# Compilation and Program Analysis (#4) : Types, and Typing MiniWhile

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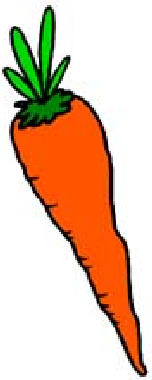
# Typing



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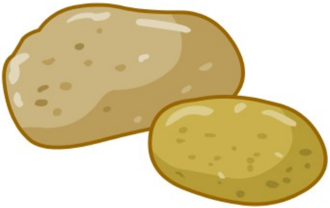
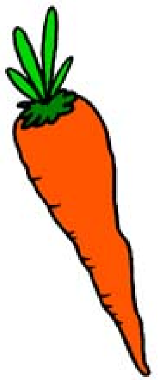


# Typing



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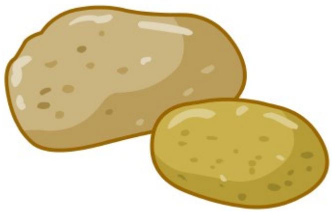

# Typing

If you write: `"5" + 37`

what do you want to obtain

- a compilation error? (OCaml)
- an exec error? (Python)
- the int 42? (Visual Basic, PHP)
- the string `"537"`? (Java)
- anything else?

and what about `37 / "5"` ?

# Typing

When is

```
e1 + e2
```

legal, and what are the semantic actions to perform ?

- Typing: an analysis that gives a type to each subexpression, and reject incoherent programs.

# When

- Dynamic typing (during exec): Lisp, PHP, Python
- Static typing (at compile time): C, Java, OCaml

▶ Here: the second one.

# Slogan

*well typed programs do not go wrong*

# Typing objectives

- Should be **decidable**.
- It should reject programs like `(1 2)` in OCaml, or `1+"toto"` in C before an actual error in the evaluation of the expression: this is **safety**.

  The type system is related to the kind of error to be detected: **operations on basic types** / method invocation (message not understood) / correct synchronisation (e.g. session types) in concurrent programs / ...
- The type system should be expressive enough and not reject too many programs. (**expressivity**)

# Principle

All sub-expressions of the program must be given a type

$$\texttt{fun}\ (x : \texttt{int}) \rightarrow \texttt{let}\ (y : \texttt{int}) = (+ :)(((x : \texttt{int}), (1 : \texttt{int})) : \texttt{int} \times \texttt{int})\ \texttt{in}$$

What does the programmer write?

- The type of all sub-expressions (like above) easy to verify, but tedious for the programmer
- Annotate only variable declarations (Pascal, C, Java, ...)

$$\texttt{fun}\ (x : \texttt{int}) \rightarrow \texttt{let}\ (y : \texttt{int}) = +(x, 1)\ \texttt{in}\ y$$

- Only annotate function parameters

$$\texttt{fun}\ (x : \texttt{int}) \rightarrow \texttt{let}\ y = +(x, 1)\ \texttt{in}\ y$$

- Annotate nothing: complete inference : Ocaml, Haskell, ...

# Properties

- correction: “yes” implies the program is well typed.
- completeness: the converse.

(optional)

- principality : The most general type is computed.

# Typing judgement

We will define how to compute **typing judgements** denoted by:

$$\Gamma \vdash e : \tau$$

and means "in environment $\Gamma$, expression $e$ has type $\tau$"

- $\Gamma$ associates a type $\Gamma(x)$ to all free variables $x$ in $e$.

# Safety = well typed programs do not go wrong

In general a type-safety property looks like this:

**Theorem (Safety)**

*If $\emptyset \vdash e : \tau$, then the reduction of $e$ is infinite or terminates with a value.*

# Typing Safety

In general, a type-safety proof is based on two lemmas:

**Lemme (progression)**

*If* $\emptyset \vdash e : \tau$*, then* $e$ *is a value or there exists* $e'$ *such that* $e \rightarrow e'$*.*

**Lemme (preservation)**

*If* $\emptyset \vdash e : \tau$ *and* $e \rightarrow e'$ *then* $\emptyset \vdash e' : \tau$*.*

This works almost the same for small-step and big-step.

# What is a good output for a type-checker?

- We do not want:

```
failwith "typing error"
```

  the origin of the problem should be clearly stated

- We keep the types for next phases.

In practice

- Input: Trees are decorated by source code lines.
- Output: Trees are decorated by types.

1. Generalities about typing
2. Imperative languages (C, Mini-While)
   - Simple Type Checking for mini-while
   - More advanced typing features
3. Type Safety
4. A bit of implementation

# Mini-While Syntax

Expressions (with boolean expression to make typing interesting):

$$
\begin{array}{rcll}
e & ::= & c & \textit{constant} \\
 & | & x & \textit{variable} \\
 & | & e + e & \textit{addition} \\
 & | & e \times e & \textit{multiplication} \\
 & | & e < e & \textit{boolean expression} \\
 & | & \ldots &
\end{array}
$$

Mini-while:

$$
\begin{array}{rcll}
S(Smt) & ::= & x := expr & \text{assign} \\
 & | & skip & \text{do nothing} \\
 & | & S_1; S_2 & \text{sequence} \\
 & | & \texttt{if } e \texttt{ then } S_1 \texttt{ else } S_2 & \text{test} \\
 & | & \texttt{while } e \texttt{ do } S \texttt{ done} & \text{loop}
\end{array}
$$

# Typing rules for expr

Here types are basic types: Int|Bool

$$\frac{}{\Gamma \vdash x : \Gamma(x)} \qquad \frac{n \in \mathbb{Z}}{\Gamma \vdash n : \texttt{int}}$$ (or $\mathrm{tt}\colon\ \mathrm{bool}, \ldots$)

$$\frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 + e_2 : \texttt{int}} \qquad \frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 < e_2 : \texttt{bool}}$$

$$\frac{\Gamma \vdash e_1 : \texttt{bool} \qquad \Gamma \vdash e_2 : \texttt{bool}}{\Gamma \vdash e_1 \wedge e_2 : \texttt{bool}} \qquad \ldots$$

# Example

**show that** $(x + 42)$ **is well typed in an environment where** $[x \mapsto int]$

# Typing rules for statements: $\Gamma \vdash S$

A statement $S$ is well-typed (there is no type for statements)

**on board!**

# Typing: an example

Considering $\Gamma = \{x \mapsto int\}$, prove that the given sequence of instructions is well typed:

```
x = 3 ;
x = x+9 ;
```

**on board!**

# Problem: how to define $\Gamma$ in mini-while? (1/2)

Possible solution: programs declare variables:

$$
\begin{array}{lcll}
P & ::= & D; S & \text{program} \\
D & ::= & var\ x : \tau \mid D; D & \text{Variable declaration}
\end{array}
$$

Suppose the operational semantics discard the variable declaration for now but $\Gamma$ is defined according to declarations: for a given program $D; S$: $\Gamma(x) = \tau \iff var\ x : \tau \in D$

# Problem: how to define $\Gamma$ in mini-while? (2/2)

$\Gamma$ can be defined as a typing rule for programs.
From declarations we infer $\Gamma : Var \rightarrow Basetype$ with the two following rules:

$$\frac{}{var\ x : t \rightarrow_d [x \mapsto t]}$$

$$\frac{D_1 \rightarrow_d \Gamma_1 \quad D_2 \rightarrow_d \Gamma_2 \quad Dom(\Gamma_1) \cap Dom(\Gamma_2) = \emptyset}{D1; D_2 \rightarrow_d \Gamma_1 \cup \Gamma_2}$$

Typing of programs can be defined as follows:

$$\frac{D \rightarrow_d \Gamma \quad \Gamma \vdash S}{\emptyset \vdash D; S}$$

# Typing a “runtime configuration”

In mini-while, initial state of big step and almost all states of SOS are not just statements, they are pairs: (statement, store). To reason on types at runtime we first need to type runtime configurations:

**Definition (Configuration typing)**

$$\Gamma \vdash (S, \sigma) \iff (\Gamma \vdash S \wedge \forall x. \emptyset \vdash \sigma(x) : \tau \iff \Gamma(x) = \tau)$$

Notes:

- $\sigma(x)$ is a value: no $\Gamma$ is needed to type it
- last part somehow says $\Gamma$ and $\sigma$ agree on the type of variables.

# Example continued

**What is the full program corresponding to the previous example?**

```
x = 3 ;
x = x+9 ;
```

**What is the initial $\sigma$ for this program? Does it agree with $\Gamma$?**

see later ...

# Hybrid expressions

What if we have `1.2 + 42` ?

- reject?
- compute a float!

▶ This is **type coercion**. We will see how to implement it during a lab.

▶ It requires a very local form of type inference.

# More complex expressions

What if we have types `pointer of bool` or `array of int`? We might want to check equivalence (for addition ...).

▶ This is called **structural equivalence** (see Dragon Book, "type equivalence"). This is solved by a basic graph traversal checking that each element are equivalent/compatible.

# Sub-typing

- A type can be more precise than another one, e.g.

$$int <: num$$

- Need additional rule to use sub-typing:

$$\frac{e : \tau \qquad \tau <: \tau'}{e : \tau'}$$

- Sometimes, rule to compose sub-types, e.g. functions or parametric types

$$\frac{e : List[\tau] \qquad \tau <: \tau'}{e : List[\tau']}$$

Note: subtyping is heavily used in OOP

# One-step type safety

**Theorem (Type correctness for big-step semantics)**

*If* $\emptyset \vdash e : \tau$ *and* $e \longrightarrow v$ *then the value* $v$ *is of the right type* $\emptyset \vdash v : \tau$*.*

Another one-step reduction is expression evaluation.

Suppose for now $\Gamma$ and $\sigma$ agree on types and we prove correctness of expression types as follows:

**Prove the following theorem (variant of type correctness)**

*Suppose* $\forall x \in vars(e).\, \emptyset \vdash \sigma(x) : \tau \iff \Gamma(x) = \tau$

*prove type correctness for* $Val(e, \sigma)$*, i.e.:*

$\Gamma \vdash e : \tau \implies \emptyset \vdash Val(e, \sigma) : \tau$

# Typing recap 1/2

$$\begin{array}{lcll} P & ::= & D; S & \text{program} \\ D & ::= & var\ x : t & \text{type declaration} \end{array}$$

From declarations we infer $\Gamma : Var \to Basetype$ with the two following rules:

$$\frac{}{var\ x : t \to_d [x \mapsto t]}$$

$$\frac{D_1 \to_d \Gamma_1 \quad D_2 \to_d \Gamma_2 \quad Dom(\Gamma_1) \cap Dom(\Gamma_2) = \emptyset}{D1; D_2 \to_d \Gamma_1 \cup \Gamma_2}$$

# Typing recap 2/2

Then a typing judgment for expressions is $\Gamma \vdash e : \tau \in Basetype$.
Statements have no type and judgement is: $\Gamma \vdash S$.

$$\frac{D \rightarrow_d \Gamma \quad \Gamma \vdash S}{\emptyset \vdash D; S} \qquad \frac{\Gamma \vdash e_1 : \texttt{int} \quad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 + e_2 : \texttt{int}} \qquad \Gamma \vdash x : \Gamma(x)$$

$$\frac{c \in \mathbf{Z}}{c : int} \qquad \frac{b \in \mathbb{B}}{c : bool} \qquad \frac{\Gamma \vdash S_1 \quad \Gamma \vdash S_2}{\Gamma \vdash S_1; S_2} \qquad \frac{\Gamma \vdash x : \tau \quad \Gamma \vdash e : \tau}{\Gamma \vdash x := e}$$

$$\frac{\Gamma \vdash e : \texttt{bool} \quad \Gamma \vdash S_1 \quad \Gamma \vdash S_2}{\Gamma \vdash \texttt{if } e \texttt{ then } S_1 \texttt{ else } S_2} \qquad \frac{\Gamma \vdash e : \texttt{bool} \quad \Gamma \vdash S}{\Gamma \vdash \texttt{while } e \texttt{ do } S \texttt{ done}}$$

Typing configurations:

$$\Gamma \vdash (S, \sigma) \iff (\Gamma \vdash S \wedge \forall x. \emptyset \vdash \sigma(x) : \tau \iff \Gamma(x) = \tau)$$

# Before getting into technical proof

What "wrong behaviours" are prevented by our type-system?

▷ We will try to prove this.

What wrong behaviours are still here?

1. Generalities about typing
2. Imperative languages (C, Mini-While)
   - Simple Type Checking for mini-while
   - More advanced typing features
3. Type Safety
   - Type safety for expressions
   - Recall the typing system for mini-while
   - Safety
4. A bit of implementation

# Safety = well typed programs do not go wrong

In case of a <u>small-step semantics</u> the proof that "well typed programs do not go wrong" relies on two lemmas:

**Well-type programs run without error**

## Lemma (progression for mini-while)

*If* $\Gamma \vdash (S, \sigma)$*, then there exists* $S', \sigma'$ *such that* $(S, \sigma) \Rightarrow (S', \sigma')$
*OR there exists* $\sigma'$ *such that* $(S, \sigma) \Rightarrow \sigma'$*.*

Note: $(S, \sigma)$ cannot be a final configuration.

**... and remain well-typed**

## Lemma (preservation)

*If* $\Gamma \vdash (S, \sigma)$ *and* $(S, \sigma) \Rightarrow (S', \sigma')$ *then* $\Gamma \vdash (S', \sigma')$*.*

Note that $\Gamma$ never changes (defined by declarations)

**Proofs!** (recall the property for expression evaluation)

# Initial Configuration

Problem: how do we start execution?

Until now $(S, \emptyset)$ was a good starting configuration.

**But $\Gamma$ and $\emptyset$ do not agree on the typing**

what is a good initial $\sigma$?

Initialisation or consider that $\sigma(x)$ is fine?

**Discussion + refer to practical session.**

# Principle

- Gamma is constructed with lexing information or parsing (variable declaration with types).
- Rules are semantic actions. The semantic actions are responsible for the evaluation order, as well as typing errors.

# Type Checking V1 : Visitor

## MuTypingVisitor.py

```
# now visit expr

def visitAtomExpr(self, ctx):
    return self.visit(ctx.atom())

def visitOrExpr(self, ctx):
    lvaltype = self.visit(ctx.expr(0))
    rvaltype = self.visit(ctx.expr(1))
    if (BaseType.Boolean == lvaltype) and (BaseType.Boolean == rvaltype):
        return BaseType.Boolean
    else:
        self._raise(ctx, 'boolean operands', lvaltype, rvaltype)
```

# In practice for mini-C (lab sessions)

- No annotation is added to the AST (everything is int or bool, no ambiguity)
- We can create associating type to variables, directly from parsing

# Conclusion

We have seen:

- The properties and principle of static typing
- A type system for miniml
- A type system for mini-while
- Type safety and how to prove it — miniml and mini while
- discussion on variable declaration and initialisation

Further discussions not really covered in the course:

- Typing functions (later in the course)
- More complex (i.e. real life) type system: sub-typing, objects, functions and sub-typing
- There exist very rich type systems (research on type system), e.g. behavioural types, ownership types, ...