Time Minimization Fuzzy Transportation Problem

K.Abdul Razak and P.Ramesh

***Abstract*—The method of fuzzy time minimization transportation problem is developed and consists of fuzzy triangular number. It is based on moving the basic feasible solution to optimal solution attained. The numerical example was illustrating the theoretical concept.**

***Index Terms*—Transportation problem, time minimization, feasible solution, optimum solution.**

I. Introduction

The cost limiting transportation problem, prominently known as Hitchock-Koopmans problem, might be communicated as of late, the issue of transportation in least time has been contemplated by Hammer 1969 [1], Gartinkel and Rao 1971 [2], Swarc 1971 [3], and so forth. In this paper, another technique for accomplishing a base time of transportation has been produced which is altogether different from other existing techniques.

In a period limiting transportation issue time grid $[t_{ij}]$ is given where t_{ij} is the season of transporting merchandise from the starting point to the goal. For any given doable arrangement $X = [X_{ij}]$ [4] fulfilling the given state of free market activity, the time of transportation is the most extreme of t_{ij}'s among the cells in which there are positive portions i.e. the time of transportation is $[\max_{ij} t_{ij} : x_{ij} > 0]$. This season of (i, j) transportation stays autonomous of the measure of the item sent in as much as $x_{ij} > 0$.

It is accepted that (i) the bearers have adequate ability to convey products from an inception to the goal in a solitary trek (ii) they begin at the same time from their separate starting points.

For circumstances where diminishing is not limiting the season of transportation might be vital are: 1. In military transportation where in the midst of crisis the season of transportation is of prime thought 2. In the transportation of short-lived merchandise, for example, new leafy foods and so on where a postponement in transportation may result in a bigger misfortune than any cost favourable position achieved by transporting at the most reduced expense. Without a doubt, numerous different circumstances can be thought of where the season of transportation is a pertinent thought. These ideas were talked about in Sharma and Kanti swarup [5] and furthermore created in a limited number of cycles. Ilija Nikolic [6] considers to add up to transportation time problem with respect to the season of the dynamic transportation courses to recognize the strategy for age of the ideal arrangement in chose cases are produced. Parametric technique was acquainted Hadi Basirzadeh [7] with tackle fuzzy transportation problem by utilizing positioning of fuzzy numbers.

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In this paper a few variations of the aggregate fuzzy transportation time problem are planned. Parametric method was used to solve problem by ranking fuzzy number. The calculations created for the assurance of their ideal arrangements are exhibited and executions are shown by methods for a numerical precedent.

II. Formulation

The time minimizing transportation problem is to decide the genuine non-negative factors $x_{ij}, i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$ fulfilling the requirements

$$\begin{aligned} &\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m \\ &\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \ldots, n \\ &\quad x_{ij} \geq 0 \end{aligned} \tag{1}$$

and minimizing $\max_{(i,j)} t_{ij} : x_{ij} > 0$, where a_i and b_j are given non-negative number and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Here the objective function is not a convex function [8].

Suppose t_{ij} the time required for transporting all x_{ij} units utilizing comparing courses (i, j) for all $i \in I$ and $j \in J$. In the writing, rather than the aggregate transportation time, regularly, is watched the "transportation proficiency" and limited as following standard:

$$F(x) = \sum_{i \in I} \sum_{j \in J} t_{ij} x_{ij} \tag{2}$$

In any case, we lean toward that in numerous genuine issues is generally normal to center to minimization just the season of dynamic transportation courses (i, j), as next goal

$$T(x) = \sum_{i \in I} \sum_{j \in J} t_{ij} h_{ij} \tag{3}$$

where h_{ij} as assistant capacity demonstrates dynamic and non dynamic transportation courses (exercises)

$$h_{ij} = \begin{cases} 1, & \text{if } x_{ij} > 0 \\ 0, & \text{if } x_{ij} < 0 \end{cases} \tag{4}$$

Demonstrative and clear case of this sort of aggregate transportation time is in a military issue where is of essential significance to investigate the aggregate time of all methods for transportation which might be presented to the peril ofthe foe assaults. This two sorts of measure of the transportation productivity (2) and (3) will be considered Variant A (straight capacity) and Variant B (nonlinear capacity) of the aggregate time transportation issue separately. On the off chance that various ideal arrangements exist with T^* as insignificant estimation of

(3), it is prescribed to improve an another criteria hold T^*, similar to the transportation proficiency (2), the season of the longest dynamic transportation task, the quantity of units on transportation activity with longest time, the aggregate transportation cost etc [9].

A. Definition

A feasible solution: A set $X = \{x_{ij}\}$ of non-negative numbers satisfying (1) is called a feasible solution [8], [10].

B. A better feasible solution

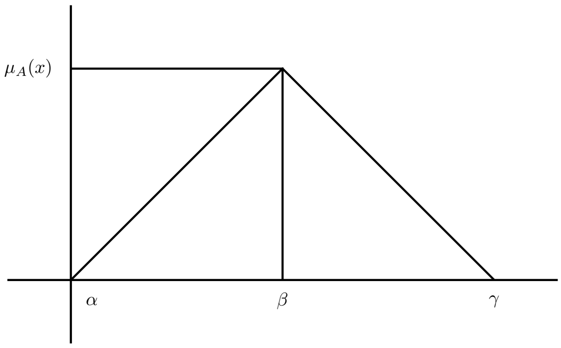
Let $X_1 = \{x_{ij}^1\}$ and $X_2 = \{x_{ij}^2\}$ be two feasible solutions of problem of transportation in minimum time: Let

$$M_1 = \{(i,j) : x_{ij}^1 > 0\},\ M_2 = \{(i,j) : x_{ij}^2 > 0\}$$
$$T_1 = \max_{(i,j)\in M_1} t_{ij}, \qquad T_2 = \max_{(i,j)\in M_2} t_{ij}$$

C. Definition: Triangular fuzzy number

A triangular fuzzy number $\check{A}$ is denoted by (α,β,γ) where α,β,γ are real numbers and its membership function $\mu_A()$ is given below

$$\mu_A(x) = \begin{cases} \dfrac{(x-\alpha)}{(\beta-\alpha)}, & \text{for } \alpha \le x \le \beta \\ \dfrac{(\gamma-x)}{(\gamma-\beta)}, & \text{for } \beta \le x \le \gamma \end{cases} \tag{5}$$



as indicated by the previously mentioned meaning of a triangular fuzzy number, Let $\check{A} = (\underline{A(r)}, \overline{A(r)}), (0 \le r \le 1)$, be a fuzzy number then the esteem $M(\check{A})$ is allocated to $\check{A}$ is determined as pursues

$$M_0^{Tri}(\check{A}) = \frac{1}{2}\int_0^1 \left\{\underline{A(r)} + \overline{A(r)}\right\} dr = \frac{1}{4}(2\beta+\alpha+\gamma)$$

which is very convenient for calculation.

III. The new methodology for fathoming Fuzzy transportation problem

Presently, we present another strategy for taking care of a fuzzy transportation problem where the transportation cost, time, supply and requests are fuzzy numbers. The fuzzy numbers in every issue might be triangular fuzzy numbers. The ideal answer for the fuzzy transportation problem can be acquired as a fresh or fuzzy shape [7].

Stage 1. Ascertain the qualities $M(.)$ for each fuzzy information, the transportation costs $\check{c}_{ij}, \tilde{t}_{ij}, \tilde{a}_i, \tilde{b}_j$ amounts which are fuzzy numbers.

Stage 2. By supplanting $M(\check{c}_{ij}), M(\tilde{a}_i)$ and $M(\tilde{b}_j)$ which are fresh amounts rather than the $\check{c}_{ij}$ and time $\tilde{t}_{ij}$ supply $\tilde{a}_i$ and request $\tilde{b}_j$ amounts which are fuzzy amounts, characterize another fresh transportation problem.

Stage 3. Take care of the new fresh transportation problem, by regular technique, what's more, and acquire the fresh ideal arrangement of the problem. Note if any arrangement of this transportation problem will contain precisely $(m+n-1)$ essential attainable arrangements. The ideal arrangement is in your grasp. In the event that you need fuzzy type of arrangement, follow the next stage.

Stage 4. Decide the areas of non-zero essential attainable arrangements in transportation scene. The premise is established traversing tree, that is there must be no less than one fundamental cell in each line and in every section of the transportation scene. Likewise, the premise, must be a tree, that is the $(m-n-1)$ essential cells ought to not contain a cycle. Consequently, there exist a few lines and segments which have just a single essential cell. By beginning from these cells, ascertain the fuzzy essential arrangements, proceed until acquire $(m+n-1)$ fundamental arrangements.

IV. Numerical Example

The accompanying precedent might be useful to clear up the proposed strategy. Consider the fuzzy transportation problem all data are taken in triangular fuzzy number in this problem. We want to solve the problem and compare the result.

TABLE I

	Bl	B2	B3	B4	B5	Supplies
Al	$(3,5,8)$	$(4,6,7)$	$(5,8,9)$	$(2,4,5)$	$(3,4,7)$	$(16,17,20)$
A2	$(2,4,5)$	$(5,6,8)$	$(3,5,8)$	$(4,5,7)$	$(4,6,7)$	$(12,16,18)$
A3	$(1,4,5)$	$(3,5,6)$	$(4,5,8)$	$(2,3,6)$	$(5,6,8)$	$(17,20,22)$
A4	$(2,3,4)$	$(5,6,8)$	$(3,6,7)$	$(4,5,7)$	$(6,8,9)$	$(14,17,21)$
Demand	$(15,16,17)$	$(10,12,14)$	$(17,18,19)$	$(11,13,15)$	$(10,11,12)$	

$$\tilde{A}_{11} = (3,5,8), M_0^{Tri}(\check{A}_{11}) = \frac{1}{4}(10+3+8) = 5.25$$
$$\tilde{A}_{12} = (4,6,7), M_0^{Tri}(\check{A}_{12}) = \frac{1}{4}(12+4+7) = 5.75$$
$$\ldots\ldots\ldots$$
$$\tilde{A}_{45} = (6,8,9), M_0^{Tri}(\check{A}_{45}) = \frac{1}{4}(16+6+9) = 7.75$$

and fuzzy supplies are

$$\tilde{a}_1 = (16,17,20),\ M_0^{Tri}(\tilde{a}_1) = \frac{1}{4}(34+16+20) = 17.5$$

the same way we find $\tilde{a}_2, \tilde{a}_3$, t and $\tilde{a}_4$
and fuzzy demands are

$$\tilde{b}_1 = \ (15,16,17)\ ,\ M_0^{Tri}(\tilde{b}_1) = \frac{1}{4}(32+15+17) = 16$$

the same way we find $\tilde{b}_2$, $\tilde{b}_3$, $\tilde{b}_4$ and $\tilde{b}_5$
and the total fuzzy supply is $\tilde{S} = (60,72,78)$ and the total fuzzy demand is $\tilde{D} = (64,70,76)$, so

$$\tilde{S} = (60,71,78)\ ,\ M_0^{Tri}(\tilde{S}) = \frac{1}{4}(142+60+78) = 70$$
$$\tilde{D} = (64,70,76)\ ,\ M_0^{Tri}(\tilde{D}) = \frac{1}{4}(140+64+76) = 70$$

Since $M_0^{Tri}(\tilde{S})$ and $M_0^{Tri}(\tilde{D})$ are balanced, so the problem is balanced.

Presently, by utilizing our technique we change the fuzzy transportation into a fresh transportation. In this way, we have the accompanying decreased fuzzy transportation problem:

TABLE II

	Bl	B2	B3	B4	B5	Supplies
Al	5.25	5.75	7.5	3.75	4.5	17.5
A2	3.75	6.25	5.25	5.25	5.75	15.5
A3	3.5	4.75	5.5	3.5	6.25	19.75
A4	3.0	6.25	5.5	5.25	7.75	17.25
Demand	16	12	18	13	11	

As appeared in the Table II, the consequence of defuzzification of the fuzzy numbers acquiring the measures which are not all number. In this way, existing of a non whole number esteem in a transportation pursues this reality that the arrangement of the fresh transportation problem isn't number. We take note of that the arrangement in a standard thing 1564 H. Basirzadeh [7] transportation problem is number, since its network is a unimodular framework. We solve the problem, we obtain the solutions are $x_{12} = 5.25, x_{13} = 1.25, x_{15} = 11, x_{23} = 15.5, x_{32} = 6.75, x_{34} = 13, x_{41} = 16, x_{43} = 1.25$ and the total crisp value of the problem is $x_0 = 302.875$

TABLE III

	Bl	B2	B3	B4	B5	Supplies
Al		5.25	1.25		11	17.5
A2			15.5			15.5
A3		6.75		13		19.75
A4	16		1.25			17.25
Demand	16	12	18	13	11	

Presently, we can come back to introductory issue and acquire the fuzzy arrangement of the fuzzy transportation problem dependent on the information of Table III.

TABLE IV

	Bl	B2	B3	B4	B5	Supplies
Al		$(4, 5, 7)$	$(-2, 2, 3)$		$(8, 11, 14)$	$(16, 17, 20)$
A2			$(12, 16, 18)$			$(12, 16, 18)$
A3		$(4, 7, 9)$		$(11, 12, 17)$		$(17, 20, 22)$
A4	$(13, 16, 19)$		$(-5, 3, 4)$			$(14, 17, 21)$
Demand	$(15, 16, 17)$	$(10, 12, 14)$	$(17, 18, 19)$	$(11, 13, 15)$	$(10, 11, 12)$	

Fuzzy optimal solution of the given problem is $\tilde{X}_{12} = (4,5,7), \tilde{X}_{13} = (-2,2,3), \tilde{X}_{15} = (8,11,14)$, $\tilde{X}_{23} = (12,16,18)$, $\tilde{X}_{32} = (4,7,9), \tilde{X}_{34} = (11,12,17)$, $\tilde{X}_{41} = (13,16,19)$, $\tilde{X}_{43} = (-5,3,4)$

The results are given in both the solutions of crisp value and fuzzy value. From this method we find the unique solution of the fuzzy time minimizing transportation problem. The optimum value of the problem is $x_0 = 302.875$

Conclusion

In this paper, a basic yet successful parametric strategy is acquainted with tackle fuzzy time minimizing transportation problem by utilizing positioning of fuzzy numbers. This technique can be utilized for a wide range of fuzzy transportation problem, regardless of whether triangular and trapezoidal fuzzy numbers with ordinary or irregular information. The new technique is an orderly strategy, simple to apply and can be used for a wide range of transportation problem whether expanded or limited objective works.

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