Chapter 1

Revision Notes

1 Fundamental Theorem of Arithmetic

Concepts Covered:
- Fundamental Theorem of Arithmetic
- Prime Factorisation method to find HCF and LCM of given numbers

- **The Fundamental Theorem of Arithmetic:** Every **composite number** can be expressed as a product of primes and this factorisation is unique, apart from the order in which the prime factors occur. The Fundamental Theorem of Arithmetic is also called Unique Factorisation Theorem.

Composite number = Product of prime numbers

[Board 2021]

Composite Numbers: A composite number is a positive integer greater than 1 that has more than two distinct factors, meaning it is divisible by 1, itself and at least one other number.

OR

Any integer greater than 1 can either be a **prime number** or can be written as a unique product of prime numbers. e.g.,

(i) $2 \times 11 = 22$ is the same as $11 \times 2 = 22$.

(ii) 6 can be written as 2×3 or 3×2, where 2 and 3 are prime numbers.

Prime Numbers: Prime numbers are natural numbers greater than 1 that have exactly two distinct factors: 1 and the number itself. This means they cannot be divided evenly by any other number. Examples of prime numbers include 2, 3, 5, 7, and 11.

Example 1

Represent 10,626 as a product of unique prime factors.

Sol. 10,626 can be represented as $2 \times 3 \times 7 \times 11 \times 23$

- By using the Fundamental Theorem of Arithmetic, we can find the HCF and LCM of given numbers (two or more). This method is also called *Prime Factorisation Method*. [Board 2023, 2018]

- **Prime Factorisation Method to find HCF and LCM.**

(i) Find all the prime factors of given numbers.

(ii) HCF of two or more numbers = Product of the smallest power of each common prime factor involved in the numbers.

(iii) LCM of two or more numbers = Product of the greatest power of each prime factor, involved in the numbers.

Fundamental Theorem of Arithmetic

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Example 2

Find the LCM and HCF of 6 and 20 by the prime factorisation method.

Sol. Here, Prime factorisation of $6 = 2^1 \times 3^1$

Prime factorisation of $20 = 2^2 \times 5^1$

$\therefore$ $\text{HCF}(6, 20) = 2^1 = 2$

and $\text{LCM}(6, 20) = 2^2 \times 3^1 \times 5^1 = 4 \times 15 = 60.$

Fundamental Facts

(1) The concept of LCM is important to solve problems related to race tracks, traffic light, etc.

(2) When pairing two objects against each other, the LCM value is useful in optimising the quantities of the given objects..

2 Irrational Numbers

Concepts Covered:
- Irrational Numbers

- **Irrational Numbers:** A number is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. For example, $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$.

- Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

Co-prime Numbers: Co-prime numbers don't need to be prime themselves; they just shouldn't share any prime factors. Any pair of consecutive integers, like 14 and 15, are always co-prime. 1 is co-prime with every number, making it the "universal friend" in the number world.

- The sum or difference of a rational and an irrational number is irrational.

- The product and quotient of a non-zero rational and an irrational number is irrational. [Board 2024, 20, 19, 18]

- **Step-by-Step Proof:** Irrationality of $\sqrt{n}$

Step 1: Assume the opposite

Assume $\sqrt{n}$ is a rational number.

Then it can be written as:

$$\sqrt{n} = \frac{a}{b}$$

where $a, b \in \mathbb{Z}$, $b \neq 0$, and HCF$(a, b) = 1$ (i.e., a and b are co-prime).

Step 2: Remove the square root

Multiply both sides by b and square both sides:

$$\sqrt{n} \cdot b = a \Rightarrow nb^2 = a^2$$

Proof Of Irrationality

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Step 3: Analyze divisibility

From $a^2 = nb^2$,

$\Rightarrow a^2$ is divisible by n,

$\Rightarrow$ So, a is also divisible by n.

Let $a = np$, where $p \in \mathbb{Z}$.

Step 4: Substitute back

Substitute $a = np$ into the equation $a^2 = n^2b^2$:

$$(np)^2 = nb^2 \Rightarrow n^2p^2 = nb^2$$

Step 5: Simplify

Divide both sides by n:

$$np^2 = b^2$$

$\Rightarrow$ This implies b^2 is divisible by n,

$\Rightarrow$ So, b is also divisible by n.

Step 6: Reach a contradiction

Since both a and b are divisible by n, this contradicts the assumption that HCF$(a, b) = 1$.

Step 7: Conclude

Therefore, our assumption is false.

Hence, $\sqrt{n}$ is irrational.

Fundamental Facts

(1) The discovery of irrational numbers is usually attributed to Pythagoras, more specifically to the Pythagorean Hippasus of metapontum who produced a proof of the irrationality of $\sqrt{2}$.

(2) Irrational numbers are numbers that cannot be expressed as the ratio of two whole numbers.

Example 1

Prove that $\sqrt{2}$ is an irrational number.

Sol. Let $\sqrt{2}$ be rational

Then, its simplest form $= \dfrac{p}{q}$

Where p and q are integers having no common factor other than 1, and $q \neq 0$

Now, $\sqrt{2} = \dfrac{p}{q}$

On squaring both sides we get

$$\Rightarrow \qquad 2 = \frac{p^2}{q^2}$$

$$2q^2 = p^2$$

$\Rightarrow$ 2 divides p^2

$\Rightarrow$ 2 divides p

($\because$ 2 is a prime and divides $p^2 \Rightarrow$ 2 divides p)

Let $p = 2r$ for some integer r

Putting $p = 2r$ in (i) we get

$$\Rightarrow \qquad 2q^2 = 4r^2$$

$$\Rightarrow \qquad q^2 = 2r^2$$

$\Rightarrow$ 2 divides p^2 ($\because$ 2 divides $2r^2$)

$\Rightarrow$ 2 divides q

($\because$ 2 is prime and divides $q^2 \Rightarrow$ 2 divides q)

Thus, 2 is a common factor of p and q. But this contradicts the fact that p and q have no common factor other than 1. Thus, contradiction arises by assuming $\sqrt{2}$ is rational.

Hence, $\sqrt{2}$ is irrational.

Chapter 2

Revision Notes

- **Polynomial:** An **Algebraic Expression** in the form of $a_n x^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$, (where n is a whole number and $a_0, a_1, a_2, ..., a_n$ are real numbers) is called a polynomial in one variable x of degree n.

Algebraic Expression: An algebraic expression is a mathematical phrase that combines numbers variables (like x or y) and operators (such as +, –, × or ÷) to represent a value. It does not include an equality or inequality sign, distinguishing it from equations.

- **Value of a Polynomial at a given point:** If $p(x)$ is a polynomial in x and 'α' is any real number, the value obtained by putting $x = \alpha$ in $p(x)$, is called the value of $p(x)$ at $x = \alpha$.

- **Zero of a Polynomial:** A real number k is said to be the zero of a polynomial $p(x)$, if $p(k) = 0$. Geometrically, the zeroes of a polynomial $p(x)$ are precisely the X-coordinates of the points, where the graph of $y = p(x)$ intersects the X-axis. **[Board, 2020]**

  (i) A linear polynomial has only one zero.

  (ii) A quadratic polynomial has at most two zeroes.

  (iii) A cubic polynomial has at most three zeroes.

  (iii) In general, a polynomial of **degree** n has at most n zeroes.

Degree: The degree of a mathematical expression refers to the highest power (exponent) of the variable in a polynomial. The degree helps classify polynomials, such as linear (degree 1), quadratic (degree 2), and cubic (degree 3).

- **Graphs of different types of Polynomials:**

➢ **Linear Polynomial:** The graph of a linear polynomial $p(x) = ax + b$ is a straight line that intersects X-axis at one point only.

➢ **Quadratic Polynomial:**

(i) The graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens upwards, if $a > 0$ and intersects the X-axis at a maximum of two distinct points.

(ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens downwards, if $a < 0$ and intersects X-axis at a maximum of two distinct points.

- **Relationship between the Zeroes and the Coefficients of a Polynomial:**

  (i) Zero of a linear polynomial

$$= \frac{(-1)^1 \text{ Constant term}}{\text{Coefficient of } x}$$

If $ax + b$ is a given linear polynomial, then zero of the linear polynomial is $\frac{-b}{a}$.

  (ii) In a quadratic polynomial, **[Board, 2021]**

Sum of zeroes of a quadratic polynomial

$$= \frac{(-1)^1 \text{ Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes of a quadratic polynomial

$$= \frac{(-1)^2 \text{ Constant term}}{\text{Coefficient of } x^2}$$

Polynomials

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If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then

$\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$, where α and β are zeroes of the quadratic polynomial.

Key Facts

Polynomials are also an essential tool in describing and predicting traffic patterns, so appropriate traffic control measures, such as traffic lights, can be implemented.

MNEMONICS

Concept 1: $\alpha.\beta = \frac{c}{a}$

Mnemonics: Amitabh Bachchan went to Canada by aeroplane.

Interpretations:

Amitabh's A ⇒ Alpha (α)

Bachchan's B ⇒ Beta (β)

Canada's C ⇒ Constant (c)

By for Divide by

Aeroplane's a ⇒ Variable (a).

Example 1

Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

Sol. Given polynomial, $x^2 + 7x + 10$ ...(i)

$\Rightarrow x^2 + 5x + 2x + 10 \Rightarrow x(x + 5) + 2(x + 5) \Rightarrow (x + 2)(x + 5)$

So, the zeroes are –5, –2

Sum of the zeroes is –7 and product of zeroes is 10

On comparing eq. (i) with $ax^2 + bx + c = 0$, we get $a = 1$, $b = 7$ and $c = 10$

Now, sum of the zeroes $= -5 - 2 = -7 = \frac{-b}{a}$

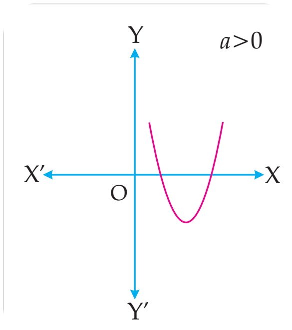
And the product of the zeroes $= (-5) \times (-2) = 10 = \frac{c}{a}$

Hence the relationship is verified.

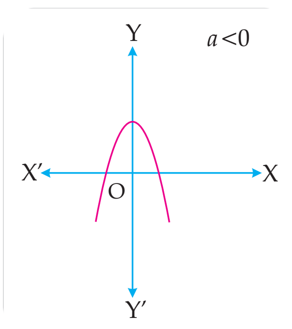
- **Discriminant of a Quadratic Polynomial:** For $f(x) = ax^2 + bx + c$, where $a \neq 0$, $b^2 - 4ac$ is called its discriminant D.

The discriminant D determines the nature of the roots/zeroes of a quadratic polynomial.

Case I: If $D > 0$, the graph of $f(x) = ax^2 + bx + c$ will intersect the X-axis at two distinct points. The x-coordinates of the points of intersection with X-axis are known as 'zeroes' of $f(x)$.

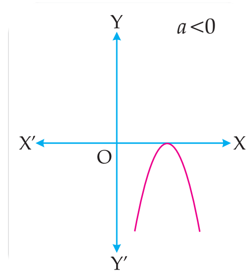


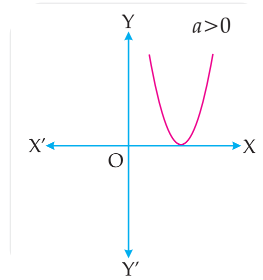
Real Roots: A polynomial of degree n can have at most n real roots. Real roots are the points where the graph of the equation intersects the x-axis.



$\therefore$ $f(x)$ will have two zeroes and we can say that roots/zeroes of the given polynomial are **real** and unequal.

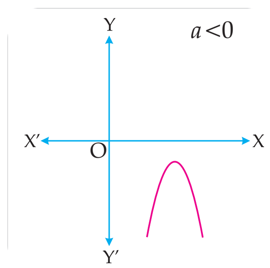
Case II: If $D = 0$, graph of $f(x) = ax^2 + bx + c$ will touch the X-axis at one point only.

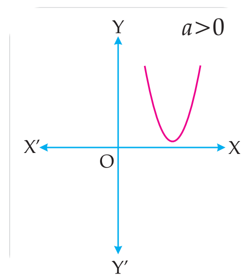




$\therefore$ $f(x)$ will have two equal "zeroes" and we can say that roots/zeroes of the given polynomial are real and equal.

Case III: If $D < 0$, the graph of $f(x) = ax^2 + bx + c$ will neither touch nor intersect the X-axis.





$\therefore$ $f(x)$ will not have any real zeroes.

Chapter 3

Revision Notes

1 Graphical Solution of Linear Equations in Two Variables

Concepts Covered:
- Solve the equations by graphical method.
- Possibilities of solutions and consistency/inconsistency.
- Conditions of unique solution/infinite number of solutions/no solution.

- **Linear equation in two variables:** An equation in the form of $ax + by + c = 0$, where a, b and c are real numbers and a and b are not zero, is called a linear equation in two **variables** x and y.

General form of a pair of linear equations in two variables is:

$$a_1x + b_1y + c_1 = 0$$

and $$a_2x + b_2y + c_2 = 0,$$

where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers, such that $a_1, b_1 \neq 0$ and $a_2, b_2 \neq 0$.

e.g., $3x - y + 7 = 0$,

and $7x + y = 3$

are linear equations in two variables x and y.

- There are two methods of solving simultaneous linear **equations** in two variables:
  (i) Graphical method and
  (ii) Algebraic methods.

[Board 2022, 2020]

Variable: A variable is a symbol used to represent an unknown value or quantity in mathematics, science, and other fields. It is typically denoted by letters such as x, y, or z. Variables allow us to create equations and expressions that describe relationships between quantities.

Equation: An equation is a mathematical statement that asserts the equality of two expressions, typically separated by an equals sign (=). It often contains variables, constants and mathematical operations like addition, subtraction, multiplication, or division.

1. **Graphical Method:**

(i) Express one variable (say y) in terms of the other variable x, $y = ax + b$, for the given equation.

(ii) Take at least two values of the dependent variable x, and find the corresponding values of the dependent variable y. Take integral values only.

(iii) Plot these values on the graph paper in order to represent these equations.

(iv) If the lines intersect at a distinct point, the point of intersection will be the unique solution for given equations. In this case, the pair of linear equations is consistent.

and, $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$, [Board 2022, 19, 16]

Graphical Method To Solve Linear Equations in two Variables

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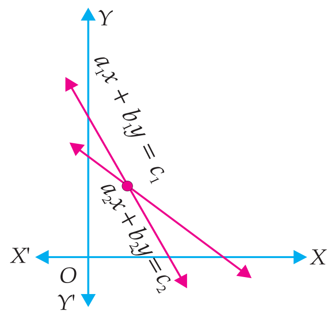
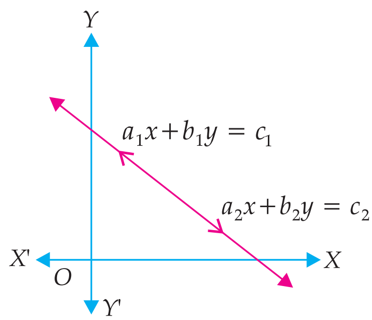


Intersecting Lines

(v) If the lines representing the linear equations coincide, then the system of equations has infinitely many solutions. In this case, the pair of linear equations is consistent and dependent.

and, $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

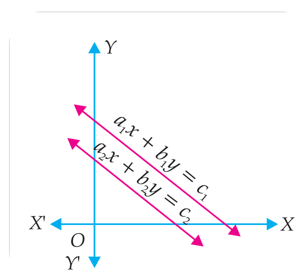


Coincident Lines

[Board 2023, 2019]

(vi) If the lines representing the pair of linear equations are parallel, then the system of equations has no solution and is called inconsistent.

and, $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$



Parallel Lines

Fundamental Facts:

(1) A linear equation in two variables is represented geometrically by a line whose points make up the collection of solutions of the equation.

In case of two variables, each solution may be interpreted as the Cartesian coordinates of a point on the Euclidean plane.

MNEMONICS

Concept: Nature of the Solution of a Pair of a Linear Equations in Two Variables. The system has a unique solution.

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

MNEMONICS: The features in Audi A_1 and A_2 are not the same as *BMW* B_1 and B_2

Interpretation:

$$A_1 \Rightarrow a_1$$
$$A_2 \Rightarrow a_2$$
$$B_1 \Rightarrow b_1$$
$$B_2 \Rightarrow b_2$$

Example 1

Champa went to a Sale to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased". Help her friends find how many pants and skirts Champa bought.

Sol. Let the number of pants be x and skirts be y.

So, the equations, formed are as:

$\Rightarrow$ $y = 2x - 2$

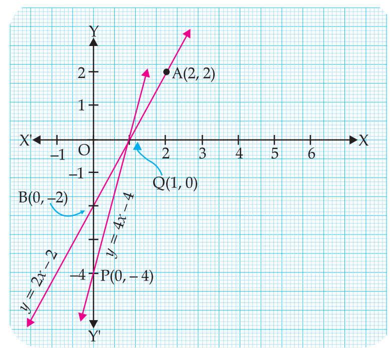
$\Rightarrow$ $y = 4x - 4$

For first equation, we have solutions as follows:

x	2	0	1
$y = 2x - 2$	2	–2	0

And for the other equation,

x	0	1
$y = 4x - 4$	–4	0



We know that, the two lines intersect at the point (1, 0).
So, $x = 1$ and $y = 0$ is the required solution.
Hence, Champa bought 1 pant and no skirts.

2 Algebraic Method to Solve Pair of Linear Equations

Concepts Covered:
- Solve the linear equations algebraically by Substitution and Elimination method.
- To solve word problems.

• **Algebraic Method:** We can solve the linear equations algebraically by **substitution method and elimination method.**

1. Substitution Method:

(i) Find the value of one variable (say y) in terms of the other variable, i.e., x from either of the equations.

(ii) Substitute this value of y in the other equation and reduce it to an equation in one variable.

(iii) Solve the equation so obtained and find the value of x.

(iv) Put this value of x in one of the equations to get the value of variable y.

2. Elimination Method:

(i) Multiply the given equations with suitable constants. Make either the x-**coefficients** or the y-coefficients of the two equations equal.

(ii) Subtract or add one equation from the other to get an equation in one variable.

(iii) Solve the equation so obtained to get the value of the variable.

(iv) Put this value in any one of the equations to get the value of the second variable.

Algebraic Methods to solve Linear Equations

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Note:

(a) If in step (ii), we obtain a true equation involving no variable, then the original pair of equations has **infinitely many solutions**.

(b) If in step (ii), we obtain a false equation involving no variable, then the original pair of equations has no solution, i.e., it is inconsistent.

Steps solving word problems

S. No.	Problem type	Steps follow
(1)	Age problems	If the problem involves finding out the ages of two persons, take the present age of one person as x and of the other as y. Then, 'a' years ago, age of the first person was '$x - a$' years and that of the second person was '$y - a$' and after 'b' years, age of the first person will be '$x + b$' years and that of the second person will be '$y + b$' years. Formulate the equations and solve them.
(2)	Problems based on numbers and digits	Let the digit in the unit's place be x and that in the ten's place be y. The two-digit number is given by $10y + x$. On interchanging the positions of the digits, the digit in the unit's place becomes y and in the ten's place becomes x. The two digit number becomes $10x + y$. Formulate the equations and then solve them.
(3)	Problems based on fractions	Let the numerator of the fraction be x and the denominator be y. Then the fraction is $\frac{x}{y}$. Formulate the linear equations based on the given conditions, and solve for x and y to find the value of the fraction.
(4)	Problems based on Distance, Speed and Time	Speed $= \frac{\text{Distance}}{\text{Time}}$ or Distance = Speed × Time or Time $= \frac{\text{Distance}}{\text{Speed}}$. **[Board, 2019]**
		To solve the problems involving the speed of the boat going downstream and upstream, let the speed of the boat in still water be x km/h and the speed of the stream be y km/h. Then, the speed of the boat downstream $= (x + y)$ km/h and the speed of the boat upstream $= (x - y)$ km/h.
(5)	Problems based on commercial Mathematics	To solve specific questions based on commercial mathematics, Let the fare for one full ticket be ₹ x and the reservation charges be ₹ y. So that one full fare $= x + y$ and one half fare $= \frac{x}{2} + y$. **[Board, 2023]** To solve the questions of profit and loss, take the cost price of the first article as ₹ x and that of the second article as ₹ y. To solve the questions based on simple interest, take the amount invested as ₹ x at some rate of interest and ₹ y at a different rate of interest as specified in the question.
(6)	Problems based on Geometry and Mensuration	Use the angle sum property of a triangle ($\angle A + \angle B + \angle C = 180°$, in case of a triangle.) **[Board, 2021]** In case of a parallelogram, opposite angles are equal and in the case of a cyclic quadrilateral, opposite angles are supplementary.

Example 2

Solve the following pair of equations by substitution method:

$7x - 15y = 2$ ...(i)

$x + 2y = 3$ ...(ii)

Sol. Using the substitution method,

From equation (ii) we have,

$\Rightarrow \quad x = 3 - 2y$ **...(iii)**

Substituting the value of x in equation (i), we get

$\Rightarrow \quad 7(3 - 2y) - 15y = 2$

$\Rightarrow \quad 21 - 14y - 15y = 2$

$\Rightarrow \quad -29y = -19$

$\Rightarrow \quad y = \frac{19}{29}$

Substituting value of y in equation (iii),

$\Rightarrow \quad x = 3 - 2\left(\frac{19}{29}\right) = \frac{49}{29}$

So, the required solution is $x = \frac{49}{29}$ and $y = \frac{19}{29}$.

Example 3

Use the elimination method to find all possible solutions of the following pair of linear equations:

$2x + 3y = 8$ ...(i)

$4x + 6y = 7$ ...(ii)

Sol. Using the elimination method,

Multiplying equation (i) by 2, we get,

$\Rightarrow \quad 4x + 6y = 16$ **...(iii)**

Now, subtracting equation (ii) from equation (iii) we have,

$\Rightarrow \quad (4x + 6y) - (4x + 6y) = 16 - 7$

$\Rightarrow 0 = 9$ (Which is a false statement)

Therefore, the following pair of linear equations has no solution.

Fundamental Fact

Algebraic expressions are the mathematical equations consisting of variables, constants, terms and coefficients.

Chapter 4

Revision Notes

- **Standard Form of a Quadratic Equation**

An equation of the form $f(x) = 0$, where $f(x)$ is a polynomial of degree 2, is called a quadratic equation.

When the terms of a **quadratic equation** are written in decreasing order of their degrees, we get the standard form of a quadratic equation as:

$\Rightarrow ax^2 + bx + c = 0, a \neq 0$

Where a, b and c are real numbers. **[Board, 2024]**

Quadratic Equation: It's a special equation, always wearing the classic disguise of $ax^2 + bx + c = 0$. The squared term, x^2, which gives it the power to create elegant parabolas—those U-shaped curves that can open up or down.

- **Solutions of Quadratic Equations**

- A quadratic **equation** in variable x is of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.
- The values of x that satisfy the equation are called the solutions, roots or zeroes of the equation.
- A real number α is said to be a solution/root or zero of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.
- A quadratic equation can be solved by the following algebraic methods:

(i) By factorisation (splitting the middle term),

(ii) Using the quadratic formula.

Equation: An equation is like a mathematical story where different characters—numbers, variables and operations—come together to find balance. It's a puzzle where two expressions meet, separated by an equal sign, declaring they are perfectly equal, no matter how different they look.

Example 1

Check whether the following equation is a quadratic equation: $(x – 2)^2 + 1 = 2x – 3$.

Sol. We have,

$$\Rightarrow \quad (x-2)^2 + 1 = 2x - 3$$
$$\Rightarrow \quad x^2 + 4 - 4x + 1 = 2x - 3$$
$$\Rightarrow \quad x^2 - 6x + 8 = 0$$

It is of the form $ax^2 + bx + c = 0$,

So, the given equation is a quadratic equation.

If $ax^2 + bx + c = 0$, where $a \neq 0$ can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

- **Method for factorisation of the equation**

$ax^2 + bx + c = 0$, where $a \neq 0$. **[Board, 2024]**

➢ Find the product of a and c, i.e., "ac".

➢ Find a pair of numbers b_1 and b_2 whose product is "ac" and whose sum is "b" (if you can't find such numbers, it can't be factorised).

➢ Split the middle term using b_1 and b_2, expressing the term bx as $b_1x \pm b_2x$. Now factorise, by grouping the pairs of terms.

- **Solution of a Quadratic Equation using Quadratic Formula**

➢ The roots of the quadratic equation $ax^2 + bx + c = 0$; $a \neq 0$ can be found using the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ **[Board, 2024]**

The above result is known as **quadratic formula** or **Sridharacharya's formula**.

Here, $b^2 – 4ac \geq 0$, for real roots.

Solution of a quadratic equation

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Example 2

Find the roots of the quadratic equation $2x^2 – 5x + 3 = 0$, by factorisation method.

Sol. Using factorisation method,

$$2x^2 - 5x + 3 = 0$$
$$2x^2 - 3x - 2x + 3 = 0$$
$$x(2x - 3) - 1(2x - 3) = 0$$
$$\Rightarrow \quad (2x - 3)(x - 1) = 0$$
$$\Rightarrow \quad x = \frac{3}{2}, 1$$

So, the roots of the quadratic equation are 1 and $\frac{3}{2}$.

Example 3

Find the roots of the quadratic equation $6x^2 – x – 2 = 0$.

Sol. On comparing with $ax^2 + bx + c = 0$

We get $a = 6$, $b = –1$ and $c = –2$

Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{1 \pm \sqrt{1 - 4(6)(-2)}}{2(6)}$$
$$x = \frac{12 \pm \sqrt{49}}{12} = \frac{1 \pm 7}{12} = \frac{2}{3}, -\frac{1}{2}$$

- The Greek mathematician **Euclid** developed a geometrical approach for finding the roots, which are solutions of quadratic equations.
- Brahmagupta (C.E. 598–665) gave an explicit formula to solve a quadratic equation of the form $ax^2 + bx + c = 0$.

- **Discriminant and Nature of Roots**

- For the quadratic equation $ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ is known as discriminant, i.e., Discriminant, $D = b^2 - 4ac$.
- Nature of roots of a quadratic equation:

  (a) If $b^2 - 4ac > 0$, the quadratic equation has two distinct real roots. **[Board, 2024]**

  (b) If $b^2 - 4ac = 0$, the quadratic equation has two equal real roots. **[Board, 2024]**

  (c) If $b^2 - 4ac < 0$, the quadratic equation has no real root.

Euclid: Often referred to as the "Father of Geometry," was a Greek mathematician who lived around 300 BCE in Alexandria, Egypt. He is best known for his work "Elements", a comprehensive compilation of mathematical knowledge at the time, which laid the foundation for modern geometry.

- The real roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}, \text{ where } b^2 - 4ac > 0.$$

- Roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are $\frac{-b}{2a}$ and $\frac{-b}{2a}$, where $b^2 - 4ac = 0$

Nature of Roots

Scan Me!

$$\text{Sum of the zeroes} = \frac{-b}{a}$$

$$\text{Product of the zeroes} = \frac{c}{a}$$

- **Quadratic identities:**

  (i) $(a + b)^2 = a^2 + 2ab + b^2$

  (ii) $(a - b)^2 = a^2 - 2ab + b^2$

  (iii) $a^2 - b^2 = (a + b)(a - b)$ **[Board, 2024]**

MNEMONICS

Concept 1: Roots of a quadratic equation using quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Mnemonics: A negative **Boy** could not decide if he did or didn't want to go to a **Radical** party. The Boy was **Square** so he **missed out** on 4 **A**wesome **C**hicks. This was all over by **2 a.m.**

Interpretations:

A negative Boy $\Rightarrow (-b)$

he did or didn't want to go $\Rightarrow (+/-)$

to a Radical party $\Rightarrow (\sqrt{\quad})$

Boy was Square $\Rightarrow (b^2)$

missed out $\Rightarrow (-)$

4 Awesome $\Rightarrow 4a$

Chicks $\Rightarrow c$

all over $\Rightarrow$ Divided by

2 a.m. $\Rightarrow 2a$

Example 4

Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

Sol. On comparing given quadratic equation with $ax^2 + bx + c = 0$

We get,

$$\Rightarrow \quad a = 3, b = -2, c = \frac{1}{3}$$

$$\text{Discriminant, } D = b^2 - 4ac$$

$$\Rightarrow \quad D = (-2)^2 - 4(3)\left(\frac{1}{3}\right)$$

$$= 4 - 4 = 0$$

As $D = 0$, the given quadratic equation has real and equal roots.

Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{0}}{2(3)}$$

$$= \frac{1}{3}, \frac{1}{3}$$

Fundamental Facts

(1) Quadratic equations are second order polynomials. This means that the highest power of their variables is two.

(2) An equation of the form of $ax^2+bx+c=0$ is called as a quadratic equation. It has two roots. Both of them may be real, equal or imaginary.

Chapter 5

Revision Notes

1 To Find the n^{th} Term of an Arithmetic Progression

Concepts Covered:
- First term and common difference of an A.P.
- Finite and infinite A.P.
- Formula for finding nth term of an A.P.

• An arithmetic progression is a sequence of numbers in which each term is obtained by adding or subtracting a fixed number to the preceding term, except the first term.

• The difference between the two successive terms of an "AP" is called the common difference and is denoted by d.

• Each number in the sequence of an arithmetic progression is called a term of the A.P.

• The arithmetic progression having **finite** number of terms is called a finite arithmetic progression.

Finite: The word finite refers to something that has limits or bounds, meaning it is not infinite or endless. It applies to anything measurable or countable, such as time, space or resources. For example, a finite lifespan means a life that eventually ends.

• The arithmetic progression having infinite number of terms is called an **infinite arithmetic progression.**

• A list of numbers $a_1, a_2, a_3,..., a_n$ is an A.P. if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, ...$,give the same value, i.e., $a_{k+1} - a_k$ is same for all different values of k.

• The general form of an A.P. is $a, a + d, a + 2d, a + 3d, ...$

• If the A.P. $a, a + d, a + 2d,..., l$ is reversed to $l, l - d, l - 2d, ..., a$, the common difference changes to the negative of the common difference of the original sequence.

MNEMONICS

Concept 1: n^{th} term of AP: $a_n = a + (n - 1)d$

Mnemonics: Nokia Offers Additional Programmes in English To Attract Positive New One Buyer Daily

Interpretations:

Nokia 'N' $\Rightarrow$ n^{th} term.
Offer 'O' $\Rightarrow$ of
Additional 'A' $\Rightarrow$ Arithmetic
Programme 'P' $\Rightarrow$ Progression
In 'I' $\Rightarrow$ is
English 'E' $\Rightarrow$ Equal
To 'T' $\Rightarrow$ To
Attract 'A' $\Rightarrow$ a
Positive 'P' $\Rightarrow$ +
New 'N' $\Rightarrow$ n
One Buyer $\Rightarrow$ –1
Daily 'D' $\Rightarrow$ d

Key Formulae

• The general (n^{th}) term of an A.P. is expressed as:
$a_n = a + (n - 1)d$. .... from the starting. **[Board 2024, 23]**
where, a is the first term and d is the common difference.

• The general (n^{th}) term of an A.P., $l, l - d, l - 2d,$........, a is given by:
$a_n = l + (n - 1)(- d) = l - (n - 1)d$ .... from the end.
where, l is the last term, d is the common difference and n is the number of terms.

Fundamental Facts

• An A.P. or arithmetic sequence is a sequence of numbers such that the difference between the consecutive terms is constant.

• If a constant is added or subtracted from each term of an A.P., the resulting sequence is also an A.P.

• If each term of an A.P. is multiplied or divided by a non-zero constant, the resulting sequence is also an A.P.

• If the terms are selected at a regular interval, the given sequence is in A.P.

• If the terms of sequence are connected with plus (+) or minus (–), the pattern is called a series.

Example: 2 + 4 + 6 + 8 + ... is a series.

• If the terms of a sequence or a series are written under specific conditions, then the sequence or series is called a progression.

Example 1

Find the 10^{th} term of the AP : 2, 7, 12, ...

Sol. We have,

$\Rightarrow$ First term, $a = 2$

$\Rightarrow$ Common difference, $d = 7 - 2 = 5$

For 10^{th} term, $n = 10$

Now, $a_n = a + (n - 1)d$

$\Rightarrow$ $a_{10} = 2 + (10 - 1)(5)$

$\Rightarrow$ $a_{10} = 2 + 45$

$= 47$

Hence, the 10^{th} term of an AP is 47.

Example 2

Determine the AP whose 3^{rd} term is 5 and 7^{th} term is 9.

Sol. Given that,

$\Rightarrow$ $a_3 = 5$ and $a_7 = 9$

So, we have

$\Rightarrow$ $a_3 = a + (3 - 1)d$

$$5 = a + 2d \quad ...(i)$$

$$\Rightarrow \quad a_7 = a + (7-1)d$$

$$9 = a + 6d \quad ...(ii)$$

On solving equations (i) and (ii),

We get, $a = 3, d = 1$

So, the AP is: 3, 4, 5, 6, 7, 8, ...

Example 3

Check whether 301 is a term of the list of numbers 5, 11, 17, 23, …

Sol. For the given AP,

$\Rightarrow$ First term, $a = 5$

$\Rightarrow$ Common difference, $d = 11 - 5 = 6$

We know that, $a_n = a + (n-1)d$

$$\Rightarrow \quad 301 = 5 + (n-1)(6)$$

$$\Rightarrow \quad 301 = 6n - 1$$

$$\Rightarrow \quad n = \frac{302}{6} = \frac{151}{3}$$

(Since, n is the number of terms, it cannot be in decimals.)

So, 301 is not a term of the given AP.

Example 4

A flower bed has 23 rose plants in the first row, 21 in the second, 19 in the third and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Sol. We know that,

The number of roses in the flower bed forms an AP as: 23, 21, 19, …, 5

So, we have,

$\Rightarrow$ First term, $a = 23$

$\Rightarrow$ Common difference, $d = 21 - 23 = -2$

$\Rightarrow$ Last term, $a_n = 5$

Let the number of rows be n

Now, $a_n = a + (n-1)d$

$$\Rightarrow \quad 5 = 23 + (n-1)(-2)$$

$$\Rightarrow \quad -18 = -2n + 2$$

$$\Rightarrow \quad -2n = -20 \quad \Rightarrow \quad n = 10$$

So, there are 10 rows in the flower bed.

2 Sum of n terms of an Arithmetic Progression

Concepts Covered

- Formula to find the sum of n terms of A.P.
- Students will be able to recall some patterns which occur in their daily life.

• Sum of n terms of an A.P. is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{[Board, 2024]}$$

where, a is the first term, d is the common difference and n is the total number of terms.

• Sum of n terms of an A.P. when the first and last terms is given.

$$S_n = \frac{n}{2}[a + l] \quad \text{[Board, 2023]}$$

where, a is the first term and l is the last term.

• Sum of natural numbers $S_n = n\left(\frac{n+1}{2}\right)$

• The n^{th} term of an A.P. is the difference of the sum of first n terms and the sum of the first $(n-1)$ terms of it, i.e.,

$a_n = S_n - S_{n-1}$.

Patterns of AP in real life: Arithmetic Progressions (AP) are everywhere around us! They show up in the equal spacing of steps on a staircase, consistent increases in savings with regular deposits, or the arrangement of seats in a theater. Even nature uses AP–like the growth of tree rings or the spacing of leaves.

Arithmetic Progression

Scan Me!

Example 5

Find the sum of the first 22 terms of the AP: 8, 3, −2, …

Sol. For the given AP,

First term, $a = 8$

Common difference, $d = 3 - 8 = -5$

Number of terms, $n = 22$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

So, $$S_{22} = \frac{22}{2}[2 \times (8) + (22-1)(-5)]$$

$$= -979$$

Hence, the sum of the first 22 terms is −979.

Example 6

How many terms of the AP: 24, 21, 18, … must be taken so that their sum is 78?

Sol. For the given AP,

First term, $a = 24$

Common difference, $d = 21 - 24 = -3$

Let the number of terms be n

Sum, $S_n = 78$

Now, $$S_n = \frac{n}{2}[2a + (n-1)d]$$

On substituting the values,

$$78 = \frac{n}{2}[2 \times (24) + (n-1)(-3)]$$

$$156 = n[48 - 3n + 3]$$

$$156 = n(51 - 3n)$$

$$3n^2 - 51n + 156 = 0$$

$$\Rightarrow \quad n^2 - 17n + 52 = 0$$

$$\Rightarrow \quad (n-4)(n-13) = 0$$

$$\Rightarrow \quad n = 4, 13$$

Both values are admissible
Hence, the number of terms taken is either 4 or 13.

Example 7

Find the sum of the first 24 terms of a list of numbers whose n^{th} term is given by:

$$a_n = 3 + 2n$$

Sol. Given that, $a_n = 3 + 2n$
For $n = 1$, $a_1 = 3 + 2(1) = 5$
For $n = 2$, $a_2 = 3 + 2(2) = 7$
For $n = 3$, $a_3 = 3 + 2(3) = 9$
And so on,
We get list of numbers in AP as: 5, 7, 9, 11, …
First term, $a = 5$
Common difference, $d = 7 - 5 = 2$
Number of terms, $n = 24$

$$S_{24} = \frac{24}{2}[2 \times (5) + (24 - 1)(2)]$$

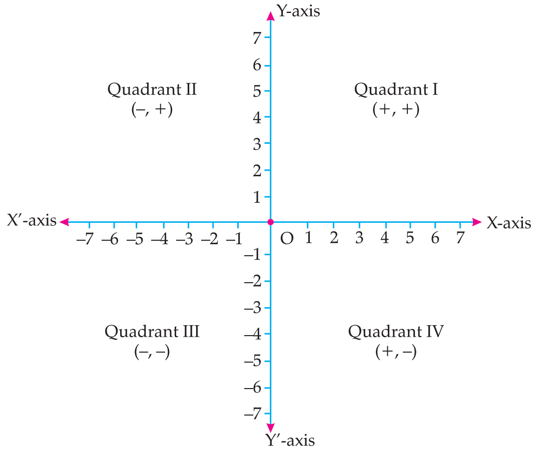
$$= 672$$

Hence, the sum of first 24 terms is 672.

Chapter 6

Revision Notes

• Two perpendicular number lines intersecting at the origin are called coordinate axes. The horizontal line is the X-axis (denoted by $X'OX$) and the vertical line is the Y-axis (denoted by $Y'OY$).



• The **point** of intersection of the X-axis and Y-axis is called the origin, denoted by O whose coordinate point is (0, 0).

• **Cartesian plane** is a plane obtained by putting the coordinate axes perpendicular to each other in the plane. It is also called coordinate plane or XY-plane.

Point: In mathematics, a "point" is like a tiny, invisible dot—so small it has no width, length, or depth. It's the most basic building block, yet it holds infinite possibilities! Points are used to define positions, connect lines, form shapes and help us measure distances. They're like the coordinates of a treasure map, marking where things begin or end! You could say a point is the "VIP" of geometry—always there, but never taking up too much space!

• The X-coordinate of a **point** is its perpendicular distance from the Y-axis. The Y-coordinate of a point is its perpendicular distance from the X-axis.

• The abscissa of a point is the X-coordinate of the point.

• The ordinate of a point is the Y-coordinate of the point.

• If the abscissa of a point is x and the ordinate of the point is y, then (x, y) are called the coordinates of the point.

• The axes divide the Cartesian plane into four parts called the quadrants (one fourth part), numbered I, II, III and IV **anti-clockwise** from OX.

Anti-clockwise: "Anti-clockwise" is the direction that goes against the flow of a clock's hands. Imagine starting from the top of a clock—if you move to the left instead of the right, you're moving anti-clockwise!

• The coordinates of a point on the X-axis are of the form $(x, 0)$ and that of the point on Y-axis are $(0, y)$.

• The sign of the coordinates depicts the quadrant in which it lies. The coordinates of a point are of the form (+, +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant.

KEY FORMULAE

• The distance between two points, i.e., $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$d = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$ **[Board, 2024]**

• The distance of a point $P(x, y)$ from origin is $\left|\sqrt{x^2 + y^2}\right|$

• Coordinates of point (x, y) which divides the line segment by joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ internally are

$$x = \left(\frac{mx_2 + nx_1}{m + n}\right)$$

and $$y = \left(\frac{my_2 + ny_1}{m + n}\right)$$

[Board, 2024]

Midpoint: Midpoint is the point that lies exactly in the middle of two other points, cutting the distance between them in half. It's a balance point, where the line is split into two equal parts. The midpoint helps create symmetry and is used in everything from geometry to navigation!

• Coordinates of the **midpoint** of the line segment obtained by joining the points (x_1, y_1) and (x_2, y_2) are:

$$x = \left(\frac{x_2 + x_1}{2}\right) \text{ and } y = \left(\frac{y_2 + y_1}{2}\right)$$ **[Board, 2024]**

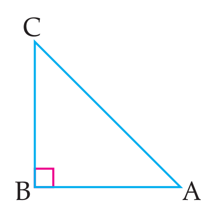
• Three points A, B and C are **collinear** if the distances AB, BC and CA are such that the sum of two distances is equal to the third.

Collinear: In geometry, "collinear" means that a set of points lies on the same straight line, just like the cars on the highway. These points are perfectly aligned, no matter how far you zoom out

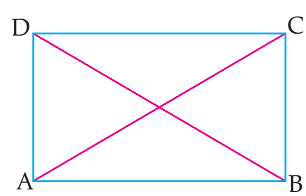
• Three points A, B and C are the vertices of an equilateral triangle if $AB = BC = CA$.

• The points A, B and C are the vertices of an isosceles triangle if $AB = BC$ or $BC = CA$ or $CA = AB$.

• Three points A, B and C are the vertices of a right triangle, if $AB^2 + BC^2 = CA^2$.



• For the given four points A, B, C and D:



1. If $AB = BC = CD = DA$; $AC = BD$, then $ABCD$ is a square.

2. If $AB = BC = CD = DA$; $AC \neq BD$, then $ABCD$ is a rhombus.

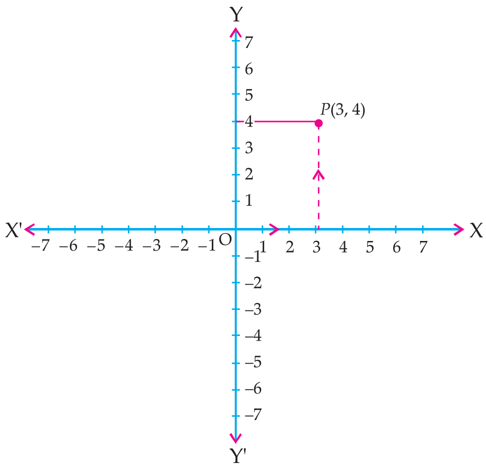
3. If $AB = CD$, $BC = DA$; $AC = BD$, then $ABCD$ is a rectangle.

4. If $AB = CD$, $BC = DA$; $AC \neq BD$, then $ABCD$ is a parallelogram.

- Diagonals of a square, rhombus, rectangle and parallelogram always bisect each other.
- Diagonals of a rhombus and square bisect each other at a right angle.
- If $x \neq y$, then $(x, y) \neq (y, x)$ and if $(x, y) = (y, x)$, then $x = y$.
- To plot a point $P(3, 4)$ in the Cartesian plane:

  (i) Move a distance of 3 units along the X-axis.

  (ii) Move a distance of 4 units along the Y-axis.



MNEMONICS

Concept: Abscissa and Ordinate

Mnemonics: *XOR* is Abstract and *YOR* is Ordinary

$X \rightarrow$ Abscissa

$Y \rightarrow$ Ordinate

The coordinate on the x-axis is known as Abscissa, and the coordinate on the y-axis is known as Ordinate.

Example 1

Do the points (3, 2), (−2, −3) and (2, 3) form a triangle? If so, name the type of triangle formed.

Sol. Let the points be $P(3, 2)$, $Q(-2, -3)$ and $R(2, 3)$.
Using the distance formula, calculate the length of the sides formed from these points as:

$$PQ = \sqrt{(-3-3)^2 + (-3-2)^2}$$

$$= \sqrt{(-5)^2 + (-5)^2} = \sqrt{50} \text{ units}$$

$$QR = \sqrt{(-2+2)^2 + (-3+3)^2}$$

$$= \sqrt{(-4)^2 + (-6)^2} = \sqrt{52} \text{ units}$$

$$PR = \sqrt{(2-3)^2 + (3-2)^2}$$

$$= \sqrt{(-1)^2 + 1^2} = \sqrt{2} \text{ units}$$

As the sum of any of these two sides is greater than the third side, so, the points P, Q and R form a triangle.

Also, $(\sqrt{50})^2 + (\sqrt{2})^2 = (\sqrt{52})^2$

(Satisfying the pythagoras' theorem)
Hence, the given points form a right-angled triangle.

Example 2

Find a relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5).

Sol. Let the point P(x, y) be equidistant from the given points
Applying the distance formula,

$$\sqrt{(x-7)^2 + (y-1)^2} = \sqrt{(x-3)^2 + (y-5)^2}$$

Squaring on both sides,

$$(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$$

$$x^2 - 14x + 49 + y^2 - 2y + 1$$

$$= x^2 - 6x + 9 + y^2 - 10y + 25$$

$$50 - 14x - 2y = 34 - 6x - 10y$$

$$8x - 8y = 16$$

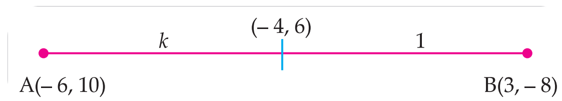
$$x - y = 2$$

This is the required relation between x and y.

Example 3

In what ratio does the point (−4, 6) divide the line segment joining the points A(−6, 10) and B(3, −8)?

Sol. Let the ratio be $k : 1$
Using the section formula,



$$-4 = \frac{3k-6}{k+1} \text{ and } 6 = \frac{-8k+10}{k+1}$$

On simplifying, $-4 = \dfrac{3k-6}{k+1}$

$$-4k - 4 = 3k - 6$$

$$7k = 2.$$

$$k = \frac{2}{7}$$

Required ratio is 2 : 7
Therefore, the point (−4, 6) divides the line segment joining the points A(− 6, 10) and B(3, − 8) in the ratio 2 : 7.

Fundamental Facts

(1) Coordinate geometry is the system of geometry where the position of points on the plane is described using an ordered pair of numbers.

(2) Coordinate geometry acts as a bridge between the Algebra and Geometry.

(3) Medians of a triangle are concurrent. The point of concurrency is called the centroid.

(4) Trisection of a line segment means dividing it into 3 equal parts, so 2 points are required for it.

Chapter 7

Revision Notes

- A triangle is one of the basic shapes of geometry. It is a polygon with three sides and three vertices/corners.
- Two figures are said to be congruent if they have the same shape and same size.
- Those figures that have the same shape but not necessarily the same size are called similar figures.
- Hence, we can say that all congruent figures are similar, but all similar figures are not congruent.
- **Similarity of Triangles:** Two triangles are similar, if:

  (i) their **corresponding** sides are **proportional**

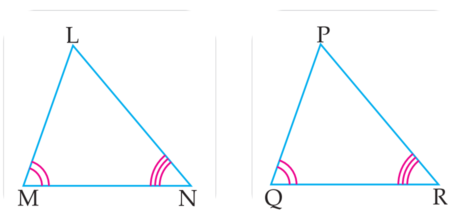
  > **Proportional:** In math, proportional is like a perfect friendship between two quantities—they grow or shrink together in harmony! If one doubles, the other doubles too; if one halves, the other follows suit."

  > **Corresponding:** Corresponding terms in equations keep the balance intact. It's all about connections—whether it's points, angles, or numbers, corresponding elements maintain their unique bond, ensuring every part has its rightful pair.

  (ii) their corresponding angles are equal.

  If ΔABC and ΔDEF are similar, then this similarity can be written as $\Delta ABC \sim \Delta DEF$.

- **Criteria for Similarity of Triangles:**



In ΔLMN and ΔPQR, if

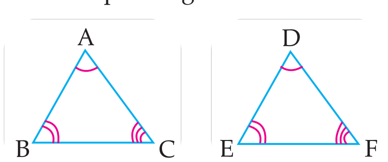
(a) $\angle L = \angle P$, $\angle M = \angle Q$ and $\angle N = \angle R$

(b) $\dfrac{LM}{PQ} = \dfrac{MN}{QR} = \dfrac{LN}{PR}$,

then $\Delta LMN \sim \Delta PQR$. **[Board, 2024]**

Basic Proportionality Theorem
Scan Me!

(i) AAA Criterion: In two triangles, if the corresponding angles are equal, then the triangles are similar and hence their corresponding sides are in the same ratio.



If ΔABC and ΔDEF are similar,

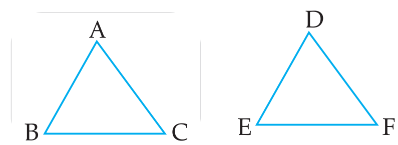
$\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.

Therefore, $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

Remark: If two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal. Therefore, AAA similarity criterion can also be stated as follows:

AA Criterion: If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

(ii) SSS Criterion: In two triangles, if the sides of one triangle are proportional to the sides of another triangle, then the two triangles are similar, and hence corresponding angles are equal.



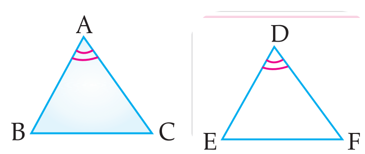
If $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD}$

$\therefore \quad \Delta ABC \sim \Delta DEF$

then $\angle A = \angle D$, $\angle B = \angle E$,

and $\angle C = \angle F$

(ii) SAS Criterion: If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the two triangles are similar.

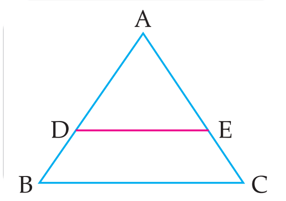


If $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ and $\angle A = \angle D$, then $\Delta ABC \sim \Delta DEF$.

Some theorems based on similarity of triangles

(i) If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, the other two sides are divided in the same ratio. It is known as the 'Basic Proportionality **Theorem**' or '**Thales' Theorem**'. **[Board, 2024]**

> **Theorem:** A theorem in math is like a puzzle that has been solved through logic and proof. Once a theorem is proven, it is considered a universal truth that applies in all situations. It's a powerful tool that mathematicians use to build new ideas, solve problems and explore the mysteries of numbers and shapes!



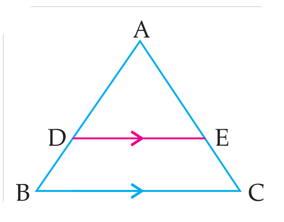
In ΔABC, let $DE \parallel BC$, then

(A) $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

(B) $\dfrac{AB}{DB} = \dfrac{AC}{EC}$

(C) $\dfrac{AD}{AB} = \dfrac{AE}{AC}$.

(ii) If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. It is the **'Converse of the Basic Proportionality Theorem'.**



If $\dfrac{AD}{DB} = \dfrac{AE}{EC}$,

then $DE \parallel BC$

MNEMONICS

Concept 1: Area of a triangle = $\dfrac{1}{2}$ × Base × height

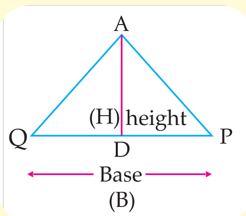
Mnemonics: 'Audi' is the product of half of 'BMW' and 'Honda'.

Interpretations:

A ⇒ Area

B ⇒ Base

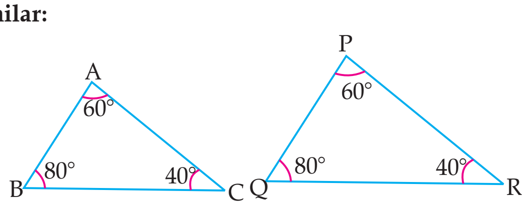
H ⇒ Height



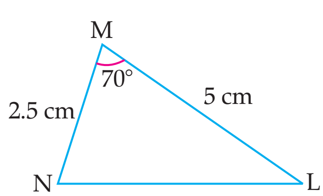
Example 1

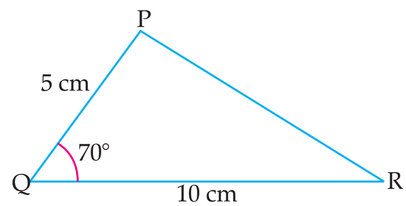
Identify which of the following pairs of triangles are similar:

(a)



(b)





Sol. (a) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

$\Delta ABC \sim \Delta PQR$ by the AA or AAA similarity rule

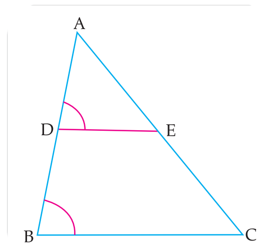
(b) $\angle M = \angle Q$

$\dfrac{MN}{PQ} = \dfrac{ML}{QR}$

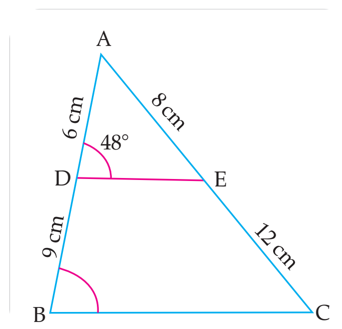
$\Delta MNL \sim \Delta PQR$, by the SAS similarity rule.

Example 2

In the figure, if $AD = 6$ cm, $DB = 9$ cm, $AE = 8$ cm, $EC = 12$ cm and $\angle ADE = 48°$, find $\angle ABC$.



Sol. It is given that,



Here, in ΔABC and ΔADE,

$$AB = (9 + 6) \text{ cm} = 15 \text{ cm}$$

Similarly,

$$AC = AE + EC$$
$$= (8 + 12) \text{ cm}$$
$$= 20 \text{ cm}$$ **[½]**

Now, $\dfrac{AD}{AB} = \dfrac{6}{15} = \dfrac{2}{5}$

and $\dfrac{AE}{AC} = \dfrac{8}{20} = \dfrac{2}{5}$

Then, $\dfrac{AD}{AB} = \dfrac{AE}{AC}$

i.e., $DE \parallel BC$ (By Converse of *BPT*)

∴ $\angle ABC = \angle ADE$

(Corresponding angles)

Hence, $\angle ABC = 48°$. **[½]**

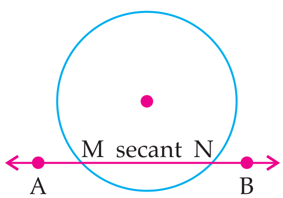
Chapter 8

Revision Notes

Problems on Circles

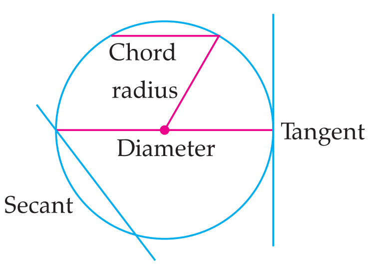
Scan Me!

- **Circle:** A circle is a round-shaped figure that has no **corners** or edges.
- **Tangent:** A tangent to a circle is a line that intersects the circle at only one point
- The common point of the circle and the tangent is called the point of contact.
- **Secant:** A line that intersects the circle at two district points is called a secant.



- A tangent to a circle is a special case of a secant when the two end points of the corresponding **chord** **coincide**.

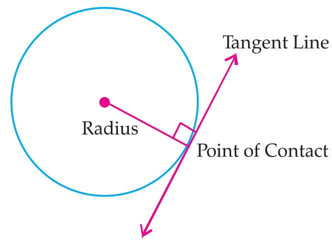
An example showcasing different terms related to the circles and their diagrammatically representation:



- There is no tangent to a circle passing through a point lying inside the circle.
- At any point on the circle, there can be only one tangent.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact. **[Board, 2024, 2022, 2020]**
- The alternate segment theorem, also known as the tangent–chord theorem, states that the angle between a tangent and a chord through a point of contact on a circle is equal to the angle in the alternate segment.

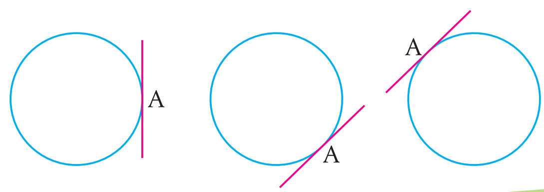
Corner: A "corner" is where two or more edges meet to create a sharp or distinct turning point. Imagine it as the meeting place where lines or surfaces have a little chat before parting ways!
In geometry, corners are also called vertices and they give shapes their structure—like the tip of a triangle, the vertex of an angle or the corners of a cube.
They're tiny spots packed with big importance, marking transitions, boundaries and the essence of shapes!

Chord: A chord in math is like a secret bridge inside a circle, connecting two points on its boundary.
It doesn't pass through the center unless it's the biggest, which we call the diameter.



- There are exactly two tangents to a circle through a point outside the circle.

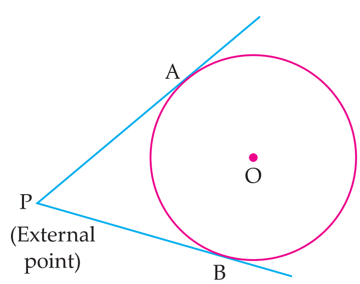
An examples for proving that at any point on the circle, there is only one tangent possible:



- The length of the segment of the tangent from the external point P and the point of contact with the circle is called the length of the tangent.
- The lengths of the tangents drawn from an external point to a circle are equal. **[Board, 2024]**

In the figure,

$$PA = PB.$$



Coincide: The term "coincide" means two objects share the exact same space or position. Imagine placing one sheet of paper perfectly on top of another so that you can't tell there are two—they "coincide."
In geometry, two lines, shapes or points coincide when they overlap completely, as if they are one and the same.
It's like a perfect harmony in space where no difference exists.
Whether it's two graphs meeting every step of the way or a point lying precisely on a curve, coincidence is about total alignment and agreement!

MNEMONICS

Concept 1: One Tomato has Two Seeds

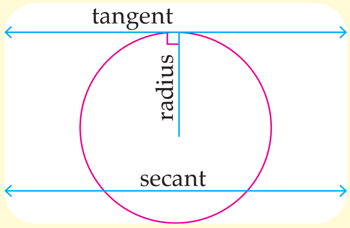
Interpretations:

One Tomato: One Point $\rightarrow$ Tangent

Two Seeds: Two Points $\rightarrow$ Secant

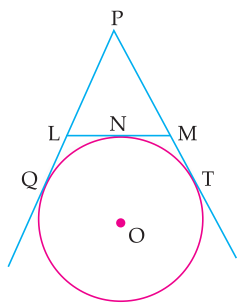
If a line touches a circle at one point, then it is known as Tangent.

If a line touches a circle at two points, then it is known as Secant.



Example 1

If PQ = 28 cm, then find the perimeter of ΔPLM.

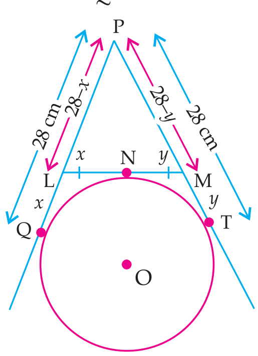


[SQP, 2021]

Sol. Given, $PQ = 28$ cm

$\therefore$ $PQ = PT$ (The lengths of the tangents from an external point are equal.)

i.e., $PQ = PT = 28$ cm



According to the figure,

Let $LQ = x$, then

$PL = (28 - x)$ cm

and let $MT = y$, then

$PM = (28 - y)$ cm

and $LM = LN + NM$

$= x + y$

$\therefore$ $LN = LQ = x$

And $NM = MT = y$

(The lengths of tangents from an external point are equal.)

Now,

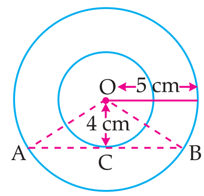
the perimeter of $\Delta PLM = PL + LM + PM$

$= (28 - x) + (x + y) + (28 - y)$

$= 28 + 28 = 56$ cm

Example 2

If the radii of two concentric circles are 4 cm and 5 cm, then find the length of each chord of one circle that is tangent to the other circle. [SQP, 2020]



Sol. In ΔOBC,

$$CO^2 + BC^2 = OB^2$$
$$4^2 + BC^2 = 5^2$$
$$16 + BC^2 = 25$$
$$BC^2 = 25 - 16$$
$$BC^2 = 9$$
$$BC = 3 \text{ cm}$$

In ΔOAC,

$$OC^2 + AC^2 = OA^2$$
$$4^2 + AC^2 = 5^2$$
$$AC^2 = 9$$
$$AC = 3 \text{ cm}$$

$\therefore$
$$AB = AC + BC$$
$$= 3 + 3$$
$$= 6 \text{ cm.}$$

Fundamental Facts

(1) The word 'tangent' comes from the Latin word '*tangere*', which means to touch and was introduced by the Danish mathematician, Thomas Fincke in 1583.

(2) The line perpendicular to the tangent and passing through the point of contact, is known as the normal.

(3) In two concentric circles, the chord of the larger circle, which touches the smaller circle is bisected at the point of contact.

Chapter 9

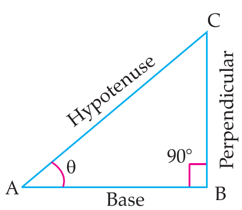
Revision Notes

1 Trigonometric Ratios and their Values

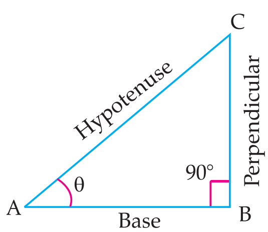
Concepts Covered:
- Six trigonometric ratios with their sides of a right angled triangle.
- Values of trigonometric ratios between 0° to 90°.

Concepts Covered:

1. In fig., a right triangle ABC, right angled at B is given and $\angle BAC = \theta$ is an acute angle. Here, side AB which is adjacent to $\angle A$ is base, side BC opposite to $\angle A$ is perpendicular and the side AC is **hypotenuse** which is opposite to the right angle B.



- The **trigonometric ratios** of $\angle A$ in right triangle ABC are defined as



sine of $\angle A = \sin\theta$

$$= \frac{\text{Perpendicular or opposite side to angle A}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

cosine of $\angle A = \cos\theta$

Hypotenuse: Imagine you're building a triangle's "superhighway" – the hypotenuse is the fast lane! In any right-angled triangle, it's the longest side, connecting the two ends of the 90-degree angle like a direct express route.

Trigonometric ratios: Trigonometric ratios are like the triangle's Instagram filters– they highlight the relationships between angles and sides in a right-angled triangle.

Trigonometric Ratios

Scan Me!

$$= \frac{\text{Base or adjacent side to angle A}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

tangent of $\angle A = \tan\theta$

$$= \frac{\text{Perpendicular or opposite side to angle A}}{\text{Base or adjacent side to angle A}} = \frac{BC}{AB}$$

cotangent of $\angle A = \cot\theta$

$$= \frac{\text{Base or adjacent side to angle A}}{\text{Perpendicular or opposite side to angle A}} = \frac{AB}{BC}$$

secant of $\angle A = \sec\theta$

$$= \frac{\text{Hypotenuse}}{\text{Base or adjacent side to angle A}} = \frac{AC}{AB}$$

cosecant of $\angle A = \operatorname{cosec}\theta$

$$= \frac{\text{Hypotenuse}}{\text{Perpendicular or opposite side to angle A}} = \frac{AC}{BC}$$

- It is clear from the above ratios that cosecant, secant and cotangent are the reciprocals of sine, cosine and tangent, respectively.

Also, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ **[Board 2024, 23, 18]**

- The trigonometric ratios of an **acute angle** in a right angled triangle express the relationship between the angle and length of its sides.

Acute angle: Acute angle is like the enthusiastic friend at a party–always small and lively, measuring less than 90 degrees.

- The value of trigonometric ratio of an angle does not depend on the size of the triangle but depends on the angle only.

$\angle A$	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined (∞)

cot A	Not defined (∞)	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined (∞)
cosec A	Not defined (∞)	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

[Board 2024, 23]

Note
defined (∞): Imagine you're trying to divide a huge, never-ending chocolate bar (∞) among zero friends. The chocolate is infinite, but there's no one to share it with. That's why mathematicians say it's not defined, a fancy way of saying, "This breaks all the rules of logic!"

• Since, the hypotenuse is the longest side in a right triangle, the value of sin A or cos A is always less than 1 (or, in particular, equal to 1)

MNEMONICS

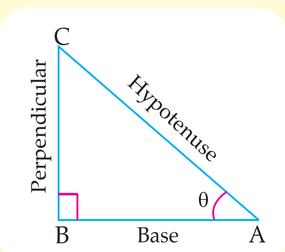
Concept: Trigonometric Ratios

In right angled ΔABC, we have

$\sin\theta = \frac{BC}{AC}$, $\cos\theta = \frac{BA}{AC}$, $\tan\theta = \frac{BC}{AB}$,

$\cot\theta = \frac{AB}{BC}$, $\sec\theta = \frac{AC}{BA}$, $\csc\theta = \frac{AC}{BC}$

sin	cos	tan
Pandit	Badri	Prasad
Har	Har	Bhole
cosec	sec	cot



Interpretation:

Here,

$$\sin\theta = \frac{\text{Pandit}}{\text{Har}} = \frac{P}{H} = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos\theta = \frac{\text{Badri}}{\text{Har}} = \frac{B}{H} = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BA}{AC}$$

$$\tan\theta = \frac{\text{Prasad}}{\text{Bhole}} = \frac{P}{B} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$$\cot\theta = \frac{\text{Bhole}}{\text{Prasad}} = \frac{B}{P} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC}$$

$$\sec\theta = \frac{\text{Har}}{\text{Badri}} = \frac{H}{B} = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{BA}$$

$$\csc\theta = \frac{\text{Har}}{\text{Pandit}} = \frac{H}{P} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC}.$$

Key Formula

$\sin\theta = \frac{P}{H}$, $\csc\theta = \frac{H}{P}$

$\cos\theta = \frac{B}{H}$, $\sec\theta = \frac{H}{B}$

$\tan\theta = \frac{P}{B}$, $\cot\theta = \frac{B}{P}$

Relation between Trigonometric Ratios:

$\sin\theta = \frac{1}{\csc\theta}$ or $\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ or $\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ or $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Fundamental Facts

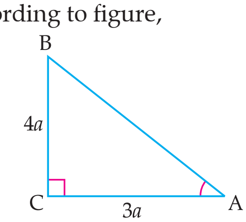
(1) The three basic functions in trigonometry are sine, cosine and tangent.

(2) Trigonometry, as the name might suggest, is all about triangles.

Example 1

Given, $\tan A = \frac{4}{3}$, find other trigonometric ratios of the angle A.

Sol. According to figure,



In right angle ΔABC

$$\tan A = \frac{4}{3} \quad \text{(given)}$$

As, $$\tan A = \frac{P}{B}$$

$$BC = 4a, AC = 3a$$

Now, in right angled triangle ABC

$$AB^2 = BC^2 + AC^2$$

(By using Pythagoras Theorem)

$$AB^2 = (4a)^2 + (3a)^2$$
$$AB^2 = 16a^2 + 9a^2$$
$$AB^2 = 25a^2$$
$$AB = 5a$$

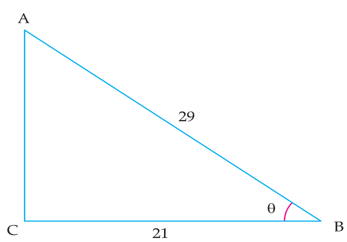
Thus, other trigonometric ratios are:

$$\sin A = \frac{P}{H} = \frac{BC}{AB} = \frac{4a}{5a} = \frac{4}{5}$$
$$\cos A = \frac{B}{H} = \frac{AC}{AB} = \frac{3a}{5a} = \frac{3}{5}$$
$$\cot A = \frac{1}{\tan A} = \frac{3}{4}$$
$$\sec A = \frac{1}{\cos A} = \frac{5}{3}$$
$$\text{cosec } A = \frac{1}{\sin A} = \frac{5}{4}.$$

Example 2

Consider ΔACB, right-angled at C in which AB = 29 units, BC = 21 units and $\angle ABC = \theta$ (see fig.). Determine the values of

(i) $\cos^2\theta + \sin^2\theta$, (ii) $\cos^2\theta - \sin^2\theta$.



Sol. In right angle ΔABC

$$AB^2 = AC^2 + CB^2 \quad \text{(By using Pythagoras Theorem)}$$
$$AC = AB^2 - CB^2$$
$$AC = \sqrt{AB^2 - CB^2}$$
$$= \sqrt{29^2 - 21^2}$$
$$= \sqrt{841 - 441}$$
$$= \sqrt{400}$$
$$AC = 20 \text{ units}$$

Thus,
$$\sin\theta = \frac{AC}{AB} = \frac{20}{29},$$
$$\cos\theta = \frac{BC}{AB} = \frac{21}{29}$$

Now,

(i) $$\cos^2\theta + \sin^2\theta = \left(\frac{21}{29}\right)^2 + \left(\frac{20}{29}\right)^2$$
$$= \frac{21^2 + 20^2}{29^2}$$
$$= \frac{441 + 400}{841}$$
$$= \frac{841}{841} = 1$$

(ii) $$\cos^2\theta - \sin^2\theta = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2$$
$$= \frac{21^2 - 20^2}{29^2}$$
$$= \frac{441 - 400}{841} = \frac{41}{841}$$

2 Trigonometric Identities

Concepts Covered:

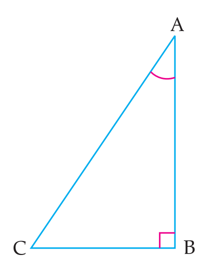
- Three important identities are: (i) $\sin^2\theta + \cos^2\theta = 1$, (ii) $1 + \tan^2\theta = \sec^2\theta$, (iii) $1 + \cot^2\theta = \text{cosec}^2\theta$.

• An equation is called an **identity** if it is true for all values of the variable(s) involved.

Identity:
An identity is like a superpower for math or even life! It's an equation that always holds true, no matter what values you plug in.

• An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle.

• In ΔABC, right-angled at B, by **Pythagoras Theorem,**



Trigonometric Identities

Scan Me!

$$AB^2 + BC^2 = AC^2 \quad \text{...(i)}$$

Dividing each term of (i) by AC^2,

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

or $$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$$

or $$(\cos A)^2 + (\sin A)^2 = 1$$

or $$\cos^2 A + \sin^2 A = 1 \quad \text{...(ii)}$$

[Board 2023, 2019]

• This is true for all values of A such that $0° \le A \le 90°$. So, this is a trigonometric identity. Now divide equation (i) by AB^2.

$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

or $$\left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2$$

or $$1 + \tan^2 A = \sec^2 A \quad \text{...(iii)}$$ **[Board 2020]**

• Is this equation true for $A = 0°$? Yes, it is. What about $A = 90°$? Well, tan A and sec A are not defined for $A = 90°$. So, eqn. (iii) is true for all values of A such that $0° \le A < 90°$.

Again, dividing eqn. (i) by BC^2

$$\frac{AB^2}{BC^2} + \frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

or $$\left(\frac{AB}{BC}\right)^2 + \left(\frac{BC}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2$$

or $$\cot^2 A + 1 = \text{cosec}^2 A \quad \text{...(iv)}$$ **[Board 2020, 2019]**

• Note that cosec A and cot A are not defined for all $A = 0°$. Therefore eqn. (iv) is true for all values of A such that $0° < A \le 90°$.

• Using these identities, we can express each trigonometric ratio in terms of other trigonometric ratios, i.e., if any one of the ratios is known, we can determine the **values** of other trigonometric ratios.

KEY FORMULAE

$\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, $\cot^2\theta + 1 = \text{cosec}^2\theta$.

Example 3

Express the ratio cos A, tan A and sec A in terms of sin A.

Sol. As we know that,

$$\cos^2 A + \sin^2 A = 1$$
$$\cos^2 A = 1 - \sin^2 A$$

$\Rightarrow$ $$\cos A = \pm\sqrt{1-\sin^2 A}$$

Hence, $$\cos A = \sqrt{1-\sin^2 A}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}}$$

And, $$\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1-\sin^2 A}}$$

Example 4

Prove that $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$, using the identity $\sec^2\theta = 1 + \tan^2\theta$.

Sol. $$\text{LHS} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

$$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta} = \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$$

$$= \frac{\{(\tan\theta + \sec\theta) - 1\}}{\{(\tan\theta - \sec\theta) + 1\}} \times \frac{\tan\theta - \sec\theta}{\tan\theta - \sec\theta}$$

$$= \frac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1}{\tan\theta - \sec\theta} = \frac{1}{\sec\theta - \tan\theta} = \text{RHS}$$

Tips & Tricks T&T

As, we have to convert L.H.S. in terms of sec θ and tan θ, divide numerator and denominator by cos θ.

Chapter 10

Revision Notes

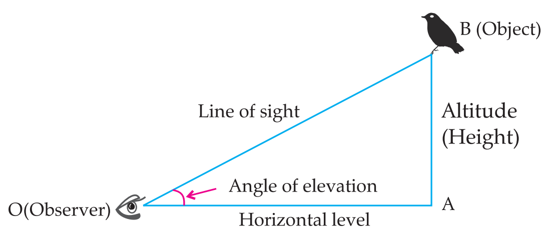
• The line of sight is the line drawn from the eye of an observer to the point on the object viewed by the observer.

Line of sight: It's the shortest distance between your eyes and the object you're focused on, connecting the observer and the target.
In geometry, it's often used to measure angles of elevation or depression, making it a key player in understanding perspective and vision.

• The angle of elevation of a point on the object being viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at a point on the object. **[Board, 2024, 23]**

Horizontal :
In mathematics, "horizontal" means lying flat like the horizon where the earth meets the sky.
It's like the calm surface of a still lake or a perfectly straight road stretching endlessly.
A horizontal line doesn't tilt; it stays steady, parallel to the x-axis on a graph.

• Line of sight, angle of elevation and altitude (height):



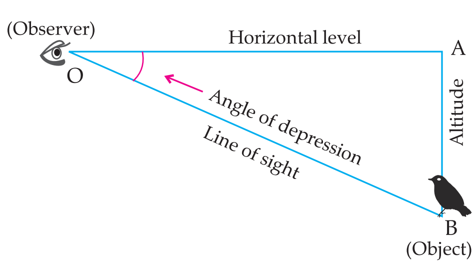
(i) $\angle AOB$ is the angle of elevation.

(ii) By height AB, means object is at point B from the point A located at the ground.

(iii) AO is the distance of the observer from the point A.

Observer :
An "observer" is like a curious traveller who watches and measures the world from a specific viewpoint.
Imagine standing at one spot and describing how objects appear to you—this perspective shapes what you see, how far things seem, or how angles form.
In geometry, physics, or calculus, the observer's position can change everything, from the way coordinates transform to how events are perceived in space and time.
It's a reminder that in math, just like in life, perspective matters!

• The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at a point on the object. **[Board, 2024, 23]**



Fundamental Facts

(1) The height of object above the water surface is equal to the depth of its image below the water surface.

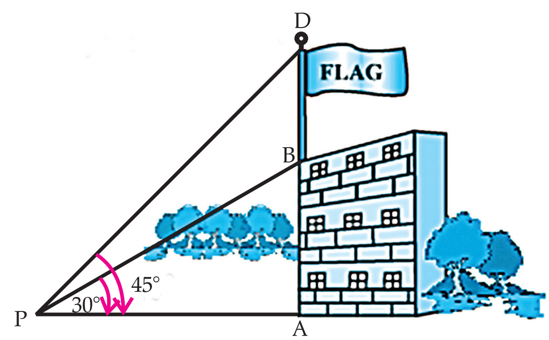
(2) The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

KEY FORMUALE

Angle of Elevation = Angle of Depression **[Board, 2024]**

Example 1

From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (Take $\sqrt{3}$ = 1.732)



Sol. Given: $AB = 10$ m

$\angle BPA = 30°$ and $\angle DPA = 45°$.

Now, in right angled ΔPBA, $\tan 30° = \dfrac{BA}{PA}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{AP}$$

$$\Rightarrow \quad AP = 10\sqrt{3} \text{ m}$$

Thus, distance of the building from point $P = 10\sqrt{3}$ m $= 17.32$ m.

Let $BD = x$ m

Thus, $AD = DB + BA = (10 + x)$ m

In right angled ΔPDA, $\tan 45° = \dfrac{AD}{AP} = \dfrac{10+x}{17.32}$

$$\Rightarrow \quad 1 = \frac{10+x}{17.32}$$

$\Rightarrow \quad 17.32 = 10 + x$

$\Rightarrow \quad x = 17.32 - 10$

$= 7.32$ m

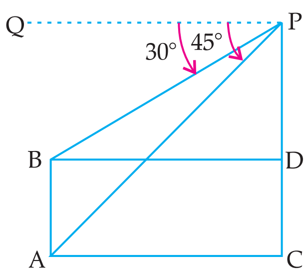
Thus, length of the flagstaff = 7.32 m.

Some Applications of Trigonometry

Scan Me!

Example 2

The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.



Sol. Given: $AB = 8$ m

$\angle QPB = 30°$ and $\angle QPA = 45°$.

To find: length of PC and AC.

As, PB is transversal of parallel lines QP and BD

$\therefore \quad \angle PBD = \angle QPB = 30°$ (alternate angles are equal)

Similarly, $\angle QPA = \angle PAC = 45°$

In right angled ΔPBD, $\tan 30° = \dfrac{PD}{BD}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{PD}{BD}$

$\Rightarrow \quad BD = \sqrt{3}PD \quad$ ...(i)

In right angled ΔPAC, $\tan 45° = \dfrac{PC}{AC}$

$\Rightarrow \quad 1 = \dfrac{PC}{AC}$

$\Rightarrow \quad AC = PC$

As, $\quad PC = PD + DC$

Thus, $\quad AC = PD + DC$

$\Rightarrow \quad BD = PD + DC$

From eq. (i) we get $\sqrt{3}PD = PD + DC$

$\Rightarrow \quad \sqrt{3}PD - PD = DC$

$\Rightarrow \quad PD(\sqrt{3}-1) = DC \quad$ (As $DC = AB = 8$ m)

$\Rightarrow \quad PD = \dfrac{8}{(\sqrt{3}-1)} = \dfrac{8}{(\sqrt{3}-1)} \times \dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$= \dfrac{8(\sqrt{3}+1)}{3-1} = 4(\sqrt{3}+1)$ m.

So, the height of multi-storeyed building

$(PC) = 4(\sqrt{3}+1)+8$

$= 4\{(\sqrt{3}+1)+2\} = 4(\sqrt{3}+3)$ m.

And, distance between the two building is also

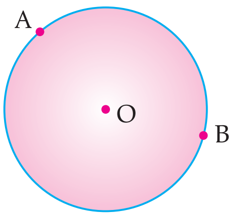
$(BD) = PD + DC$

$= 4(\sqrt{3}+1)+8 = 4(\sqrt{3}+3)$ m.

Chapter 11

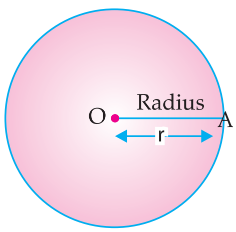
Revision Notes

- A circle is a collection of all points in a **plane** which are at a constant distance from a fixed point in the same plane.



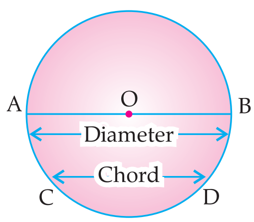
Plane: A plane is like an endless sheet of paper that stretches infinitely in all directions. It has no thickness and is perfectly flat, making it the ultimate "flat surface."

- A **line segment** joining the centre of the circle to a point on the circumference of the circle is called its radius.

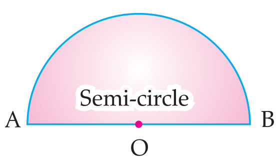


line segment: A line segment is like a perfectly straight path connecting two points, but unlike a line, it has a clear beginning and end.

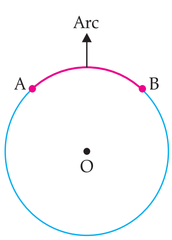
- A line segment joining any two points of a circle is called a chord. A chord passing through the centre of circle is called its diameter. A diameter is the longest chord of the circle. Here *AB* is a diameter.



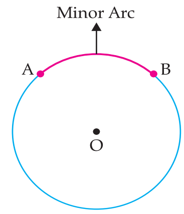
- A diameter of a circle divides a circle into two equal arcs, each known as a semi-circle.



- A part of a circumference of circle is called an arc.



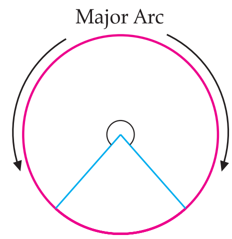
- An arc of a circle whose length is less than that of a semi-circle of the same circle is called a minor arc.



Areas related to circles

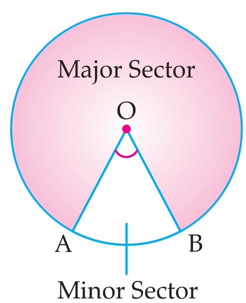
Scan Me!

- An arc of a circle whose length is greater than that of a semi-circle of the same circle is called a major arc.

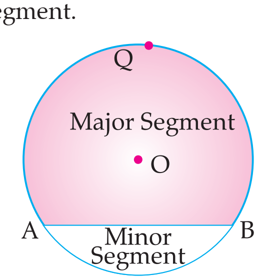


- The region bounded by an arc of a circle and two **radii** at its end points is called a sector.

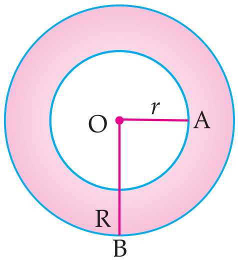
Radii: Radii (plural of radius) are straight lines that connect the centre of a circle or sphere to any point on its boundary. A radius is like a magical line that connects the heart of a circle (its centre) to any point on its edge. It's the backbone of the circle, quietly maintaining its perfect roundness. No matter where you go around the circle, the radius is always the same length Together, all the radii create the boundary of the circle, giving it its flawless symmetry.



- A chord divides the interior of a circle into two parts, each called a segment.



- Circles having the same centre but different radii are called concentric circles.



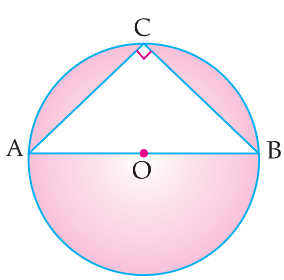
- Two circles (or arcs) are said to be **congruent** if on placing one over the other cover each other completely.

Congruent:
In mathematics, "congruent" means two shapes are like twins—they may have been moved, flipped or rotated, but they are still exactly the same in size and shape.
Congruence shows us that appearances may change, but the essence remains the same!

- The distance around the circle or the length of a circle is called its circumference or perimeter.
- The mid-point of the hypotenuse of a right triangle is equidistant from the vertices of the triangle.
- **Angle** subtended at the **circumference** by a diameter is always a right angle.

Angle:
Angle is the story-teller of shapes and directions. It's formed when two rays meet at a common point, whispering secrets about how wide or narrow their embrace is.
Angles come in all personalities—acute, right, obtuse, or even reflexive—each with its unique charm. They help us navigate the world, from building bridges to mapping stars, proving that even the smallest turn can create the biggest impact!

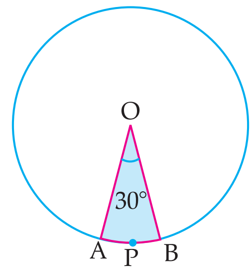
Circumference:
The circumference of a circle is like the circle's "journey"—the distance you travel if you walk around its edge once. Think of it as the circle's perimeter, wrapping all the way around.
It's tied to the magical number π, with a formula C = 2πr, where r is the radius



- Angle described by minute hand in 60 minutes is 360°.
- Angle described by hour hand in 12 hours is 360°.

Example 1

Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector (Use $\pi = 3.14$).



Sol. Given that,
radius of circle, $r = 4$ cm
Minor sector angle, $\theta = 30°$
We know that,
area of minor sector, $A_1 = \dfrac{\pi r^2\theta}{360°}$

$$A_1 = 3.14 \times (4)^2 \times \frac{30°}{360°}$$

$$= 3.14 \times 16 \times \frac{1}{12}$$

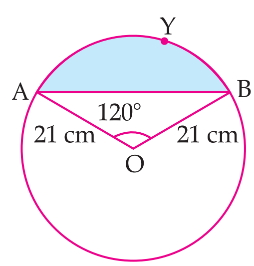
$$= 4.19 \text{ cm}^2$$

Now, area of corresponding major sector,
A_2 = Area of Circle – Area of Minor Sector

$A_2 = \pi r^2 - A_1$
$A_2 = 3.14 \times (4)^2 - 4.19$
$A_2 = 50.24 - 4.19$
$A_2 = 46.05 \text{ cm}^2$

Example 2

Find the area of the segment *AYB* shown in fig, if radius of the circle is 21 cm and $\angle AOB = 120°$. (Use $\pi = \dfrac{22}{7}$)



Sol. Given that, Radius of circle, r = 21 cm
$\angle AOB = 120°$
We know that,
Area of segment *AYB*, $A = \dfrac{\pi r^2\theta}{360°} - \dfrac{1}{2}r^2\sin\theta$
On putting the values,
Area of segment *AYB*

$$= \pi \times (21)^2 \times \frac{120°}{360°} - \frac{1}{2} \times (21)^2 \times \sin 120°$$

$$= \frac{22}{7} \times 441 \times \frac{1}{3} - \frac{1}{2} \times 441 \times \sin(90° + 30°)$$

$$= 22 \times 63 \times \frac{1}{3} - \frac{1}{2} \times 441 \times \cos 30°$$

$$\{\sin(90 + \theta) = \cos\theta\}$$

$$= 22 \times 21 - \frac{1}{2} \times 441 \times \frac{\sqrt{3}}{2}$$

$$= 462 - 220.5 \times 0.866$$

$$= 462 - 190.953 = 271.047 \text{ cm}^2$$

Key FORMULAE

1. Circumference (perimeter) of a circle = πd or $2\pi r$, where d is diameter and r is the radius of the circle.

2. Area of a circle = πr^2.

3. Area of a semi-circle = $\frac{1}{2}\pi r^2$. **[Board, 2023]**

4. Perimeter of a semi-circle = $\pi r + 2r = (\pi + 2)r$ **[Board, 2023]**

5. Area of a ring or an annulus = $\pi(R + r)(R - r)$. where R is the outer radius and r is the inner radius.

6. Length of arc, $l = \frac{2\pi r\theta}{360°}$ or $\frac{\pi r\theta}{180°}$, where θ is the angle subtended at centre by the arc. **[Board, 2023]**

7. Area of a sector = $\frac{\pi r^2\theta}{360°}$ or area of sector = $\frac{1}{2}(l \times r)$, where l is the length of arc. **[Board, 2023]**

8. Area of minor segment = $\frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2 \sin\theta$

9. Area of major segment = Area of the circle – Area of minor segment = πr^2 – Area of minor segment.

10. If a chord subtends a right angle at the centre, then area of the corresponding segment $= \left[\frac{\pi}{4} - \frac{1}{2}\right]r^2$

11. If a chord subtends an angle of 60° at the centre, then area of the corresponding segment $= \left[\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right]r^2$

12. Distance moved by a wheel in 1 revolution = Circumference of the wheel.

13. Number of revolutions (made by a wheel) in one minute $= \frac{\text{Distance moved in 1 minute}}{\text{Circumference}}$

14. Perimeter of a sector $= \frac{\pi r\theta}{180°} + 2r$

MNEMONICS

Concept : π **value**

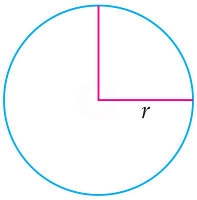
"Can i have a small container of coffee" this mnemonic help us in getting the value of π = 3.14159 ....

	CAN	I	HAVE	A	SMALL	CONTAINER	OF	COFFEE
No. of Letters →	↓ 3	↓ 1	↓ 4	↓ 1	↓ 5	↓ 9	↓ 2	↓ 6

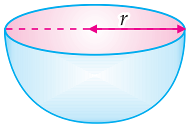
Chapter 12

Revision Notes

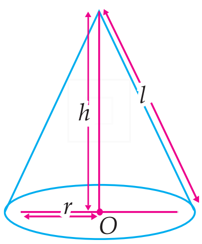
- A sphere is a perfectly round geometrical object in **three-dimensional space**.



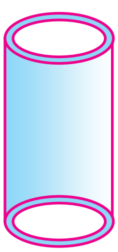
- A hemisphere is half of a sphere.



- A cone is a three dimensional geometric shape tapers smoothly from a flat base to a point called the apex or vertex.



- A cylinder is a solid or a **hollow** object that has a circular base and a circular top of the same size.



Surface area and volume of combination of solids

Scan Me!

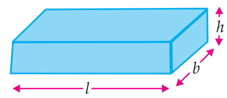
Three dimensional space: Imagine a giant invisible box where everything in the universe happens! This magical box has three directions you can move: left-right (x-axis), forward-backward (y-axis) and up-down (z-axis). Unlike flat 2D paper, 3D space gives depth, letting objects exist with height, width and depth.

Hollow: In the world of mathematics, particularly in 3D geometry, "hollow" is a term that breathes life into shapes by giving them an interior void—a space to dream within their boundaries. Imagine a sphere that isn't solid but instead cradles an empty core, or a cylinder that's all about the shell and nothing about the filler.

Curve Surface Area: Imagine the curved surface area as the cosy sweater that hugs a 3D shape! It's the soft, smooth part of the object that wraps around, leaving the flat faces out of the picture. For a cone, think of it as the ice cream cone's cone (without the scoop!) and for a cylinder, it's like the label wrapping around a can.

Key Formulae

- **Cuboid:**



Here, l is length, b is breadth and h is height of the cuboid.

Lateral surface area or area of four walls $= 2(l + b)h$

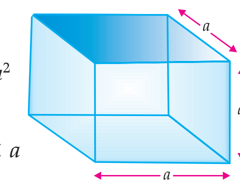
Total surface area $= 2(lb + bh + hl)$

Volume $= l \times b \times h$

Diagonal $= \sqrt{l^2 + b^2 + h^2}$ **[Board 2020, 2017]**

- **Cube:**

Here, a is edge of cube.



Lateral surface area or area of four walls $= 4 \times a^2$

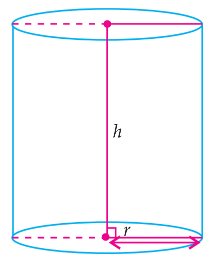
Total surface area $= 6 \times a^2$

Volume $= a^3$

Diagonal of a cube $= \sqrt{3} \times a$

- **Right Circular Cylinder**:

Here, r is the radius of base and h is the height of the right circular cylinder.



Area of base or top face $= \pi r^2$

Area of curved surface or **curved surface area**

$= $ perimeter of the base $\times$ height

$= 2\pi rh$

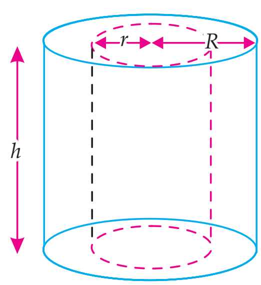
Total surface area (including both ends)

$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

Volume $=$ (Area of the base $\times$ height) $= \pi r^2 h$

[Board 2023, 2020, 2019]

- Right Circular Hollow Cylinder:



$R =$ outer radius
$r =$ inner radius
$h =$ height of the cylinder

Here, R and r are the external and internal radii and h is the height of the right circular hollow cylinder.

Total surface area $=$ External surface area $+$ Internal surface area $+$ Area of brim

Fundamental Facts

The surface area of a solid object is a measure of the total area that the surface of the object occupies.

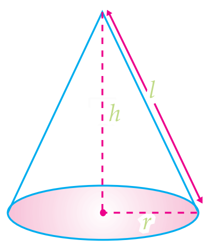
$= (2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)$
$= [2\pi h(R + r) + 2\pi(R^2 - r^2)]$
$= [2\pi(R + r)(h + R - r)]$

Curved surface area $= (2\pi Rh + 2\pi rh) = 2\pi h(R + r)$

Volume of the material used = (External volume) – (Internal volume)

$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$

- **Right Circular Cone:**



Here, r, h and l are the radius, vertical height and slant height respectively of the right circular cone.

Slant height, $l = \sqrt{h^2 + r^2}$

Area of curved surface $= \pi rl$

$= \pi r\sqrt{h^2 + r^2}$ **[Board, 2022]**

Total surface area= Area of curved surface + Area of base

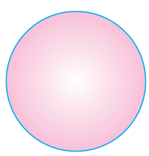
$= \pi rl + \pi r^2$
$= \pi r(l + r)$

Volume $= \frac{1}{3}\pi r^2 h$ **[Board, 2023, 2020]**

- **Sphere:**

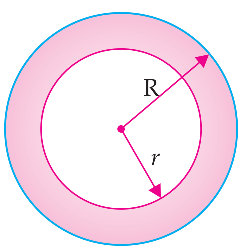
Surface area $= 4\pi r^2$

Volume $= \frac{4}{3}\pi r^3$



Here, r is the radius of the sphere.

- **Spherical Shell:**



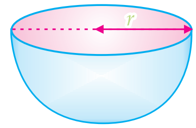
Here, R and r are the external and internal radii of the **spherical shell**.

Surface area (outer)= $4\pi R^2$

Volume of material

$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(R^3 - r^3)$

- **Hemisphere:**



Here, r is the radius of the hemisphere.

Area of curved surface $= 2\pi r^2$

Total surface area = Area of curved surface + Area of base

$= 2\pi r^2 + \pi r^2$
$= 3\pi r^2$

Volume $= \frac{2}{3}\pi r^3$ **[Board, 2019]**

MNEMONICS

Mnemonics: Circles Can Be Round or Straight!

Interpretations: For a cylinder, we remember the shape of a "can" or cylindrical container that can hold liquid, sphere is symmetrical, like a ball in the Sky!

Interpretations: For a sphere, we remember the shape's symmetry and its ball-like appearance.

$\frac{\text{Sphere}}{2} = \text{Hemisphere}$

Half of Sphere is Hemisphere

Volume of Sphere $= \frac{4}{3}\pi r^3$

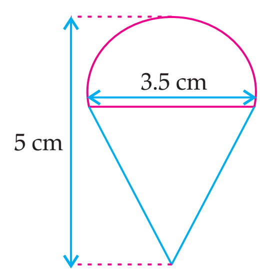
Volume of Hemisphere

$= \frac{\text{Volume of Sphere}}{2} = \frac{2}{3}\pi r^3$

Example 1

Rasheed got a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour.

(Take $\pi = \frac{22}{7}$)



Sol. Given that, diameter of top = 3.5 cm

Radius of top, $r = 1.75$ cm

Area that Rasheed has to colour, A

= CSA of Cone + CSA of hemisphere

$$A = \pi rl + 2\pi r^2$$

Now, height of cone,

$$h = 5 - 1.75 = 3.25 \text{ cm}$$

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{(1.75)^2 + (3.25)^2}$$

$$= \sqrt{13.625} = 3.7 \text{ cm}$$

We have, $A = \pi r(l + 2r)$

$$= \frac{22}{7} \times 1.75 \times (3.7 + 3.5)$$

$$= \frac{22}{7} \times 1.75 \times 7.2$$

$$= \frac{277.2}{7}$$

$$= 39.6 \text{ cm}^2$$

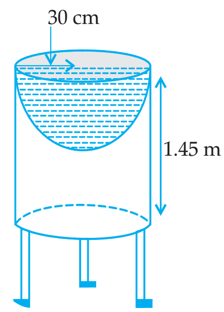
Example 2

Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath. (Use $\pi = \frac{22}{7}$)

Sol. Given that, height of cylinder,

$$h = 1.45 \text{ m} = 145 \text{ cm}$$

Radius of cylinder, $r = 30$ cm



Total surface area of bird bath

= CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 30 \times (145 + 30)$$

$$= \frac{44}{7} \times 30 \times 175$$

$$= \frac{231000}{7}$$

$$= 33000 \text{ cm}^2$$

$$= 3.3 \text{ m}^2$$

Hence, total surface area of bird bath = 3.3 m².

Chapter 13

Revision Notes

- Statistics deals with the collection, presentation and analysis of numerical data.
- Three measures of **central tendency** are:
  (i) Mean, **(ii)** Median and **(iii)** Mode.
- **Mean:** Mean in statistics is the average value of a set of numbers, calculated by summing all values and dividing by the total count of values.

i.e., $\text{Mean} = \dfrac{\text{Sum of all observations}}{\text{No. of observations}}$

[Board, 2024]

Central Tendency: Central tendency is like finding the "centre" of a data set, the point that centre best represents the entire collection of numbers. The three common measures of central tendency are the mean, median and mode. The mean is the average, the median is the middle value, and the mode is the most frequent value.

Mean is calculated using two different methods:

(i) Direct method

(ii) Assumed mean method

The choice of the method depends on the numerical values of x_i and f_i

STATISTICS

Scan Me!

- **Median:** It is defined as the middle most or the central value of the variable in a set of observations, when the observations are arranged either in ascending or descending order of their magnitudes.

It divides the arranged series in two equal parts i.e., 50% of the observations lie below the median and the remaining are above the median.

[Board, 2024]

- **Mode:** Mode is the observation which occurred maximum times. In **ungrouped data**, mode is the observation having maximum frequency. In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. To find the mode of grouped data, locate the class with the maximum frequency. This class is known as the modal class. The mode of the data is a value inside the modal class. **[Board, 2024]**

Ungrouped Data: Ungrouped data is like a collection of individual puzzle pieces, each standing alone without any organisation. It's a set of raw numbers, values or observations that haven't been grouped into categories or ranges. Imagine a list of students' ages: 18, 22, 20, 19, 25.

Key Formulae

- **Mean:**

(a) For Ungrouped Data:

If n observations $x_1, x_2, ..., x_n$ are given, then their arithmetic mean is given by :

$$\frac{x_1 + x_2 + ... + x_n}{n}$$

(b) For Grouped Data:

To find the mean of grouped data, it is assumed that the frequency of each class-interval is centred around its mid-point. The methods to find mean are:

(i) Direct Method:

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

where the x_i (class mark) is the mid-point of the i^{th} class interval and f_i is the corresponding frequency.

$$\text{class mark} = \frac{\text{upper limit} + \text{lower limit}}{2}$$

(ii) Assumed Mean Method or Short-cut Method:

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

where, A is the assumed mean and $d_i = x_i - A$ are the deviations of x_i from A for each i.

Grouped Data: Grouped data is like organising a messy pile of information into neat, manageable categories. Imagine you have a bunch of test scores scattered everywhere, but instead of looking at each score individually, you group them into ranges—like 0–10, 11–20 and so on.

- Median of **Grouped Data:**

First find **cumulative frequencies** of all classes which is N and then calculate, $\frac{N}{2}$ and the class in which $\frac{N}{2}$ lies. This class is known as the median class. Median of the given distribution lies in this class.

Median of the grouped data can be calculated using the formula:

$$\text{Median } (M_e) = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$$

where l = lower limit of median class,

f = frequency of median class,

N = number of observations,

$c.f.$ = cumulative frequency of the class preceding the median class,

h = class-size or width of the class-interval.

Cumulative Frequency: Cumulative frequency is like a running total in statistics. Imagine you're counting how many people in a group are taller than a certain height. Instead of just counting once, you keep adding to the total as you go along, creating a cumulative count.

- **Mode of Grouped Data:**

Mode of the grouped data can be calculated by using the formula:

$$\text{Mode } (M) = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where, l = lower limit of the modal class,

h = width or size of the class-interval,

f_1 = frequency of the modal class,

f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class succeeding the modal class.

- **Empirical relation between mean, median and mode:**

(i) Mode = 3 median – 2 mean **[SQP, 2023–24]**

(ii) $\text{Median} = \frac{1}{3}\text{ mode} + \frac{2}{3}\text{ mean}$

(iii) $\text{Mean} = \frac{3}{2}\text{ median} - \frac{1}{2}\text{ mode}$

Fundamental Facts

- The mean is the average of a data set.
- The mode is the most common number in a data set.
- The median is the middle of the set of numbers.

- Class intervals should be continuous while calculating mean and median for grouped data.
- Cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class.

MNEMONICS

Concept - Relation between mean, median and mode
"3 Mediators Minus 2 Managers Make the Mood"

3 Mediators = 3 Median
2 Managers = 2 Mean
Mood = Mode
Thus, Mode = 3 Median – 2 Mean.

Example 1

The marks obtained by 30 students of Class X of a certain school in a Mathematics paper consisting of 100 marks are presented in table below. Find the mean of the marks obtained by the students by following two methods:

(i) Direct Method **(ii) Assumed Method**

Marks obtained (x_i)	10	20	30	40	50	56	60	70	72	80	88	92	95
Number of students (f_i)	1	1	3	4	3	2	4	4	1	1	2	3	1

Sol. **(a)** First convert the ungrouped data into grouped data by forming class-interval.
(b) Secondly students falling in any upper class-limit would be considered in the next class.
Here we take $a = 47.5$, $h = 15$, then $d_i = x_i - 47.5$

Class-Interval	f_i	x_i	$d_i = x_i - a$	$f_i x_i$	$f_i d_i$
10 – 25	2	17.5	–30	35	– 60
25 – 40	3	32.5	–15	97.5	– 45
40 – 55	7	47.5	0	332.5	0
55 – 70	6	62.5	15	375	90
70 – 85	6	77.5	30	465	180
85 – 100	6	92.5	45	555	270
	$\Sigma f_i = 30$			$\Sigma f_i x_i = 1860$	$\Sigma f_i d_i = 435$

Now, **(i)** Using Direct Method: $\bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{1860}{30} = 62$

(ii) Using Assumed Method: $\bar{x} = a + \dfrac{\Sigma f_i d_i}{\Sigma f_i} = 47.5 + \dfrac{435}{30} = 47.5 + 14.5 = 62$

Example 2

A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household:

Family size	1 – 3	3 – 5	5 – 7	7 – 9	9 – 11
Number of families	7	8	2	2	1

Find the mode of this data.

Sol. As, maximum class frequency is 8 here and the corresponding class to this frequency is 3 – 5.
So, the modal class is 3 – 5.
So, here lower limit (l) of modal class = 3,
Class size (h) = 2
Frequency (f_0) = 7, $f_1 = 8$, $f_2 = 2$

Now, mode $= l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

$$= 3+\left(\frac{8-7}{2\times 8-7-2}\right)\times 2$$

$$= 3+\frac{1}{7}\times 2 = 3+\frac{2}{7} = 3.286$$

Therefore, the mode of the data = 3.286.

Example 3

A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained:

Height (in cm)	Less than 140	Less than 145	Less than 150	Less than 155	Less than 160	Less than 165
Number of girls	4	11	29	40	46	51

Find the median height of girls.

Sol. As, the given distribution is of less than type so, change it into class intervals having 140,145 ……. as upper class limit.

Also numbers of girls are in increasing order therefore, it is cumulative type of frequency so change it into frequency as shown in table:

Class interval	Below 140	140 – 145	145 – 150	150 – 155	155 – 160	160 – 165
Frequency	4	7	18	11	6	5
Cumulative frequency	4	11	29	40	46	51

$$\text{Median} = l+\left(\frac{\frac{N}{2}-cf}{f}\right)\times h$$

Here, $N = 51$

So, $\frac{N}{2} = \frac{51}{2} = 25.5$

$\therefore$ Median class = 145 – 150

lower limit (l) = 145

cf = 11 (cumulative frequency of preceding class)

f = 18

class size (h) = 5

$$\text{Thus, Median} = 145+\left(\frac{25.5-11}{18}\right)\times 5$$

$$= 145+\frac{72.5}{18} = 149.03$$

Hence, median height of the girls is 149.03.

Chapter 14

Revision Notes

- Probability is a branch of mathematics that deals with calculating the likelihood of a given event's occurrence.
- A **random experiment** is an experiment or a process for which the **outcome** cannot be predicted with certainty. e.g.,
  (i) tossing a coin, **(ii)** throwing a dice, **(iii)** selecting a card and **(iv)** selecting an object etc.

Probability Part 1

Scan Me!

- Outcome associated with an experiment is called an event. e.g., **(i)** Getting a head on tossing a coin, **(ii)** getting a face card when a card is drawn from a pack of 52 cards.
- The events whose probability is one are called sure/certain events.
- The events whose probability is zero are called impossible events.
- An event with only one possible outcome is called an elementary event.
- In a given experiment, if two or more events are equally likely to occur or have equal probabilities, then they are called equally likely events.
- Probability of an event always lies between 0 and 1.
- Probability can never be negative and more than one.
- A pack of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. Each suit consists of an ace, one king, one queen, one jack and 9 other cards numbered from 2 to 10. Four suits are spades, hearts, diamonds and clubs.
- King, queen and jack are face cards.
- The sum of the probabilities of all elementary events of an experiment is 1.
- Two events A and B are said to be complementary to each other if the sum of their probabilities is 1.

Probability Part 2

Scan Me!

- Probability of an event E, denoted as $P(E)$, is given by:

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total possible number of outcomes}}$$

[Board 2024]

For an event E, $P(\overline{E}) = 1 - P(E)$, where the event $\overline{E}$ representing 'not E' is the complement of the event E.

[Board 2024]

- For A and B two possible outcomes of an event,
  (i) If $P(A) > P(B)$, then event A is more likely to occur than event B.
  (ii) If $P(A) = P(B)$, then events A and B are equally likely to occur.

Random Experiment: A random experiment is like rolling the dice of uncertainty! It's an action or process where you know all the possible outcomes, but you can't predict which one will happen in any single try. For example, tossing a coin, rolling a dice or drawing a card from a shuffled deck – you know the possibilities (heads or tails, numbers 1 to 6 or 52 cards), but the result remains a mystery until it happens.

Outcome: An outcome is the final destination of possibilities—a snapshot of what happens when the dice of life roll or the gears of action turn.

Know the Facts

- The experimental or empirical probability of an event is based on what has actually happened while the theoretical probability of an event attempts to predict what will happen on the basis of certain assumptions.
- As the number of trials in an experiment go on increasing, we may expect the experimental and theoretical probabilities to be nearly the same.
- When we speak of a coin, we assume it to be 'fair' i.e., it is symmetrical so that there is no reason for it to come down more often on one side than the other. We call this property of the coin as being 'unbiased'.
- In the case of experiment we assume that the experiments have equally likely outcomes.
- A deck of playing cards consists of 4 suits : spades (♠), hearts (♥), diamonds (♦) and clubs (♣). Clubs and spades are black while hearts and diamonds are red in colour.

[Board 2024]

Fundamental Fact: By the phrase 'random toss', we means that the coin is allowed to fall freely without any bias or interference.

Key Formula

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Number of all possible outcomes of the experiment}}$$

$0 \le P(E) \le 1$

MNEMONICS

Zero < Probability < One

The probability of any event always lies between 0 and 1 (inclusive).

For example,

Probability of a sure event is 1 and that of an impossible event is 0.

Example 1

Suppose we throw a dice once.

(i) What is the probability of getting a number greater than 4?

(ii) What is the probability of getting a number less than or equal to 4?

Sol. Total number of possible outcomes $n(s) = 6$

(i) Number greater than 4 on dice = 5, 6

$\therefore$ Number of favourable outcomes $n(E) = 2$

So, required probability $= \frac{n(E)}{n(S)}$

$$= \frac{2}{6} = \frac{1}{3}$$

(ii) Number less than or equal to 4 on dice = 1, 2, 3, 4.

$\therefore$ Number of favourable outcomes $n(E) = 4$

So, required probability $= \frac{n(E)}{n(S)}$

$$= \frac{4}{6} = \frac{2}{3}$$

Example 2

One card is drawn from a well-shuffled deck of 52 playing cards. Calculate the probability that the card will

(i) be an ace,

(ii) not be an ace.

Sol. Total number of possible outcomes $n(S) = 52$.

(i) Number of ace in pack of cards = 4

$\therefore$ Number of favourable outcomes $n(E) = 4$

So, required probability $= \frac{n(E)}{n(S)}$

$$= \frac{4}{52} = \frac{1}{13}$$

(ii) Number of cards without ace = 52 – 4 = 48

$\therefore$ Number of favourable outcomes $n(E) = 48$

So, required probability $= \frac{n(E)}{n(S)}$

$$= \frac{48}{52}$$

$$= \frac{12}{13}$$

Example 3

Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?

Sol. Let probability of Sangeeta's winning the match $= P(S)$

And, probability of Reshma winning the match $= P(R)$

Given, $P(S) = 0.62$

So, $P(R) = 1 - P(S)$

[$\because$ $P(S)$ and $P(R)$ are complementary]

$= 1 - 0.62 = 0.38$

Thus, probability of Reshma winning the match $P(R) = 0.38$