

CAP Companion Sheet

Mini-while abstract syntax

Expressions:

$e \in \mathcal{E} ::=$	n	<i>integer constant</i>
	$tt \mid ff$	<i>boolean constant</i>
	x	<i>variable</i>
	$e + e$	<i>addition</i>
	$e \times e$	<i>multiplication</i>
	$e \text{ or } e$	<i>or</i>
	$e \text{ and } e$	<i>and</i>
	$e == e \mid e < e \mid \dots$	<i>tests</i>
	\dots	

Statements:

$S \in \mathcal{Stm} ::=$	$x := e$	<i>assign</i>
	$skip$	<i>do nothing</i>
	$S_1; S_2$	<i>sequence</i>
	$\text{if } b \text{ then } S_1 \text{ else } S_2$	<i>test</i>
	$\text{while } b \text{ do } S \text{ done}$	<i>loop</i>

Typing and static semantic for mini-while

We add declarations for the language:

$P ::=$	$D; S$	<i>program</i>
$D ::=$	$\text{var } x : \tau \mid D; D$	<i>type declaration</i>

From declarations we infer $\Gamma : \text{Var} \rightarrow \text{Basetype}$ with the two following rules:

$$\frac{}{\text{var } x : \tau \rightarrow_d [x \mapsto \tau]}$$

$$\frac{D_1 \rightarrow_d \Gamma_1 \quad D_2 \rightarrow_d \Gamma_2 \quad \text{Dom}(\Gamma_1) \cap \text{Dom}(\Gamma_2) = \emptyset}{D_1; D_2 \rightarrow_d \Gamma_1 \cup \Gamma_2}$$

Then a typing judgment for expressions is $\Gamma \vdash e : \tau \in \text{Basetype}$.

Statements and programs have no type.

$$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}} \quad \frac{}{\Gamma \vdash x : \Gamma(x)} \quad \frac{\Gamma \vdash S_1 \quad \Gamma \vdash S_2}{\Gamma \vdash S_1; S_2}$$

$$\frac{\Gamma \vdash x : \tau \quad \Gamma \vdash e : \tau}{\Gamma \vdash x := e} \quad \frac{\Gamma \vdash b : \text{bool} \quad \Gamma \vdash S_1 \quad \Gamma \vdash S_2}{\Gamma \vdash \text{if } b \text{ then } S_1 \text{ else } S_2}$$

$$\frac{\Gamma \vdash b : \text{bool} \quad \Gamma \vdash S}{\Gamma \vdash \text{while } b \text{ do } S \text{ done}} \quad \frac{D \rightarrow_d \Gamma \quad \Gamma \vdash S}{\emptyset \vdash D; S}$$

(...and other similar rules)

State $\sigma : \text{Var} \rightarrow \text{Value}$ ($\text{Value} = \mathbb{Z} \cup \mathbb{B}$).

Typing configurations:

$$\Gamma \vdash (S, \sigma) \iff (\Gamma \vdash S \wedge \forall x \tau, \emptyset \vdash \sigma(x) : \tau \iff \Gamma(x) = \tau)$$

Operational semantics for mini-while

Evaluation

$$\begin{aligned} \text{Val} : \mathcal{E} &\rightarrow \text{State} \rightarrow \text{Value} \\ \text{Val}(n, \sigma) &= \text{value}(n) \\ \text{Val}(x, \sigma) &= \sigma(x) \\ \text{Val}(e_1 + e_2, \sigma) &= \text{Val}(e_1, \sigma) + \text{Val}(e_2, \sigma) \\ &\dots \end{aligned}$$

Small step $(\text{Stm}, \text{State}) \Rightarrow (\text{Stm}, \text{State})$ or $(\text{Stm}, \text{State}) \Rightarrow \text{State}$

$$(x := e, \sigma) \Rightarrow \sigma[x \mapsto \text{Val}(e, \sigma)] \quad (\text{skip}, \sigma) \Rightarrow \sigma \quad \frac{(S_1, \sigma) \Rightarrow \sigma'}{((S_1; S_2), \sigma) \Rightarrow (S_2, \sigma')}$$

$$\frac{(S_1, \sigma) \Rightarrow (S'_1, \sigma')}{((S_1; S_2), \sigma) \Rightarrow (S'_1; S_2, \sigma')} \quad \frac{\text{Val}(b, \sigma) = tt}{(\text{if } b \text{ then } S_1 \text{ else } S_2, \sigma) \Rightarrow (S_1, \sigma)}$$

$$\frac{\text{Val}(b, \sigma) = ff}{(\text{if } b \text{ then } S_1 \text{ else } S_2, \sigma) \Rightarrow (S_2, \sigma)}$$

$$(\text{while } b \text{ do } S \text{ done}, \sigma) \Rightarrow (\text{if } b \text{ then } (S; \text{while } b \text{ do } S \text{ done}) \text{ else skip}, \sigma)$$