Quadratic Equations: Learn Concepts, Solved Examples, & Preparation Strategies

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Quadratic equations in algebra is the most frequently used topic. It is a very important topic from the perspective of competitive and entrance examinations. Quadratic equation is the basics of algebra and one must be familiar with it to be able to score well in the algebra section of any exam. Once the basics are clear, the quadratic equation becomes very easy. However, continuous practice is the key to success in quadratic.

In this article, we will describe a quadratic equation in one variable and cover the different methods of finding the roots of a quadratic equation in one variable, the nature of the roots, some solved examples, and word problems.

History of Quadratic Equations

Quadratic equations were first solved by the ancient Babylonian mathematicians. Later, Indian mathematician, Brahmagupta in around A.D. 598 – 665 gave an explicit formula to solve a quadratic equation in one variable of the form $ax^2 + bx + c = 0$ where a is non-zero. Further in A.D. 1025, Sridhara Acharya derived a formula for finding the roots of a quadratic equation, by the method of completing the roots.

When you've finished with Quadratic Equations, you can read learn how to find roots of quadratic equations concepts in depth here!

What are Quadratic Equations?

When the value of a polynomial of the form $ax^2 + bx + c$ is equal to zero, then it is known as a quadratic equation. That is, $ax^2 + bx + c = 0$ is a quadratic equation in one variable that is x, where a, b, c are real numbers and a is a non-zero. The equation $ax^2 + bx + c = 0$ is the **general** or **standard form**.

#Tip: For example: $x^2 + 5x + 12 = 0$ and $5x^2 - 4x - 16 = 0$ are quadratic equations in the variable x.

In examinations, you must simplify a given equation before deciding whether it is quadratic or not.

How to Solve a Quadratic Equation

There are two ways to solve a quadratic equation:

1. Factorisation Method
2. Using Formula- Sridharacharya Formula

discuss both these methods in more detail along with some examples.

Factorisation Method

A quadratic equation in one variable can be solved by factorising the equation, i.e., by making factors out of the equation.

Consider the equation $x^2 - 7x + 12 = 0$, it can also be written as (x-3)(x-4)=0. Now this factorised equation can be very easily solved using a property of real numbers called the **zero-product rule**.

Zero-product Rule

If a and b are two real numbers or algebraic expressions and if ab=0, then either a=0or b=0 or both a=0 and b=0 .Using this zero-product rule, the solutions of the equation (x-3)(x-4)=0 can be obtained by keeping each factor equal to zero and then solving the equation for x. Hence, we get:

x-3=0 or x-4=0

x=3 or x=4.

Hence, the roots of the equation are 3,4.

Solving a quadratic equation by Factorisation
Procedure:

1. If there are any fractions, clear them.
2. Write the equation in the standard form, i.e., $ax^2 + bx + c = 0$
3. Factorise L.H.S into a product of two linear factors
4. Use the zero-product rule, i.e., put each linear factor equal to zero.
5. Solve the resulting linear equations.

#Tip- check your answer by substituting the value in the original equation.

Solved Examples

Solve the following equations
Example 1: $ax^2 = 3x$
Solution: Given $ax^2 = 3x$
$\Rightarrow 2x^2 - 3x = 0 \quad (writing as\, ax^2 + bx + c = 0)$
$\Rightarrow x(2x - 3) = 0 \quad (factorising left side)$
$\Rightarrow x = 0 or 2x - 3 = 0 \quad (zero - product rule)$
$\Rightarrow x = 0 or x = \frac{3}{2}$

Solution: Given(x+3) (x-3)=40

x2-9=40 x2-9-40=0
$x2 - 49 = 0(writingas\, ax^2 + bx + c = 0)$

(x+7) (x-7)=0 (factorising left side)

x+7=0 or x-7=0 (zero-product rule)

x=-7 or x=7.

Hence, the roots of the given equation are -7, 7.

Example 3. $\sqrt{2x + 9} = 13 - x$
Solution: Given $\sqrt{2x + 9} = 13 - x$
Squaring both sides, we get
$2x + 9 = (13 - x)^2$
$\Rightarrow 2x + 9 = 169 - 26x + x^2$
$\Rightarrow -x^2 + 28x - 160 = 0$
$\Rightarrow x^2 - 28x + 160 = 0$
$\Rightarrow (x - 8)(x - 20) = 0 \quad (factorisingleftside)$
$\Rightarrow x - 8 = 0orx - 20 = 0 \quad (zero - productrule)$
$\Rightarrow x = 8orx = 20$

But x=20 does not satisfy the given equation, so it is rejected. Hence, the root of the given equation is 8.

#Tip- in questions like example 3, when squaring of the equation is done, then the roots of the final equation must be verified to determine whether they are roots of the original equation as well or not.

• Sridharacharya Formula

Solving a quadratic equation using the Sridharacharya formula can be tricky in terms of calculations but it is comparatively faster.

Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solving a Quadratic Equation using the Sridharacharya Formula Procedure:
Step1. If the equation consists of any fractions or square roots, then clear them
Step2. Write the equation in standard form, i.e., $ax^2 + bx + c = 0, a \neq 0$
Step3. Compare the equation formed in step 2 with the standard equation
Step4. Use the Sridharacharya formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solved Examples

Solution: Comparing the given equation with $ax^2 + bx + c = 0$, we get

$\Rightarrow a = 1, b = -4, c = 1$

$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4.1.1}}{2 \times 1} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$

$\Rightarrow x = 2 \pm \sqrt{3}$

Hence, the roots of the given equation are $2 + \sqrt{3}, 2 - \sqrt{3}$.

Example 2. $3x^2 - 4x - 4 = 0$

Solution: Comparing the given equation with $ax^2 + bx + c = 0$, we get

$\Rightarrow a = 3, b = -4, c = -4$

$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4.3.(-4)}}{2 \times 3} = \frac{4 \pm \sqrt{16 + 48}}{6} = \frac{4 \pm \sqrt{64}}{6} = \frac{4 \pm 8}{6}$

$\Rightarrow \frac{4+8}{6}, \frac{4-8}{6} = \frac{12}{6}, \frac{-4}{6} = 2, -\frac{2}{3}$.

Hence, the roots of the given equation are $2, -\frac{2}{3}$.

Example 3. $2x^2 + \sqrt{7}x - 7 = 0$

Solution: Comparing the given equation with $ax^2 + bx + c = 0$, we get

$\Rightarrow a = 2, b = \sqrt{7}, c = -7$

$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-\sqrt{7} \pm \sqrt{(\sqrt{7})^2 - 4.2.(\sqrt{7})}}{2 \times 2} = \frac{\sqrt{7} \pm \sqrt{63}}{4}$

$\Rightarrow x = \frac{\sqrt{7} \pm 3\sqrt{7}}{4} = \frac{2\sqrt{7}}{4}, \frac{-4\sqrt{7}}{4} = \frac{\sqrt{7}}{2}, -\sqrt{7}$

Hence, the roots of the given equation are $\frac{\sqrt{7}}{2}, -\sqrt{7}$.

Word Problems on Quadratic Equation

A lot of daily life problems can be solved using quadratic equations. To solve any kind of word problem, first, you must translate the words into an algebraic equation and then determine which method to apply to solve the equation so formed.

Solving Word Problems

Word problems are not specific to any one type. There can be innumerable types of word problems. Hence, there is no fixed technique or method to solve word problems. It's all about logic and practice. However, if you keep the following points in mind, then it will be easy for you to solve any kind of word problem.

1. First, you must carefully read and then reread the question and see what the question is asking for and what quantity is to be found.
2. Write whatever information is given in the question.

find the final answer.

#Tip- Check the answers by substituting them in the original equation to see if they satisfy the equation or not.

If you've learned Quadratic Equations, you can move on to learn about Average formulas in details here!

Solved Examples

The following sample word problems are the types that usually are a part of the Quantitative Aptitude section of various competitive examinations.

Example1. Nine times a certain number is equal to five less than twice the square of the number. Find the number.

Solution: Let the required whole number be x.

According to the question,
$\Rightarrow 9x = 2x^2 - 5 \Rightarrow 2x^2 - 9x - 5 = 0$
$\Rightarrow (x-5)(2x+1) = 0$
$\Rightarrow x - 5 = 0$ or $2x + 1 = 0$
$\Rightarrow x = 5$ or $x = -\frac{1}{2}$

Since x is supposed to be a whole number, the answer, i.e., the required whole number is 5.

Example2. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.
Solution: Let the natural number be x.

According to the question,
$\Rightarrow x^2 - 84 = 3(x+8)$
$\Rightarrow x^2 - 3x - 108 = 0 \Rightarrow x^2 - 12x + 9x - 108 = 0$
$\Rightarrow x(x-12) + 9(x-12) = 0 \Rightarrow (x-12)(x+9) = 0$
$\Rightarrow x - 12 = 0$ or $x + 9 = 0 \Rightarrow x = 12$ or $x = -9$.

Since, x is supposed to be a natural number, we will not consider x=-9. Hence, the required natural number is 12.

Example3. The sum of two natural numbers is 8 . If the sum of their reciprocals is $\frac{8}{15}$
Solution: Let the numbers be x and 8-x where $x \in N$ and $x < 8$.
According to the question

$\Rightarrow \frac{1}{x} + \frac{1}{8-x} = \frac{8}{15}$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow (x-3)(x-5) = 0$

$\Rightarrow x - 3 = 0$ or $x - 5 = 0$

$\Rightarrow x = 3$ or $x = 5$.

When x=3, then the numbers are 3,8-3 i.e. 3,5.

When x=5, then the numbers are 5,8-5 i.e. 5,3.

Hence, the required numbers are 3,5 .

We hope you found this article useful for your preparations. You can contact us if you have any doubts regarding this topic or any related topics. You can also download the Testbook App, which is absolutely free to practice more questions of quadratic equations. You can also attempt mock test series on the app for your preparation for competitive exams.

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Quadratic Equations FAQs

Q.1 What is a quadratic equation?

Ans.1 The equation of the form $ax^2 + bx + c = 0$ is a quadratic equation in one variable that is x, where a, b, c are real numbers and a is a non-zero.

Q.2 How to solve a quadratic equation?

Ans.2 A quadratic equation in one variable can be solved using two methods: Factorisation method and Using Sridharacharya formula.

Q.3 What is the Sridharacharya formula?

Ans.3 The Sridharacharya formula is: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Q.4 What is the Sridharacharya formula used for?

Ans.4 The Sridharacharya formula is used to find the roots of a quadratic equation.

Q.5 What is the zero-product rule?