

CAP— Compilation (#6) : SSA

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Master 1, ENS de Lyon et Dpt Info, Lyon1

2025-2026



1 SSA Control Flow Graph

2 LAB: CFG + SSA

3 Exercises

Credits

Source <http://homepages.dcc.ufmg.br/~fernando/classes/dcc888/ementa/slides/StaticSingleAssignment.pdf>

- The SSA book (collective)
- Modern Compiler Implementation in C/Java/ML (Andrew Appel)
- Fernando Magno Quintao Pereira's course
<https://www.youtube.com/user/pronesto/videos>
- Adrian Sampson's course
<https://www.cs.cornell.edu/courses/cs6120/2020fa/>

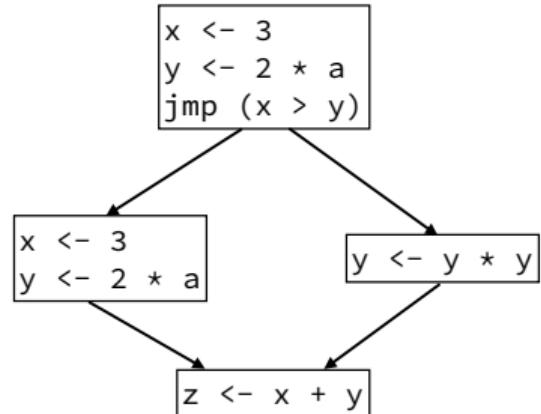
Motivation: It's all about information

Compilers alternate between two tasks:

1. *computing* some information (invariants) of the program
2. *using* this information to justify some program transformations

Dataflow analyses associate facts to every program point:

- * a fact is *associated* to a *definition-site* of a variable
- * a fact is *exploited* at a *use-site* of a variable



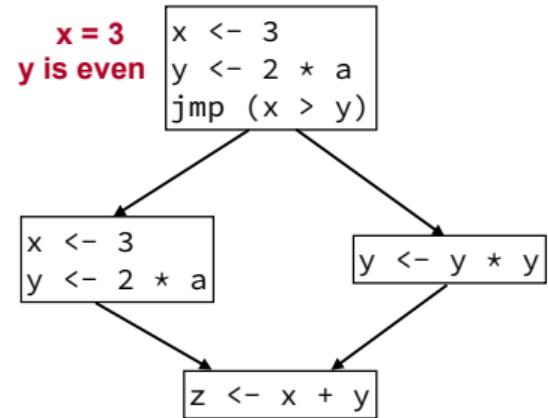
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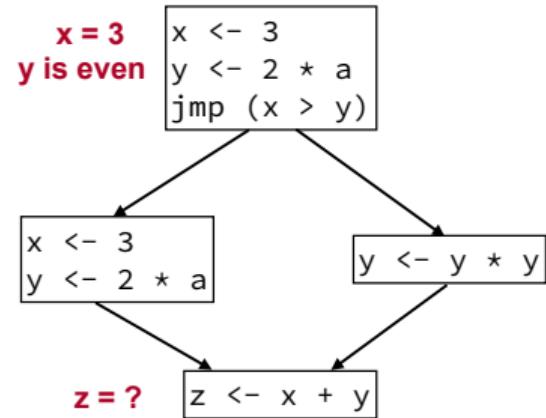
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What do we know about x and y ?

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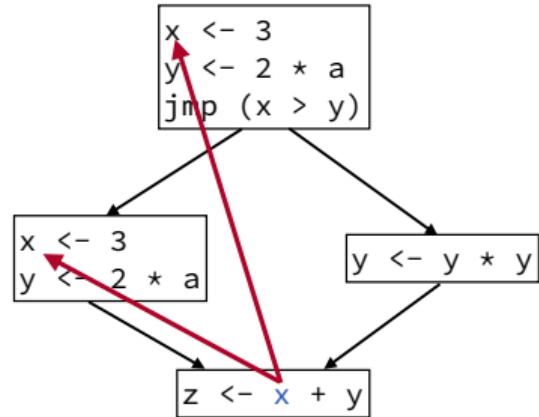
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One solution: use a data-structure, the *def-use and use-def chains*

M def and **N** use of a variable: $O(N * M)$ space and time



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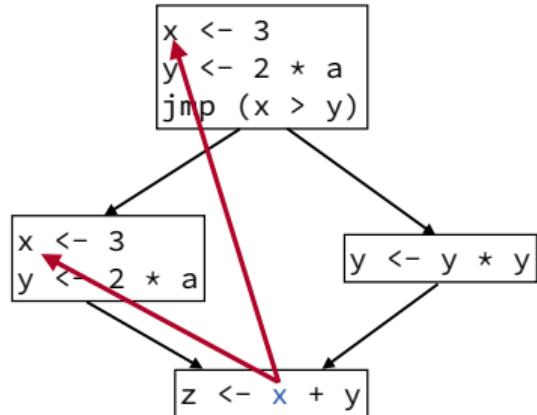
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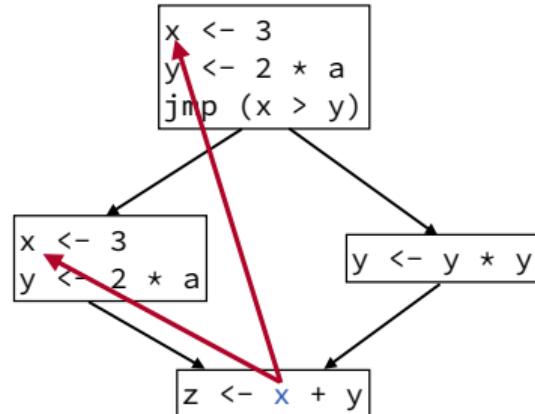
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We want to enforce an invariant by construction: we want an *intermediate representation*

Single Static Assignment (SSA)

Each variable has **exactly one definition** in the syntax¹

Use-def chains are explicit in the syntax of the program -> Many optimizations are simplified

¹ Dynamically, it can be defined many times: it is not “Single Assignment”!

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Introduced in 1988:

“Global value numbers and redundant computations” by Rosen, Wegman and Zadeck

Used in most modern compilers: GCC, llvm, HotSpot...

We will consider here more specifically Control Flow Graphs in SSA form

¹ Dynamically, it can be defined many times: it is not “Single Assignment”!

Converting to SSA form: informally

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Converting straight code

bl:

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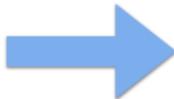
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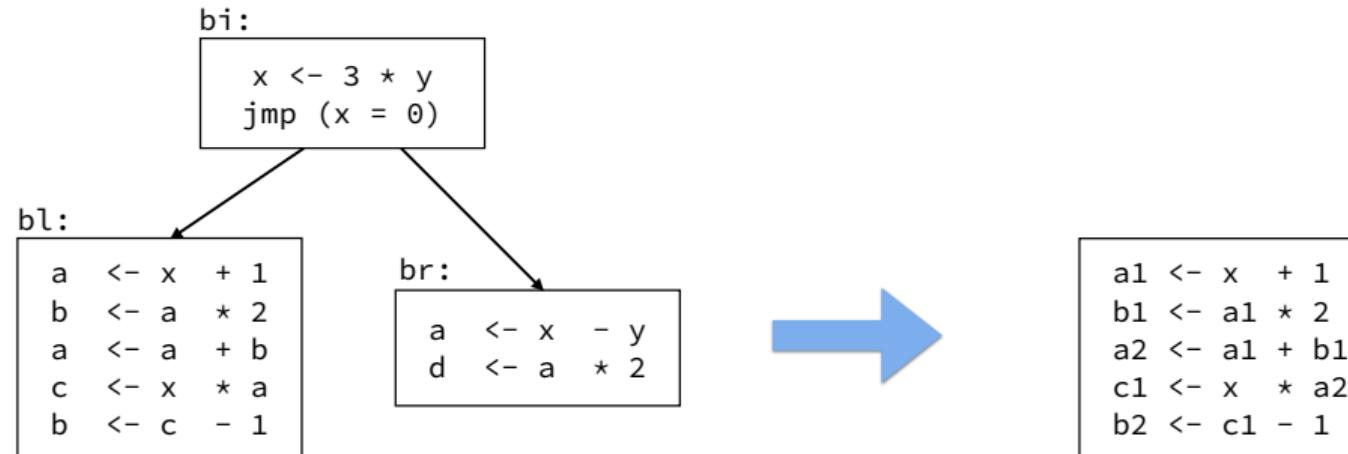
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Rule 2: use the most recent definition at each use-site

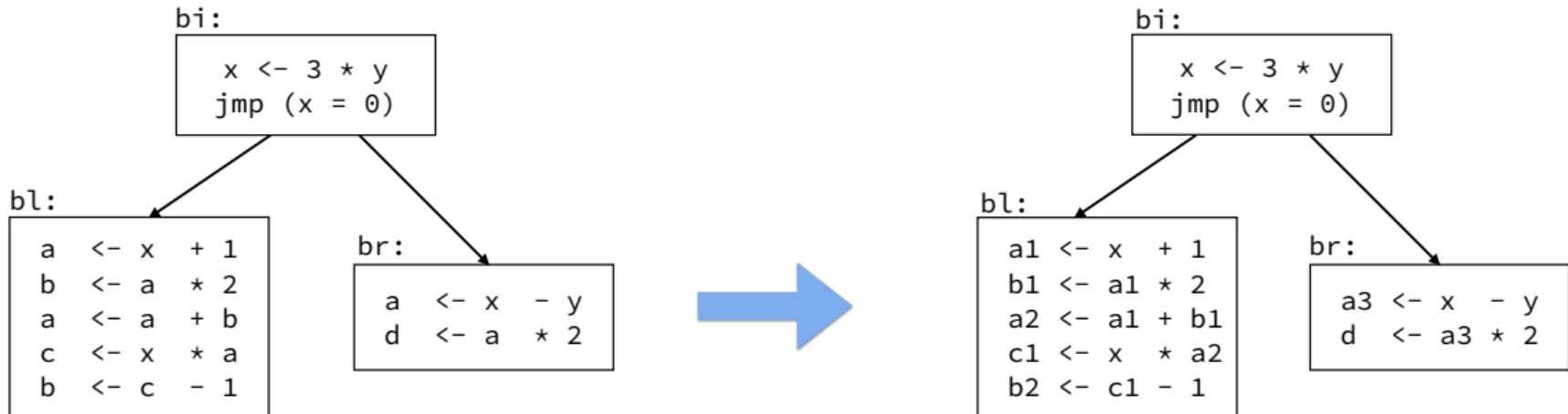
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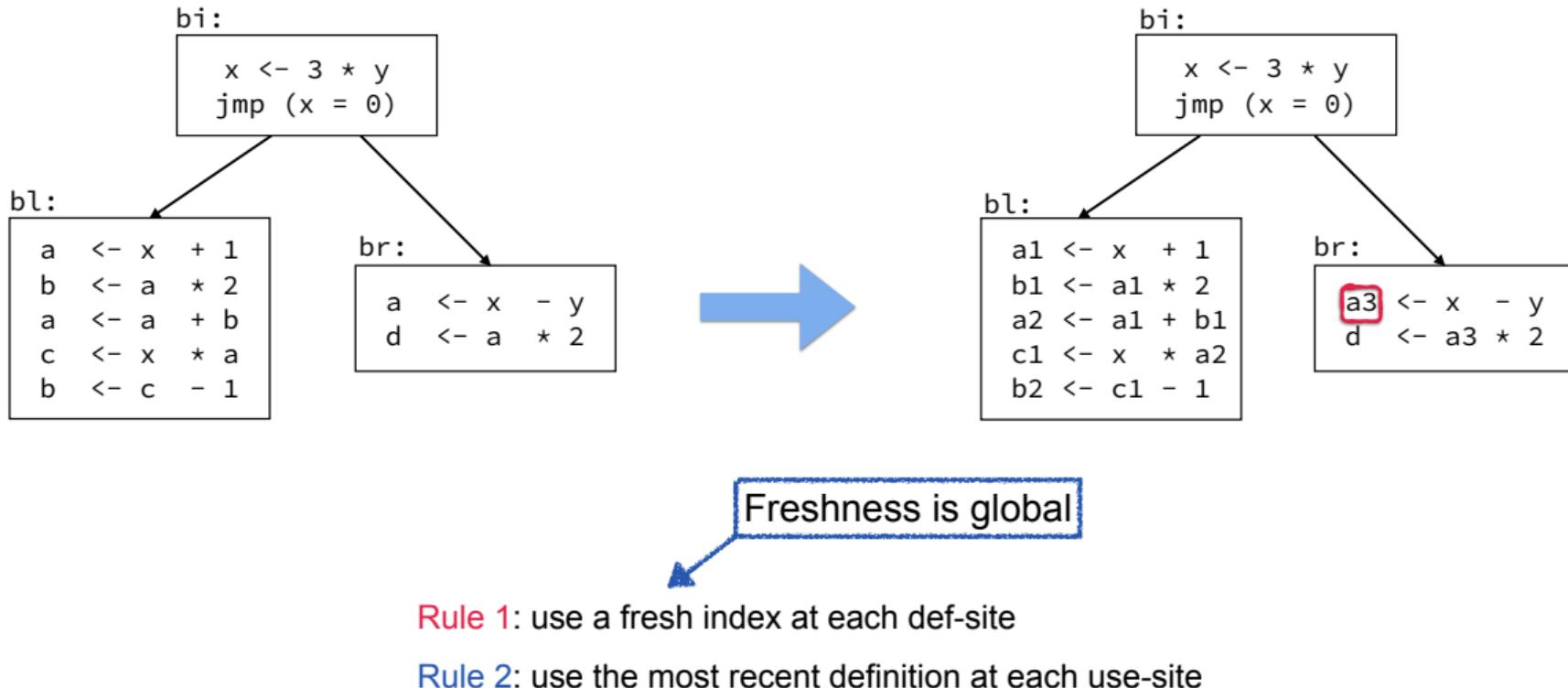
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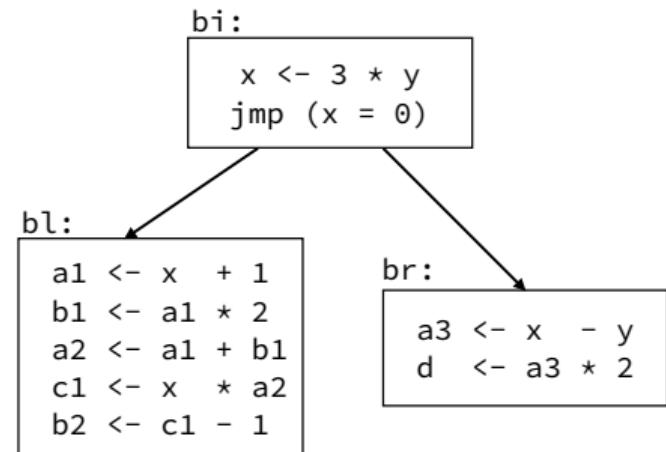
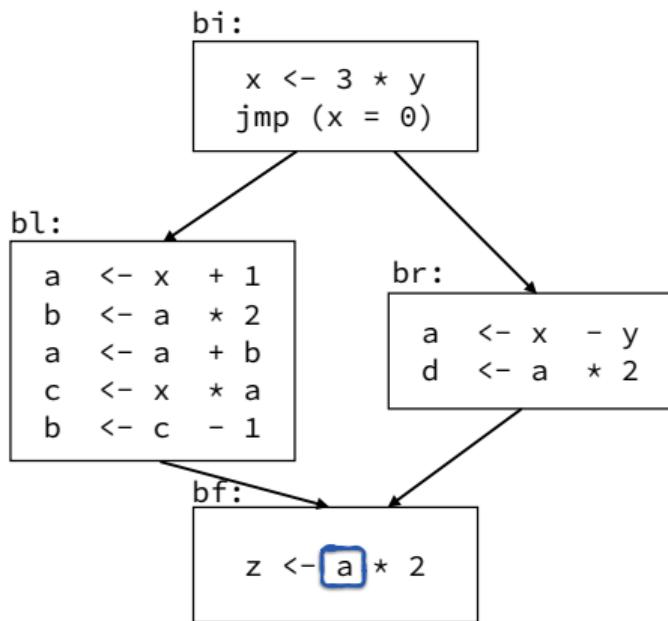
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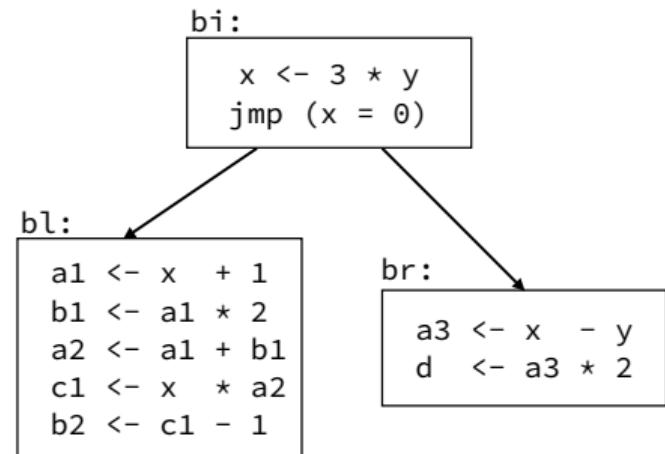
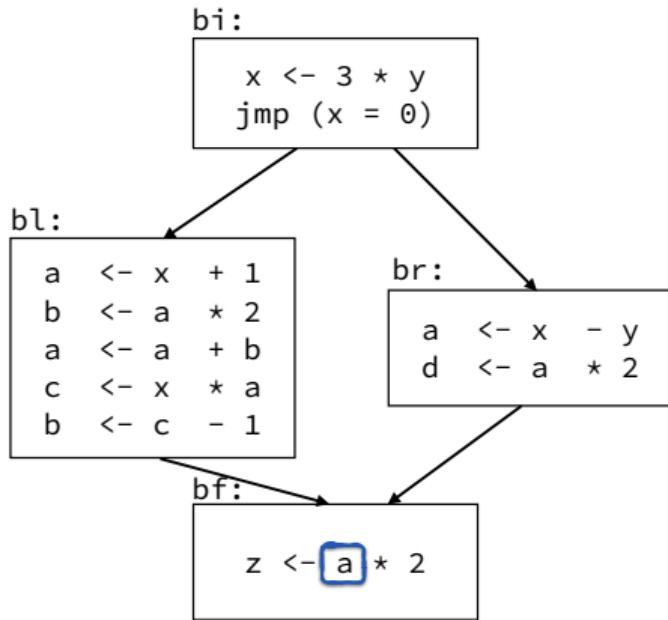
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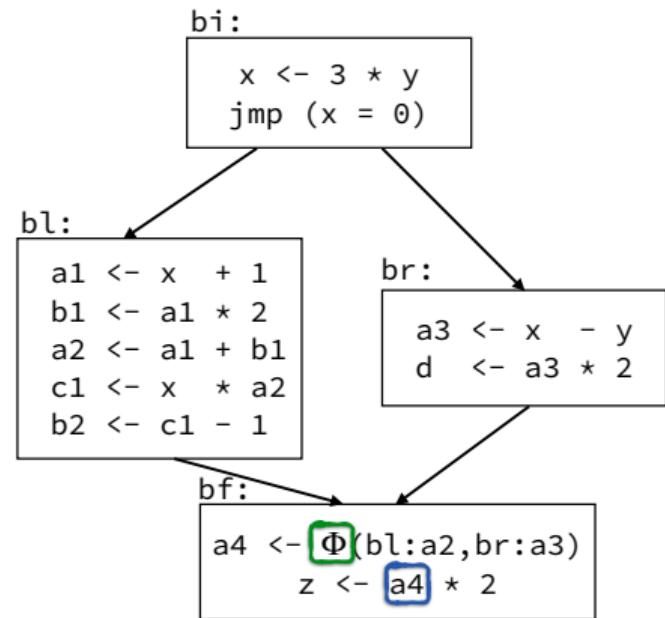
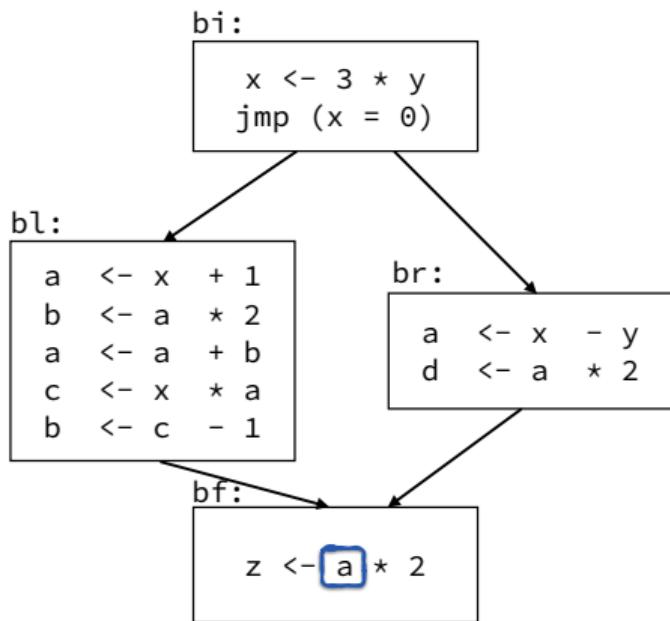


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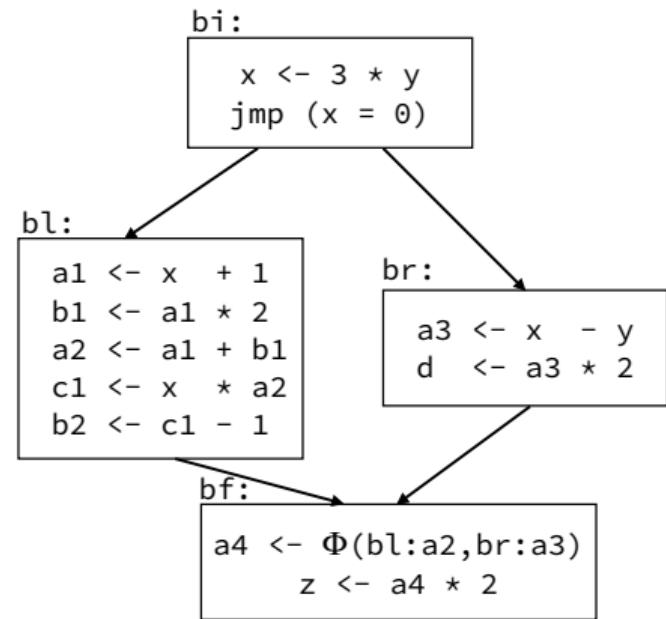
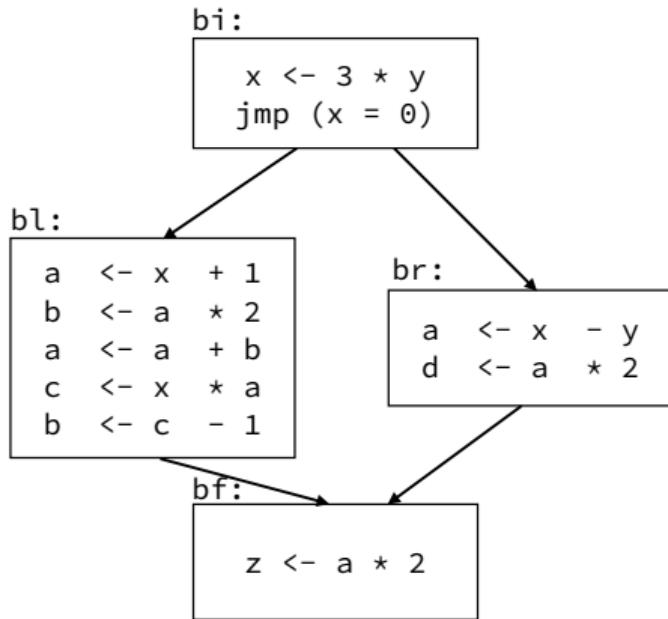


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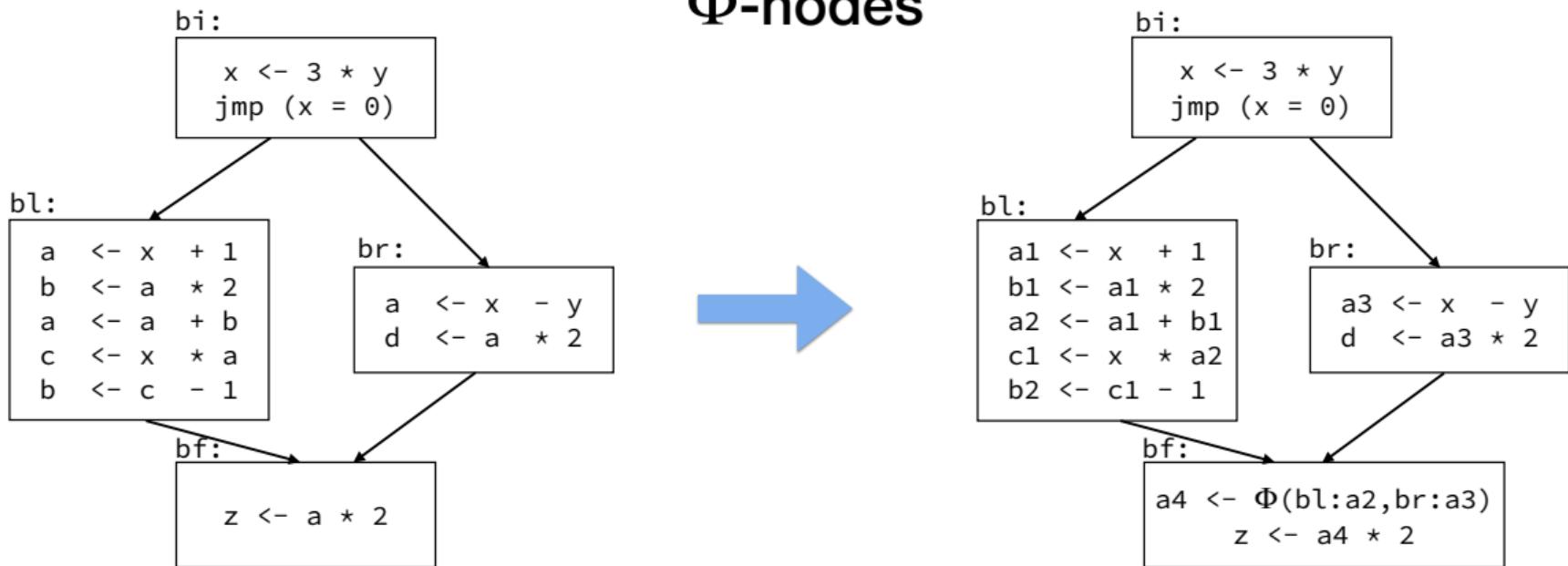
Rule 3: at merge points, introduce Φ -nodes

Converting merging points: Φ-nodes



Goal: to decide when to introduce Φ-nodes

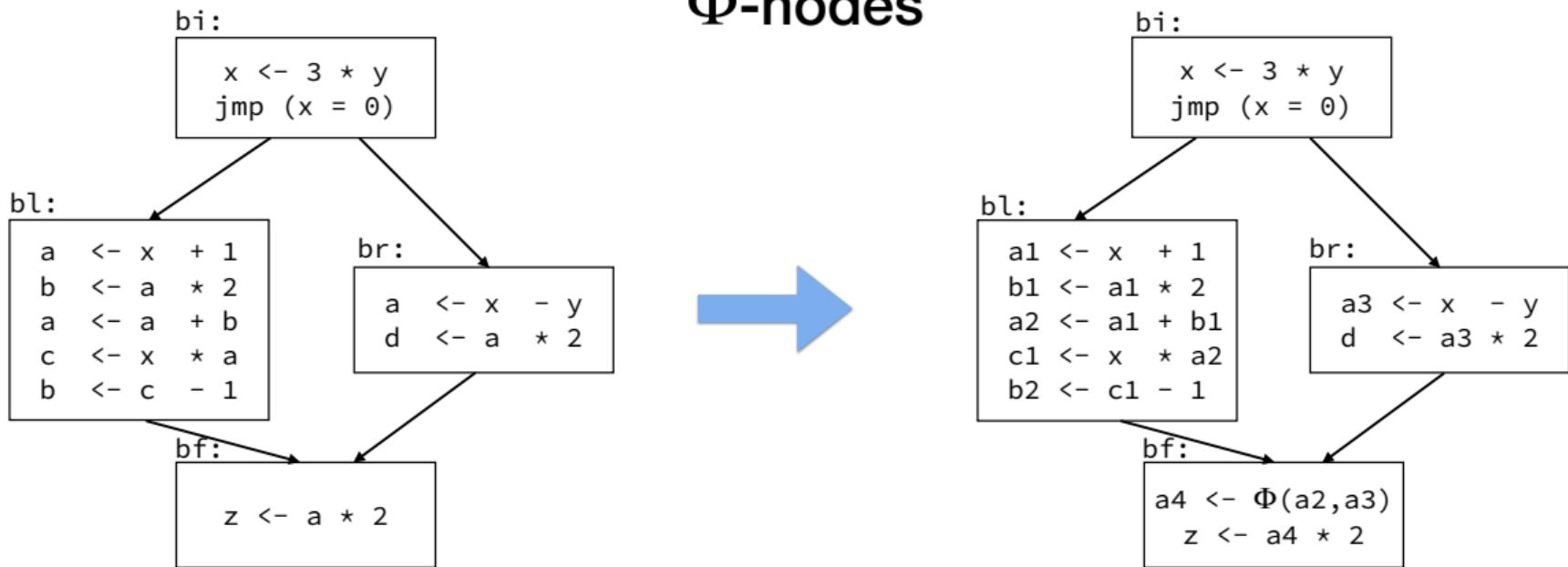
Converting merging points: Φ-nodes



Goal: to decide when to introduce Φ -nodes

One per variable at every join point?

Converting merging points: Φ-nodes



Goal: to decide when to introduce *few* Φ -nodes

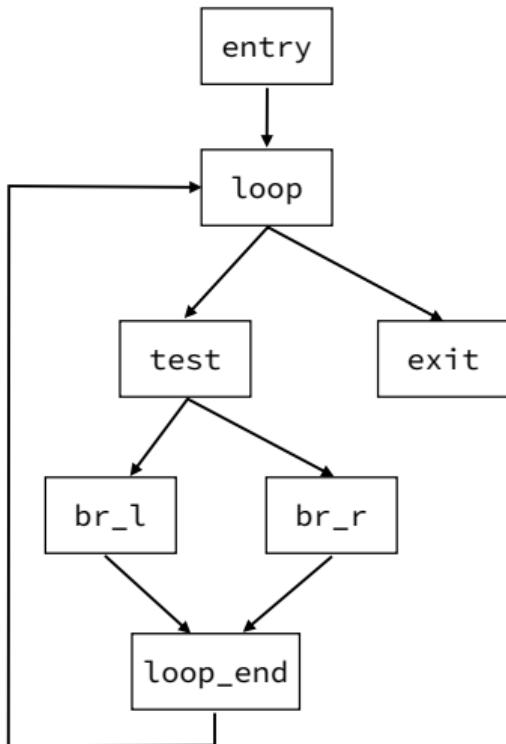
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Converting to SSA form: an algorithm

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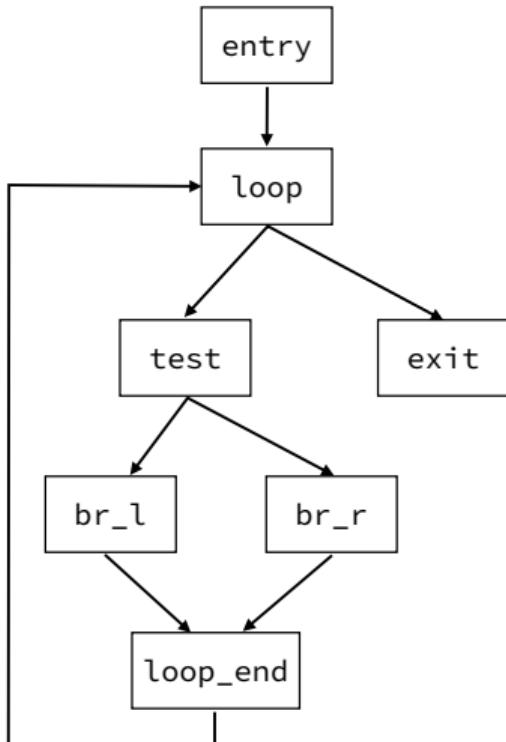
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The domination relation



A dominates B if any path from entry to B contains A

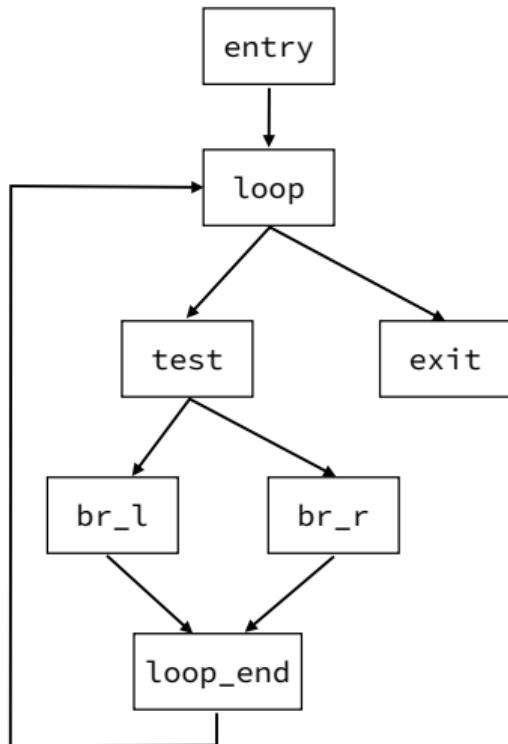
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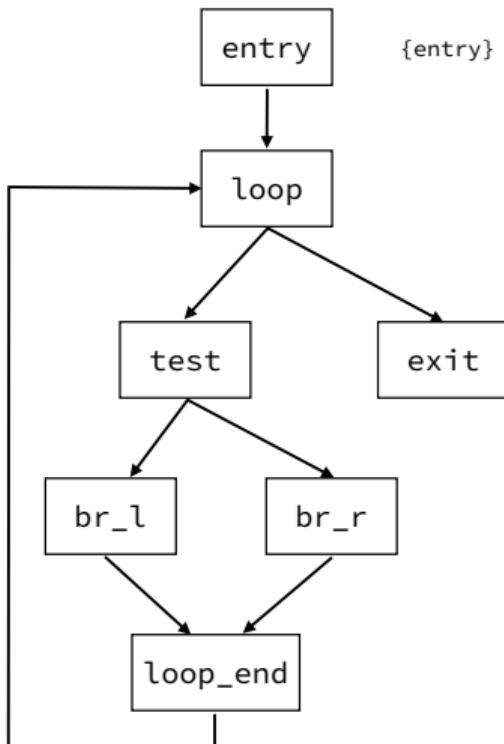
Can you annotate the nodes with their dominators?

The domination relation



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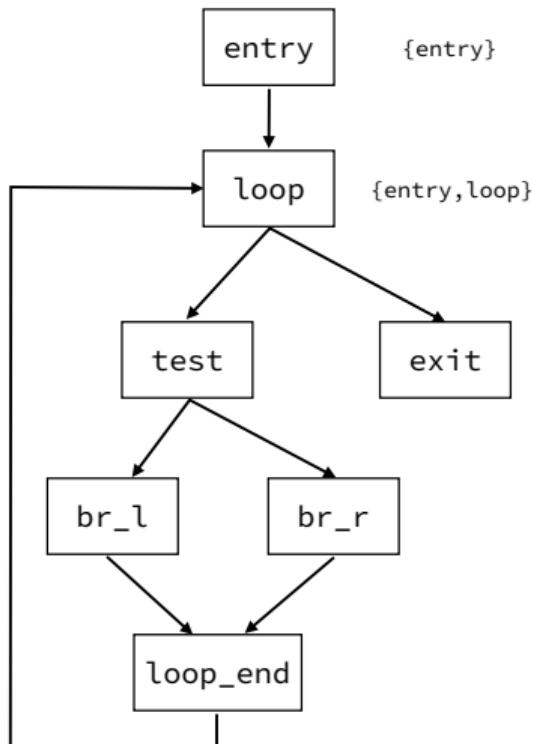
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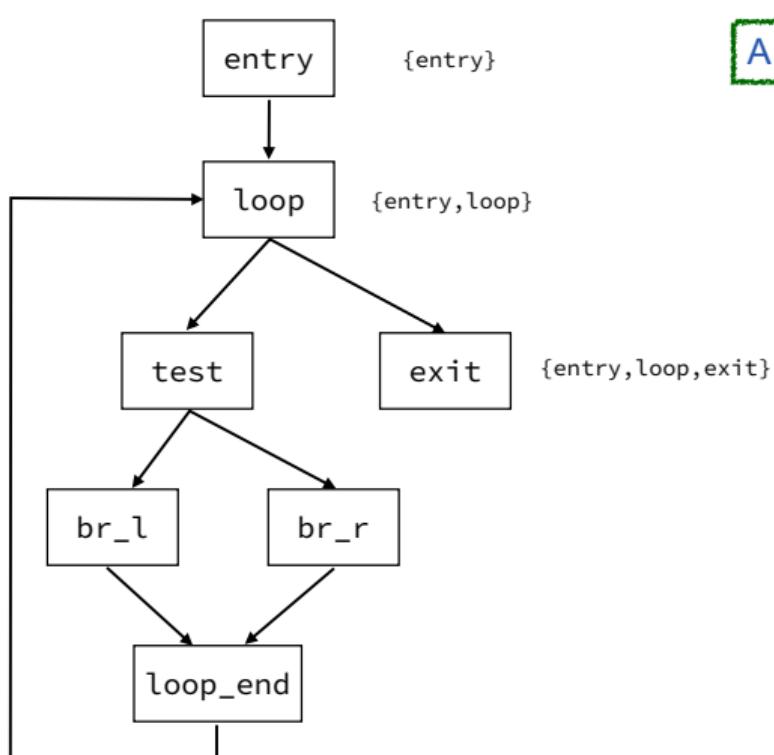
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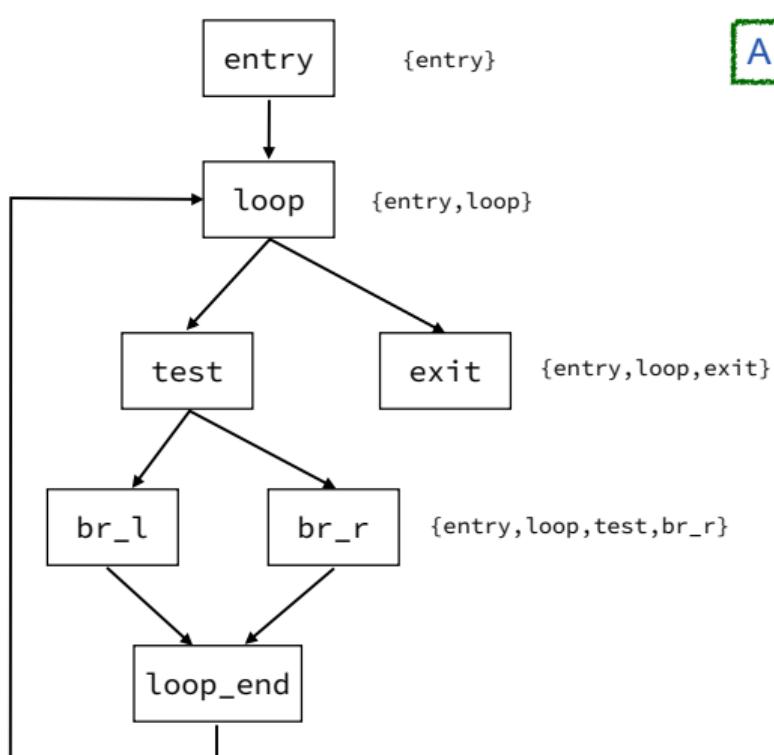
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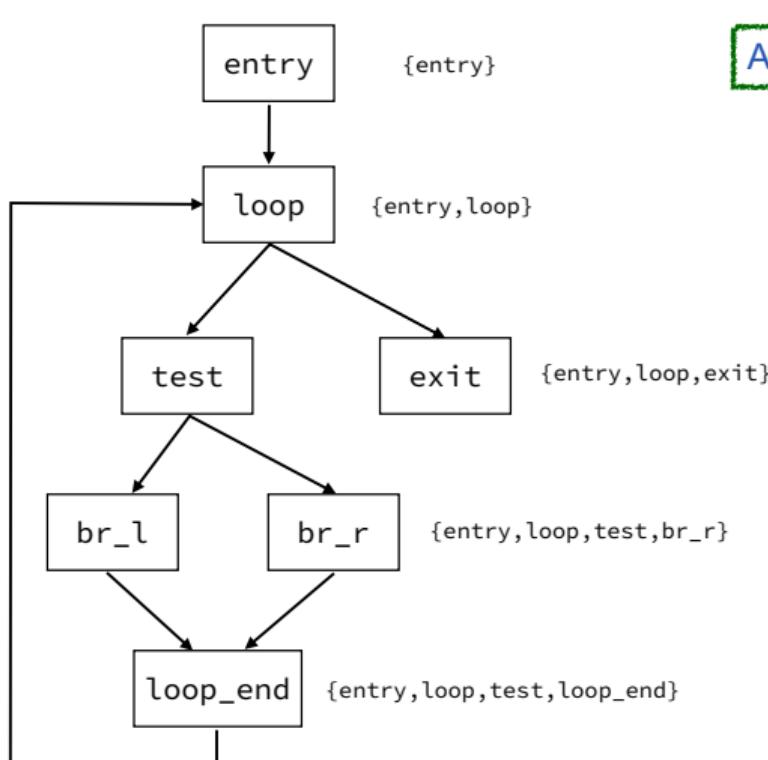
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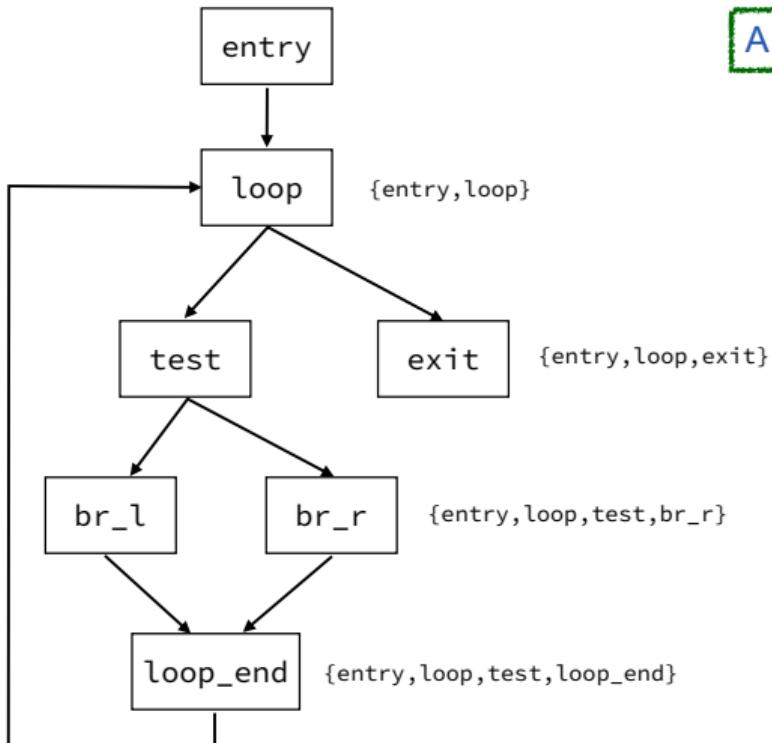
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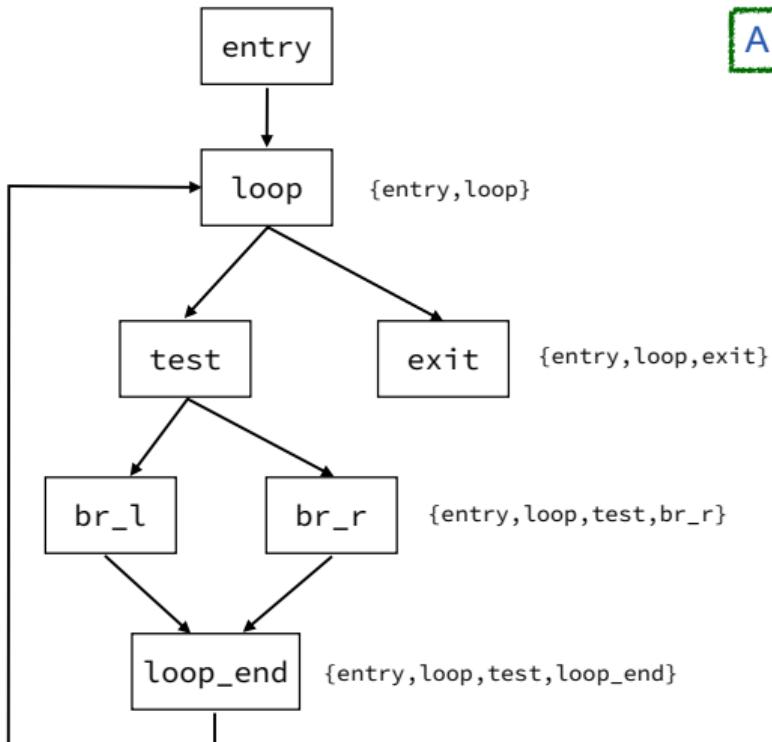
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A strictly dominates B if A dominates B and A is not B

The *domination tree* stores the domination relation
A is parent to B if:

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- A does not strictly dominate any C that strictly dominates B

The domination relation



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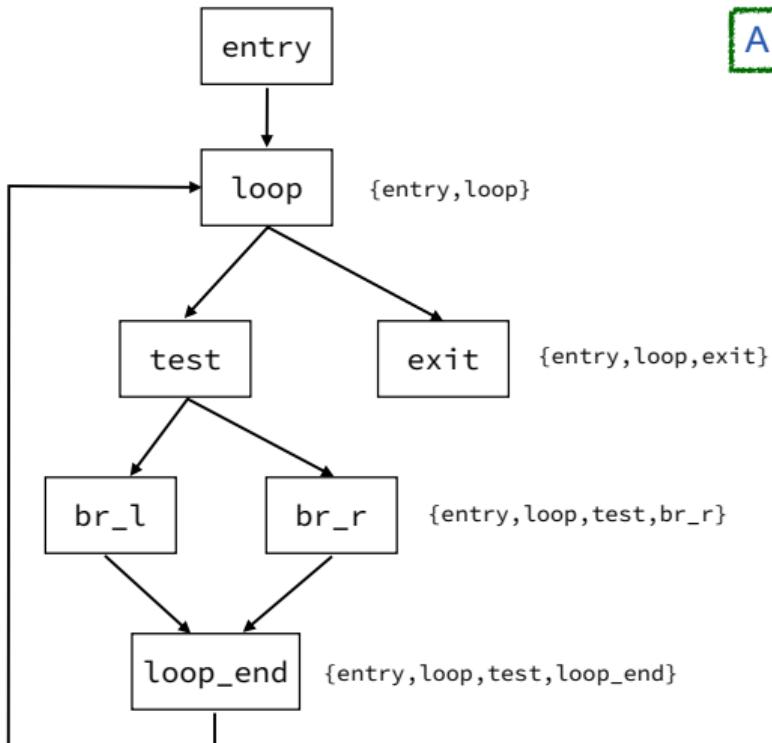
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Can you build the domination tree
 of the CFG to the left?

The domination relation



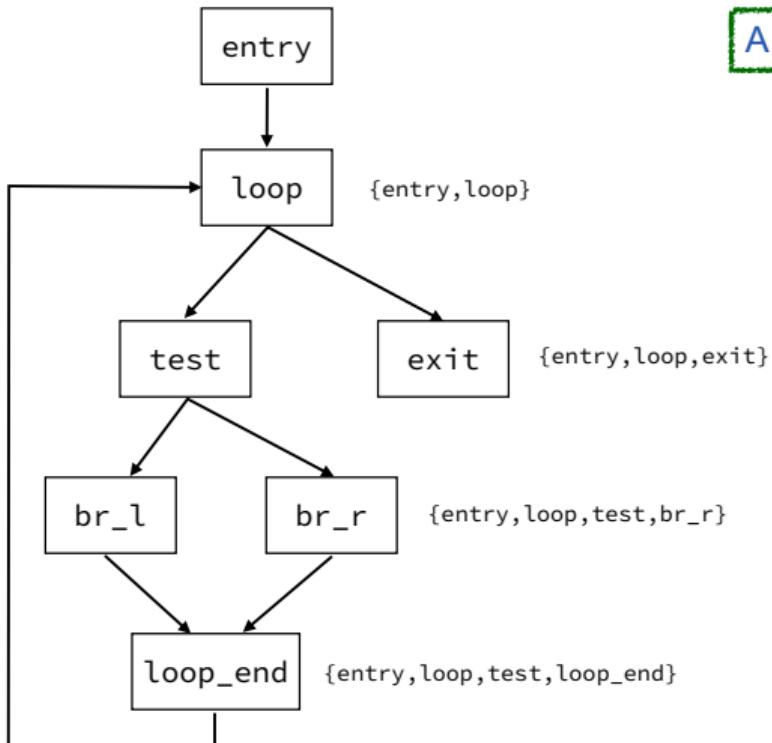
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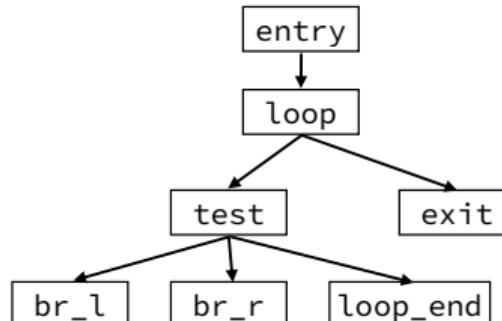


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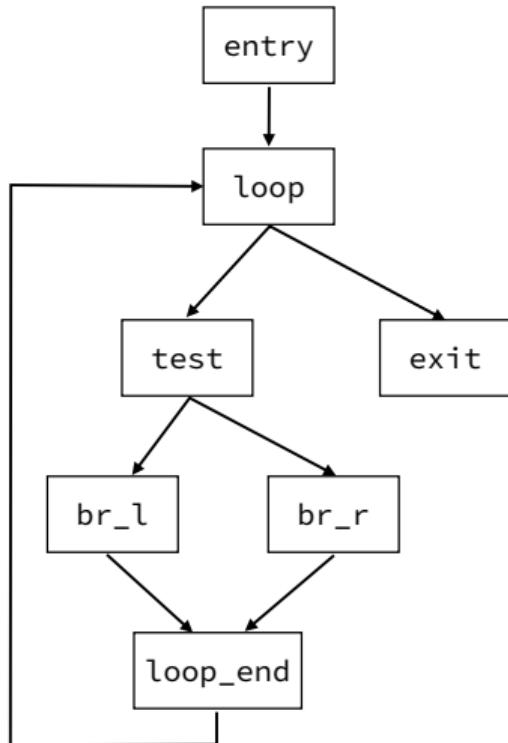
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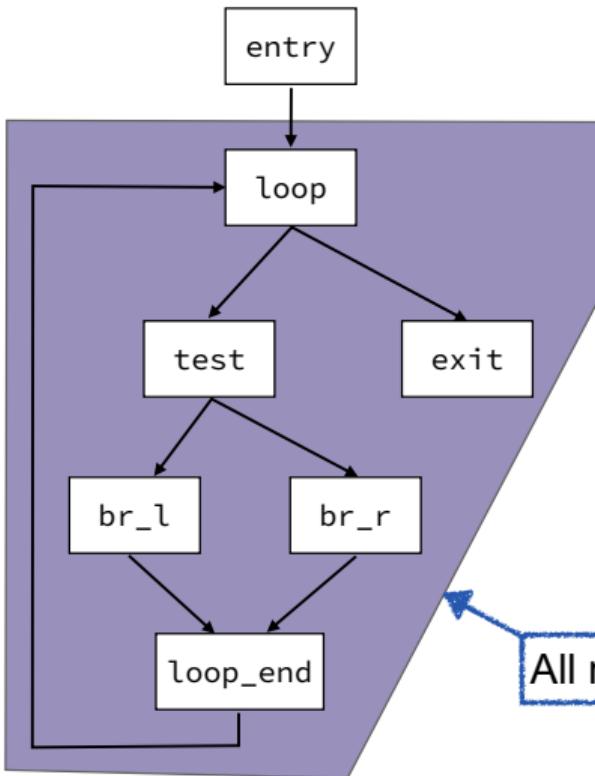
Dominance frontier



B belongs to A's *dominance frontier* if:

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Dominance frontier

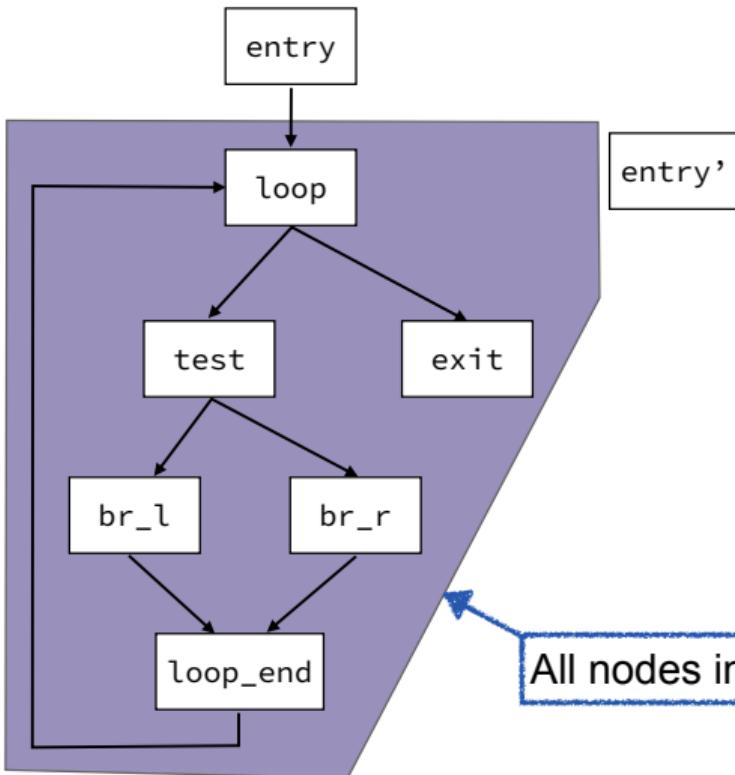


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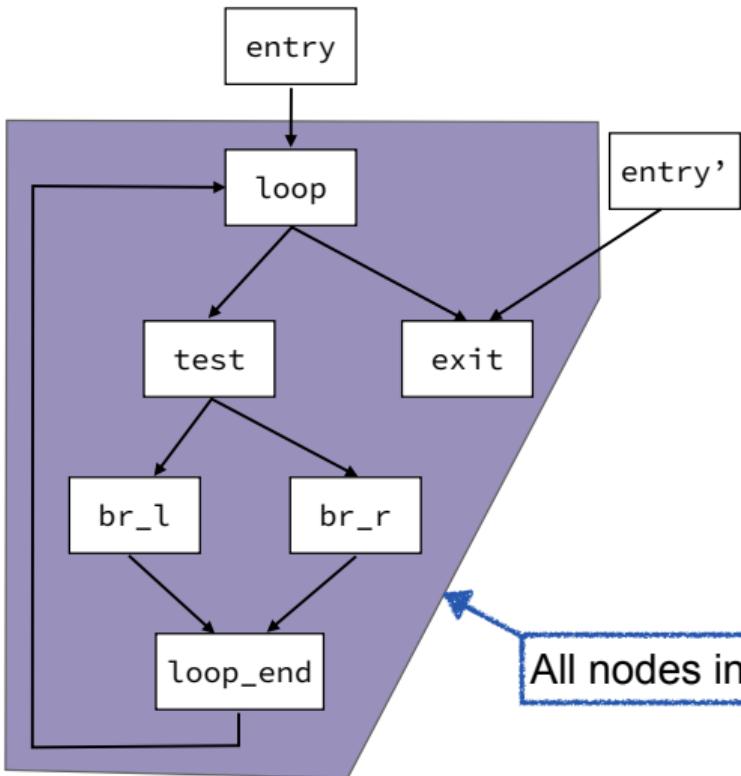


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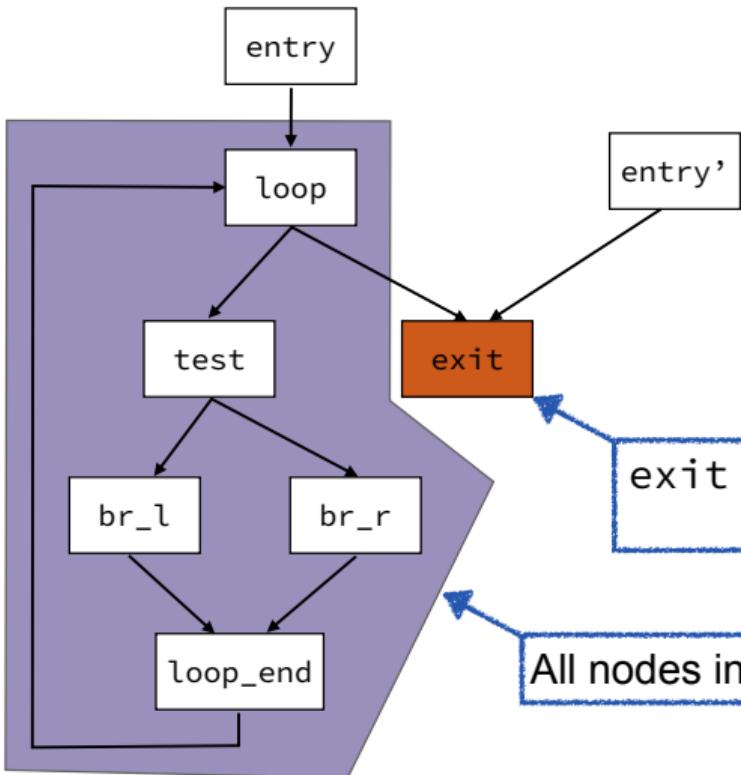
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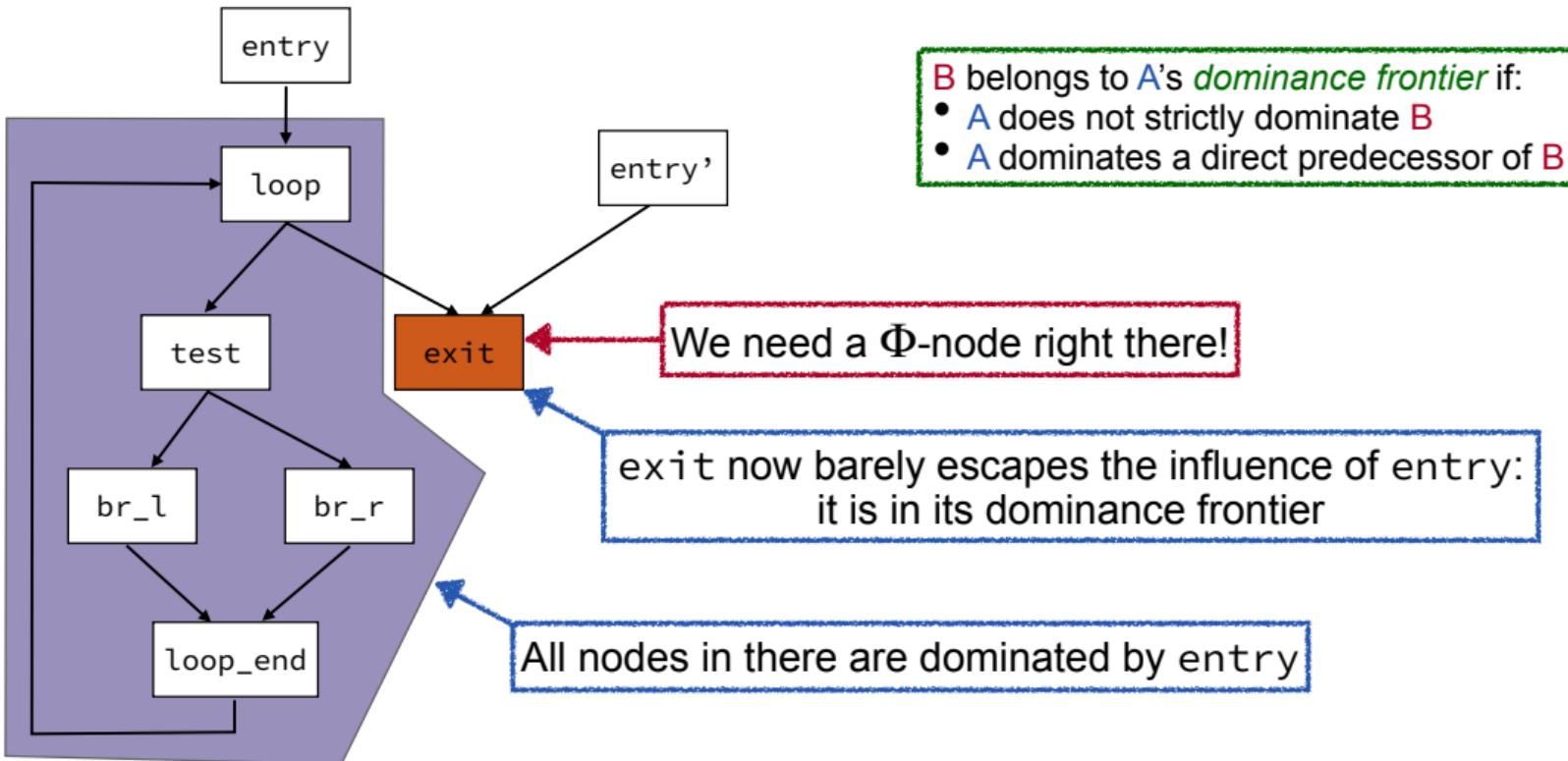
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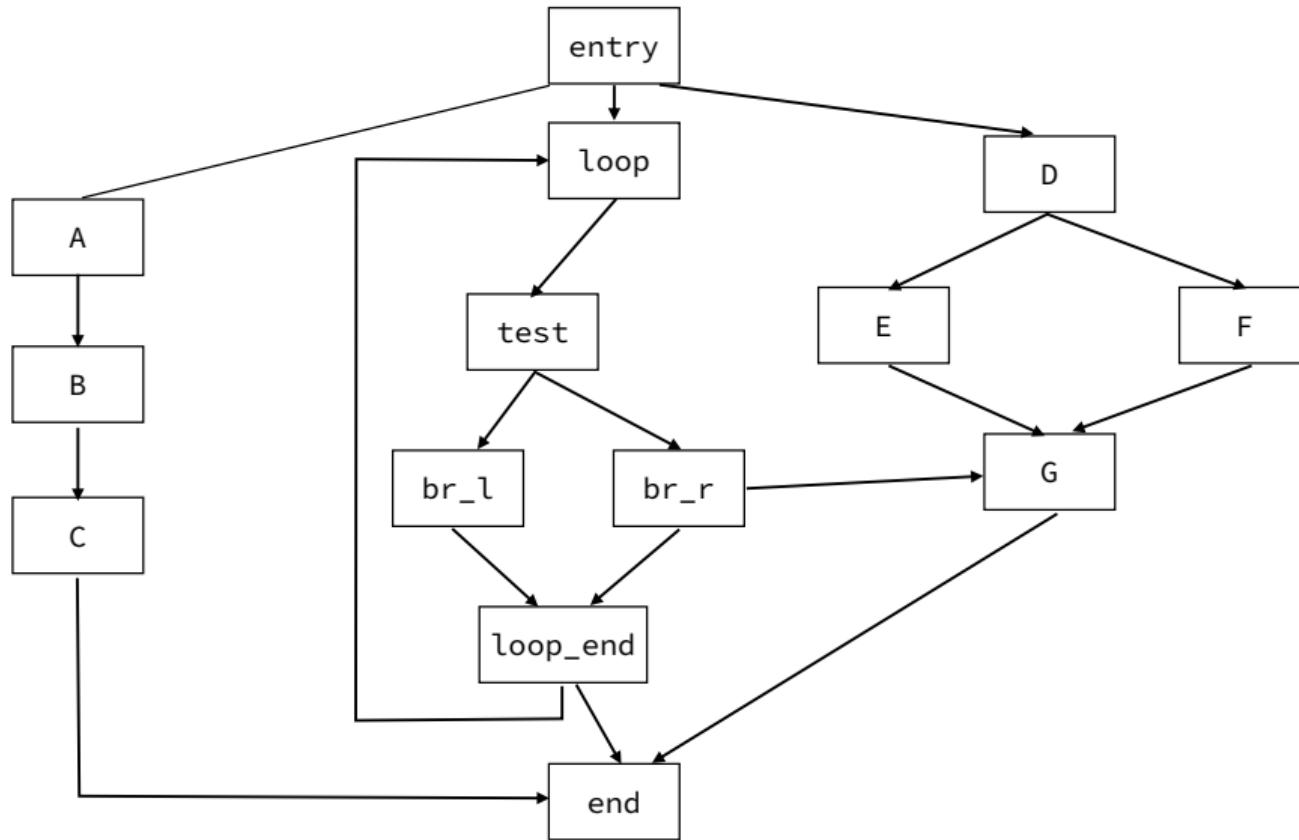
exit now barely escapes the influence of entry:
it is in its dominance frontier

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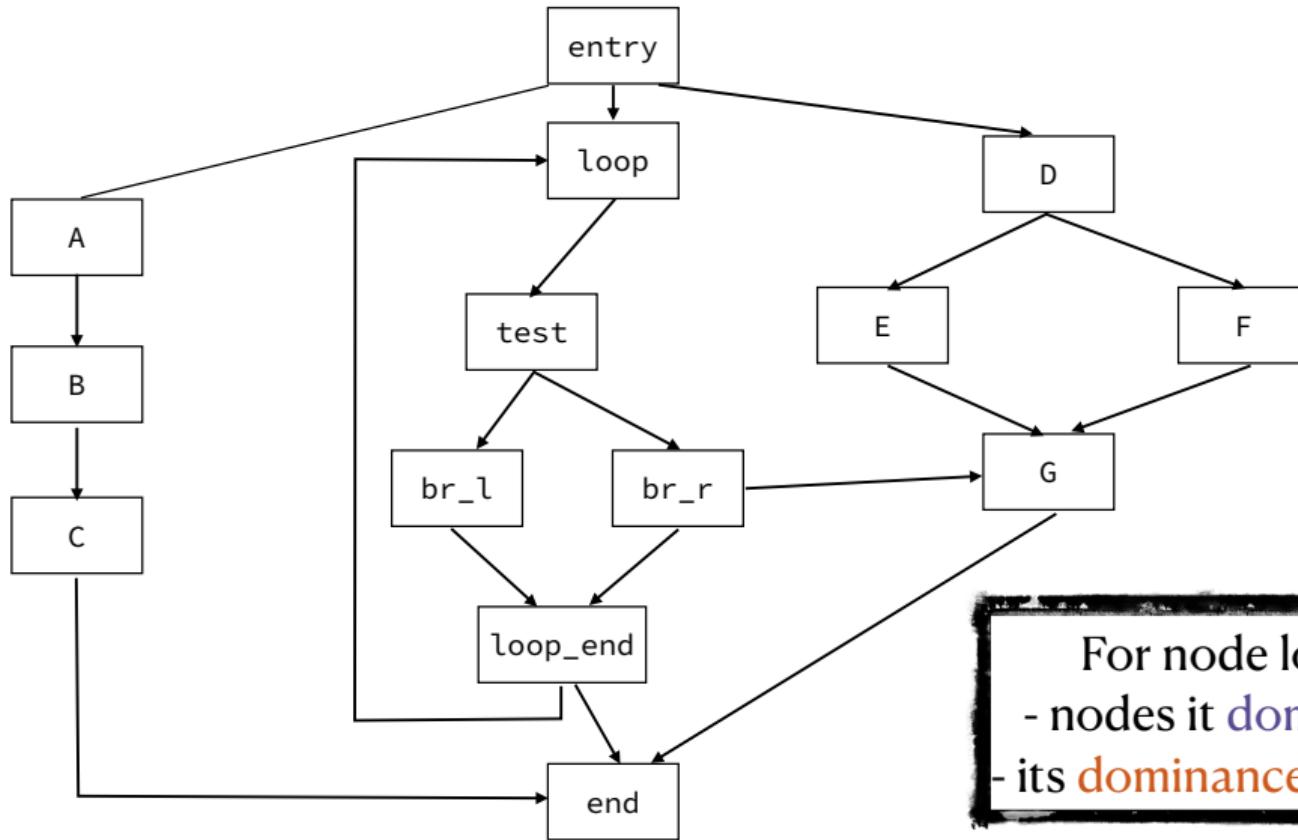
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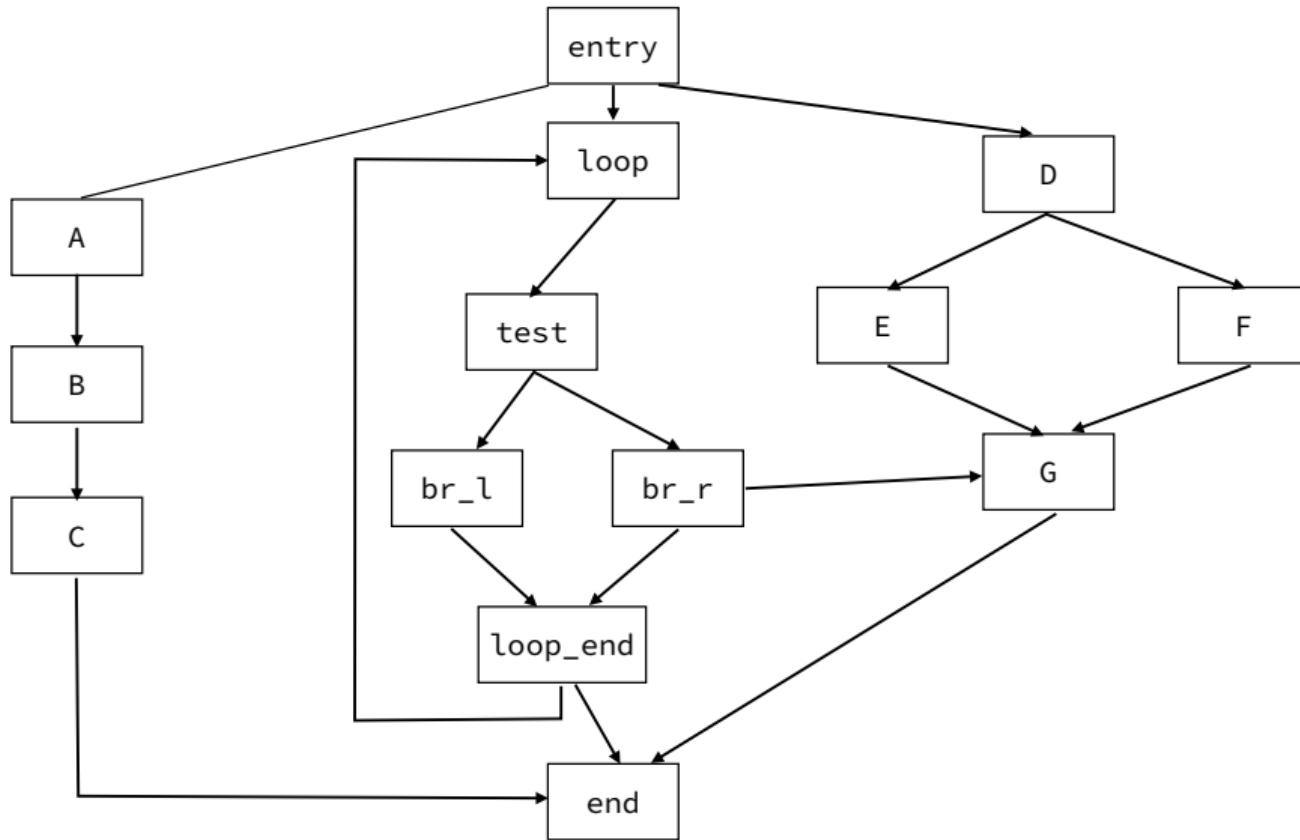
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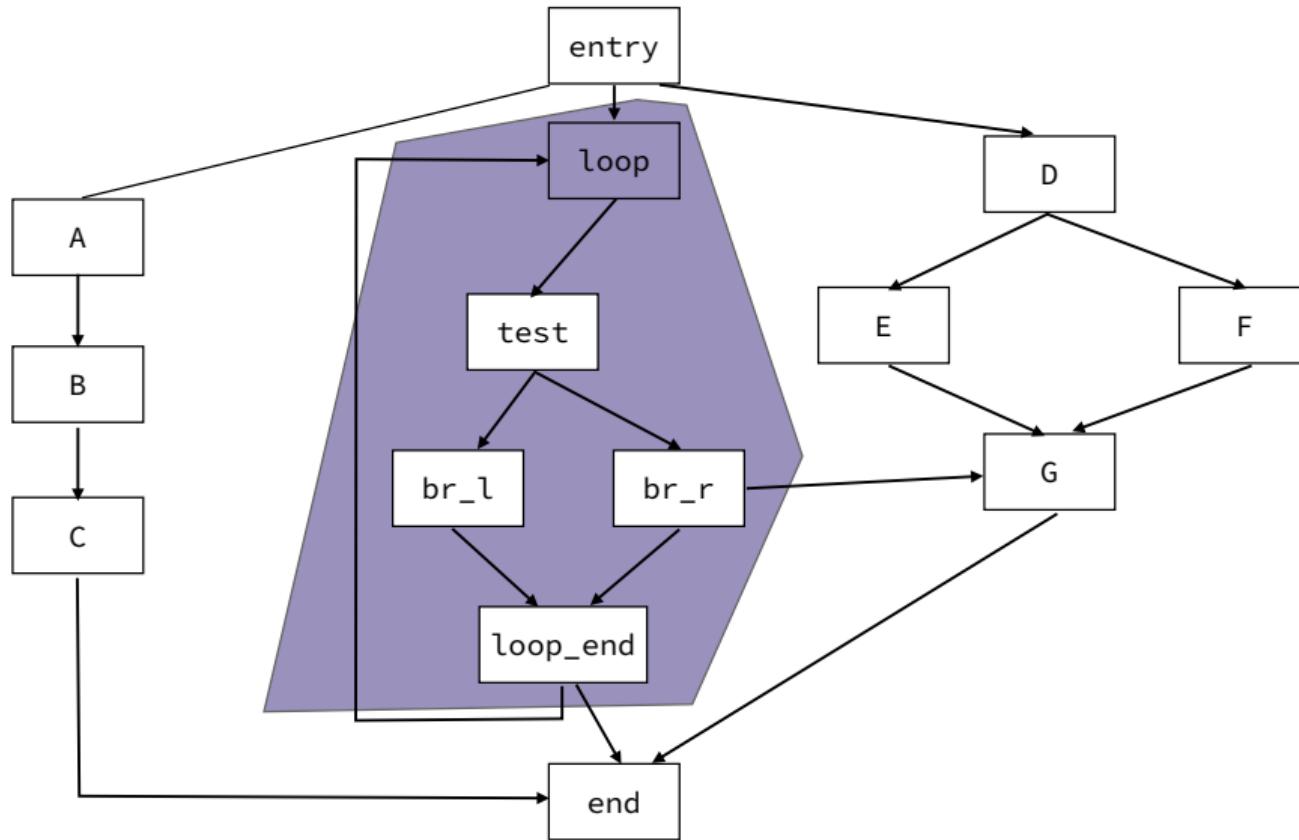
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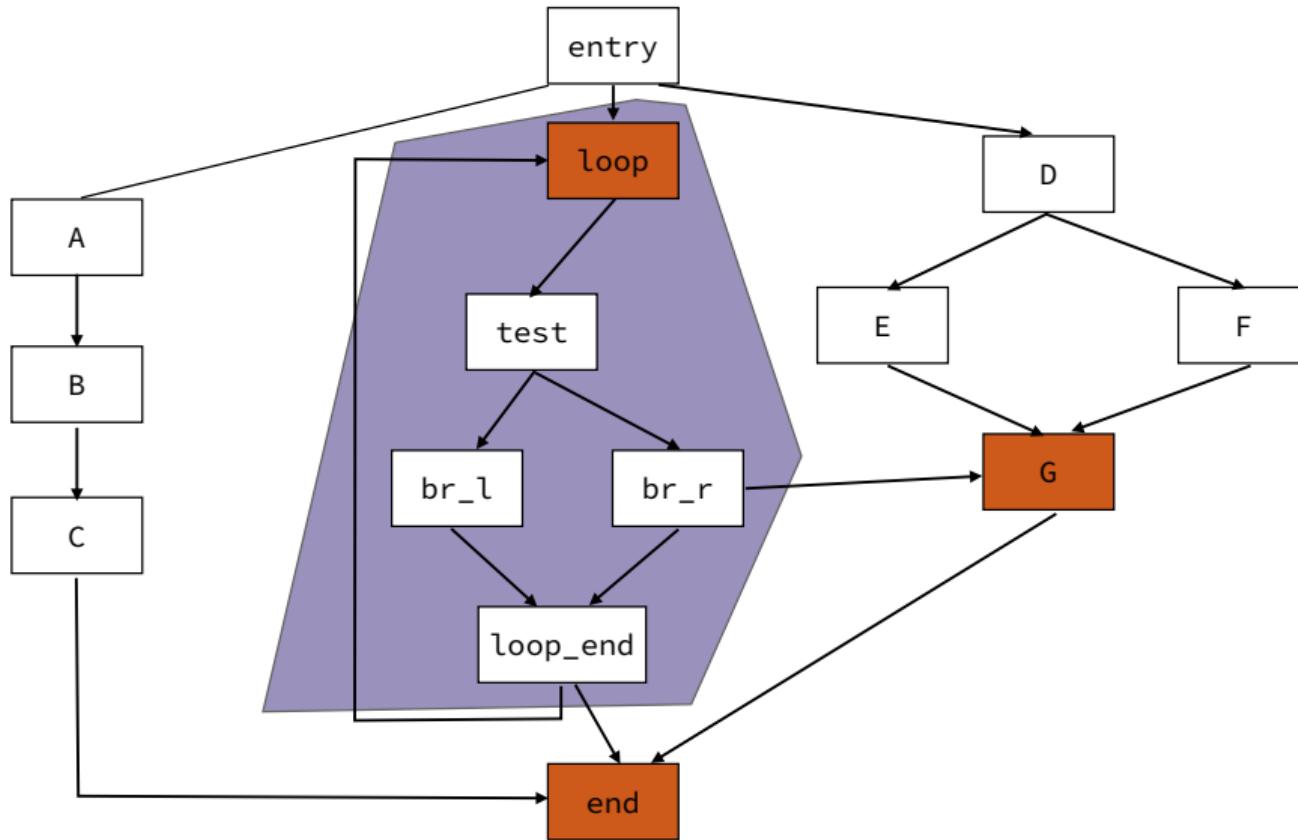
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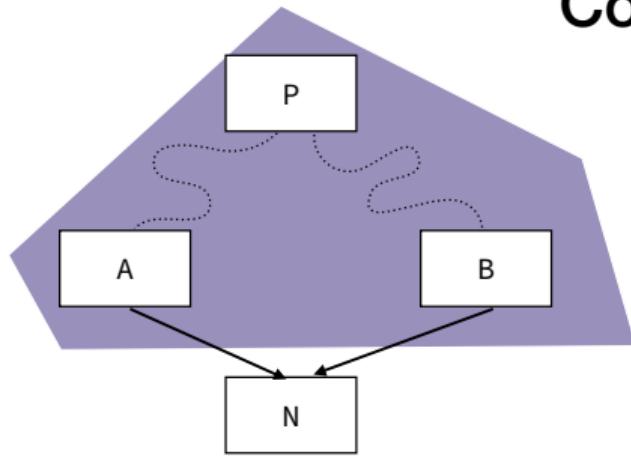
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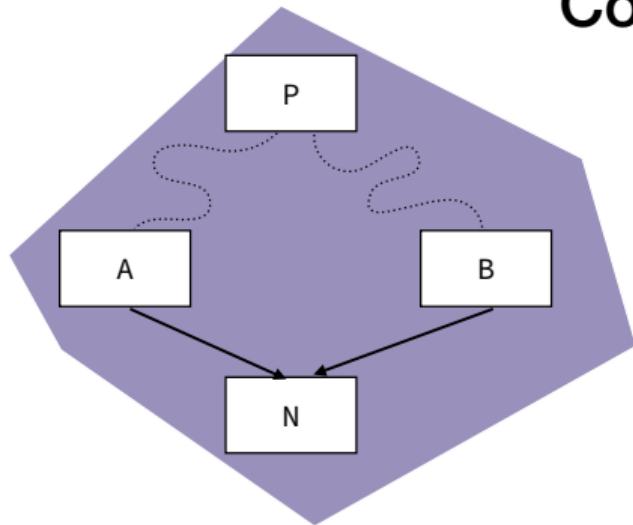


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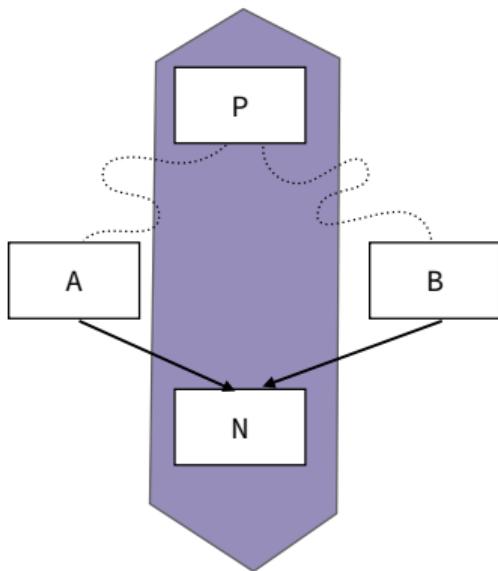
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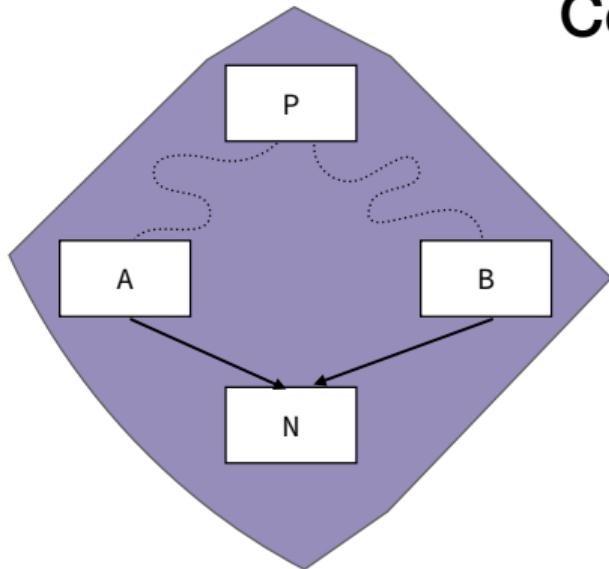
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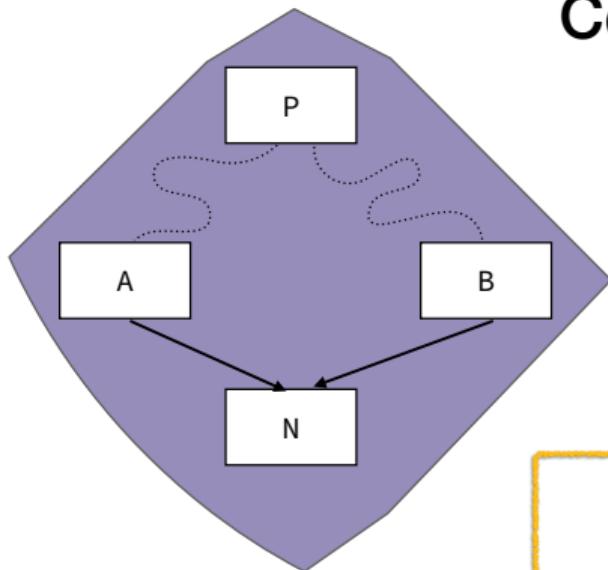
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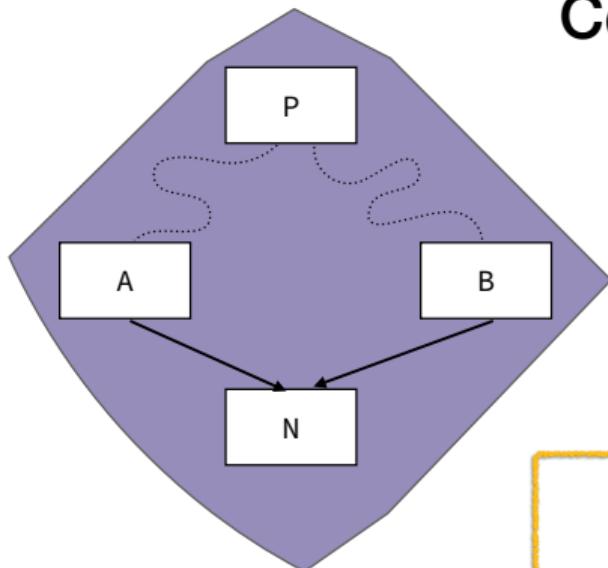
Let $D[n]$ be the set of nodes dominating n

$$D[\text{entry}] \triangleq \{\text{entry}\}$$

$$D[n] \triangleq \{n\} \cup \left(\bigcap_{p \in \text{pred}(n)} D[p] \right)$$

As is traditional, this system of equations
can be solved by iteration¹

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Complexity?

¹: For a more efficient algorithm, see Lengauer and Tarjan's 1979

“A fast algorithm for finding dominators in a flowgraph”

Computing the dominance frontier

G : ambient cfg

DT: Dominance Tree of G

DF: map from nodes to sets of nodes

`computeDF(n) ::=`

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“Obvious”, immediate frontier

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`S <- {y | y successor of n in G but not in DT}`

`for c in children(n) in DT:`

`computeDF(c)`

`for each w in DF[c]:`

`if n does not dominate w:`

`S <- S U {w}`

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The rest of the frontier is inherited
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`computeDF(n) ::=`

$S \leftarrow \{y \mid y \text{ successor of } n \text{ in } G \text{ but not in } DT\}$

for c in $\text{children}(n)$ in DT :

`computeDF(c)`

 for each w in $DF[c]$:

 if n does not dominate w :

$S \leftarrow S \cup \{w\}$

$DF[n] \leftarrow S$

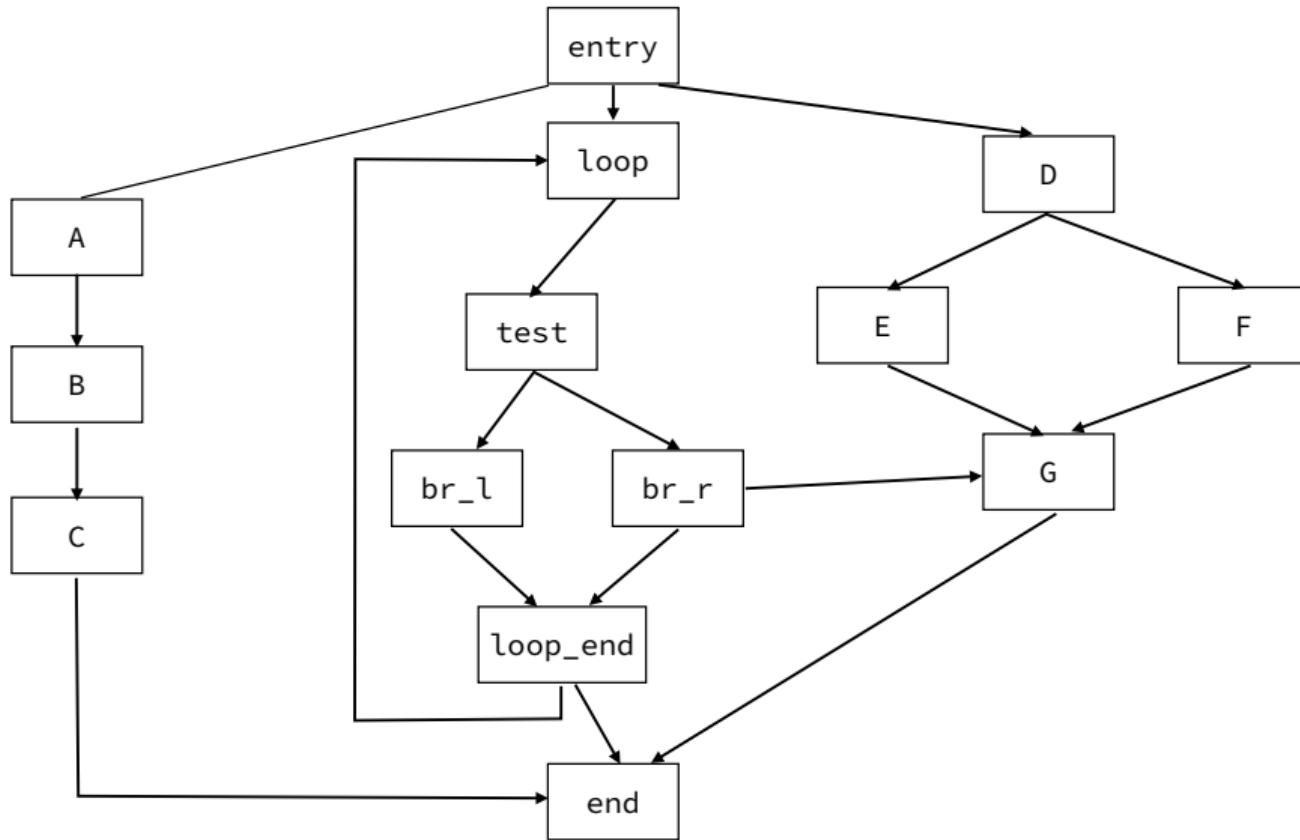
`DF ::= computeDF(entry)`

“Obvious”, immediate frontier

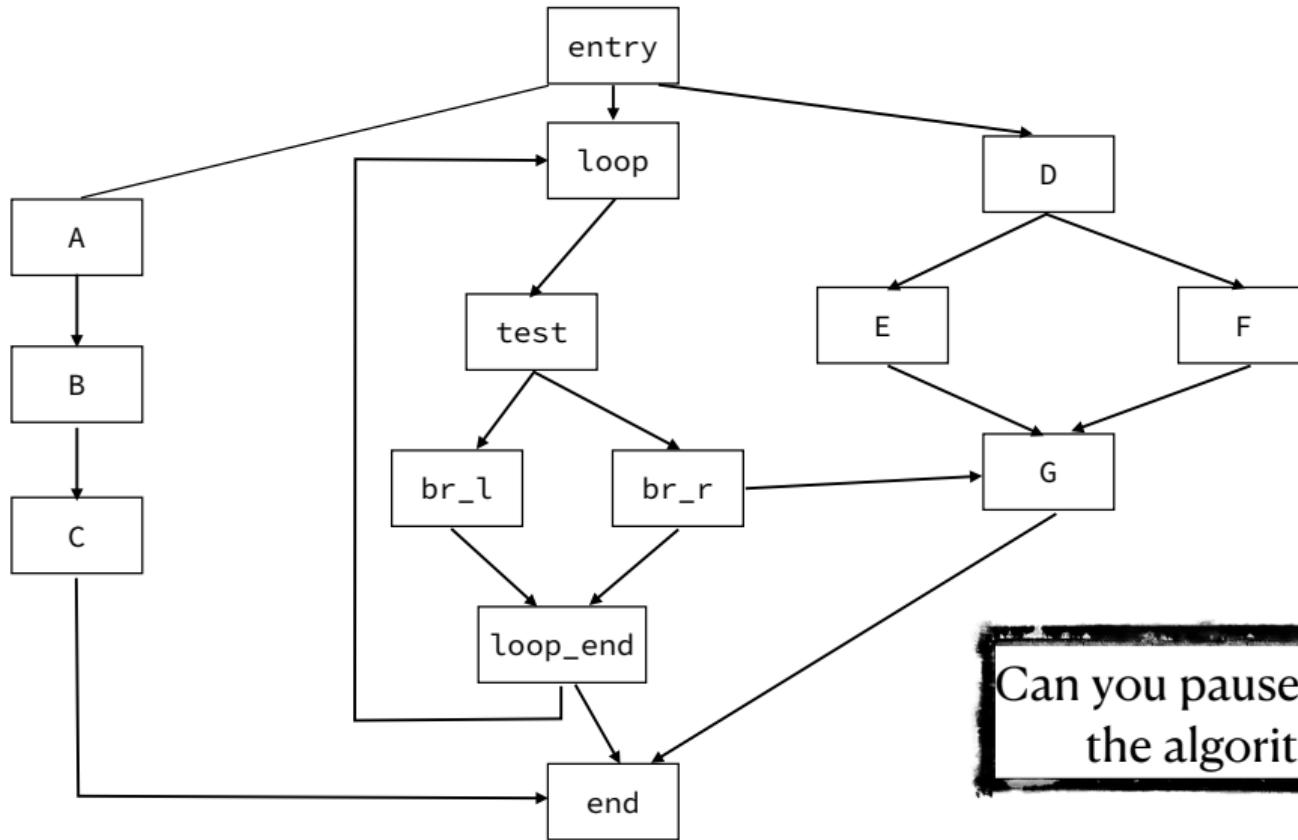
The rest of the frontier is inherited
from the other children

We kickstart the pass from the entry

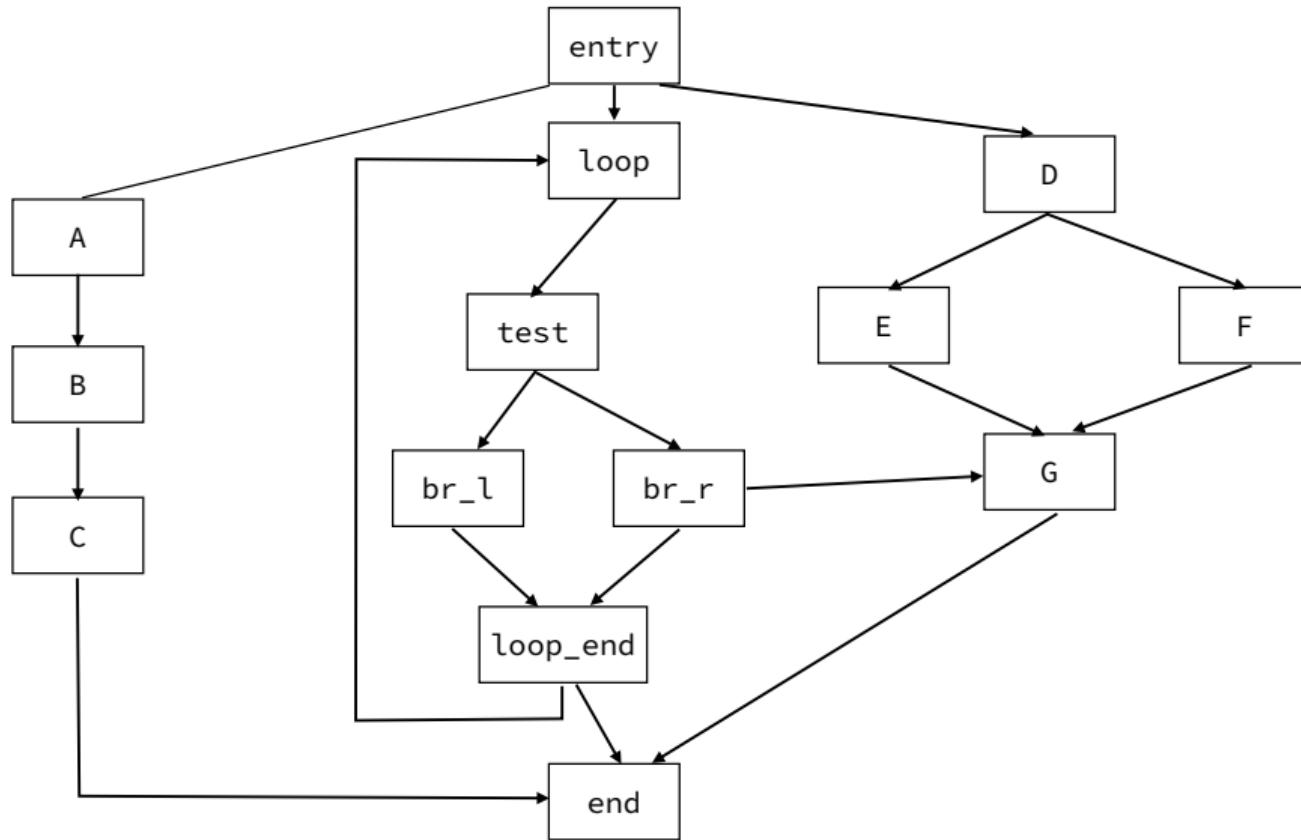
Computing the dominance frontier



Computing the dominance frontier



Computing the dominance frontier

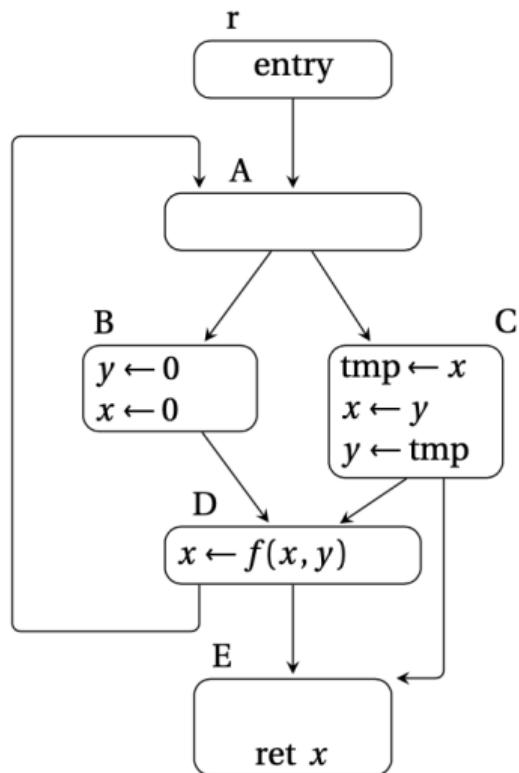


Taking stock

We want to convert a cfg to SSA-form

- The key difficulty is to figure out *where* exactly Φ -nodes are needed
- We observed *the dominance frontier* of a node seems to be the right notion
- We saw how to construct the dominance frontier,
based on the construction of *the dominance tree*

We can now turn to the construction!



Inserting Φ -nodes

Insert-phi ::=

```
for x in Vars:  
    for d in Defs(x):  
        for b in DF(d):  
            if there are no  $\Phi$ -node associated to x in b:  
                add one such  $\Phi$ -node  
                add b to Defs(x)
```

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Blocks containing at least one def-site of x

A Φ -node is a new definition site!

Inserting Φ -nodes

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```

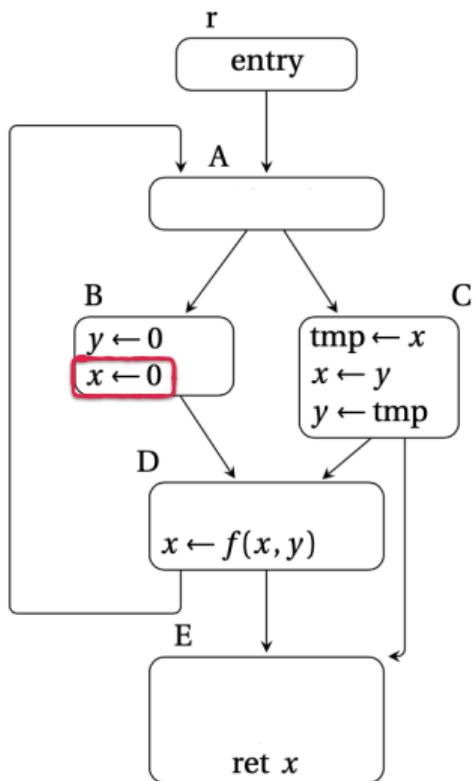
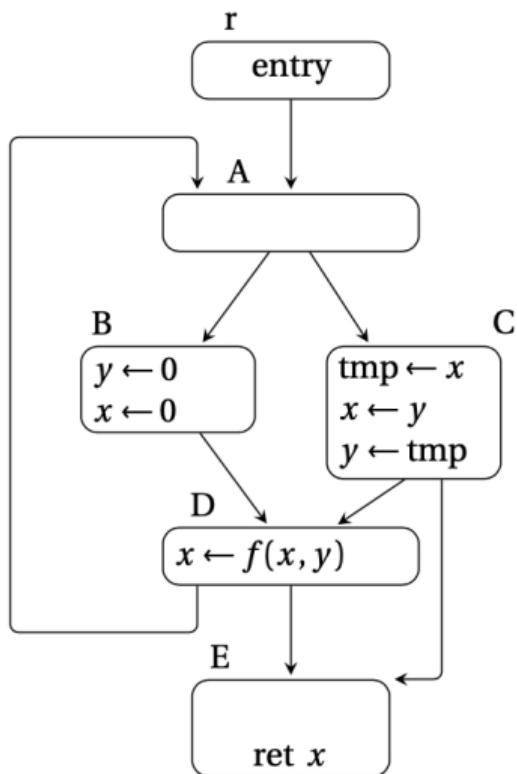
```
    for  $x$  in Vars:  
        for  $d$  in Defs( $x$ ):  
            for  $b$  in DF( $d$ ):  
                if there are no  $\Phi$ -node associated to  $x$  in  $b$ :  
                    add one such  $\Phi$ -node  
                    add  $b$  to Defs( $x$ )
```

We have not yet renamed: x can have several def-sites

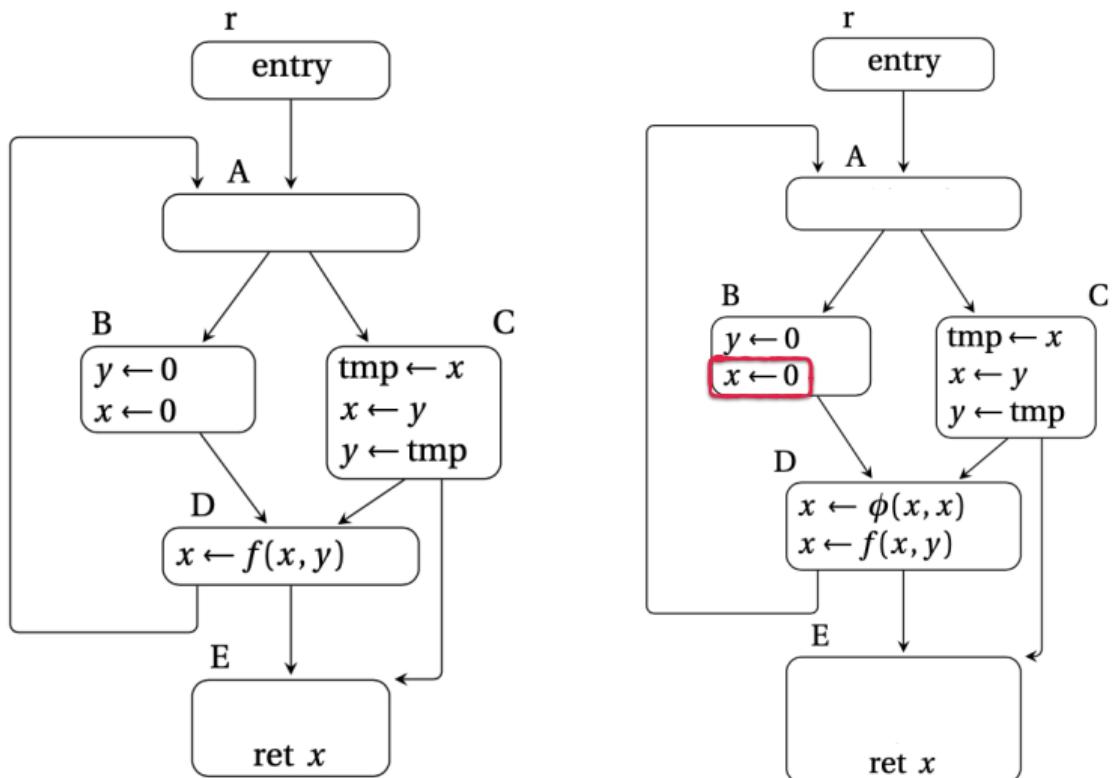
Blocks containing at least one def-site of x

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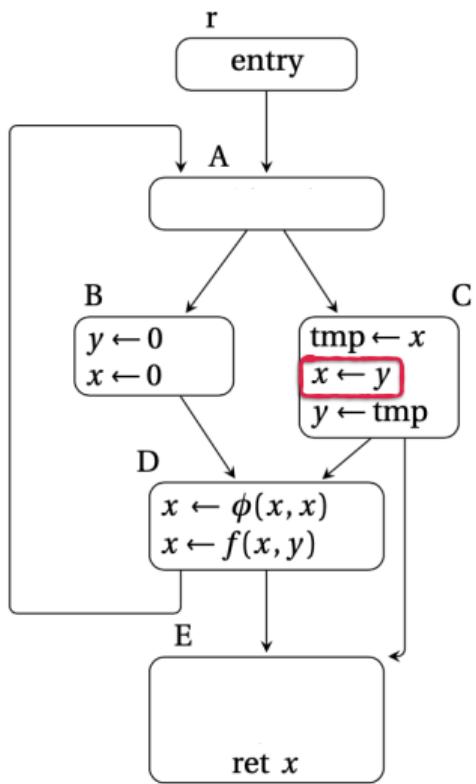
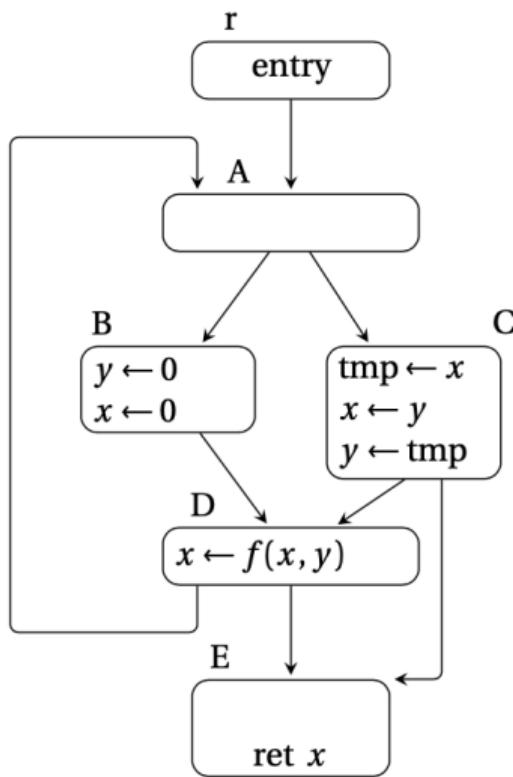
Convince yourself it converges!



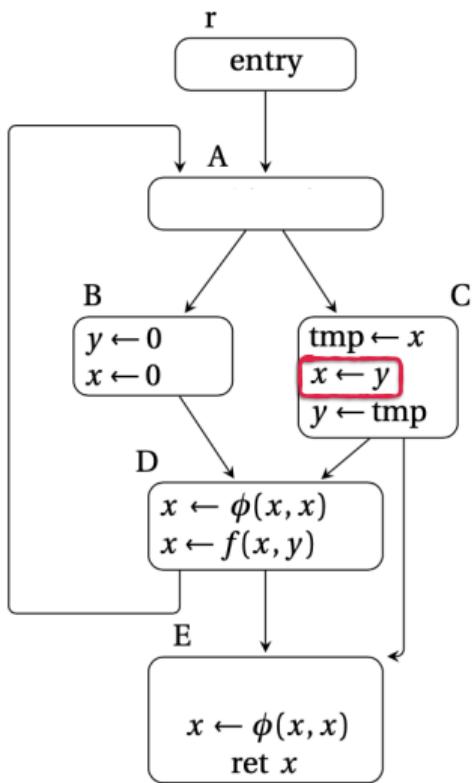
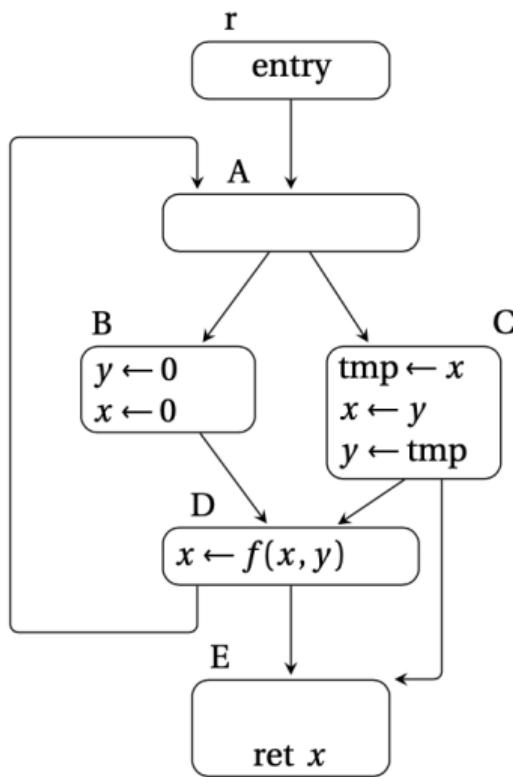
Exemple taken from the SSA book



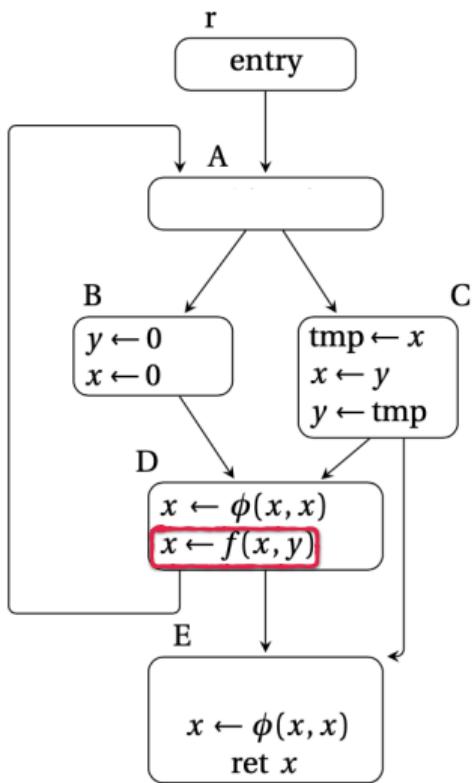
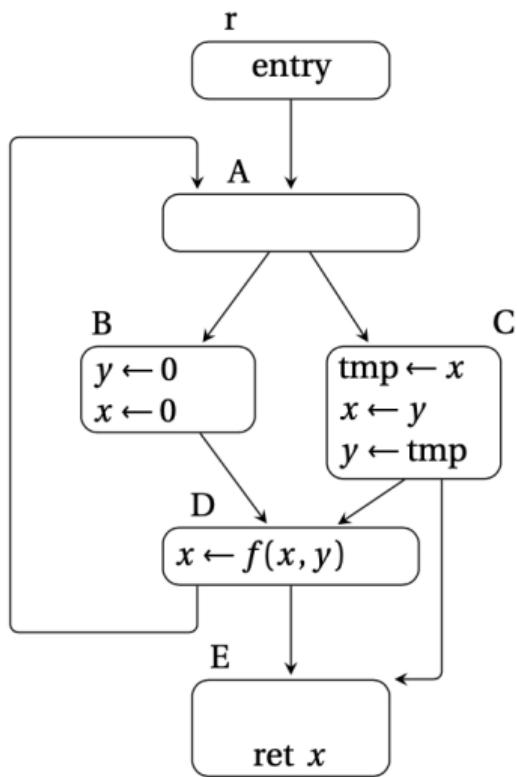
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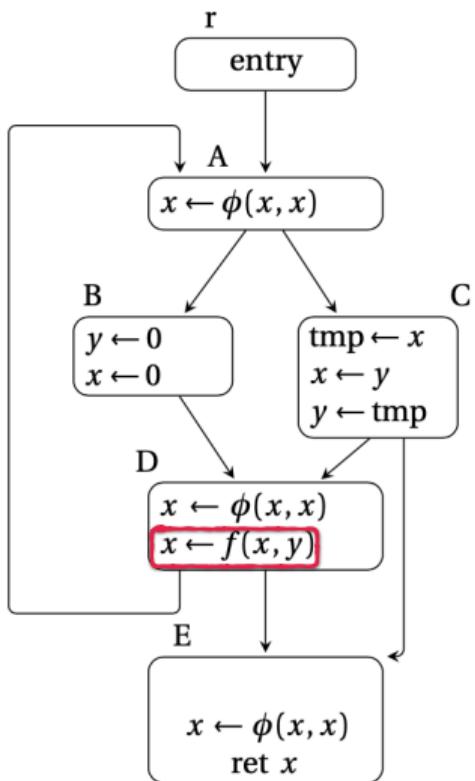
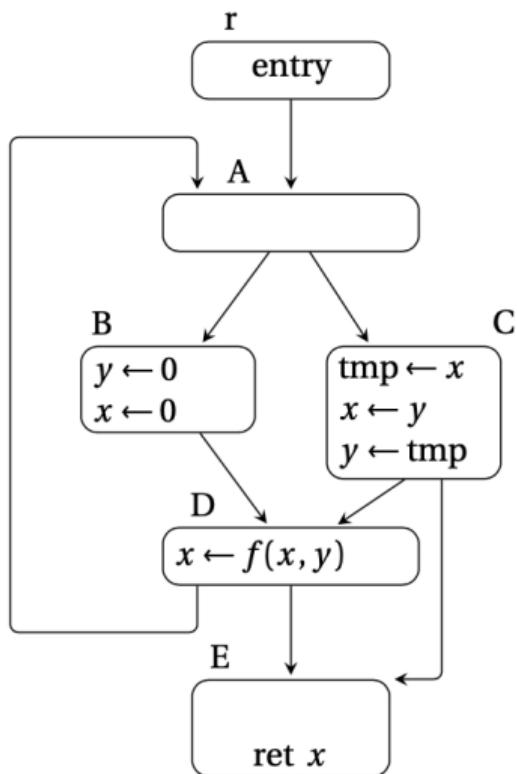
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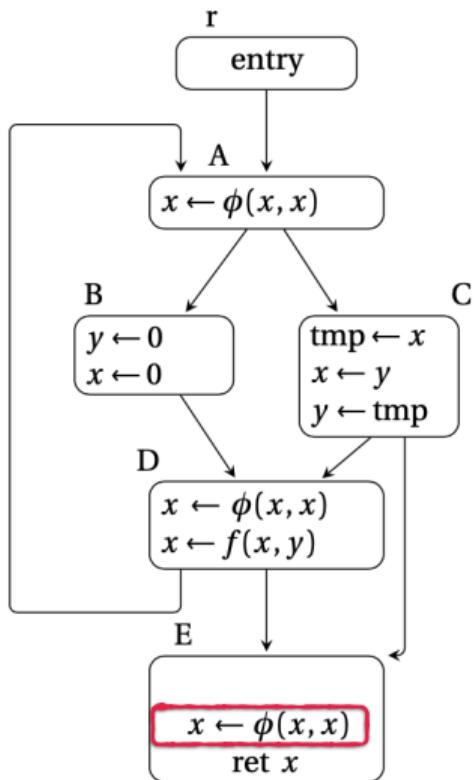
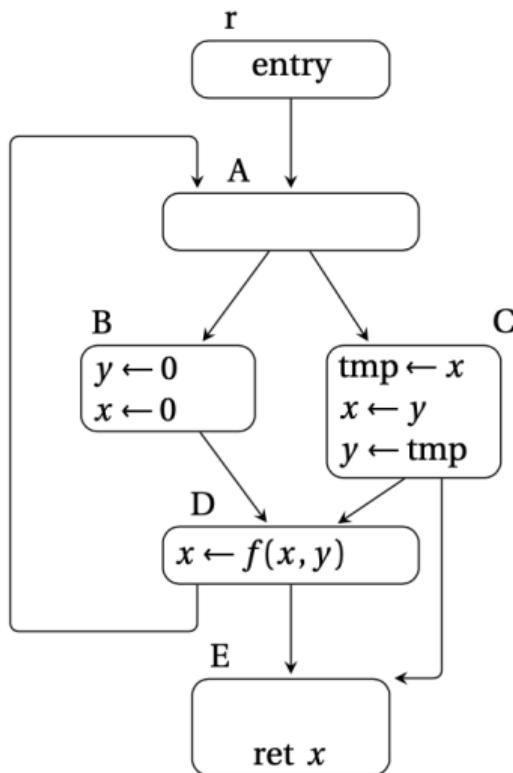


Exemple taken from the SSA book

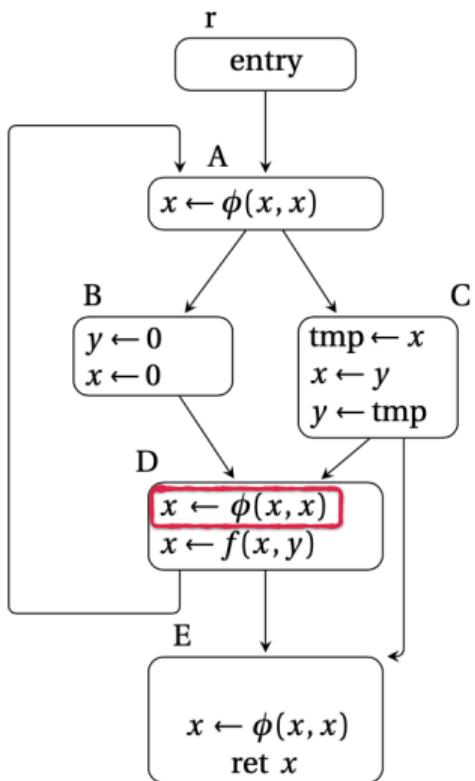
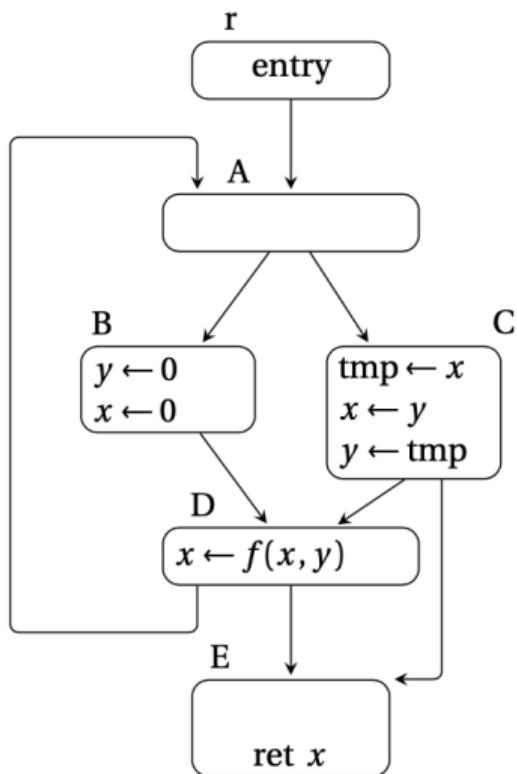


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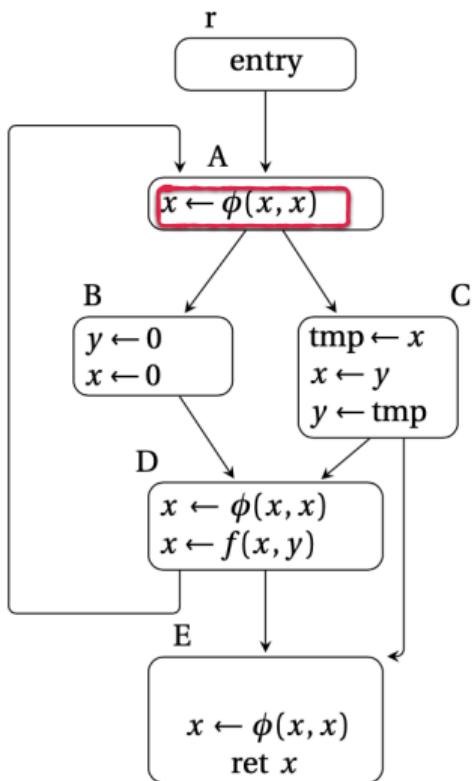
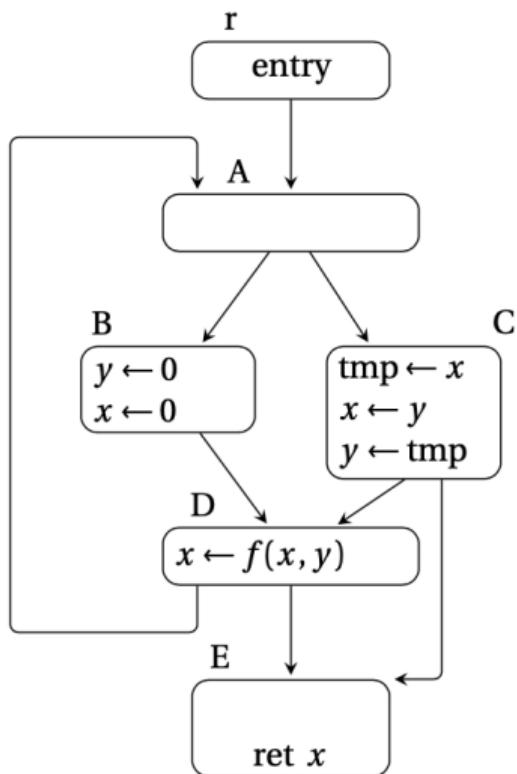


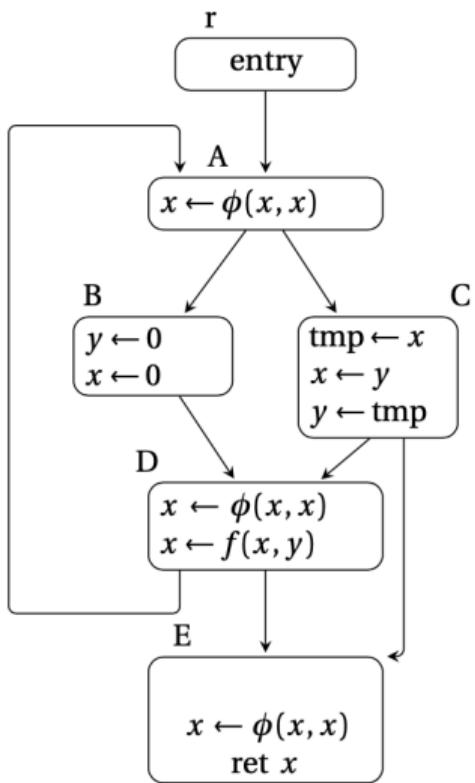
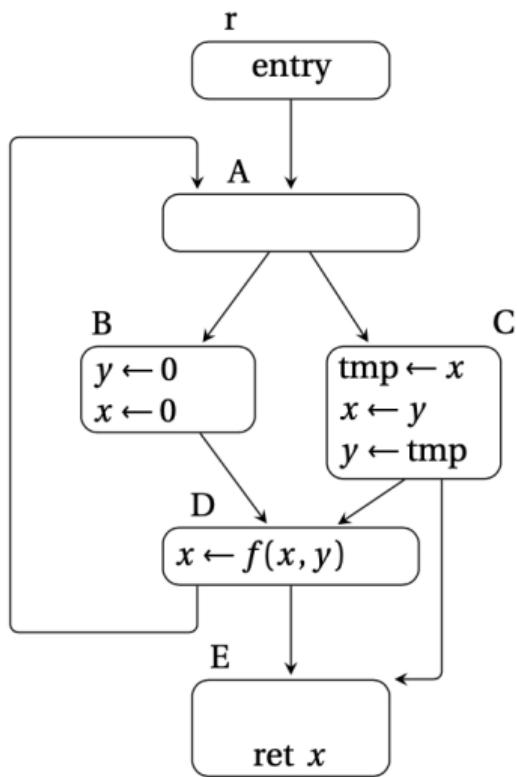


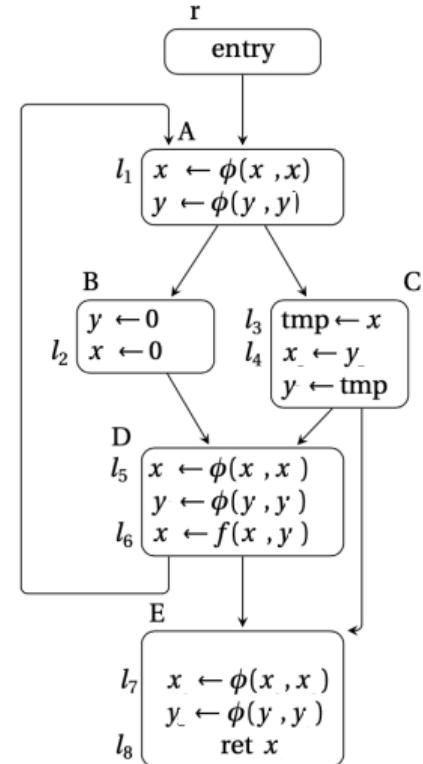
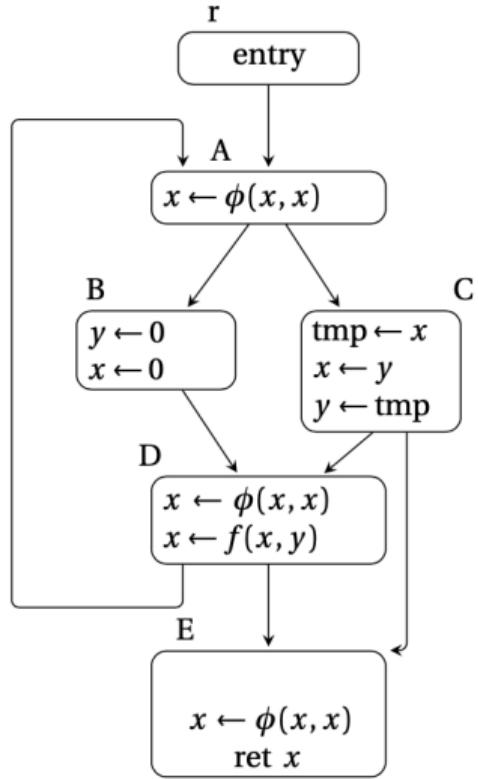
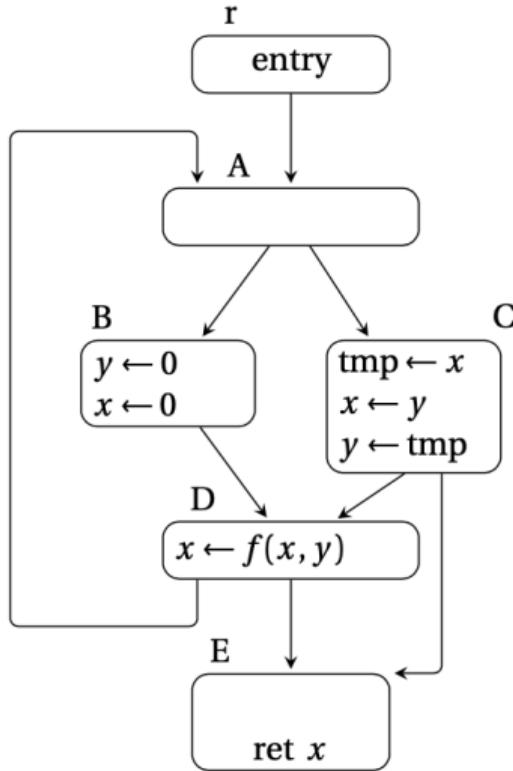
Exemple taken from the SSA book



Exemple taken from the SSA book







Renaming variables

`stack[x]` : for each variable, we maintain a stack of names ("x_i")

`rename_aux(block) ::=`

`rename() ::= rename_aux(entry)`

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rename_aux(block) ::=  
  for ins := y <- e in instr(block):  
    for each var x in e, replace x by stack[x]  
    generate a fresh name y' for y  
    push y' on top of stack[y]
```

```
rename() ::= rename_aux(entry)
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  for each s successor of block:  
    for each Φ-node p of s:  
      if x is read coming from block, replace x with stack[x]
```

```
rename() ::= rename_aux(entry)
```

Renaming variables

`stack[x]` : for each variable, we maintain a stack of names ("x_i")

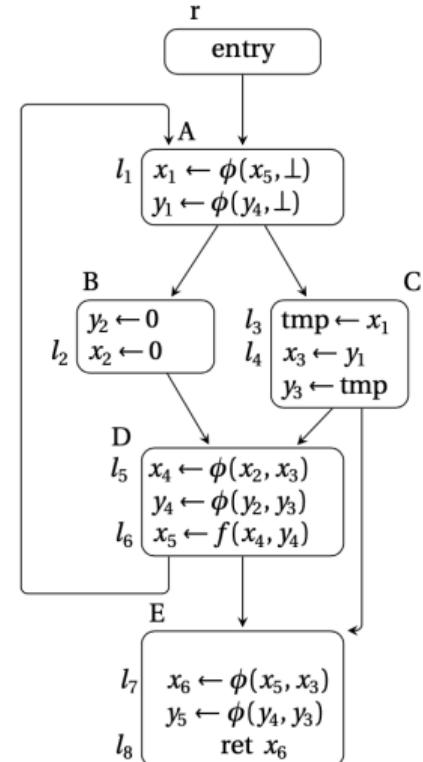
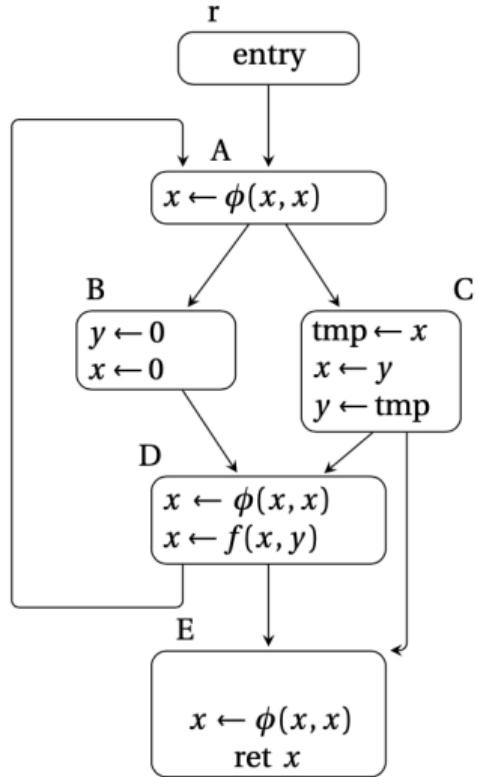
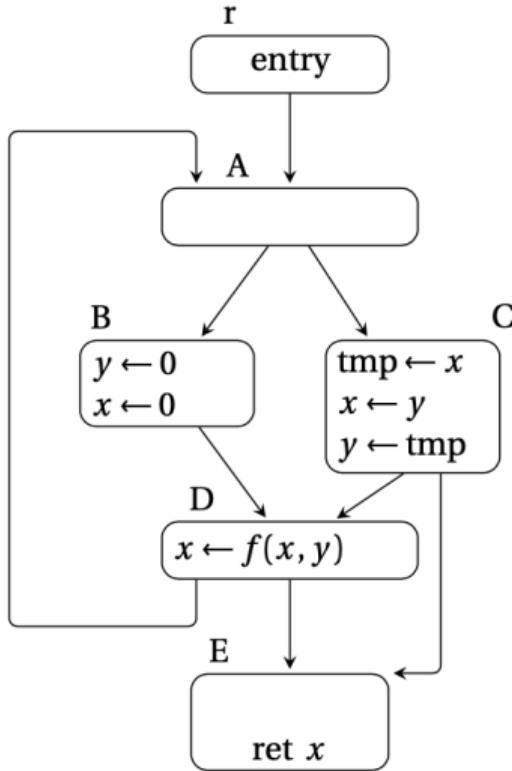
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      if x is read coming from block, replace x with stack[x]  
  for each successor b of block in the DT:  
    rename_aux(b)
```

`rename()` ::= `rename_aux(entry)`

Renaming variables

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      if x is read coming from block, replace x with stack[x]  
  for each successor b of block in the DT:  
    rename_aux(b)  
  pop from stack all variables introduced in this function call  
  
rename() ::= rename_aux(entry)
```



Exemple taken from the SSA book

Converting out of SSA form

From SSA to machine code

Processors do not support Φ -nodes, we need to compile them away!

From SSA to machine code

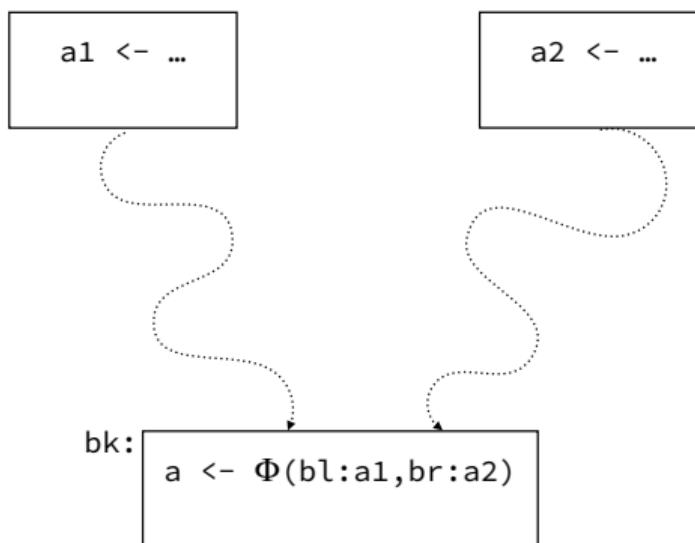
Processors do not support Φ -nodes, we need to compile them away!

bk:

```
a <-  $\Phi(bl:a1, br:a2)$ 
```

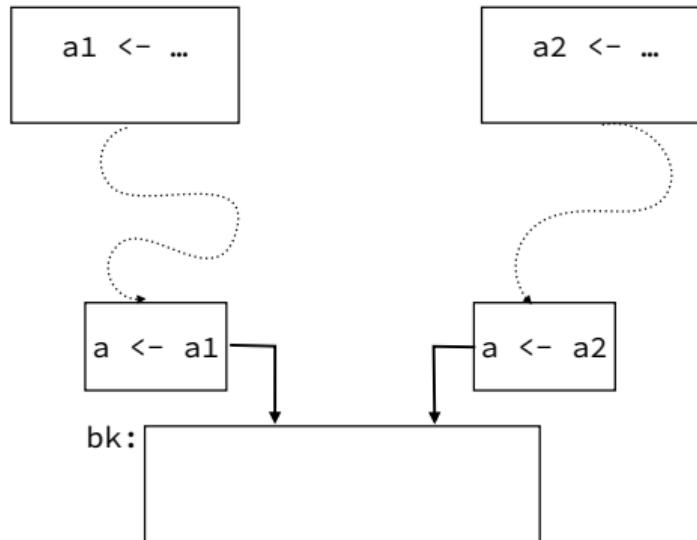
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From SSA to machine code

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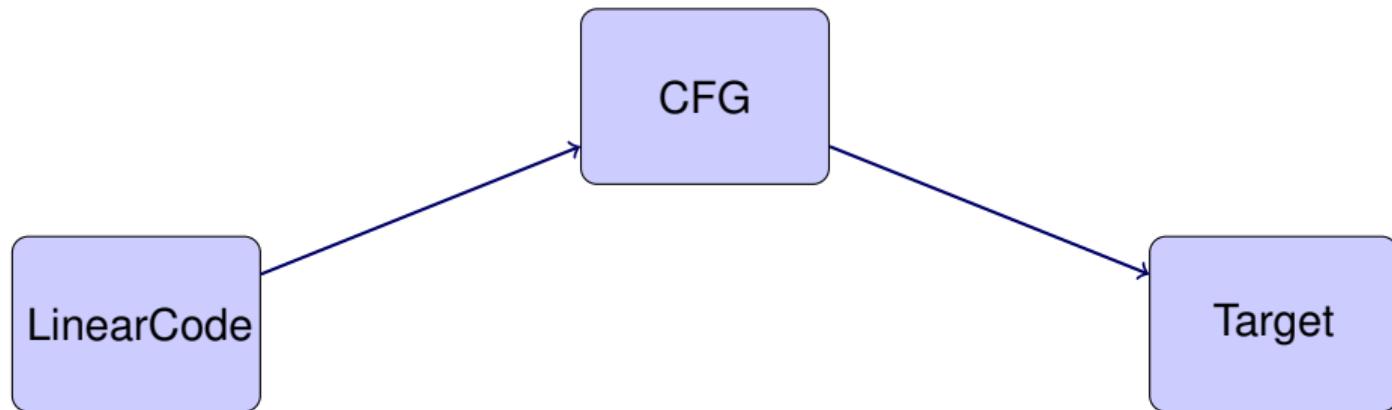


A good register allocator should then take care of eliminating needlessly introduced mov

- 1 SSA Control Flow Graph
- 2 LAB: CFG + SSA
- 3 Exercises

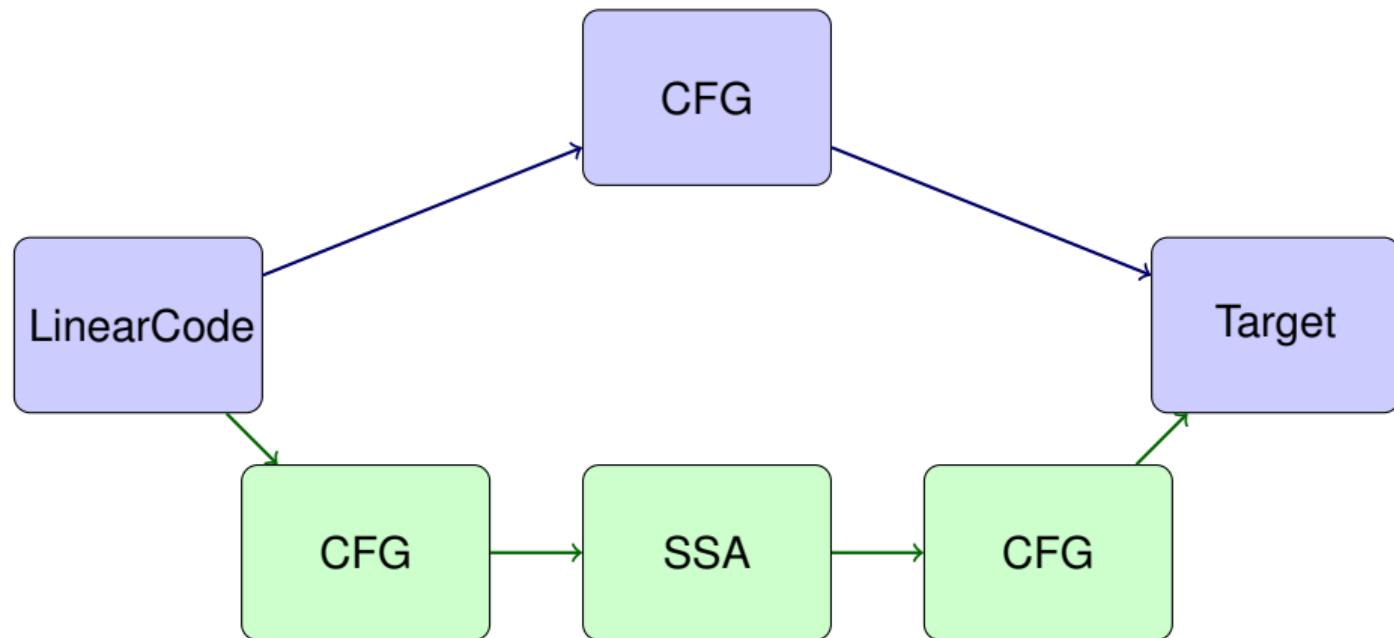
Code Generation

In previous labs



Code Generation

In previous labs



In next labs

Steps

- ① Implement SSA entry (dominance frontier and ϕ -insertion)
- ② Implement SSA exit

- 1 SSA Control Flow Graph
- 2 LAB: CFG + SSA
- 3 Exercises

To SSA and back again

```
i=1; j=1; k=0;  
while (k < 100) {  
    if (j < 20) {  
        j=i;  
        k=k+1;  
    } {  
        j=k;  
        k=k+2;  
    }  
    return j;
```

(Exercise taken from Fernando Pereira)

- ➊ Draw the CFG
- ➋ Compute the Dominance Tree and the Frontier
- ➌ Convert to SSA
- ➍ Convert out of SSA

To SSA and back again

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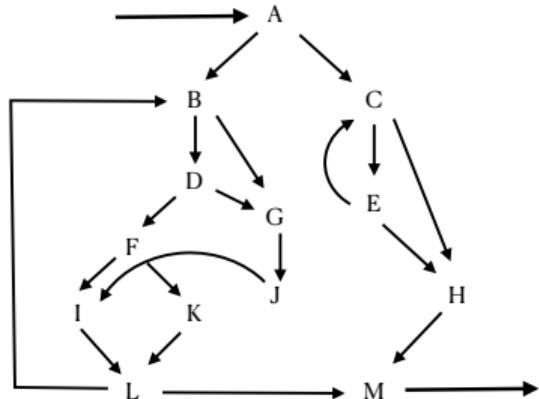
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```

Testing dominance in constant time

We wish to test in constant time whether a given node dominates another. We assume that we have already computed the dominance tree, and allow ourselves to this end a little pre-processing.

Q1. Draw the dominance tree of the graph on the right



Q2. Write an instrumented depth-first traversal labeling each node of the dominance tree with two numbers:

- N: the order in which that node was visited
- A: the maximum N among the node's descendants

Q3. Prove that these annotations can be used to test dominance in constant time.

Summary

1 SSA Control Flow Graph

2 LAB: CFG + SSA

3 Exercises