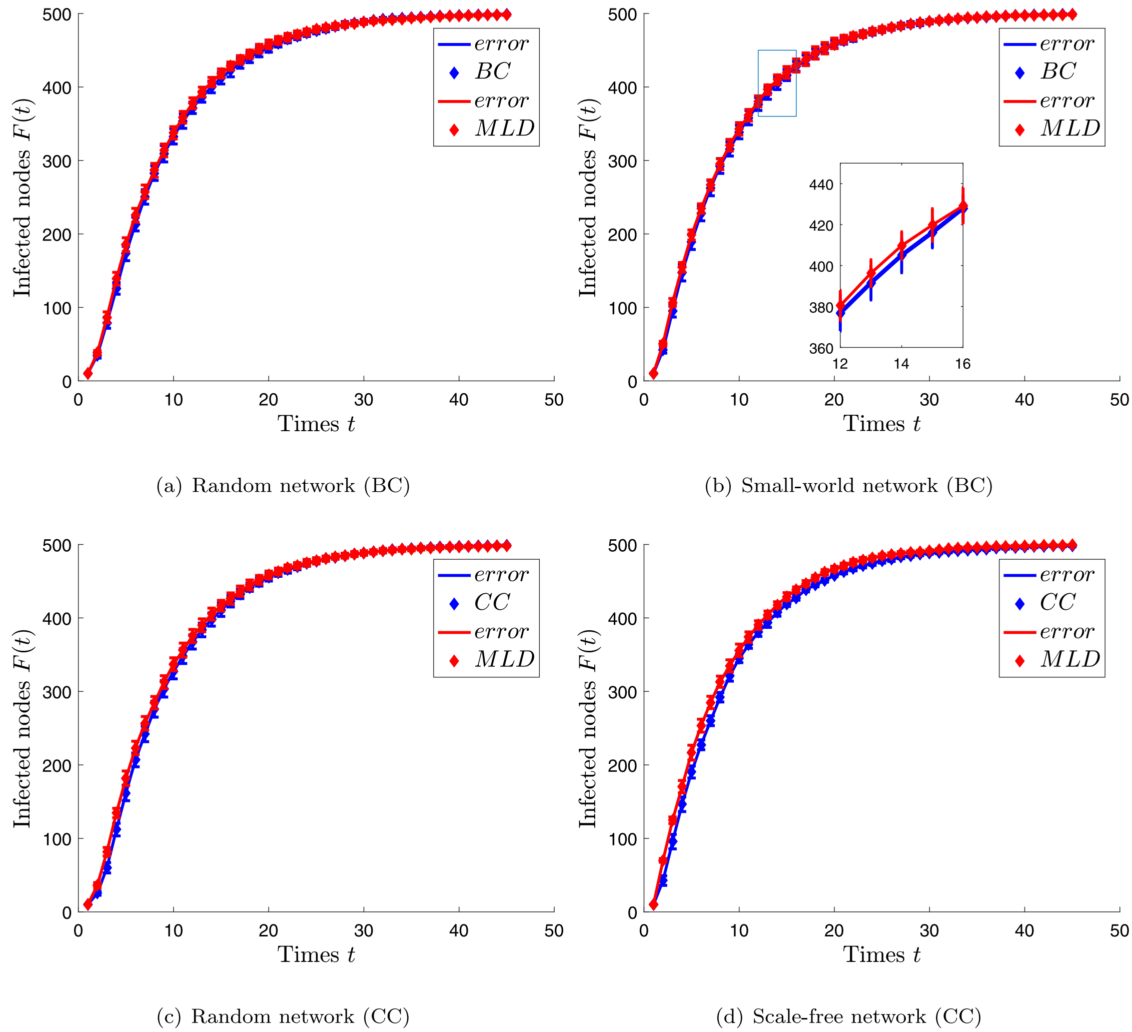
Step 3 When the infection process begins, these susceptible nodes can be affected by their neighbor nodes with a given probability (spreading ability) $\lambda = (1/2)^{\beta}$ in each time t. In addition, the total number of susceptible nodes and infectious nodes equals $|N|$ at any time t.

Step 4 Once the susceptible node is infected into infectious state, it cannot return to the susceptible state, i.e. it is the irreversible process.

Step 5 The number of infectious nodes $F(t)$ would continue to increase over time t until all nodes are infected.

These initial nodes with higher infection ability would infect the entire network as early as possible, so the number of infectious node $F(t)$ can be an effective indicator to show the infection ability of initial nodes, i.e. the importance of initial nodes. More infectious nodes in each time t are, higher infectious ability these initial nodes are, more important these initial nodes are. BC and CC consider the nodes' distance from the selected node, and the top-10 nodes are more dissimilar with MLD. Thus, BC and CC are selected as the comparison method in this section. In these networks, all results $F(t)$ would average the results of 30 SI experiments with $\beta = 3$, the results in real-world networks are shown in Figs. 6 and 7, and the results of theoretical networks are shown in Fig. 8.

Observing from the comparison with BC of real-world networks in Fig. 6, the infection ability of top-10 nodes obtained by MLD is more effective than the nodes obtained by BC. In Karate network shown in Fig. 6(a), the number of infectious nodes obtained by MLD in the mid-term is clearly higher than the number of nodes obtained by BC. From Figs. 6(b) and 6(c), the performance of MLD is slightly better than BC in Political blogs network and Email network, and the difference can be seen



(a) Random network (BC)

(b) Small-world network (BC)

(c) Random network (CC)

(d) Scale-free network (CC)

Fig. 8. The number of infectious nodes $F(t)$ with different initial nodes obtained by different methods in three theoretical complex networks. The details of these figures are enlarged to facilitate observation. The high $F(t)$ in each time t means the strong infectious ability of these initial nodes.

from the enlarged figure in the middle propagation process. In Power network (Fig. 6(d)), the infection ability of top-10 nodes obtained by MLD is significantly superior to BC, which can be observed from the whole process. The result in Power network shows MLD is more reasonable than BC even there is no same nodes in the top-10 node list. Thus, MLD is superior to BC which can be supported by the comparison experiment in Fig. 6.

Then, CC is compared with MLD in Fig. 7, and the number of infectious nodes $F(t)$ continue to increase overall time t. In Karate network shown in Fig. 7(a), the infection ability of initial nodes obtained by MLD is clearly superior to CC, and it can be seen that $F(t)$ obtained by MLD is larger than $F(t)$ obtained by CC from the whole process. In Jazz network shown in Fig. 7(b), the performance of MLD is better than CC which can be seen from the early and middle propagation process in SI model. In USAir network shown in Fig. 7(c), the infection ability of initial nodes obtained by MLD is superior to these nodes obtained by CC, and it can be seen from the middle and late term of SI model, $F(t)$ obtained by MLD is larger than $F(t)$ obtained by CC in this term. For Power network in Fig. 7(d), the performance of MLD is far better than CC, which is similar to Fig. 6(d). Thus, the top-10 nodes' infection ability obtained by MLD is stronger than other methods with the completely different top-10 node lists in Power network. So, the comparison between CC and MLD shows that the nodes obtained by MLD have better infection ability.

Lastly, BC and CC are compared with MLD in theoretical networks in Fig. 8. In Random network (Fig. 8(a)), it can be found that MLD spreads faster than BC in the early and middle stages. The initial nodes' infection ability obtained by MLD is better than BC in Small-world network (Fig. 8(b)), which can be seen in the enlarged curve. In addition, even the individuation of BC is high in theoretical networks in Experiment II, the infection ability of top-10 nodes obtained by MLD is superior to BC, which shows the effectiveness of MLD. The comparison between CC and MLD is shown in Random (Fig. 8(c)) and Scale-free network (Fig. 8(d)). The number of infectious nodes obtained by MLD is bigger than the number obtained by CC in these two theoretical networks, which means the superiority of this proposed method even if no same nodes in the top-10 nodes obtained by these two methods.

In conclusion, this proposed method MLD is compared with BC and CC in six real-world and three theoretical complex networks. Most results show that this proposed method has a superiority performance, and some times the performance of MLD is close to the comparison method. The infection ability of top-10 nodes obtained by MLD is better than other methods in Power network even if the top-10 nodes obtained by different methods are completely different. In theoretical networks, although BC has the

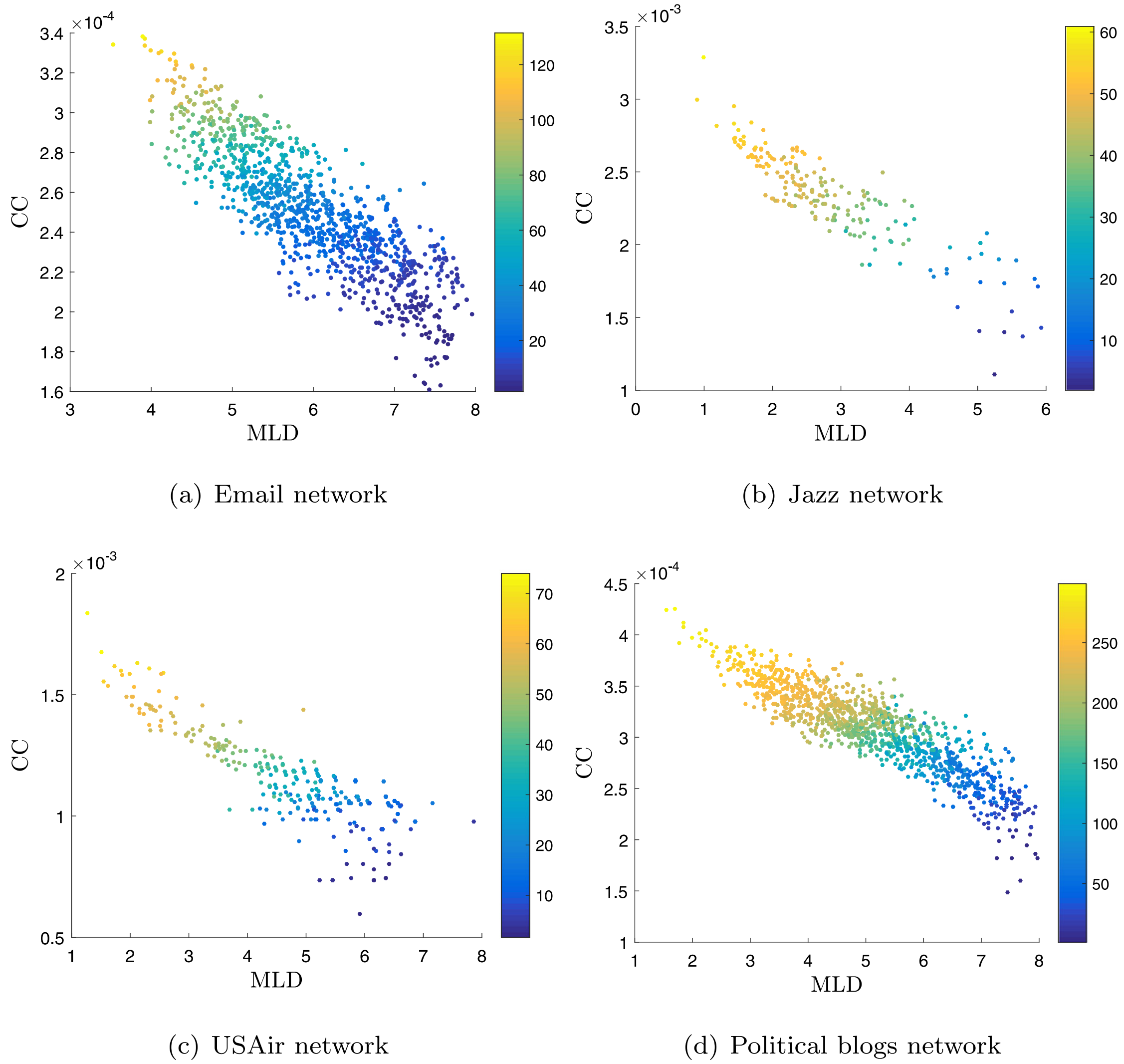


Fig. 9. The relationship between MLD and CC when $\lambda = 0.05$ in real-world networks. The value on the horizontal and vertical axes means the value obtained by MLD and CC respectively, and the color of point means the infectious ability obtained by SI model.

highest individuation, MLD has better performance, i.e. stronger infection ability than BC in the SI model.

3.6. Experiment IV: The relationship between different methods

Then, the values obtained by different methods and the infectious ability obtained by SI model are compared in this section. Because BC has lots of nodes with the same value which would cause unusual relationships between BC and MLD, the comparison methods are chosen as CC and DC in real-world networks, and they are shown in Fig. 9 (CC vs. MLD) and Fig. 10 (DC vs. MLD). Meantime, to show the reason why BC has the highest individuation in theoretical networks, BC is chosen as the comparison method in Fig. 11.

One point in the relationship graph represents one node in the network, the value of axis means the value obtained by different methods, and the color of points shows the infectious ability of this node obtained by SI model, i.e. the number of infected nodes ($F(10)$) in 10 steps. The infectious ability of nodes is obtained by averaging 50 independent experiments results when $\lambda = 0.05$. The positive correlation means these nodes would have a large value obtained by the comparison method and MLD, and negative correlation is the opposite.

Observing from Fig. 9, CC and MLD is a negative correlation, and their relationship is linear which can give similar rank results between these two methods. In addition, the value obtained by CC is relatively small than other methods (small order of magnitude) which cannot clearly show the difference in nodes' importance.

Observing from Fig. 10, the correlation between DC and MLD is also negative, which means the node with large MLD would have small DC. What is more, it can be found that there are lots of nodes with small degree centrality, which is because of the scale-free property of the complex network. Thus, there would be lots of nodes with small DC that cannot correctly identify importance. However, MLD can overcome this shortcoming, because the MLD of nodes would be more scattered which can give each node with unique value and obtain a relatively reasonable rank list.

The correlation between different methods in theoretical networks can be observed from Fig. 11. From the comparison with BC, it can be found there are lots of nodes that are small and close. This means although BC has high individuation in theoretical networks, it is concentrated in the lower rank and the values are extremely close. From the comparison with DC, it can be found

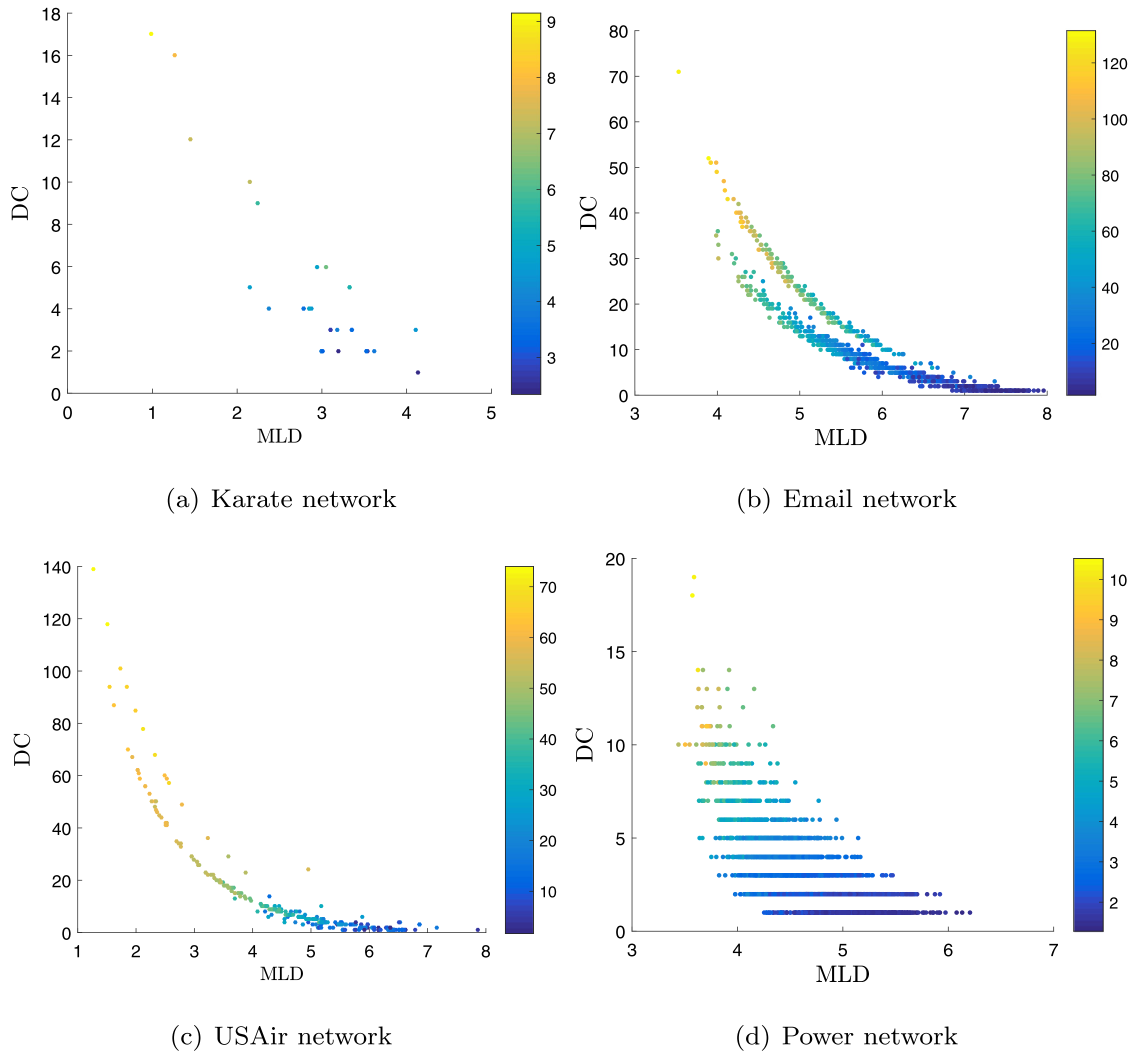


Fig. 10. The relationship between MLD and DC when $\lambda = 0.05$ in real-world networks. The value on the horizontal and vertical axes means the value obtained by MLD and DC respectively, and the color of point means the infectious ability obtained by SI model.

that DC and MLD is a negative correlation, and their relationship is linear. More detail, it can be found lots of nodes have the same value, which is not beneficial for importance ranking. In contrast, MLD can overcome this shortcoming and give node unique value.

Overall speaking, this proposed method would be different from existing methods, which is a negative correlated with existing methods. In addition, this proposed method can give a more reasonable rank list because it can identify the importance of nodes with close values obtained by existing methods.

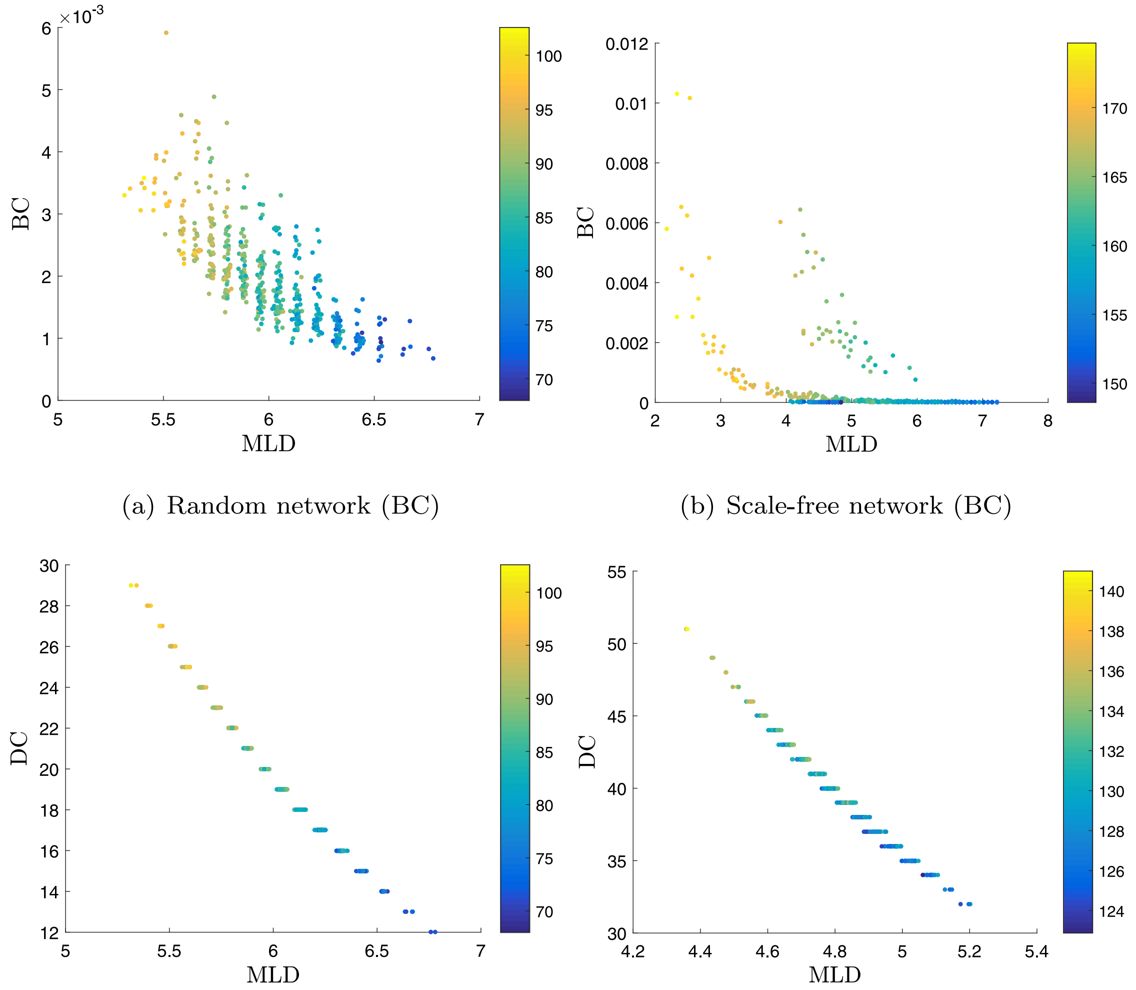
4. Conclusion

In this paper, a novel method is proposed to identify the influential nodes based on the multi-local dimension in the complex networks. Different from previous methods, this proposed method is a more general method, because it can degenerate to local information dimension and variant of local dimension with the different chosen weighting coefficient q. In addition, this proposed method is a negative correlated with existing methods which means the influential nodes would have small values of MLD and large values of existing centrality methods. Comparing with the existing centrality methods, this proposed method can effectively identify the influential nodes in the network and give a reasonable rank to these nodes, which can overcome the limitations of previous methods.

However, this proposed method can still be improved to meet the high requirements in this field. For instance, there are still some nodes with the same value of MLD, and the ranking of these nodes is relative top, which can mislead to form the correct node importance rank. Thus, in further research, the consideration factors of this method can be changed, which can demonstrate the property of the network more specifically.

CRediT authorship contribution statement

Tao Wen: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Resources, Software, Visualization, Writing - original draft. **Danilo Pelusi:** Validation, Writing - review & editing. **Yong Deng:** Funding acquisition, Project administration, Supervision, Validation, Writing - review & editing.



(a) Random network (BC) (b) Scale-free network (BC)

(c) Random network (DC) (d) Small-world network (DC)

Fig. 11. The relationship between two methods when $\lambda = 0.05$ in theoretical complex networks. The value on the horizontal and vertical axes means the value obtained by two methods respectively, and the color of point means the infectious ability obtained by SI model.

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References

[1] X. Guan, Z. Xu, Q.-S. Jia, K. Liu, Y. Zhou, Cyber-physical model for efficient and secured operation of cpes or energy internet, Sci. China Inf. Sci. 61 (11) (2018).
[2] H. Fujita, A. Gaeta, V. Loia, F. Orciuoli, Hypotheses analysis and assessment in counter-terrorism activities: a method based on owa and fuzzy probabilistic rough sets, IEEE Trans. Fuzzy Syst. (2019) 1.
[3] H.C. Yang, Y.J. liu, Q. Wan, Y. Deng, A bio-inspired optimal network division method, Physica A 527 (2019) 121259.
[4] H. Fujita, A. Gaeta, V. Loia, F. Orciuoli, Resilience analysis of critical infrastructures: A cognitive approach based on granular computing, IEEE Trans. Cybern. 49 (5) (2019) 1835–1848.
[5] H. Fujita, A. Gaeta, V. Loia, F. Orciuoli, Improving awareness in early stages of security analysis: A zone partition method based on grc, Appl. Intell. 49 (3) (2019) 1063–1077.
[6] S. Xu, W. Jiang, X. Deng, Y. Shou, A modified physarum-inspired model for the user equilibrium traffic assignment problem, Appl. Math. Model. 55 (2018) 340–353.
[7] K.Y. Wu, M.A. Esfahani, S.H. Yuan, H. Wang, Tdpp-net: Achieving three-dimensional path planning via a deep neural network architecture, Neurocomputing 357 (2019) 151–162.
[8] H.C. Yang, Y. Deng, R. Mayne, A bio-inspired network design method for intelligent transportation, Int. J. Unconv. Comput. 14 (3) (2019) 197–208.
[9] E. Rosenberg, Minimal partition coverings and generalized dimensions of a complex network, Phys. Lett. A 381 (19) (2017) 1659–1664.
[10] L.K. Gallos, S. Havlin, H.E. Stanley, N.H. Fefferman, Propinquity drives the emergence of network structure and density, Proc. Natl. Acad. Sci. USA 116 (41) (2019) 20360–20365.
[11] C. Song, L.K. Gallos, S. Havlin, H.A. Makse, How to calculate the fractal dimension of a complex network: the box covering algorithm, J. Stat. Mech. Theory Exp. 2007 (03) (2007) P03006.
[12] C.M. Song, S. Havlin, H.A. Makse, Self-similarity of complex networks, Nature 433 (7024) (2005) 392–395.
[13] T. Wen, S.Y. Duan, W. Jiang, Node similarity measuring in complex networks with relative entropy, Commun. Nonlinear Sci. Numer. Simul. 78 (2019) 104867.
[14] J. Wu, F. Chiclana, H. Fujita, E. Herrera-Viedma, A visual interaction consensus model for social network group decision making with trust propagation, Knowl.-Based Syst. 122 (2017) 39–50.
[15] L.Y. Lu, T. Zhou, Link prediction in complex networks: A survey, Physica A 390 (6) (2011) 1150–1170.
[16] J. Sun, H. Li, H. Fujita, B.B. Fu, W.G. Ai, Class-imbalanced dynamic financial distress prediction based on adaboost-svm ensemble combined with smote and time weighting, Inf. Fusion 56 (2020) 128–144.
[17] Z. Wang, M. Jusup, R.W. Wang, L. Shi, Y. Iwasa, Y. Moreno, J. Kurths, Onymity promotes cooperation in social dilemma experiments, Sci. Adv. 3 (3) (2017) 7.

[18] Z.-X. Tan, J.M. Koh, E.V. Koonin, K.H. Cheong, Predator dormancy is a stable adaptive strategy due to parrondo's paradox, Adv. Sci. 7 (3) (2020) 1901559.

[19] Z. Wang, C.T. Bauch, S. Bhattacharyya, A. d'Onofrio, P. Manfredi, M. Perc, N. Perra, M. Salathe, D.W. Zhao, Statistical physics of vaccination, Phys. Rep.-Rev. Sect. Phys. Lett. 664 (2016) 1–113.

[20] M. Perc, J.J. Jordan, D.G. Rand, Z. Wang, S. Boccaletti, A. Szolnoki, Statistical physics of human cooperation, Physics Reports-Review Section of Physics Letters 687 (2017) 1–51.

[21] T. Wen, M.X. Song, W. Jiang, Evaluating topological vulnerability based on fuzzy fractal dimension, Int. J. Fuzzy Syst. 20 (6) (2018) 1956–1967.

[22] Y. Liu, Q. Liu, C. Xie, F. Wei, Reliability assessment for multi-state systems with state transition dependency, Reliab. Eng. Syst. Saf. 188 (2019) 276–288.

[23] T. Wen, Y. Deng, The vulnerability of communities in complex networks: An entropy approach, Reliab. Eng. Syst. Saf. 196 (2020) 106782.

[24] L.Y. Lu, D.B. Chen, X.L. Ren, Q.M. Zhang, Y.C. Zhang, T. Zhou, Vital nodes identification in complex networks, Phys. Rep.-Rev. Sect. Phys. Lett. 650 (2016) 1–63.

[25] K.H. Cheong, J.M. Koh, M.C. Jones, Paradoxical survival: Examining the parrondo effect across biology, BioEssays 41 (6) (2019) 1900027.

[26] N. Capuano, F. Chiclana, H. Fujita, E. Herrera-Viedma, V. Loia, Fuzzy group decision making with incomplete information guided by social influence, IEEE Trans. Fuzzy Syst. 26 (3) (2018) 1704–1718.

[27] H. Liao, M.S. Mariani, M. Medo, Y.C. Zhang, M.Y. Zhou, Ranking in evolving complex networks, Phys. Rep.-Rev. Sect. Phys. Lett. 689 (2017) 1–54.

[28] T. Xiahou, Y. Liu, T. Jiang, Extended composite importance measures for multi-state systems with epistemic uncertainty of state assignment, Mech. Syst. Signal Process. 109 (2018) 305–329.

[29] F. Iannelli, M.S. Mariani, I.M. Sokolov, Influencers identification in complex networks through reaction-diffusion dynamics, Phys. Rev. E 98 (6) (2018) 15.

[30] M.E.J. Newman, A measure of betweenness centrality based on random walks, Social Networks 27 (1) (2005) 39–54.

[31] L.C. Freeman, Centrality in social networks conceptual clarification, Social Networks 1 (3) (1979) 215–239.

[32] M.E.J. Newman, The structure and function of complex networks, SIAM Rev. 45 (2) (2003) 167–256.

[33] S. Brin, L. Page, The anatomy of a large-scale hypertextual web search engine, Comput. Netw. Isdn Syst. 30 (1–7) (1998) 107–117.

[34] Y.C. Wang, S.S. Wang, Y. Deng, A modified efficiency centrality to identify influential nodes in weighted networks, Pramana-J. Phys. 92 (4) (2019) 11.

[35] S.L. Zhang, M. Medo, L.Y. Lu, M.S. Mariani, The long-term impact of ranking algorithms in growing networks, Inform. Sci. 488 (2019) 257–271.

[36] S. Srinivas, C. Rajendran, Community detection and influential node identification in complex networks using mathematical programming, Expert Syst. Appl. 135 (2019) 296–312.

[37] Y.C. Dong, Q.B. Zha, H.J. Zhang, G. Kou, H. Fujita, F. Chiclana, E. Herrera-Viedma, Consensus reaching in social network group decision making: Research paradigms and challenges, Knowl.-Based Syst. 162 (2018) 3–13.

[38] D. Pelusi, R. Mascella, L. Tallini, A fuzzy gravitational search algorithm to design optimal iir filters, Energies 11 (4) (2018) 18.

[39] D. Pelusi, R. Mascella, L. Tallini, J. Nayak, B. Naik, A. Abraham, Neural network and fuzzy system for the tuning of gravitational search algorithm parameters, Expert Syst. Appl. 102 (2018) 234–244.

[40] S.S. Feng, L. Wang, S.W. Sun, C.Y. Xia, Synchronization properties of interconnected network based on the vital node, Nonlinear Dynam. 93 (2) (2018) 335–347.

[41] F.Y. Meng, J. Tang, H. Fujita, Consistency-based algorithms for decision-making with interval fuzzy preference relations, IEEE Trans. Fuzzy Syst. 27 (10) (2019) 2052–2066.

[42] W. Jiang, A correlation coefficient for belief functions, Internat. J. Approx. Reason. 103 (2018) 94–106.

[43] D. Pelusi, On redundancy reduction of non-recursive second-order spectral-null codes, IEEE Access 7 (2019) 112171–112179.

[44] W. Jiang, Y. Cao, X.Y. Deng, A novel Z-network model based on Bayesian network and Z-number, IEEE Trans. Fuzzy Syst. (2019) http://dx.doi.org/10.1109/TFUZZ.2019.2918999.

[45] H. Fujita, D. Cimr, Computer aided detection for fibrillations and flutters using deep convolutional neural network, Inform. Sci. 486 (2019) 231–239.

[46] J.X. Tang, R.S. Zhang, Y.B. Yao, Z.L. Zhao, P. Wang, H. Li, J.L. Yuan, Maximizing the spread of influence via the collective intelligence of discrete bat algorithm, Knowl.-Based Syst. 160 (2018) 88–103.

[47] Q.H. Shi, C. Wang, J.W. Chen, Y. Feng, C. Chen, Location driven influence maximization: Online spread via offline deployment, Knowl.-Based Syst. 166 (2019) 30–41.

[48] L.G. Fei, Q. Zhang, Y. Deng, Identifying influential nodes in complex networks based on the inverse-square law, Physica A 512 (2018) 1044–1059.

[49] Z. Li, T. Ren, X. Ma, S. Liu, Y. Zhang, T. Zhou, Identifying influential spreaders by gravity model, Sci. Rep. 9 (2019).

[50] A. Zareie, A. Sheikhahmadi, K. Khamforoosh, Influence maximization in social networks based on TOPSIS, Expert Syst. Appl. 108 (2018) 96–107.

[51] H.M. Mo, Y. Deng, Identifying node importance based on evidence theory in complex networks, Physica A 529 (2019) 11.

[52] H. Fujita, D. Cimr, Decision support system for arrhythmia prediction using convolutional neural network structure without preprocessing, Appl. Intell. (2019) 1–9.

[53] A. Zareie, A. Sheikhahmadi, A. Fatemi, Influential nodes ranking in complex networks: An entropy-based approach, Chaos Solitons Fractals 104 (2017) 485–494.

[54] A. Sheikhahmadi, M.A. Nematbakhsh, A. Zareie, Identification of influential users by neighbors in online social networks, Physica A 486 (2017) 517–534.

[55] W. Zhang, J. Yang, X.Y. Ding, X.M. Zou, H.Y. Han, Q.C. Zhao, Groups make nodes powerful: Identifying influential nodes in social networks based on social conformity theory and community features, Expert Syst. Appl. 125 (2019) 249–258.

[56] L. Zuo, T. Xiahou, Y. Liu, Evidential network-based failure analysis for systems suffering common cause failure and model parameter uncertainty, Proc. Inst. Mech. Eng. C 233 (6) (2019) 2225–2235.

[57] L. Zuo, T. Xiahou, Y. Liu, Reliability assessment of systems subject to interval-valued probabilistic common cause failure by evidential networks, J. Intell. Fuzzy Systems 36 (4) (2019) 3711–3723.

[58] G. Del Ferraro, A. Moreno, B. Min, F. Morone, U. Perez-Ramirez, L. Perez-Cervera, L.C. Parra, A. Holodny, S. Canals, H.A. Makse, Finding influential nodes for integration in brain networks using optimal percolation theory, Nature Commun. 9 (2018) 12.

[59] F. Morone, H.A. Makse, Influence maximization in complex networks through optimal percolation, Nature 524 (7563) (2015) 65–U122.

[60] A. Sheikhahmadi, M.A. Nematbakhsh, Identification of multi-spreader users in social networks for viral marketing, J. Inf. Sci. 43 (3) (2017) 412–423.

[61] J. Wu, Q. Sun, H. Fujita, F. Chiclana, An attitudinal consensus degree to control the feedback mechanism in group decision making with different adjustment cost, Knowl.-Based Syst. 164 (2019) 265–273.

[62] J.W. Lai, K.H. Cheong, Parrondo's paradox from classical to quantum: A review, Nonlinear Dynam. (2020) 1–13.

[63] F. Riquelme, P. Gonzalez-Cantergiani, X. Molinero, M. Serna, Centrality measure in social networks based on linear threshold model, Knowl.-Based Syst. 140 (2018) 92–102.

[64] E. Rosenberg, Maximal entropy coverings and the information dimension of a complex network, Phys. Lett. A 381 (6) (2017) 574–580.

[65] E. Rosenberg, Non-monotonicity of the generalized dimensions of a complex network, Phys. Lett. A 381 (28) (2017) 2222–2229.

[66] T. Wen, W. Jiang, An information dimension of weighted complex networks, Physica A 501 (2018) 388–399.

[67] J. Pu, X.W. Chen, D.J. Wei, Q. Liu, Y. Deng, Identifying influential nodes based on local dimension, Epl 107 (1) (2014) 6.

[68] T. Bian, Y. Deng, Identifying influential nodes in complex networks: A node information dimension approach, Chaos 28 (2018) 043109.

[69] T. Wen, W. Jiang, Identifying influential nodes based on fuzzy local dimension in complex networks, Chaos Solitons Fractals 119 (2019) 332–342.

[70] T. Wen, Y. Deng, Identification of influencers in complex networks by local information dimensionality, Inform. Sci. 512 (2020) 549–562.

[71] E.W. Dijkstra, A note on two problems in connexion with graphs, Numer. Math. 1 (1) (1959) 269–271.

[72] F.N. Silva, L.F. Costa, Local dimension of complex networks, arXiv preprint, vol. arXiv, p. 1209.2476, 2013.

[73] E. Ben-Naim, H. Frauenfelder, Z. Toroczkai, Complex Networks, Springer Berlin Heidelberg, 2004, p. xviii,520.