COLLEGE OF SCIENCE AND TECHNOLOGY
SCHOOL OF SCIENCES
DEPARTMENT OF MATHEMATICS

Assignment I
Partial Differential equations
Doctoral Program in Mathematics

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Question 1:

Consider the hyperbolic equation with variable coefficients

$$\rho\left(x\right)\frac{\partial^2 u\left(t,x\right)}{\partial t^2}=div_x\left(p(x)grad_x u\right)-q(x)u\left(t,x\right) \tag{0.1}$$

in the domain

$$Q=\{(x,t):x\in\Omega\subset\mathbb{R}^n,t>0\}$$

with initial conditions

$$u\left(t,x\right)\Big|_{t=0}=\varphi(x),\frac{\partial u}{\partial t}|_{t=0}=\psi\left(x\right)$$

(0.2)

and Boundary conditions

$$\alpha\left(x\right)u\left(t,x\right)+\beta\left(x\right)\frac{\partial u}{\partial\eta}|_{\partial\Omega}=0(0.3)$$

Functions $p(x),q(x),\rho(x),\alpha(x),\beta(x)$ are smooth enough
(For example: $p((x)\in C^1\left(\Omega\right)\ ,\ \alpha,\beta,q,\rho\in C\left(\Omega\right)$
Let $p(x)\geq p_0>0,\ \rho(x)\geq\rho_0>0,\ q(x)\geq 0,\ \alpha(x)\geq 0,\ \beta(x)\geq 0,\ \alpha^2+\beta^2>0$

Question:

Prove the existence and uniqueness of the solution (0.1),(),(0.3)

Question 2:

Consider the system of partial differential equations PDEs

$$\frac{\partial^{m_i}u_i\left(x,t\right)}{\partial t^{m_i}}=F\left(t,x,u_1,u_2,\cdots,u_N,\cdots,\frac{\partial^{|\alpha|}u_j}{\partial t^{\alpha_0}\cdot\partial x_1^{\alpha_1}\cdots\partial x_n^{\alpha_n}}\right) \tag{0.4}$$

where $x=(x_1,\cdots,x_n)\in\mathbb{R}^n$, $|\alpha|=\alpha_1+\cdots+\alpha_n$, $\alpha\leq n_j$, $\alpha_0<n_j$, $i,j=1,2,\cdots,N$

With the system (0.4), we consider the initial conditions

$$\frac{\partial^k u_i}{\partial t^k}=\varphi_i^k\left(x\right),k=0,1,\cdots,n-1. \tag{0.5}$$

Here $\varphi_i^k\left(x\right)$ are defined in a given domain $\Omega\in\mathbb{R}^n$ on the hyper surface $\{t=t_0\}$

The problem (0.4) , (0.5) is called cauchy problem.

Question:

Prove the existence and uniqueness of the solution of the cauchy problem (0.4) , (0.5) about the point $\{t=t_0\}$ in the class of analytic solution given that the coefficients and initial data are analytic functions.