

$$x_t = -\frac{1}{3}x_{t-1} - \frac{1}{4}w_{t-2} + w_t$$

Sup 1 a)

$$(1 + \frac{1}{3}B)x_t = (1 + \frac{1}{4}B^2)w_t \quad \text{Causal because } \phi(z) = (1 + \frac{1}{3}z) > 0, z = -3, \text{ outside unit disk}$$

$$(1 + \frac{1}{3}z)(1 + \gamma_1 z + \gamma_2 z^2 + \dots + \gamma_j z^j + \dots) = 1 - \frac{1}{4}z^2$$

$$\rightarrow 1 + (\gamma_1 + \frac{1}{3})z + (\gamma_2 + \frac{1}{3}\gamma_1)z^2 + \dots + (\gamma_j + \frac{1}{3}\gamma_{j-1})z^j + \dots = 1 - \frac{1}{4}z^2$$

$$\gamma_1 + \frac{1}{3} = 0 \quad \gamma_1 = -\frac{1}{3}$$

$$\gamma_2 + \frac{1}{3}\gamma_1 = -\frac{1}{4} \quad \gamma_2 = \frac{5}{36}$$

$$x_t = w_t + \frac{1}{12} \sum_{j=1}^{\infty} \left(\frac{1}{3}\right)^{j-1} w_{t-j}$$

$$\gamma_3 = \frac{-5}{36 \cdot 3}$$

$$\gamma_4 = \frac{5}{36 \cdot 9}$$

$$\gamma_j + \frac{1}{3}\gamma_{j-1} = 0 \text{ for } j > 2 \rightarrow \gamma_j = \frac{5}{12} \left(\frac{1}{3}\right)^{j-1} \text{ for } j \geq 2$$

b) $(1 - \frac{1}{4}z) = 0, \theta(z)$ where $z = 4$, outside of 1

$$(1 - \frac{1}{4}z^2)(1 + \pi_1 z + \pi_2 z^2 + \dots + \pi_3 z^3 + \dots) = 1 + \frac{1}{3}z$$

$$\pi_1 = \frac{1}{3} \quad \pi_2 = -\frac{1}{12} \quad \pi_3 = \frac{1}{48} \quad \pi_4 = -\frac{1}{48 \cdot 4}$$

$$1 + \pi_1 z + (\pi_2 - \frac{1}{4})z^2 + (\pi_3 - \frac{1}{4}\pi_2)z^3 + \dots = 1 + \frac{1}{3}z$$

$$\pi_1 = \frac{1}{3}$$

$$\pi_2 - \frac{1}{4}\pi_1 = 0 \quad \pi_2 = \frac{1}{12}$$

$$\pi_j = \frac{1}{3} \left(-\frac{1}{4}\right)^{j-1}$$