Probability: Meaning, Types, Terms, Theorems, Formulas and Solved Examples

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What is probability? Probability means the chances of occurrence of an event, mathematically defined as the ratio of the number of favourable cases to the total number of cases. Let's begin with some useful and common terms of Probability to get familiar with the concept of mathematics.

With this article on Probability learn more about probability definition in math with types of probability, various terms involved with formulas, theorems, laws and solved examples.

Learn the various concepts of the Binomial Theorem here.

Probability Terms and Definitions

- **Sample Point:** It denotes one of the possible outcomes. For example, in a deck of cards 4 of hearts is a sample point, similarly, the queen of clubs is a sample point.
- **Experiment or Trial:** A series of trials where the results are always unpredictable. For example, the tossing of a coin, picking a card from a deck of cards, rolling dice, etc.
- **Random Experiment:** This is an action where the result is uncertain, that is an experiment whose outcome cannot be predicted with certainty. For example, the throwing of dice or a coin, or selecting a card from a group of cards, etc.
- **Sample Space:** The set of all probable outcomes is termed sample space, It is denoted by S.
- **Favourable outcomes:** The favourable outcomes as the name signifies are the outcomes in which an individual is interested.
- **Event:** The possible subsets of the sample space linked with a random experiment are termed as the event of an experiment. Events are categorized based on their occurrence as follows;
- **Equally likely events:** Any set of events is said to be equally likely if they have an equal possibility to occur. Consider an example: When a dice is thrown the six(occurrences of 1 to 6) events are equally likely.
- **Exhaustive events:** A set of events is said to be exhaustive if at least one of them surely occurs whenever the experiment is executed. Consider the same above example of tossing the coin, here all the events from 1 to 6 are exhaustive.
- **Mutually exclusive events:** A set of events is said to be mutually exclusive if the occurrence of one of the events excludes/prevents the possibility of the occurrence of any additional event. Mutually exclusive events happen in the identical sample space. Suppose if A and B both are mutually exclusive events in the same sample space then $P(A \cap B) = 0$.

sample spaces. Suppose if A and B both are independent events in different sample spaces then P(A∩B) = P(A)P(B).

- **Dependent Events:** If the occurrence of one event influences or affects the probability of another event then the two events are said to be dependent. Now that we are familiar with the definitions and terms of probability let's head towards the concept of probability, its definition, and other related concepts.
- **Complimentary event:** This type of event denotes the non-happening of events. The complement of an event P is the event, not P (or P').
- **Impossible Event:** The event that cannot happen is called an impossible event. For example, in tossing a coin it is impossible to get both head and tail at an equal time.

Now that we are familiar with the definitions and terms of probability in math, let's head towards the concept of probability, its definition, and other related concepts.

Definition of Probability

The mathematical definition of probability of an event is defined as the ratio of the number of cases in its favor to the total number of cases. Consider S as a sample space and E be an event such that n(S) = n, n(E) = m and each outcome is equally likely. Then; $p\left(E\right)=\frac{n(E)}{n(S)}=\frac{m}{n}=\frac{No.\ of\ favourable\ outcomes\ of\ E}{Total\ number\ of\ possible\ outcomes}$

Check out this article on Number Systems.

Types of Probability

There are three important classes of probabilities:

- Theoretical Probability
- Experimental Probability
- Axiomatic Probability

Let us learn about three of them in the section:

Theoretical Probability

Theoretical Probability is based on the potential chances of something happening. The type of probability is principally based on the logic behind probability. For example, if a coin is tossed, the theoretical probability of getting a head or a tail will be ½ or 0.5.

Experimental Probability

a coin is tossed 8 times and the head is recorded 4 times then, the experimental probability for heads is 4/8 or 1/2.

Axiomatic Probability

In axiomatic probability, a set of commands or assumptions are set which fits all types. These axioms are set by Kolmogorov and are called Kolmogorov's **three axioms.** With the axiomatic method of probability, the chances of existence or non-existence of the events can be quantified.

Also, read about Relations and Functions here.

Complementary Events

The possibility that there will be simply two outcomes that state that an event will occur or will not occur. For example, a person will visit or not visit your residence, getting a government job or not receiving a job, etc. are examples of complementary events. The complement of an event occurring is exactly the reverse of the probability of it not occurring. Some more additional examples are:

- It will rain or not rain tomorrow.
- The trainee will pass the examination or not pass.
- You win the quiz or you won't win.

Probability Density Function

The Probability Density Function (PDF) is the probability function that is outlined for the density of a continuous random variable existing within a certain range of values. The PDF explains the normal distribution and how mean and deviation exist. The standard normal distribution is applied to build a database of statistics, which are frequently used in science to describe the real-valued variables, whose distribution is not identified.

Know more about Sequences and Series here.

Formulas of Probability

In probability examples one thing that helps a lot are the formulas and theorem as probability sometimes gets a little confusing, so next will look at the formulas;

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- If A and B are mutually exclusive events i.e $A \cap B = \phi$, then $P(A \cup B) = P(A) + P(B)$.
- If A, B, and C are any three events then the formula for addition is given by;
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Conditional Probability

Let A and B be any two events correlated with a random experiment. Then, the probability of occurrence of an event A with the condition that B has already occurred such that the probability of B is not equal to zero (P(B) ≠ 0), is called the conditional probability and denoted by $P(A|B)$.
$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ Similarly, $P(B \mid A) = \frac{P(A \cap B)}{P(A)},\ where\ P(A) \neq 0$

Check more topics of Mathematics here.

Key points for Conditional Probability

- Consider If E is the complement of the event E, then $P\left(E\right) = 1 - P(E)$
- For any event A, 0 ≤ P(A) ≤ 1, this is the range of probability.
- If A and B are independent events, then $P(B|A) = P(B) \because P(A \cap B) = P(A) \times P(B)$

Points to remember for Probability

If A and B are any two events in a random experiment then;
P(A ∩ B) = P(A | B) P(B), if P(B) ≠ 0.
P(A ∩ B) = P(B | A) P(A), if P(A) ≠ 0.

Important Probability Theorem and Distributions

Let us check out some of the important probability related theorems like the law of total probability, Bayes theorem, Binomial distribution and more in this section.

Law of Total Probability

Consider E1, E2, ….., En is n mutually exclusive and exhaustive events associated with a random experiment. If A is an event that occurs with E1 or E2 or … or En, then
$P(A) = \sum_{i=1}^{n} P(E_i) \times P(A \mid E_i)$

i.e P(A) = P(E1) × P(A | E1) + P(E2) × P(A | E2) + …. + P(En) × P(A |En).

Learn more about Lines of Regression here.

Baye’s Theorem

Consider E1, E2, ….., En is n mutually exclusive and exhaustive events associated with a random experiment and let S be the sample space. Let A be an event that occurs together with any one of E1 or E2 or … or En such that P(A) ≠ 0. Then

If the probability of success of an event in a particular trial is p and that of the probability of failure is q = 1 – p, then the probability of exactly x successful event in n independent experiments is given by:
$P\left(x:\ n,\ p\right) = {}^{n}C_{x} \times p^{x} \times q^{n-x},\ where\ x = 1, 2, \ldots.., n$

Mean and Variance of Binomial Distribution

The mean and variance of a binomial distribution B(n, p) is given by:Mean: $X = E\left(x\right) = np$
$Variance: Var(x) = npq$
$StandardDeviation = \sqrt{(}Var(x)) = \sqrt{(}npq)$

Check out this article on Arithmetic Mean.

Some Concept of Mean, Median, and Mode

Mean is defined as the average value/expected value. Median is the central part of the data /observation after the arrangement. The mode is related to the highest occurred part in the provided observation. The relationship between mean, median, and mode is given by,

Mode = 3 Median – 2 Mean.

Probability Solved Problems

Learn more about probability with solved probability examples:

Example 1: Two dice are thrown simultaneously. Find the probability that the total of the numbers appearing on them is a prime number?

Solution: The sample space for the two dice thrown simultaneously is :

S = {(a, b)} {where a = 1, 2, ….., 6 and b = 1, 2, ……, 6}

⇒ n (S) = 36

Let A be a subset of S defined as A = {(a, b) | a + b is a prime no.

A = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)}

⇒ n (A) = 15

As we know that,

$P\left(A\right) = \frac{n(A)}{n(S)}$
$P\left(A\right) = \frac{n(A)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

Let p represent the probability of getting 6 = 1 / 6

and q represent the probability of not getting 6, that is = 1 – (1 / 6) = 5 / 6

As we know that, according to binomial distribution:

$$P(x:\ n,\ p) = ^n C_x \times p^x \times q^{n-x},\ where\ x = 1, 2, \ldots.., n$$

Here we get, n = 4, x = 2, p = 1 / 6 and q = 5 / 6.

Hence the probability of getting exactly 2 sixes when a fair dice is rolled 4 times is given by;

$$b(x:\ n,\ p) = ^4 C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^2 = \frac{25}{216}$$

Example 3: In how many ways can the letters of the word PROBABILITY be arranged so that no two vowels are together?

Solution:

Concept:

The number of ways in which r objects can be arranged in n places is $^nP_r = \frac{n!}{(n-r)!}$.

The number of ways in which n objects, out of which p, q, r etc. are of the same type, can be arranged is: $\frac{n!}{p!q!r!\ldots}$.

The number of ways in which r distinct objects can be selected from a group of n distinct objects is $^nC_r = \frac{n!}{r!(n-r)!}$.

n! = 1 × 2 × 3 × ... × n.

0! = 1.

If some objects cannot be together, then first arrange the remaining objects, and then arrange the objects which cannot be together in the empty spaces between them.

Calculation:

The word PROBABILITY has 11 letters out of which 4 (O, A, I, I) are vowels and 7 (P, R, B, B, L, T, Y) are consonants.

Since no two vowels can be together, we will first arrange the remaining 7 consonants.

The consonant can be arranged in $\frac{7!}{2!} = 2520 ways.$

Now, the 4 vowels can be arranged in any of the 8 spaces between the consonants, as illustrated by a _ below:

_ P _ R _ B _ B _ L _ T _ Y _.

∴ The total number of possible arrangements = 2520 × 70 × 12 = 21,16,800.

Example 4: A bag contains 3 red and 2 yellow marbles. A marble is drawn at random. What is the probability of drawing a red marble?

Solution:

There are a total of 5 marbles in the bag.

The total numbers of ways of selecting one marble out = $^5C_1 = 5ways.$

Let E be the event of selecting a red marble.

Since, the bag contains 3 red marbles,

∴ one red marble can be drawn from 3 red = $^3C_1 = 3ways.$

∴ $P(E) = \frac{n(E)}{n(S)} = \frac{3}{5}.$

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Probability FAQs

Q.1 Two dice are thrown simultaneously. Find the probability that the total of the numbers appearing on them is a prime number.
Ans.1

Let A be a subset of S defined as A = {(a, b) | a + b is a prime no.

A = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)}

⇒ n (A) = 15

As we know that,

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

Q.2 If a fair die is rolled 4 times, then find the probability of exactly 2 sixes?

Ans.2

Given, A fair die is rolled 4 times.

Let p represent the probability of getting 6 = 1 / 6

and q represent the probability of not getting 6, that is = 1 – (1 / 6) = 5 / 6

As we know that, according to binomial distribution:

$$P(x:\ n,\ p) =^n C_x \times p^x \times q^{n-x},\ where\ x = 1, 2, \ldots\ldots, n$$

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Hence the probability of getting exactly 2 sixes when a fair dice is rolled 4 times is given by;

$$b(x:\ n,\ p) =^4 C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^2 = \frac{25}{216}$$

Q.3 What does a complimentary event imply?

Ans.3

A complimentary event means the non-happening of an event. If B is an event then its complimentary event is not B (or B').

Q.4 What does an impossible event imply?

Ans.4

As the name signifies, an impossible event is an event that cannot happen. For example in the tossing of a coin getting both head and tail at the same time is not possible.

Q.5 Is there some range for probability?

Ans.5

Yes, probability can only range between 0 to 1