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ABSTRACT

In this paper, emotions are classified into four types, namely, respect for the strong, envying the strong, sympathy for the weak, and bullying the weak. The corresponding relationship between the four emotion types and the two behaviors of competition and cooperation is then defined. The payoff matrices of the game based on emotions are obtained and the evolutionary dynamics of the four emotion types in a finite population based on the Moran process are studied. Next, we derive the absorption probabilities of a 4×4 symmetric evolutionary game of the population. The influence of the payoff parameters and the natural selection intensity on the result of the group evolution are then analyzed. The calculations indicate that there are differences in the absorption probabilities of the four absorption states of the system. At a steady state, individuals of the types envying the strong and bullying the weak have the highest probability of occupying the entire population, and individuals of the type respect for the strong and sympathy for the weak have the lowest one. By comparing the level of cooperation and average payoffs at a steady state, we observe that the level of cooperation and average payoffs based on the proposed model are better than those of the prisoner's dilemma game with two behaviors. Therefore, emotional evolution can promote cooperation and achieve better group fitness.

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Existing analysis on the occurrence and evolution of cooperative behavior is mostly based on individual rationality, but the hypothesis of individual rationality is unable to justify the sociological experimental results from the ultimatum game. Human emotions are rich and diverse, and diverse emotions are triggered by different situations. Emotions, for instance, empathy, bullying, respect, and jealousy, are generated based on different social situations. The information of social comparisons will affect the individual's emotional evolution and behavioral strategies accordingly. Based on the Moran process, we have analyzed the emotional evolution mechanism in social comparative situations. We have also studied the evolution of cooperation from an emotional perspective, and our study is able to shed some light on the "emotions" and "reasons" behind a certain social phenomenon.

I. INTRODUCTION

The evolutionary game in social dilemmas is a topic of immense interest. Recently, Benjamin and Sarkar use thermodynamic limits to study the trigger mechanism of cooperative behavior in the game of public goods. They have discovered that rewards and punishments trigger cooperation, while costs inhibit cooperation.¹ Benjamin and Dash analyze the thermodynamic sensitivity in the social dilemma of an unlimited population and obtain the evolutionary tendency of the strategy by observing the net change in the proportion of players in a certain strategy.² Wu *et al.*³ introduce dynamic environment and preference selection mechanism in the online prisoner's dilemma game, and find that even under severe temptation, the introduction of preference selection related to the evolutionary environment is an effective facilitator of battle. Liu *et al.*⁴ propose

that the combined use of pro-social punishment and exclusion strategies can maintain a sufficiently high level of public cooperation. Chen *et al.*⁵ propose that self-organizing interdependence can promote the evolution of cooperation in interdependent networks. All of the above examples are based on the game model of individual rationality. However, how can we model the game individuals to be more realistic like in real world conditions?

The interactions between human beings are full of emotions. Driven by different emotions, people will make certain behavioral choices and obtain different benefits. Experimental studies have suggested that emotions affect decision-making in many different ways. Tooby and Cosmides believe that the reason why emotional traits have adaptive advantages is that emotions affect the cost and benefit tradeoffs of individuals and allow individuals to optimize their decisions.⁶ Leith and Baumeister have discovered that embarrassment increases the preference for a long-shot (high-risk, high-payoff) lottery over a low-risk, low-payoff one. Anger has a similar effect. However, when the participants experience happiness or sadness, they tried to avoid the high-risk and high-payoff strategies.⁷ De Waal has discovered that primates have empathy, and this is related to their group lifestyles. It was concluded that emotional evolution promotes the generation of prosocial behavior.⁸ Nakahashi and Ohtsuki⁹ investigated the case when the group size is relatively large; individuals with the ability of emotional contagion will weaken their sensitivity to the environment and yield larger benefits to them.

Cognitive neuroscience research based on brain function activities has noticed that negative emotion and emotional responses are important factors affecting individual cooperation.¹⁰ Nowak *et al.*¹¹ noted that “although competition is a natural characteristic of natural selection, the following ‘charity’ attributes are essential in direct and indirect reciprocal winner strategies: hope, generosity, and magnanimity.” Therefore, the evolutionary mechanism of cooperation studying from an emotional perspective will enrich people’s cognition of the emergence of cooperative behavior.^{12–17} Based on the perspective of social comparison theory, the occurrence and termination of competition and cooperation among individuals may result from the relative comparison of the individuals’ own fitness and the fitness of others. Thus, this generates endogenous emotions of sympathy, indifference, bullying, respect, fear, and jealousy and the corresponding behaviors. Szolnoki *et al.*^{18,19} investigated individuals with two emotional parameters (envying the strong and sympathy for the weak) and established a quantitative mapping relationship between emotional characteristics and cooperation strategies. They proposed that imitating emotion instead of strategies in spatial games will elevate social welfare and envy is an important inhibitor of cooperative behavior. Based on the ultimatum game model, Wang *et al.*²⁰ and Ye *et al.*²¹ classified the emotions into four types: generous, mean, kind, and greedy by defining two parameters: the pay level and the level of net payoffs. A quantitative mapping relationship between emotional characteristics and game payoffs was also established. Individuals with fairness and moderate goodness are found to be better for survival and a fair solution cannot guarantee high payoffs for the group. The kindness of moderately negative expectation of net payoffs is the reason for group cooperation and high payoffs. Xie *et al.*²² classified emotions into four types, namely, respect, sympathy, bullying, and jealousy, and used a

genetic algorithm to simulate the evolution of the population. The results show that the diversity of emotions can effectively promote cooperation. The emergence of sympathy and respect has more evolutionary advantages. Wang *et al.*²³ further expanded emotions into six types and adopted a spatial prisoner’s dilemma game with voluntary participation. The evolutionary game results based on the lattice network have shown that as the temptation to betray grows, the social system gradually changes from a benign atmosphere of “respect and love” to a malignant state of “fearing and bullying.”

The above dynamics analysis of evolutionary games is based on numerical simulation methods, for instance, the Monte Carlo simulation. Regarding the theoretical analysis method of the evolutionary game under a limited group, the random evolutionary game model is generally used. The three commonly used random processes are the paired comparison process, the Moran process, and the Wright–Fisher process, among which the Moran process is the most widely studied and applied in asynchronous update mechanism. In Ref. 24, population evolution dynamics were analyzed based on the Moran process. Since randomness can render a certain phenotype in the population to reach absorption state, the absorption probability thus becomes an important indicator of the evolution dynamics. In addition, the influence of natural selection intensity on the evolution dynamics is also considered. Taylor *et al.*²⁵ derived the theoretical solution to the absorption probability on the basis of a 2×2 symmetric group evolutionary game. In Ref. 26, the evolutionary game with three strategies was studied and the recursive relationship of absorption probability was established. The global absorption probability and local absorption probability were defined. The conditions for a single strategy to invade the whole population were also discussed.

In this paper, emotions are classified into four types, namely, respect and envying for the strong, sympathy, and bullying for the weak. Based on the Moran process, the evolution of the four emotion types is studied. The absorption probabilities of a 4×4 symmetric group evolutionary game are deduced. The cooperation level and the average payoffs of the stable evolution are also elucidated. Finally, the effects of the payoff parameters and natural selection intensity on the results of group evolution are discussed.

II. MODEL BASED ON AN EMOTIONAL GAME

A. Fitness calculation

We define the corresponding relationship between four emotion types (sympathy, respect, bullying, and envying) and two behaviors (competition and cooperation) as follows: sympathy and respect correspond to cooperation, whereas bullying and envying correspond to competition. The four emotion types in the population represent: respect for the strong and sympathy for the weak (type A), sympathy for the weak and envying the strong (type B), respect for the strong and bullying the weak (type C), and envying the strong and bullying the weak (type D). Thus, the relationships between the four emotion types and the two behaviors are shown in Fig. 1.

The game processes are in the following: the population is fully mixed and the game is played randomly in pairs (without considering playing the game with itself). Based on the average field hypothesis, individuals in the population have equal probability of being relatively strong and relatively weak. The individual payoff

matrix of the four emotion types mentioned above is shown in Table I (the given values are the payoffs of the game corresponding to the row player). Among them, the parameter R is called the reward for cooperation between the two parties; the parameter J is called the penalty of mutual competition; the parameter T is called the temptation of successful competition; the parameter F is called the cost of cooperation failure. Take the game between an individual in type A and an individual in type B for instance. Since these two individuals have equal opportunities to be the strong and the weak, we will discuss individually: (1) when the individual in type A is the strong and the individual in type B is the weak, then the individual in type A sympathizes for the weak (cooperation) and the individual in type B envies the strong (competition). Thus, the individual in type A gains F , the individual in type B gains T . (2) When the individual in type A is the weak and the individual in

type B is the strong, then the individual in type A respects for the strong (cooperation) and the individual in type B sympathizes the weak (cooperation). Thus, the individual in type A gains R , the individual in type B also gains R . To sum up, the game payoff of the individual in type A is $(F + R)/2$, and that of the individual in type B is $(T + R)/2$.

For a finite population size N , let i represent the number of individuals in type A, j represent the number of individuals in type B, k represent the number of individuals in type C, and $N - i - j - k$ represent the number of individuals in type D. The system state is defined as a three-dimensional array (i, j, k) , where i , j , and k satisfy the following conditions: $0 \leq i \leq N$, $0 \leq j \leq N$, $0 \leq k \leq N$ and $i + j + k \leq N$. According to Ref. 27, the fitness of individuals in type A, type B, type C, and type D can be defined as

$$f_{i,j,k} = 1 - \omega + \omega \frac{(i-1)R + j \frac{F+R}{2} + k \frac{F+R}{2} + (N-i-j-k)F}{N-1} \quad (1 \leq i \leq N), \quad (1)$$

$$g_{i,j,k} = 1 - \omega + \omega \frac{i \frac{T+R}{2} + (j-1) \frac{F+T}{2} + k \frac{R+J}{2} + (N-i-j-k) \frac{F+J}{2}}{N-1} \quad (1 \leq j \leq N), \quad (2)$$

$$h_{i,j,k} = 1 - \omega + \omega \frac{i \frac{T+R}{2} + j \frac{R+J}{2} + (k-1) \frac{F+T}{2} + (N-i-j-k) \frac{F+J}{2}}{N-1} \quad (1 \leq k \leq N), \quad (3)$$

$$l_{i,j,k} = 1 - \omega + \omega \frac{iT + j \frac{T+J}{2} + k \frac{T+J}{2} + (N-i-j-k-1)J}{N-1} \quad (1 \leq N-i-j-k \leq N), \quad (4)$$

where $f_{i,j,k}$ is the fitness of individuals in type A, when i is 0, $f_{i,j,k}$ is 0; $g_{i,j,k}$ is the fitness of individuals in type B, when j is 0, $g_{i,j,k}$ is 0; $h_{i,j,k}$ is the fitness of individuals in type C, when k is 0, $h_{i,j,k}$ is 0; $l_{i,j,k}$ is the

fitness of individuals in type D, when $N - i - j - k$ is 0, $l_{i,j,k}$ is 0. In addition, $\omega \in [0, 1]$ represents the intensity of natural selection, that is, the contribution of game payoffs to fitness.

B. Description of the system states

Since all of the system states are represented by three-dimensional arrays, for the convenience of description, the index numbers of the elements in the system states are specified in the following.

- When the state parameters of system elements are $i = 0$ and $j = 0$, the state index number of the element is k ($0 \leq k \leq N$).

TABLE I. The payoff matrix of the game.

	A	B	C	D
A	R	$(R+F)/2$	$(R+F)/2$	F
B	$(T+R)/2$	$(T+F)/2$	$(R+J)/2$	$(F+J)/2$
C	$(T+R)/2$	$(R+J)/2$	$(T+F)/2$	$(F+J)/2$
D	T	$(T+J)/2$	$(T+J)/2$	J

FIG. 1. The relationships between emotion types and behaviors.

- (2) When the state parameters of system elements are $i = 0$ and $j \neq 0$ ($1 \leq j \leq N$), the state index number of the element is $N + 1 + k + \sum_2^j (N - j + 2)$ ($0 \leq k \leq N - j$).
- (3) When the state parameters of system elements are $1 \leq i < N$ and $0 \leq j \leq N - 1$, the state index number of the element is $N + 1 + k + \sum_{j=1}^N (N - j + 1) + \sum_1^i (N - i - j + 2)$ ($0 \leq k \leq N - i - j$).
- (4) When the state parameters of system elements are $i = N$, the state index number of the element is $N + 1 + \sum_{j=1}^N (N - j + 1) + \sum_{i=1}^{N-1} \sum_{j=1}^{N-i+1} j = (N + 1)(N + 2)(N + 3)/6 - 1$.

According to the above state numbering rules, the numbers composed by i, j, k ($0 \leq i + j + k \leq N$) are ordered from the smallest number to the largest number. When the state parameter of system element i is N , the element's state serial number is $(N + 1)(N + 2)(N + 3)/6 - 1$. Since the numbers start from zero, the total number of state elements in the system state set S is $N_S = (N + 1)(N + 2)(N + 3)/6$.

C. Moran process

For a finite population, the Moran process corresponding to the evolution of emotion types represents that an individual is selected for replication in accordance with the probability proportional to fitness at each time step, and a randomly selected individual of the population is then replaced. Therefore, the Markov process can be used to describe the change rule of the system states. In each time step, the specific evolution of the system states is shown as follows:

- (1) the number i of individuals in type A increases (or decreases) by one correspondingly, which corresponds to three situations. In case 1, the number j of individuals in type B decreases (or increases) by one, and the number of individuals in type C and type D remains unchanged. In case 2, the number k of individuals in type C decreases (or increases) by one, and the number of individuals in type B and type D remains unchanged. In case 3, the number $N - i - j - k$ of individuals in type D decreases (or increases) by one, and the number of individuals in type B and type C remains unchanged. The transition probabilities corresponding to the above situations are shown in Eqs. (5)–(10),

$${}_{k,k}^{jj-1} P_{i,i+1} = \frac{if_{i,j,k}}{if_{i,j,k} + jg_{i,j,k} + kh_{i,j,k} + (N - i - j - k)l_{i,j,k}} \cdot \frac{j}{N}, \quad (5)$$

$${}_{k,k-1}^{jj} P_{i,i+1} = \frac{if_{i,j,k}}{if_{i,j,k} + jg_{i,j,k} + kh_{i,j,k} + (N - i - j - k)l_{i,j,k}} \cdot \frac{k}{N}, \quad (6)$$

$${}_{k,k}^{jj-1} P_{i,i+1} = \frac{if_{i,j,k}}{if_{i,j,k} + jg_{i,j,k} + kh_{i,j,k} + (N - i - j - k)l_{i,j,k}} \cdot \frac{N - i - j - k}{N}, \quad (7)$$

$${}_{k,k}^{jj+1} P_{i,i-1} = \frac{jg_{i,j,k}}{if_{i,j,k} + jg_{i,j,k} + kh_{i,j,k} + (N - i - j - k)l_{i,j,k}} \cdot \frac{i}{N}, \quad (8)$$

$${}_{k,k+1}^{jj} P_{i,i-1} = \frac{kh_{i,j,k}}{if_{i,j,k} + jg_{i,j,k} + kh_{i,j,k} + (N - i - j - k)l_{i,j,k}} \cdot \frac{i}{N}, \quad (9)$$

$${}_{k,k}^{jj} P_{i,i-1} = \frac{(N - i - j - k)l_{i,j,k}}{if_{i,j,k} + jg_{i,j,k} + kh_{i,j,k} + (N - i - j - k)l_{i,j,k}} \cdot \frac{i}{N}. \quad (10)$$

- (2) The number i of individuals in type A remains unchanged, corresponding to seven cases. In case 1, the number j of individuals in type B increases by one, the number k of individuals in type C decreases by one, and the number $N - i - j - k$ of individuals in type D remains unchanged. The corresponding transition probability is shown in Eq. (11). In case 2, the number j of individuals in type B increases by one, the number k of individuals in type C remains unchanged, and the number $N - i - j - k$ of individuals in type D decreases by one. The corresponding transition probability is shown in Eq. (12). In case 3, the number j of individuals in type B decreases by one, the number k of individuals in type C increases by one, and the number $N - i - j - k$ of individuals in type D remains unchanged. The corresponding transition probability is shown in Eq. (13). In case 4, the number j of individuals in type B decreases by one, the number k of individuals in type C remains unchanged, and the number $N - i - j - k$ of individuals in type D increases by one. The corresponding transition probability is shown in Eq. (14). In case 5, the number j of individuals in type B remains unchanged, the number k of individuals in type C increases by one, and the number $N - i - j - k$ of individuals in type D decreases by one. The corresponding transition probability is shown in Eq. (15). In case 6, the number j of individuals in type B remains unchanged, the number k of individuals in type C decreases by one, and the number $N - i - j - k$ of individuals in type D increases by one. The corresponding transition probability is shown in Eq. (16). In case 7, all of them remain unchanged. The corresponding transition probability is shown in Eq. (17),

$${}_{k,k}^{jj+1} P_{i,i} = \frac{jg_{i,j,k}}{if_{i,j,k} + jg_{i,j,k} + kh_{i,j,k} + (N - i - j - k)l_{i,j,k}} \cdot \frac{k}{N}, \quad (11)$$

$${}_{k,k}^{jj+1} P_{i,i} = \frac{jg_{i,j,k}}{if_{i,j,k} + jg_{i,j,k} + kh_{i,j,k} + (N - i - j - k)l_{i,j,k}} \cdot \frac{N - i - j - k}{N}, \quad (12)$$

$${}_{k,k+1}^{jj-1} P_{i,i} = \frac{kh_{i,j,k}}{if_{i,j,k} + jg_{i,j,k} + kh_{i,j,k} + (N - i - j - k)l_{i,j,k}} \cdot \frac{j}{N}, \quad (13)$$

$${}_{k,k}^{jj-1} P_{i,i} = \frac{(N - i - j - k)l_{i,j,k}}{if_{i,j,k} + jg_{i,j,k} + kh_{i,j,k} + (N - i - j - k)l_{i,j,k}} \cdot \frac{j}{N}, \quad (14)$$

$$\begin{aligned} {}_{k,k+1}^{jj}P_{i,i} &= \frac{k h_{i,j,k}}{i f_{i,j,k} + j g_{i,j,k} + k h_{i,j,k} + (N - i - j - k) l_{i,j,k}} \\ &\cdot \frac{N - i - j - k}{N}, \end{aligned} \quad (15)$$

$$\begin{aligned} {}_{k,k-1}^{jj}P_{i,i} &= \frac{(N - i - j - k) l_{i,j,k}}{i f_{i,j,k} + j g_{i,j,k} + k h_{i,j,k} + (N - i - j - k) l_{i,j,k}} \cdot \frac{k}{N}, \end{aligned} \quad (16)$$

$$\begin{aligned} {}_{k,k}^{jj}P_{i,i} &= \frac{i f_{i,j,k}}{i f_{i,j,k} + j g_{i,j,k} + k h_{i,j,k} + (N - i - j - k) l_{i,j,k}} \cdot \frac{i}{N} \\ &+ \frac{j g_{i,j,k}}{i f_{i,j,k} + j g_{i,j,k} + k h_{i,j,k} + (N - i - j - k) l_{i,j,k}} \cdot \frac{j}{N} \\ &+ \frac{k h_{i,j,k}}{i f_{i,j,k} + j g_{i,j,k} + k h_{i,j,k} + (N - i - j - k) l_{i,j,k}} \cdot \frac{k}{N} \\ &+ \frac{(N - i - j - k) l_{i,j,k}}{i f_{i,j,k} + j g_{i,j,k} + k h_{i,j,k} + (N - i - j - k) l_{i,j,k}} \\ &\cdot \frac{N - i - j - k}{N}, \end{aligned} \quad (17)$$

where ${}_{k,k}^{jj-1}P_{i,i+1}$ is the transition probability from state (i, j, k) to state $(i + 1, j - 1, k)$, and other symbols have similar meanings.

D. Transition probability matrix

We use Eqs. (5)–(17) and the numbering rules described in Sec. II B to obtain the corresponding transition probabilities. The transition probability matrix P is

$$P = [p_{a,b}]_{a,b \in S} = \begin{bmatrix} p_{0,0} & p_{0,1} & \cdots & p_{0,N_S-1} \\ p_{1,0} & p_{1,1} & \cdots & p_{1,N_S-1} \\ \vdots & \vdots & \ddots & \cdots \\ p_{N_S-1,0} & p_{N_S-1,1} & \cdots & p_{N_S-1,N_S-1} \end{bmatrix},$$

which satisfies $p_{a,b} \geq 0, a, b \in S; \sum_{b \in S} p_{a,b} = 1, a \in S$.

Let $P = [p_{a,b}]_{a,b \in S}$ represent the transition probability matrix of Markov chain $X_n, n \geq 0$. If the non-negative sequence π_b satisfies (1) $\sum_{b \in S} \pi_b = 1$, (2) $\pi_b = \sum_a \pi_a p_{a,b}, b \in S$, then π_b is called the stationary distribution probability of Markov chain $X_n, n \geq 0$. The condition (2) can be rewritten as matrix form,

$$\pi = \pi P, \quad (18)$$

where $\pi = [\pi_0, \pi_1, \dots, \pi_{N_S-1}]$.

According to Eqs. (5)–(17), combined with the numbering rules in Sec. II B, the transition probabilities of the four states $S_0 (i = 0, j = 0, k = 0)$, $S_N (i = 0, j = 0, k = N)$, $S_{(N+1)(N+2)/2-1} (i = 0, j = N, k = 0)$ and $S_{N_S-1} (i = N, j = 0, k = 0)$ can be calculated as follows:

$$p_{0,0} = {}_{0,0}^{0,0}P_{0,0} = \frac{N^2 \cdot l_{0,0,0}}{N \cdot l_{0,0,0}} \cdot \frac{1}{N} = 1, \quad (19)$$

$$p_{N,N} = {}_{N,N}^{0,0}P_{0,0} = \frac{N^2 \cdot h_{0,0,N}}{N \cdot h_{0,0,N}} \cdot \frac{1}{N} = 1, \quad (20)$$

$$p_{(N+1)(N+2)/2-1,(N+1)(N+2)/2-1} = {}_{0,0}^{N,N}P_{0,0} = \frac{N^2 \cdot g_{0,N,0}}{N \cdot g_{0,N,0}} \cdot \frac{1}{N} = 1, \quad (21)$$

$$p_{N_S-1,N_S-1} = {}_{0,0}^{N_S-1,N_S-1}P_{0,0} = \frac{N^2 \cdot f_{N_S-1,N_S-1}}{N \cdot f_{N_S-1,N_S-1}} \cdot \frac{1}{N} = 1. \quad (22)$$

At the same time, the probabilities of these four states reaching other states are 0. Since $p_{0,0}, p_{N,N}, p_{(N+1)(N+2)/2-1,(N+1)(N+2)/2-1}, p_{N_S-1,N_S-1}$ are the elements on the diagonal of the transition probability matrix P , we substitute them into Eq. (18) to expand. Then, the following four equations in the system arise: $\pi_0 = \pi_0$, $\pi_N = \pi_N$, $\pi_{(N+1)(N+2)/2-1} = \pi_{(N+1)(N+2)/2-1}$, and $\pi_{N_S-1} = \pi_{N_S-1}$. As the four equations are naturally established, the transition probability matrix P is a non-full rank matrix. The rank of the matrix is $N_S - 4$. The states $S_0 (i = 0, j = 0, k = 0)$, $S_N (i = 0, j = 0, k = N)$, $S_{(N+1)(N+2)/2-1} (i = 0, j = N, k = 0)$, and $S_{N_S-1} (i = N, j = 0, k = 0)$ are four absorption states of the system.

III. ABSORPTION PROBABILITIES

In this paper, we take the case of $N = 4$ to further study the absorption states of the system. The parameters are set to $R = 1$, $J = 0$, $1 \leq T \leq 2$, $-1 \leq F \leq 0$. According to the calculation method in Sec. II, the total number of the elements in the state set S is 35, where state S_0 , state S_4 , state S_{14} , and state S_{34} are four absorption states. Starting from any initial state, the system will eventually be absorbed by one of these four absorption states through multiple state transitions.

A. Self-circulation simplification of transition probabilities

According to the calculation method in Sec. II, the states and the corresponding probabilities that all 35 initial states reach after one-step transition are presented in Appendix A in the [supplementary material](#). It can be found that some states (such as state S_1) can reach their own state through one-step transition. Therefore, $S_1 \rightarrow S_0, S_1 \rightarrow S_1 \rightarrow S_0, \dots, S_1 \rightarrow S_1 \rightarrow S_1 \rightarrow \dots \rightarrow S_0$ are collectively denoted as $S_1 \rightarrow S_0$ (self-circulation process of state S_1). Then, the probability of $S_1 \rightarrow S_0$ is $p_{1,0} + p_{1,0} \cdot p_{1,1} + p_{1,0} \cdot p_{1,1}^2 + \dots + p_{1,0} \cdot p_{1,1}^n = p_{1,0} \cdot \lim_{n \rightarrow \infty} \frac{1-p_{1,1}^n}{1-p_{1,1}} = \frac{p_{1,0}}{1-p_{1,1}}$.

We substitute the parameters T , F , ω and obtain the probability of $S_1 \rightarrow S_0$ with $(T\omega - 6\omega + 6)/(3F\omega - 12\omega + T\omega + 12)$. The same calculation is carried out for other self-circulation situations. The one-step transition to the states and the corresponding probabilities after the self-circulation simplification are shown in Appendix B in the [supplementary material](#).

B. Theoretical derivation of absorption probabilities

The analysis results show that starting from each initial state (excluding the absorption states), there are three situations in the final absorption states:

- (1) Two absorption states: starting from 18 kinds of initial states, that is, $S_1, S_2, S_3, S_5, S_8, S_9, S_{11}, S_{12}, S_{13}, S_{15}, S_{18}, S_{24}, S_{25}, S_{27}, S_{30}, S_{31}, S_{32}$, and S_{33} , each can only be absorbed by two of the four absorption states.

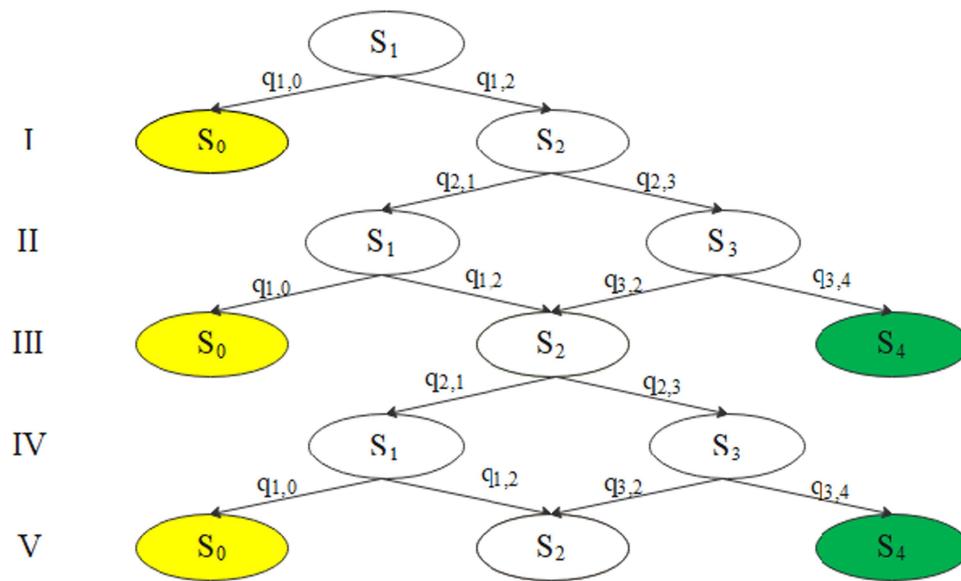


FIG. 2. The five-step random transition process starting from the initial state S_1 .

- (2) Three absorption states: starting from 12 kinds of initial states, that is, $S_6, S_7, S_{10}, S_{16}, S_{17}, S_{19}, S_{21}, S_{22}, S_{23}, S_{26}, S_{28}$, and S_{29} , each can only be absorbed by three of the four absorption states.
- (3) Four absorption states: starting from one initial state S_{20} , it can be absorbed by S_0, S_4, S_{14} , and S_{34} .

1. Two kinds of absorption states

Through the analysis, it is observed that states S_1, S_2 , and S_3 are absorbed by S_0 and S_4 ; states S_5, S_9 , and S_{12} are absorbed by S_0 and S_{14} ; states S_{15}, S_{25} , and S_{31} are absorbed by S_0 and S_{34} ; states S_8, S_{11} , and S_{13} are absorbed by S_4 and S_{14} ; states S_{18}, S_{27} , and S_{32} are absorbed by S_4 and S_{34} ; states S_{24}, S_{30} , and S_{33} are absorbed by S_{14} and S_{34} . As an illustrative example, we examine the initial state S_1 and derive its absorption probabilities by S_0 and S_4 . Figure 2 presents the five-step random transition process starting from the initial state S_1

(including arrival states and the corresponding probabilities). State S_1 can only reach three states after the three-step transition, that is, S_0, S_2 , and S_4 . Let $q_{S_u \rightarrow S_v}$ represent the probability from state S_u to state S_v after three steps. Then, the corresponding probabilities from state S_1 to states S_0, S_2 , and S_4 after three steps are $q_{S_1 \rightarrow S_0} = q_{1,2} \cdot q_{2,1} \cdot q_{1,0}$, $q_{S_1 \rightarrow S_2} = q_{1,2} \cdot (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2})$, and $q_{S_1 \rightarrow S_4} = q_{1,2} \cdot q_{2,3} \cdot q_{3,4}$.

Starting from state S_2 , which is arrived by the above three-step random transition, states S_0, S_2 , and S_4 are still reached after another two-step random transition. So, state S_1 can reach states S_0, S_2 , and S_4 in five-step random transition. The corresponding probabilities are $q_{S_1 \rightarrow S_2} \cdot q_{2,1} \cdot q_{1,0}, q_{S_1 \rightarrow S_2} \cdot (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2}),$ and $q_{S_1 \rightarrow S_2} \cdot q_{2,3} \cdot q_{3,4}$.

It can be noticed that if the initial state is S_1 , the cycle from state S_2 to states (S_0, S_2 , and S_4) will continue after five steps or more. In addition, the states after three-step random transition, that is, $S_2 \rightarrow$

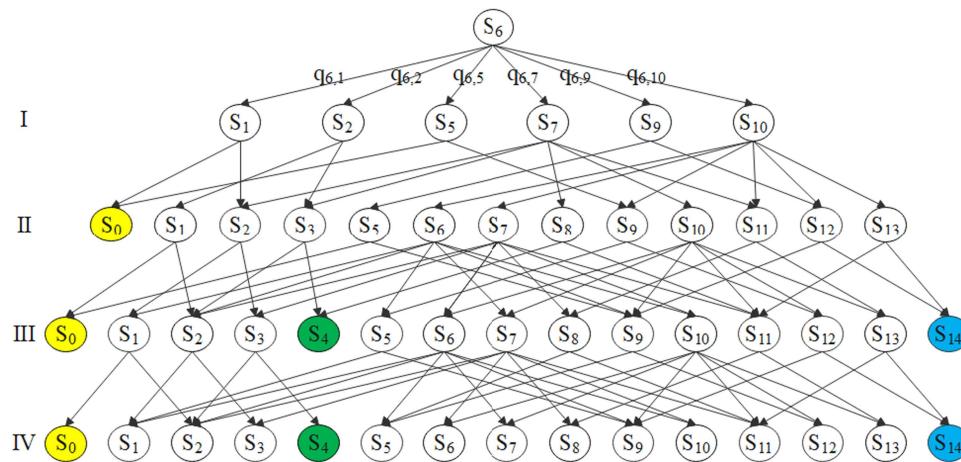


FIG. 3. The four-step random transition process starting from the initial state S_6 .

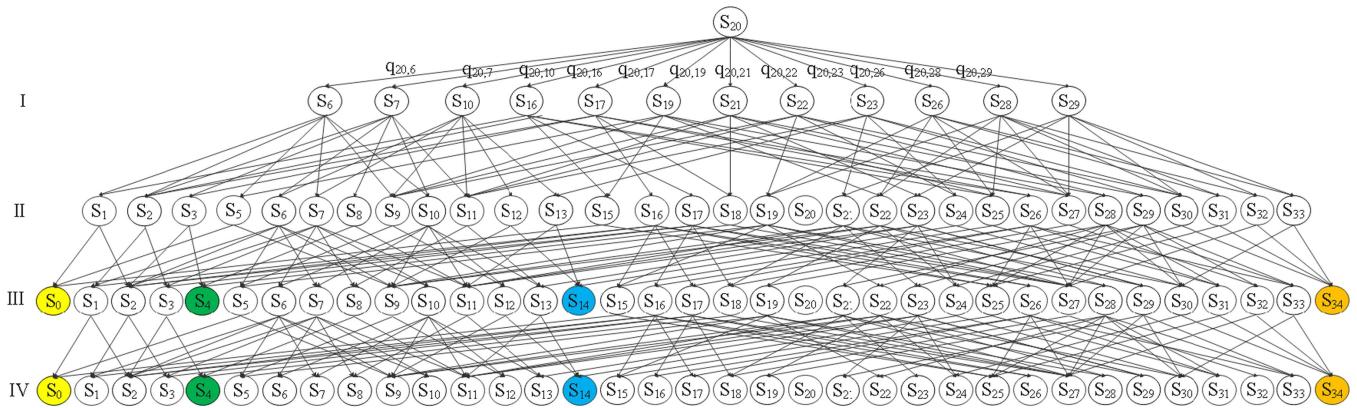


FIG. 4. The four-step random transition process starting from the initial state S_{20} .

$S_0, S_2 \rightarrow S_2 \rightarrow S_0, \dots, S_2 \rightarrow S_2 \rightarrow \dots \rightarrow S_0$ are collectively denoted as $\bar{S}_2 \rightarrow S_0$ (self-circulation process of state S_2). Then, its probability is $q_{S_1 \rightarrow S_2} \cdot [q_{2,1} \cdot q_{1,0} + q_{2,1} \cdot q_{1,0} \cdot (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2}) + q_{2,1} \cdot q_{1,0} \cdot (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2})^2 + \dots + q_{2,1} \cdot q_{1,0} \cdot (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2})^n + \dots] = q_{S_1 \rightarrow S_2} \cdot q_{2,1} \cdot q_{1,0} \cdot \lim_{n \rightarrow \infty} \frac{1 - (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2})^n}{1 - (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2})}$

$$= \frac{q_{S_1 \rightarrow S_2} \cdot q_{2,1} \cdot q_{1,0}}{q_{2,1} \cdot q_{1,0} + q_{2,3} \cdot q_{3,4}}.$$

Similarly, $S_2 \rightarrow S_4, S_2 \rightarrow S_2 \rightarrow S_4, \dots, S_2 \rightarrow S_2 \rightarrow \dots \rightarrow S_4$ are collectively denoted as $\bar{S}_2 \rightarrow S_4$. Then, its probability is $q_{S_1 \rightarrow S_2} \cdot [q_{2,3} \cdot q_{3,4} + q_{2,3} \cdot q_{3,4} \cdot (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2}) + q_{2,3} \cdot q_{3,4} \cdot (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2})^2 + \dots + q_{2,3} \cdot q_{3,4} \cdot (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2})^n + \dots] = q_{S_1 \rightarrow S_2} \cdot q_{2,3} \cdot q_{3,4} \cdot \lim_{n \rightarrow \infty} \frac{1 - (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2})^n}{1 - (q_{2,1} \cdot q_{1,2} + q_{2,3} \cdot q_{3,2})}$

$$= \frac{q_{S_1 \rightarrow S_2} \cdot q_{2,3} \cdot q_{3,4}}{q_{2,1} \cdot q_{1,0} + q_{2,3} \cdot q_{3,4}}.$$

According to the above analysis, the probability $\lambda_{S_1 \rightarrow S_0}$ of the initial state S_1 being absorbed by S_0 (the probability of reaching absorption state S_0 after one step + the probability of reaching absorption state S_0 after three steps + the probability of reaching absorption state S_0 after five steps) is $\lambda_{S_1 \rightarrow S_0} = q_{1,0} + q_{1,2} \cdot q_{2,1} \cdot q_{1,0} + \frac{q_{S_1 \rightarrow S_2} \cdot q_{2,1} \cdot q_{1,0}}{q_{2,1} \cdot q_{1,0} + q_{2,3} \cdot q_{3,4}}$.

The probability $\lambda_{S_1 \rightarrow S_4}$ of the initial state S_1 being absorbed by S_4 (the probability of reaching absorption state S_4 after three steps + the probability of reaching absorption state S_4 after five steps) is $\lambda_{S_1 \rightarrow S_4} = q_{1,2} \cdot q_{2,3} \cdot q_{3,4} + \frac{q_{S_1 \rightarrow S_2} \cdot q_{2,3} \cdot q_{3,4}}{q_{2,1} \cdot q_{1,0} + q_{2,3} \cdot q_{3,4}}$.

Starting from $S_2, S_3, S_5, S_8, S_9, S_{11}, S_{12}, S_{13}, S_{15}, S_{18}, S_{24}, S_{25}, S_{27}, S_{30}, S_{31}, S_{32}$, and S_{33} , the probabilities of being absorbed by the corresponding absorption states can be calculated in the same way. They are detailed in Appendix C in the [supplementary material](#) for clarity.

2. Three kinds of absorption states

Through the analysis, we notice that states S_6, S_7 , and S_{10} are absorbed by S_0, S_4 , and S_{14} ; states S_{16}, S_{17} , and S_{26} are absorbed by S_0, S_4 , and S_{34} ; states S_{19}, S_{22} , and S_{28} are absorbed by S_0, S_{14} , and S_{34} ; states S_{21}, S_{23} , and S_{29} are absorbed by S_4, S_{14} , and S_{34} . Take the initial state S_6 as an example and then we derive its absorption

probabilities. [Figure 3](#) presents the four-step random transition process from the initial state S_6 .

It can be observed from [Fig. 3](#) that state S_6 can reach state $S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{13}$, and S_{14} after the three-step transition. According to the state transition paths shown in [Fig. 3](#) and combined with the corresponding state transition probabilities, the corresponding three-step arrival probabilities $q_{S_6 \rightarrow S_v}$ can be obtained. For example, the probability from state S_6 to state S_1 after three-step transition is $q_{S_6 \rightarrow S_1} = q_{6,1} \cdot q_{1,2} \cdot q_{2,1} + q_{6,7} \cdot q_{7,2} \cdot q_{2,1} + q_{6,7} \cdot q_{7,6} \cdot q_{6,1} + q_{6,10} \cdot q_{10,6} \cdot q_{6,1}$. Other arrival probabilities can be obtained in the same way, and the probabilities from state S_6 to states S_0, S_4, S_{14} after three-step transition are $q_{S_6 \rightarrow S_0} = q_{6,2} \cdot q_{2,1} \cdot q_{1,0} + q_{6,9} \cdot q_{9,5} \cdot q_{5,0}; q_{S_6 \rightarrow S_4} = q_{6,2} \cdot q_{2,3} \cdot q_{3,4} + q_{6,7} \cdot q_{7,3} \cdot q_{3,4} + q_{6,7} \cdot q_{7,8} \cdot q_{8,4}; q_{S_6 \rightarrow S_{14}} = q_{6,9} \cdot q_{9,12} \cdot q_{12,14} + q_{6,10} \cdot q_{10,12} \cdot q_{12,14} + q_{6,10} \cdot q_{10,13} \cdot q_{13,14}$.

[Figure 3](#) presents that starting from all the non-absorption states reached by the three-step transition, if we perform another one-step random transition, then 15 states $S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{13}$, and S_{14} can be reached. So, we get the arrival probabilities after the four-step random transition. For example, the probability from state S_6 to state S_1 after the four-step random transition is $q_{S_6 \rightarrow S_1} = q_{S_6 \rightarrow S_2} \cdot q_{2,1} + q_{S_6 \rightarrow S_6} \cdot q_{6,1}$. Other arrival probabilities can be obtained in the same way.

Let q_{S_u} represent the sum of the probabilities that the state S_u reaches all the non-absorption states through three-step random transition. Then, $q_{S_6} = q_{S_6 \rightarrow S_1} + q_{S_6 \rightarrow S_2} + q_{S_6 \rightarrow S_3} + q_{S_6 \rightarrow S_5} + q_{S_6 \rightarrow S_6} + q_{S_6 \rightarrow S_7} + q_{S_6 \rightarrow S_8} + q_{S_6 \rightarrow S_9} + q_{S_6 \rightarrow S_{10}} + q_{S_6 \rightarrow S_{11}} + q_{S_6 \rightarrow S_{12}} + q_{S_6 \rightarrow S_{13}}$. Let r_{S_u} represent the sum of the probabilities that the state S_u reaches all the non-absorption states through four-step random transition. Then, $r_{S_6} = q_{S_6 \rightarrow S_2} \cdot q_{2,1} + q_{S_6 \rightarrow S_6} \cdot q_{6,1} + q_{S_6 \rightarrow S_1} \cdot q_{1,2} + q_{S_6 \rightarrow S_3} \cdot q_{3,2} + q_{S_6 \rightarrow S_6} \cdot q_{6,2} + q_{S_6 \rightarrow S_7} \cdot q_{7,2} + q_{S_6 \rightarrow S_2} \cdot q_{2,3} + q_{S_6 \rightarrow S_6} \cdot q_{6,3} + q_{S_6 \rightarrow S_9} \cdot q_{9,5} + q_{S_6 \rightarrow S_6} \cdot q_{6,5} + q_{S_6 \rightarrow S_{10}} \cdot q_{10,6} + q_{S_6 \rightarrow S_7} \cdot q_{7,6} + q_{S_6 \rightarrow S_6} \cdot q_{6,7} + q_{S_6 \rightarrow S_{10}} \cdot q_{10,7} + q_{S_6 \rightarrow S_{11}} \cdot q_{11,8} + q_{S_6 \rightarrow S_7} \cdot q_{7,8} + q_{S_6 \rightarrow S_6} \cdot q_{6,9} + q_{S_6 \rightarrow S_{10}} \cdot q_{10,9} + q_{S_6 \rightarrow S_5} \cdot q_{5,9} + q_{S_6 \rightarrow S_{12}} \cdot q_{12,9} + q_{S_6 \rightarrow S_6} \cdot q_{6,10} + q_{S_6 \rightarrow S_7} \cdot q_{7,10} + q_{S_6 \rightarrow S_8} \cdot q_{8,11} + q_{S_6 \rightarrow S_{10}} \cdot q_{10,11} + q_{S_6 \rightarrow S_7} \cdot q_{7,11} + q_{S_6}$

$$\rightarrow s_{13} \cdot q_{13,11} + q_{s_6 \rightarrow s_9} \cdot q_{9,12} + q_{s_6 \rightarrow s_{10}} \cdot q_{10,12} + q_{s_6 \rightarrow s_{10}} \cdot q_{10,13} + q_{s_6 \rightarrow s_{11}} \cdot q_{11,13}.$$

Let S_L represent the set of non-absorption states $S_1, S_2, S_3, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}$, and S_{13} . After four steps or more, the cycle will continue from state S_L to states $(S_0, S_L, S_4, \text{ and } S_{14})$. States $S_L \rightarrow S_0, S_L \rightarrow S_L \rightarrow S_0, \dots, S_L \rightarrow S_L \rightarrow S_L \rightarrow \dots \rightarrow S_0$ are collectively denoted as $\bar{S}_L \rightarrow S_0$. After four steps, the probability of $\bar{S}_L \rightarrow S_0$ is $q_{s_6 \rightarrow s_1} \cdot q_{1,0} + q_{s_6 \rightarrow s_5} \cdot q_{5,0}) / (q_{s_6} - r_{s_6})$. Similarly, the probability of $\bar{S}_L \rightarrow S_4$ is $q_{s_6 \rightarrow s_3} \cdot (q_{s_6 \rightarrow s_3} \cdot q_{3,4} + q_{s_6 \rightarrow s_8} \cdot q_{8,4}) / (q_{s_6} - r_{s_6})$, the probability of $\bar{S}_L \rightarrow S_{14}$ is $q_{s_6 \rightarrow s_{12}} \cdot (q_{s_6 \rightarrow s_{12}} \cdot q_{12,14} + q_{s_6 \rightarrow s_{13}} \cdot q_{13,14}) / (q_{s_6} - r_{s_6})$.

So, the probabilities of initial state S_6 being absorbed by S_0, S_4 , and S_{14} (the probability of reaching absorption states after two steps + the probability of reaching absorption states after three steps + the probability of reaching absorption states after four steps) are $\lambda_{s_6 \rightarrow s_0} = q_{6,1} \cdot q_{1,0} + q_{6,5} \cdot q_{5,0} + q_{s_6 \rightarrow s_0} + q_{s_6 \rightarrow s_1} \cdot (q_{s_6 \rightarrow s_1} \cdot q_{1,0} + q_{s_6 \rightarrow s_5} \cdot q_{5,0}) / (q_{s_6} - r_{s_6})$, $\lambda_{s_6 \rightarrow s_4} = q_{s_6 \rightarrow s_4} + q_{s_6 \rightarrow s_3} \cdot (q_{s_6 \rightarrow s_3} \cdot q_{3,4} + q_{s_6 \rightarrow s_8} \cdot q_{8,4}) / (q_{s_6} - r_{s_6})$, and $\lambda_{s_6 \rightarrow s_{14}} = q_{s_6 \rightarrow s_{12}} \cdot (q_{s_6 \rightarrow s_{12}} \cdot q_{12,14} + q_{s_6 \rightarrow s_{13}} \cdot q_{13,14}) / (q_{s_6} - r_{s_6})$.

$$q_{3,4} + q_{s_6 \rightarrow s_8} \cdot q_{8,4}) / (q_{s_6} - r_{s_6}), \text{ and } \lambda_{s_6 \rightarrow s_{14}} = q_{s_6 \rightarrow s_{14}} + q_{s_6} \cdot (q_{s_6 \rightarrow s_{12}} \cdot q_{12,14} + q_{s_6 \rightarrow s_{13}} \cdot q_{13,14}) / (q_{s_6} - r_{s_6}).$$

Similarly, we can calculate the probabilities of being absorbed by the corresponding three absorption states starting from the initial states $S_7, S_{10}, S_{16}, S_{17}, S_{19}, S_{21}, S_{22}, S_{23}, S_{26}, S_{28}$, and S_{29} . They are detailed in Appendix D in the [supplementary material](#) for clarity.

3. Four kinds of absorption states

[Figure 4](#) presents the four-step random transition process starting from the initial state S_{20} . We observe that state S_{20} can reach all states except itself of the system after three steps or more random transition. According to the state transition paths shown in [Fig. 4](#) and combined with the corresponding state transition probabilities, the corresponding three-step arrival probabilities $q_{S_{20} \rightarrow S_i}$ can be obtained. Let $q_{S_{20}}$ represent the sum of the probabilities of S_{20} reaching all non-absorption states after three steps, and $r_{S_{20}}$

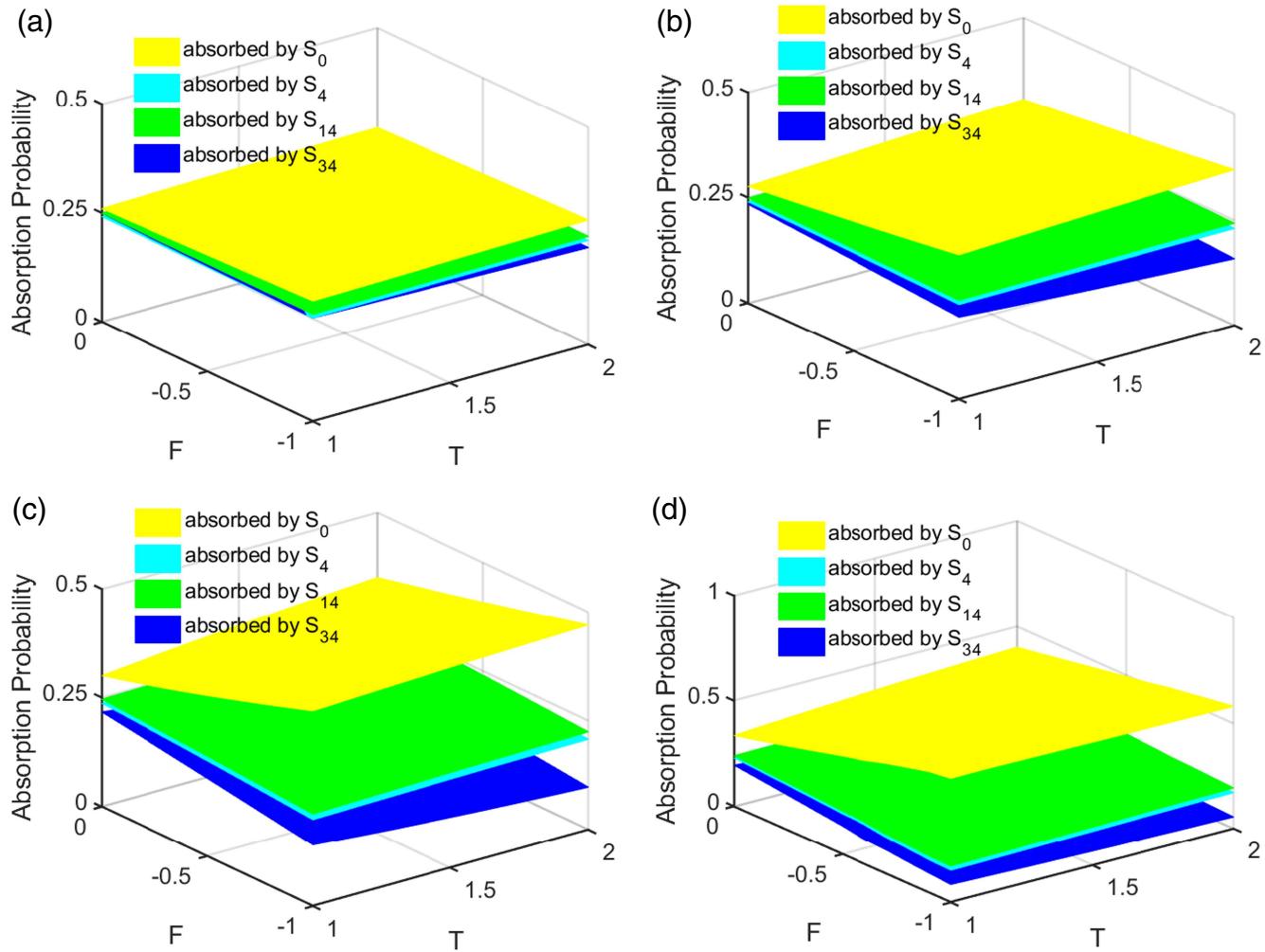


FIG. 5. The absorption probabilities of the system: (a) $\omega = 0.1$, (b) $\omega = 0.3$, (c) $\omega = 0.5$, and (d) $\omega = 0.7$.

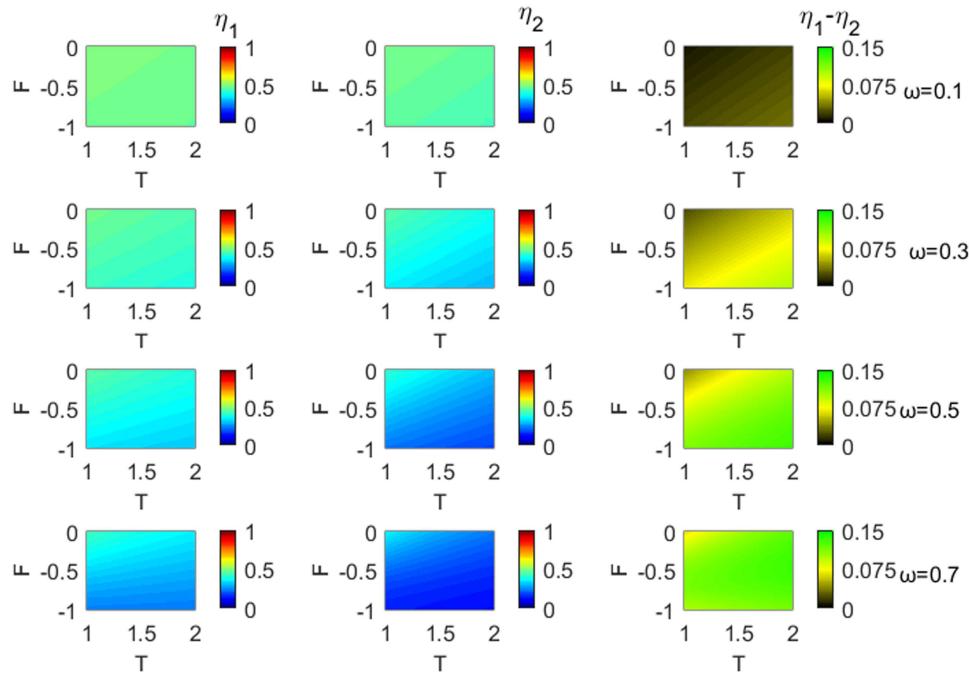


FIG. 6. The cooperation level at steady state (η_1 is the cooperation level of the emotional model in this paper, η_2 is the cooperation level of the prisoner's dilemma game with two behaviors, and $\eta_1 - \eta_2$ is the difference of the cooperation level between the two models).

represent the sum of the probabilities of S_{20} reaching all non-absorption states after four steps. The set of all non-absorbent states is denoted as S_Q . After the four-step transition or more, the cycle will continue from state S_Q to states $(S_0, S_Q, S_4, S_{14}, S_{34})$. The set $S_Q \rightarrow S_0, S_Q \rightarrow S_Q, \dots, S_Q \rightarrow S_Q \rightarrow \dots \rightarrow S_0$ is denoted as $\bar{S}_Q \rightarrow S_0$, and its probability is $q_{S_{20}} \cdot (q_{S_{20} \rightarrow S_1} \cdot q_{1,0} + q_{S_{20} \rightarrow S_5} \cdot q_{5,0} + q_{S_{20} \rightarrow S_{15}} \cdot q_{15,0}) / (q_{S_{20}} - r_{S_{20}})$. Similarly, the probability of $\bar{S}_Q \rightarrow S_4$ is $q_{S_{20}} \cdot (q_{S_{20} \rightarrow S_3} \cdot q_{3,4} + q_{S_{20} \rightarrow S_8} \cdot q_{8,4} + q_{S_{20} \rightarrow S_{18}} \cdot q_{18,4}) / (q_{S_{20}} - r_{S_{20}})$; the probability of $\bar{S}_Q \rightarrow S_{14}$ is $q_{S_{20}} \cdot (q_{S_{20} \rightarrow S_{12}} \cdot q_{12,14} + q_{S_{20} \rightarrow S_{13}} \cdot q_{13,14} + q_{S_{20} \rightarrow S_{24}} \cdot q_{24,14}) / (q_{S_{20}} - r_{S_{20}})$; and the probability of $\bar{S}_Q \rightarrow S_{34}$ is $q_{S_{20}} \cdot (q_{S_{20} \rightarrow S_{31}} \cdot q_{31,34} + q_{S_{20} \rightarrow S_{32}} \cdot q_{32,34} + q_{S_{20} \rightarrow S_{33}} \cdot q_{33,34}) / (q_{S_{20}} - r_{S_{20}})$.

Therefore, the probabilities of state S_{20} being absorbed by S_0, S_4, S_{14} , and S_{34} (the probability of reaching the absorption states after three steps + the probability of reaching the absorption states after four steps) are $\lambda_{S_{20} \rightarrow S_0} = q_{S_{20} \rightarrow S_0} + q_{S_{20}} \cdot (q_{S_{20} \rightarrow S_1} \cdot q_{1,0} + q_{S_{20} \rightarrow S_5} \cdot q_{5,0} + q_{S_{20} \rightarrow S_{15}} \cdot q_{15,0}) / (q_{S_{20}} - r_{S_{20}})$, $\lambda_{S_{20} \rightarrow S_4} = q_{S_{20} \rightarrow S_4} + q_{S_{20}} \cdot (q_{S_{20} \rightarrow S_3} \cdot q_{3,4} + q_{S_{20} \rightarrow S_8} \cdot q_{8,4} + q_{S_{20} \rightarrow S_{18}} \cdot q_{18,4}) / (q_{S_{20}} - r_{S_{20}})$, $\lambda_{S_{20} \rightarrow S_{14}} = q_{S_{20} \rightarrow S_{14}} + q_{S_{20}} \cdot (q_{S_{20} \rightarrow S_{12}} \cdot q_{12,14} + q_{S_{20} \rightarrow S_{13}} \cdot q_{13,14} + q_{S_{20} \rightarrow S_{24}} \cdot q_{24,14}) / (q_{S_{20}} - r_{S_{20}})$, and $\lambda_{S_{20} \rightarrow S_{34}} = q_{S_{20} \rightarrow S_{34}} + q_{S_{20}} \cdot (q_{S_{20} \rightarrow S_{31}} \cdot q_{31,34} + q_{S_{20} \rightarrow S_{32}} \cdot q_{32,34} + q_{S_{20} \rightarrow S_{33}} \cdot q_{33,34}) / (q_{S_{20}} - r_{S_{20}})$.

IV. RESULTS AND ANALYSIS

A. Absorption probabilities of the system

The probabilities being absorbed by the four absorption states from each initial state are deduced above and the absorption probabilities of the system are defined as below. Let b_{S_i} represent the occurrence probability of the initial state S_i ($i = 0, 1, \dots, 34$). For example, the state S_{29} , that is, $(i, j, k) = (2, 1, 1)$, the corresponding

occurrence probability is $b_{S_{29}} = \frac{C_4^2 \cdot C_1^1}{4^4}$. The probabilities being absorbed by four absorption states are

$$a_j = \sum_{i=0}^{34} b_{S_i} \cdot \lambda_{S_i \rightarrow S_j} \quad (j = 0, 4, 14, 34). \quad (23)$$

Figure 5 presents the absorption probabilities of the system. Therefore, in a mixed finite population with four emotion types, three types will gradually die out as the system evolves. That is to say, there is no coexistence of emotion types in a finite population. No matter how the emotion types of individuals in the initial population are distributed, in the end, individuals with only one emotion type can survive the evolution process. The absorption state $S_0(0, 0, 0)$ corresponds to type D, namely, envying the strong and bullying the weak. The absorption state $S_4(0, 0, 4)$ corresponds to type C, that is, respect for the strong and bullying the weak. The absorption state $S_{14}(0, 4, 0)$ corresponds to type B, that is, envying the strong and sympathy for the weak. The absorption state $S_{34}(4, 0, 0)$ corresponds to type A, that is, respect for the strong and sympathy for the weak. Among them, type A belongs to the positive one, type D belongs to the negative one, and types B and C belong to the neutral one. We observe from Fig. 5: (1) the probability being absorbed by type D > the probability being absorbed by type B > the probability being absorbed by type C > the probability being absorbed by type A; (2) with the increase of the selection intensity ω , the probability being absorbed by positive type A gradually decreases, the probability being absorbed by negative type D gradually increases, and the probability being absorbed by neutral types B and C remains basically unchanged; and (3) with the increase of the temptation of successful competition T and the decrease of the cost of cooperation failure F , the probability being absorbed by positive type A gradually

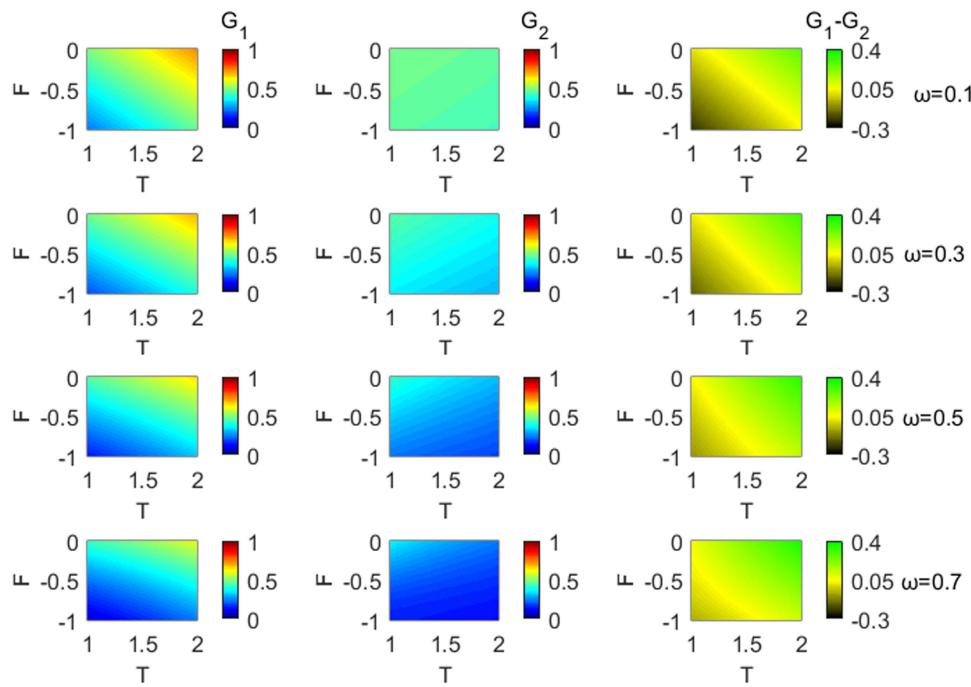


FIG. 7. The average payoffs at steady state (G_1 is the average payoff of the emotional model in this paper, G_2 is the average payoff of the prisoner's dilemma game with two behaviors, and $G_1 - G_2$ is the difference of the average payoffs between the two models).

decreases, the probability being absorbed by negative type D gradually increases, and the probability being absorbed by neutral type B and C decreases slightly.

B. Cooperation level and average payoffs

Since the steady state of system evolution must be one of the four absorption states, according to the corresponding relationship between the four emotion types and the two behaviors (cooperation and competition), the cooperation level η and the average payoffs G in the steady state of the system are defined as follows:

$$\eta = 0.5 \cdot a_4 + 0.5 \cdot a_{14} + a_{34}, \quad (24)$$

$$G = a_0 \cdot J + a_4 \cdot \frac{F+T}{2} + a_{14} \cdot \frac{F+T}{2} + a_{34} \cdot R. \quad (25)$$

For comparison, according to Ref. 25, the absorption probabilities of the prisoner's dilemma game with two behaviors in the finite population are

$$a_C = \frac{\sum_{i=0}^N C_N^i \cdot \theta_i}{2^N}, \quad (26)$$

$$a_D = \frac{\sum_{i=0}^N C_N^i \cdot (1 - \theta_i)}{2^N}, \quad (27)$$

where θ_i represents the absorption probability from i cooperative individuals to the final state $i = N$. a_C is the probability being absorbed by cooperation and a_D is the probability being absorbed by competition. According to the definitions of the cooperation level and the average payoffs, the cooperation level of the prisoner's

dilemma game with two behaviors is a_C , and the average payoffs of the system is $a_C \cdot R$.

We take $N = 4$, $R = 1$, and $J = 0$ as an example. The cooperation level and average payoffs at steady state are shown in Figs. 6 and 7. We observe that: (1) a smaller ω , a smaller T , and a larger F will yield a higher system cooperation level; (2) regardless of the values of F , T , and ω , the cooperation level of the emotional model is higher than that of the prisoner's dilemma game with two behaviors. In addition, with the increase of ω , the difference between the two becomes larger; and (3) in most areas of the parameters, the average payoff of the emotional model is higher than the one in the prisoner's dilemma game with two behaviors, and as the values of ω , T and F increase, the difference of average payoffs between the two models increases.

V. CONCLUSIONS

Based on the Moran process, the evolutionary dynamics of the four emotion types are analyzed in a finite population, revealing that the system has four absorption states. That is, the steady state of the final evolution occurs when individuals with a certain emotion type occupy the entire population, while the other three emotion types gradually die out.

For the absorption probability of the finite population evolutionary game with two behaviors, a theoretical solution was given in Ref. 25. For the absorption probability of the finite population evolutionary game with four emotion types, a recursive relationship of the absorption probability was presented in Ref. 26. However, the existing methods can only deal with situations where only the main diagonal and the adjacent two sub-diagonal elements in the transition probability matrix have values, while other elements are zero.

In this paper, a finite population evolutionary game based on the Moran process is proposed. According to the specific Markov process of the state transition (the transition probability matrix does not have the above limitations), a theoretical calculation method of absorption probability is proposed. We consider the population size $N = 4$ and the results suggest that: (1) starting from different initial states, the final absorption states of the system and the corresponding absorption probabilities are different; (2) the absorption probabilities of the four absorption states of the system are different. Furthermore, individuals with type D (envying the strong and bullying the weak) have the highest probability of occupying the entire population in the evolutionary steady state, individuals with type A (respect for the strong and sympathy for the weak) have the lowest probability of occupying the entire population in the evolutionary steady state. Therefore, without considering mutation, individuals with type D have a certain evolutionary advantage; (3) a high selection intensity ω , large T and small F will promote the system to be absorbed by type D. At the same time, based on the Moran process, the absorption probabilities of the prisoner's dilemma game with two behaviors are also deduced. The calculations are consistent with the theoretical solutions given in Ref. 25, which are introduced in Appendix E in the [supplementary material](#).

In the emotional model, there are four emotion types: type A, type B, type C, and type D, which correspond to two behaviors of cooperation and competition. Specifically, type A corresponds to cooperation, type B and type C correspond to cooperation and competition, and type D corresponds to competition. Therefore, the emotional game model is a mixed behavior model. In addition, the prisoner's dilemma game model with two behaviors only has two absorption states, while the emotional model has four absorption states with diversity. The mixed behavior and diversity may promote cooperation. By comparing the cooperation level and the average payoffs when the evolution reaches a steady state, it is found that the cooperation level and the average payoffs based on the emotional model are better than the ones in the prisoner's dilemma game with two behaviors. This shows that emotional evolution can promote group cooperation and obtain better group adaptation. This conclusion is consistent with the simulation results based on the network evolutionary game in Refs. 18 and 19.

In the derivations of the absorption probabilities, the non-absorption states are analyzed as a whole in the calculation of state transition paths and corresponding probabilities. There is no distinction between each micro-specific state transition path. Therefore, it is an approximate analysis method similar to the mean field. In addition, with the increase in the population size N , the complexity of the calculations and analysis will scale up accordingly.

SUPPLEMENTARY MATERIAL

See the [supplementary material](#) for the derivation about the transition probabilities.

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DATA AVAILABILITY

The data that support the findings of this study are available within the article and its [supplementary material](#).

REFERENCES

- ¹C. Benjamin and S. Sarkar, "Triggers for cooperative behavior in the thermodynamic limit: A case study in public goods game," *Chaos* **29**, 053131 (2019).
- ²C. Benjamin and A. Dash, "Thermodynamic susceptibility as a measure of cooperative behavior in social dilemmas," *Chaos* **30**, 093117 (2020).
- ³Y. Wu, Z. Zhang, M. Yan, and S. Zhang, "Environmental feedback promotes the evolution of cooperation in the structured populations," *Chaos* **29**, 113101 (2019).
- ⁴L. Liu, S. Wang, X. Chen, and M. Perc, "Evolutionary dynamics in the public goods games with switching between punishment and exclusion," *Chaos* **28**, 103105 (2018).
- ⁵C. Chen, X. Hu, and S. Chen, "Self-organized interdependence among populations promotes cooperation by means of coevolution," *Chaos* **29**, 013139 (2019).
- ⁶J. Tooby and L. Cosmides, "The past explains the present: Emotional adaptations and the structure of ancestral environments," *Evol. Sociobiol.* **11**, 375–424 (1990).
- ⁷K. P. Leith and R. F. Baumeister, "Why do bad moods increase self-defeating behavior? Emotion, risk taking, and self-regulation," *J. Pers. Soc. Psychol.* **71**, 1250–1267 (1996).
- ⁸F. B. M. De Waal, "The antiquity of empathy," *Science* **336**, 874–876 (2012).
- ⁹W. Nakashishi and H. Ohtsuki, "Evolution of emotional contagion in group-living animals," *J. Theor. Biol.* **440**, 12–20 (2018).
- ¹⁰A. G. Sanfey, J. K. Rilling, J. A. Aronson, L. E. Nystrom, and J. D. Cohen, "The neural basis of economic decision-making in the ultimatum game," *Science* **300**, 1755–1758 (2003).
- ¹¹M. A. Nowak, R. Highfield, and E. Mensch, *Supercooperators* (Canongate Books Ltd, 2011).
- ¹²J. W. Lai and K. H. Cheong, "Social dynamics and Parrondo's paradox: A narrative review," *Nonlinear Dyn.* **101**, 1–20 (2020).
- ¹³J. M. Koh and K. H. Cheong, "Emergent pre-eminence of selfishness: An anomalous Parrondo perspective," *Nonlinear Dyn.* **98**, 943–951 (2019).
- ¹⁴J. M. Koh and K. H. Cheong, "New doubly-anomalous Parrondo's games suggest emergent sustainability and inequality," *Nonlinear Dyn.* **96**, 257–266 (2019).
- ¹⁵C. Wang, J. M. Koh, K. H. Cheong, and N.-G. Xie, "Progressive information polarization in a complex-network entropic social dynamics model," *IEEE Access* **7**, 35394–35404 (2019).
- ¹⁶Z. X. Tan and K. H. Cheong, "Cross-issue solidarity and truth convergence in opinion dynamics," *J. Phys. A: Math. Theor.* **51**, 355101 (2018).
- ¹⁷Y. Ye, X. R. Hang, J. M. Koh, J. A. Miszczak, K. H. Cheong, and N.-G. Xie, "Passive network evolution promotes group welfare in complex networks," *Chaos, Solitons Fractals* **130**, 109464 (2020).
- ¹⁸A. Szolnoki, N. G. Xie, C. Wang, and M. Perc, "Imitating emotions instead of strategies in spatial games elevates social welfare," *Europhys. Lett.* **96**, 38002 (2011).
- ¹⁹A. Szolnoki, N. Xie, Y. Ye, and M. Perc, "Evolution of emotions on networks leads to the evolution of cooperation in social dilemmas," *Phys. Rev. E* **87**, 042805 (2013).
- ²⁰L. Wang, S. Ye, M. C. Jones, Y. Ye, M. Wang, and N. Xie, "The evolutionary analysis of the ultimatum game based on the net-profit decision," *Physica A* **430**, 32–38 (2015).
- ²¹S. Ye, L. Wang, M. C. Jones, Y. Ye, M. Wang, and N. Xie, "Effect of network topology on the evolutionary ultimatum game based on the net-profit decision," *Eur. Phys. J. B* **89**, 93 (2016).

- ²²N. Xie, K. Zhen, C. Wang, Y. Ye, and L. Wang, “Evolution of cooperation driven by the diversity of emotions,” *Connect. Sci.* **27**, 89–101 (2015).
- ²³L. Wang, S. Ye, K. H. Cheong, W. Bao, and N. Xie, “The role of emotions in spatial prisoner’s dilemma game with voluntary participation,” *Physica A* **490**, 1396–1407 (2018).
- ²⁴P. M. Altrock, A. Traulsen, and M. A. Nowak, “Evolutionary games on cycles with strong selection,” *Phys. Rev. E* **95**, 022407 (2017).
- ²⁵C. Taylor, D. Fudenberg, A. Sasaki, and M. A. Nowak, “Evolutionary game dynamics in finite populations,” *Bull. Math. Biol.* **66**, 1621–1644 (2004).
- ²⁶J. Wang, F. Fu, L. Wang, and G. Xie, “Evolutionary game dynamics with three strategies in finite populations,” [arXiv:physics/0701315](https://arxiv.org/abs/physics/0701315) (2007).
- ²⁷M. A. Nowak, A. Sasaki, C. Taylor, and D. Fudenberg, “Emergence of cooperation and evolutionary stability in finite populations,” *Nature* **428**, 646 (2004).