Supplementary Material for: Deep Learning Model Compression with Rank Reduction in Tensor Decomposition

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I. Introduction

This supplemental materials contain all detailed proofs in the original paper.

A. Proof of Proposition 1.

Proof. Mathematically, the convolution using im2col can be expressed as

$$\mathcal{Y}_{(1)} = \mathcal{W}_{(1)} \cdot \text{im2col}(\mathcal{X}), \tag{1}$$

where $\text{im2col}(\mathcal{X}) \in \mathbb{R}^{chw \times h_o w_o}$ and $\mathcal{W}_{(1)} \in \mathbb{R}^{q \times chw}$ is mode-1 unfold of tensor $\mathcal{W}$.

The kernel $\mathcal{W}$ with multilinear rank of (r_1, r_2, r_3, r_4) can be decomposed as $\mathcal{W} = \mathcal{G} \times_1 \boldsymbol{U}_1 \times_2 \boldsymbol{U}_2 \times_3 \boldsymbol{U}_3 \times_4 \boldsymbol{U}_4$. Then, we can express the mode-1 unfolding of $\mathcal{W}$ as

$$\mathcal{W}_{(1)} = \boldsymbol{U}_1 \mathcal{G}_{(1)} (\boldsymbol{U}_2 \otimes \boldsymbol{U}_3 \otimes \boldsymbol{U}_4)^\intercal.$$

By plugging in Eq. (1), we prove the proposition. $\mathcal{Y}_{(1)} = \boldsymbol{U}_1 \mathcal{G}_{(1)} (\boldsymbol{U}_2 \otimes \boldsymbol{U}_3 \otimes \boldsymbol{U}_4)^\intercal \cdot \text{im2col}(\mathcal{X})$. Q.E.D.

B. Proof of Proposition 2.

Proof. Since $\mathcal{W}$ has multilinear rank of $(r_1, r_2, 1, 1)$, it can be decomposed as $\mathcal{W} = \mathcal{G} \times_1 \boldsymbol{U}_1 \times_2 \boldsymbol{U}_2 \times_3 \boldsymbol{U}_3 \times_4 \boldsymbol{U}_4$ with $\mathcal{G} \in \mathbb{R}^{r_1 \times r_2 \times 1 \times 1}$, $\boldsymbol{U}_1 \in \mathbb{R}^{q \times r_1}$, $\boldsymbol{U}_2 \in \mathbb{R}^{c \times r_2}$, $\boldsymbol{U}_3 \in \mathbb{R}^{1 \times 1}$, and $\boldsymbol{U}_4 \in \mathbb{R}^{1 \times 1}$, we can see $\boldsymbol{U}_3$ and $\boldsymbol{U}_4$ are essentially scalar. By setting them to 1, the decomposition can be simplified as

$$\mathcal{W} = \mathcal{G} \times_1 \boldsymbol{U}_1 \times_2 \boldsymbol{U}_2.$$

After tensor mode-1 unfolding and apply Eq. 2, we have $\boldsymbol{y} = \boldsymbol{U}_1 \mathcal{G}_{(1)} \boldsymbol{U}_2^\intercal \boldsymbol{x}$. Q.E.D.

C. Proof of Lemma 1.

Proof. We summarize the update rule of the proposed scheme in the following.

$$\begin{aligned}
\mathcal{W}^t &= g(\mathcal{H}^t), \\
\hat{\mathcal{W}}^t &= \mathcal{W}^t - \eta_1 \nabla l(\mathcal{W}^t), \\
\mathcal{H}^{t+1} &= c(\hat{\mathcal{W}}^t) - \eta_2 \nabla l(c(\hat{\mathcal{W}}^t)), \\
\mathcal{W}^{t+1} &= g(\mathcal{H}^{t+1}).
\end{aligned} \tag{2}$$

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By rewriting the update rule in Eq. (2) as

$$\mathcal{W}^{t+1} = \mathcal{W}^t - \eta_1 \nabla l(\mathcal{W}^t) + \mathcal{E}^t,$$

where $\mathcal{E}^t = g(\mathcal{H}^{t+1}) - \mathcal{W}^t + \eta_1 \nabla l(\mathcal{W}^t)$ denotes the low-rank update error on the t-th iteration.

Then, we can bound the low-rank update error $\mathcal{E}^t$. We first explicitly derive $\|\mathcal{E}^t\|_F$ as the following.

$$\begin{aligned}
\|\mathcal{E}^t\|_F &= \|\mathcal{E}^t_{(1)}\|_F \\
&= \|g(\mathcal{H}^{t+1})_{(1)} - \mathcal{W}^t_{(1)} + \eta_1 \nabla l(\mathcal{W}^t)_{(1)}\|_F.
\end{aligned}$$

Let

$$\begin{aligned}
& g(\mathcal{H}^{t+1})_{(1)} \\
=& g\left(c(\hat{\mathcal{W}}^t) - \eta_2 \nabla l(c(\hat{\mathcal{W}}^t))\right)_{(1)} \\
=& \left(\hat{\boldsymbol{U}}^t_1 - \eta_2 \nabla l(\hat{\boldsymbol{U}}^t_1)\right)\left(\hat{\mathcal{G}}^t - \eta_2 \nabla l(\hat{\mathcal{G}}^t)\right)_{(1)} \\
& \left(\left(\hat{\boldsymbol{U}}^t_2 - \eta_2 \nabla l(\hat{\boldsymbol{U}}^t_2)\right) \otimes \left(\hat{\boldsymbol{U}}^t_3 - \eta_2 \nabla l(\hat{\boldsymbol{U}}^t_3)\right) \otimes \right. \\
& \left. \left(\hat{\boldsymbol{U}}^t_4 - \eta_2 \nabla l(\hat{\boldsymbol{U}}^t_4)\right)\right)^\intercal \\
=& \hat{\boldsymbol{U}}^t_1 \hat{\mathcal{G}}^t_{(1)} (\hat{\boldsymbol{U}}^t_2 \otimes \hat{\boldsymbol{U}}^t_3 \otimes \hat{\boldsymbol{U}}^t_4)^\intercal + \boldsymbol{R}^t \\
=& g(c(\hat{\mathcal{W}}^t))_{(1)} + \boldsymbol{E}^t - \boldsymbol{E}^t + \boldsymbol{R}^t \\
=& \hat{\mathcal{W}}^t_{(1)} + \boldsymbol{R}^t - \boldsymbol{E}^t \\
=& \mathcal{W}^t_{(1)} - \eta_1 \nabla l(\mathcal{W}^t)_{(1)} + \boldsymbol{R}^t - \boldsymbol{E}^t,
\end{aligned}$$

where $\boldsymbol{E}^t = \hat{\mathcal{W}}^t_{(1)} - g(c(\hat{\mathcal{W}}^t))_{(1)}$, and

$$\begin{aligned}
\boldsymbol{R}^t =& - \eta_2 \nabla l(\hat{\boldsymbol{U}}^t_1) \hat{\mathcal{G}}^t_{(1)} (\hat{\boldsymbol{U}}^t_2 \otimes \hat{\boldsymbol{U}}^t_3 \otimes \hat{\boldsymbol{U}}^t_4)^\intercal - ... \\
& + \eta_2^2 \nabla l(\hat{\boldsymbol{U}}^t_1) \nabla l(\hat{\mathcal{G}}^t)_{(1)} (\hat{\boldsymbol{U}}^t_2 \otimes \hat{\boldsymbol{U}}^t_3 \otimes \hat{\boldsymbol{U}}^t_4)^\intercal + ... \\
& - \eta_2^3 \nabla l(\hat{\boldsymbol{U}}^t_1) \nabla l(\hat{\mathcal{G}}^t)_{(1)} (\nabla l(\hat{\boldsymbol{U}}^t_2) \otimes \hat{\boldsymbol{U}}^t_3 \otimes \hat{\boldsymbol{U}}^t_4)^\intercal - ... \\
& + \eta_2^4 \nabla l(\hat{\boldsymbol{U}}^t_1) \nabla l(\hat{\mathcal{G}}^t)_{(1)} (\nabla l(\hat{\boldsymbol{U}}^t_2) \otimes \nabla l(\hat{\boldsymbol{U}}^t_3) \otimes \hat{\boldsymbol{U}}^t_4)^\intercal + ... \\
& - \eta_2^5 \nabla l(\hat{\boldsymbol{U}}^t_1) \nabla l(\hat{\mathcal{G}}^t)_{(1)} (\nabla l(\hat{\boldsymbol{U}}^t_2) \otimes \nabla l(\hat{\boldsymbol{U}}^t_3) \otimes \nabla l(\hat{\boldsymbol{U}}^t_4))^\intercal,
\end{aligned}$$

consists of 31 terms that are the permutation of low-rank weights and their gradients. Then, plugging it back, we have

$$\|\mathcal{E}^t_{(1)}\|_F = \|\boldsymbol{R}^t - \boldsymbol{E}^t\|_F \leq \|\boldsymbol{R}^t\|_F + \|\boldsymbol{E}^t\|_F.$$

According to Equation (8) in original paper, $\|\boldsymbol{E}^t\|_F$ is bounded that

$$\|\boldsymbol{E}^t\|_F \leq (1-\rho)\|\hat{\mathcal{W}}^t\|_F \leq (1-\rho)\varphi.$$

Using the assumption (2) and (3), if $0 \leq \eta_2 \leq 1$, $\mathbb{E}\left[\|\boldsymbol{R}^t\right]\|_F$ is bounded that

$$
\begin{aligned}
&\mathbb{E}\left[\|\boldsymbol{R}^t\|_F\right] \\
\leq&\eta_2 \Big(\|\nabla l(\hat{\boldsymbol{U}}_1^t)\|_2 \|\hat{\boldsymbol{\mathcal{G}}}_{(1)}^t\|_F \|\hat{\boldsymbol{U}}_2^t\|_2 \|\hat{\boldsymbol{U}}_3^t\|_2 \|\hat{\boldsymbol{U}}_4^t\|_2 + ... \\
&+ \|\nabla l(\hat{\boldsymbol{U}}_1^t)\|_2 \|\nabla l(\hat{\boldsymbol{\mathcal{G}}}^t)_{(1)}\|_F \|\hat{\boldsymbol{U}}_2^t\|_2 \|\hat{\boldsymbol{U}}_3^t\|_2 \|\hat{\boldsymbol{U}}_4^t\|_2 + ... \\
&+ \|\nabla l(\hat{\boldsymbol{U}}_1^t)\|_2 \|\nabla l(\hat{\boldsymbol{\mathcal{G}}}^t)_{(1)}\|_F \|\nabla l(\hat{\boldsymbol{U}}_2^t)\|_2 \|\hat{\boldsymbol{U}}_3^t\|_2 \|\hat{\boldsymbol{U}}_4^t\|_2 + ... \\
&+ \|\nabla l(\hat{\boldsymbol{U}}_1^t)\|_2 \|\nabla l(\hat{\boldsymbol{\mathcal{G}}}^t)_{(1)}\|_F \|\nabla l(\hat{\boldsymbol{U}}_2^t)\|_2 \|\nabla l(\hat{\boldsymbol{U}}_3^t)\|_2 \|\hat{\boldsymbol{U}}_4^t\|_2 + ... \\
&+ \|\nabla l(\hat{\boldsymbol{U}}_1^t)\|_2 \|\nabla l(\hat{\boldsymbol{\mathcal{G}}}^t)_{(1)}\|_F \\
&\quad \|\nabla l(\hat{\boldsymbol{U}}_2^t)\|_2 \|\nabla l(\hat{\boldsymbol{U}}_3^t)\|_2 \|\nabla l(\hat{\boldsymbol{U}}_4^t)\|_2 \Big) \\
\leq&\eta_2 \left(\varphi \left(\sum_{i=1}^{4} \tbinom{4}{i} 2^i \right) + G_2 \left(\sum_{i=0}^{4} \tbinom{4}{i} 2^i \right) \right) \\
=&\eta_2 \left(80\varphi + 81 G_2\right),
\end{aligned}
$$

where the matrix norm inequality that $\|\boldsymbol{AB}\|_F \leq \|\boldsymbol{A}\|_2 \|\boldsymbol{B}\|_F$ is applied. Then, by taking the expectation, it yields

$$
\mathbb{E}\left[\|\boldsymbol{\mathcal{E}}^t\|_F\right] \leq \eta_2 \left(80\varphi + 81G_2\right) + (1-\rho)\varphi
$$

Q.E.D.

D. Proof of Theorem 1

Proof. The proof outline is consistent with [1]. Consider the following.

$$
\begin{aligned}
&\mathbb{E}\left[\|\boldsymbol{\mathcal{W}}^{t+1} - \boldsymbol{\mathcal{W}}^*\|_F^2\right] \\
=&\|\boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*\|_F^2 - 2\mathbb{E}\langle \boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*, \eta_1 \nabla l(\boldsymbol{\mathcal{W}}^t) - \boldsymbol{\mathcal{E}}^t \rangle \\
&+ \mathbb{E}\|\eta_1 \nabla l(\boldsymbol{\mathcal{W}}^t) - \boldsymbol{\mathcal{E}}^t\|_F^2 \\
=&\|\boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*\|_F^2 - 2\eta_1 \langle \boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*, \nabla \mathcal{L}(\boldsymbol{\mathcal{W}}^t) \rangle \\
&+ 2\mathbb{E}\langle \boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*, \boldsymbol{\mathcal{E}}^t \rangle + \eta_1^2 \mathbb{E}\|\nabla l(\boldsymbol{\mathcal{W}}^t)\|_F^2 \\
&+ \mathbb{E}\|\boldsymbol{\mathcal{E}}^t\|_F^2 - 2\mathbb{E}\langle \nabla l(\boldsymbol{\mathcal{W}}^t), \boldsymbol{\mathcal{E}}^t \rangle \\
\leq&\|\boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*\|_F^2 - 2\eta_1 \langle \boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*, \nabla \mathcal{L}(\boldsymbol{\mathcal{W}}^t) \rangle \\
&+ 2\mathbb{E}\|\boldsymbol{\mathcal{E}}^t\|_F \|\boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*\|_F + 2\mathbb{E}\|\boldsymbol{\mathcal{E}}^t\|_F \|\nabla l(\boldsymbol{\mathcal{W}}^t)\|_F \\
&+ \eta_1^2 \mathbb{E}\|\nabla l(\boldsymbol{\mathcal{W}}^t)\|_F^2 + \mathbb{E}\|\boldsymbol{\mathcal{E}}^t\|_F^2 \\
\leq&2\|\boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*\|_F^2 - 2\eta_1 \langle \boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*, \nabla \mathcal{L}(\boldsymbol{\mathcal{W}}^t) \rangle \\
&+ \|\nabla l(\boldsymbol{\mathcal{W}}^t)\|_F^2 + \eta_1^2 \mathbb{E}\|\nabla l(\boldsymbol{\mathcal{W}}^t)\|_F^2 + 3\mathbb{E}\|\boldsymbol{\mathcal{E}}^t\|_F^2 \\
\leq&2\|\boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*\|_F^2 - 2\eta_1 \langle \boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*, \nabla \mathcal{L}(\boldsymbol{\mathcal{W}}^t) \rangle \\
&+ \eta_1^2 G_1^2 + 3\eta_2^2 a^2 + G_1^2,
\end{aligned}
$$

where Lemma 1 is applied and $a = 81G_2 + 80\varphi + \frac{1}{\eta_2}(1-\rho)\varphi$. Using the assumption that $\mathcal{L}$ is μ-strongly convex, we have

$$
\begin{aligned}
&\mathbb{E}\left[\|\boldsymbol{\mathcal{W}}^{t+1} - \boldsymbol{\mathcal{W}}^*\|_F^2\right] \\
\leq&(2 - \eta_1 L)\|\boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*\|_F^2 - 2\eta_1 \left(\mathcal{L}(\boldsymbol{\mathcal{W}}^t) - \mathcal{L}(\boldsymbol{\mathcal{W}}^*)\right) \\
&+ \eta_1^2 G_1^2 + 3\eta_2^2 a^2 + G_1^2, \\
\Rightarrow\ & 2\eta_1 \left(\mathcal{L}(\boldsymbol{\mathcal{W}}^t) - \mathcal{L}(\boldsymbol{\mathcal{W}}^*)\right) \\
\leq&(2 - \eta_1 L)\mathbb{E}\left[\|\boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*\|_F^2\right] - \mathbb{E}\left[\|\boldsymbol{\mathcal{W}}^{t+1} - \boldsymbol{\mathcal{W}}^*\|_F^2\right] \\
&+ \eta_1^2 G_1^2 + 3\eta_2^2 a^2 + G_1^2, \\
\Rightarrow\ & \mathbb{E}\left(\mathcal{L}(\boldsymbol{\mathcal{W}}^t) - \mathcal{L}(\boldsymbol{\mathcal{W}}^*)\right) \\
\leq&(\frac{2}{2\eta_1} - \frac{\mu}{2})\mathbb{E}\|\boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*\|_F^2 - \frac{1}{2\eta_1}\mathbb{E}\left[\|\boldsymbol{\mathcal{W}}^{t+1} - \boldsymbol{\mathcal{W}}^*\|_F^2\right] \\
&+ \frac{\eta_1}{2} G_1^2 + \frac{3\eta_2^2}{2\eta_1} a^2 + \frac{1}{2\eta_1} G_1^2,
\end{aligned}
$$

If $\eta_1 = \frac{1}{1/2^t + \mu}$, and $\rho = 1 - \eta_2$, we have

$$
\begin{aligned}
&\mathbb{E}\left(\mathcal{L}(\boldsymbol{\mathcal{W}}^t) - \mathcal{L}(\boldsymbol{\mathcal{W}}^*)\right) \leq \left(\frac{1}{(2)^t} + \frac{\mu}{2}\right)\mathbb{E}\|\boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*\|_F^2 \\
&\quad - \left(\frac{1}{2^{(t+1)}} + \frac{\mu}{2}\right)\mathbb{E}\|\boldsymbol{\mathcal{W}}^{t+1} - \boldsymbol{\mathcal{W}}^*\|_F^2 + \frac{1}{4/2^{(t+1)} + 2\mu} G_1^2 \\
&\quad + \left(\frac{1}{(2)^t} + \mu\right)(\frac{3}{2}\eta_2^2 b^2 + G_1^2) \\
&\leq \left(\frac{1}{(2)^t} + \frac{\mu}{2}\right)\mathbb{E}\|\boldsymbol{\mathcal{W}}^t - \boldsymbol{\mathcal{W}}^*\|_F^2 \\
&\quad - \left(\frac{1}{2^{(t+1)}} + \frac{\mu}{2}\right)\mathbb{E}\|\boldsymbol{\mathcal{W}}^{t+1} - \boldsymbol{\mathcal{W}}^*\|_F^2 + \frac{1}{2\mu} G_1^2 \\
&\quad + \left((\frac{1}{2})^t + \mu\right)(\frac{3}{2}\eta_2^2 b^2 + \frac{1}{2}G_1^2).
\end{aligned}
$$

where $b = 81(G_2 + \varphi)$. Then, setting $\eta_2 = (\frac{1}{\sqrt{C}})^t$ and applying the telescope sum from $t = 0$ to T, we have

$$
\begin{aligned}
&\frac{1}{T}\sum_{t=0}^{T}\mathbb{E}\left(\mathcal{L}(\boldsymbol{\mathcal{W}}^t) - \mathcal{L}(\boldsymbol{\mathcal{W}}^*)\right) \\
\leq&\frac{1}{T}\sum_{t=0}^{T}(\frac{3}{2}\eta_2^2 b^2 + \frac{1}{2}G_1^2)\left(\left(\frac{1}{2}\right)^t + \mu\right) + \frac{c}{T} \\
&- \left(\frac{1}{2^{(T+1)}} + \frac{\mu}{2}\right)\mathbb{E}\|\boldsymbol{\mathcal{W}}^{T+1} - \boldsymbol{\mathcal{W}}^*\|_F^2 + \frac{1}{2\mu}G_1^2 \\
\leq&\frac{1}{2T}\sum_{t=0}^{T}\left(\frac{3}{2}\eta_2^2 b^2 + \frac{1}{2}G_1^2\right)^2 + \frac{1}{2T}\sum_{t=0}^{T}\left(\left(\frac{1}{2}\right)^t + \mu\right)^2 \\
&+ \frac{c}{T} + \frac{1}{2\mu}G_1^2 \\
=&\frac{9b^4}{8T}\sum_{t=0}^{T}\left(\frac{1}{C^2}\right)^t + \frac{3b^2 G_1^2}{4T}\left(\frac{1}{C}\right)^t \\
&+ \frac{1}{2T}\left(\sum_{t=0}^{T}(\frac{1}{4})^t + 2\mu\sum_{t=0}^{T}(\frac{1}{2})^t + 2c\right) + \frac{G_1^4}{8} + \frac{\mu^2}{2} + \frac{1}{2\mu}G_1^2 \\
\leq&\frac{1}{2T}\left(4\mu + \frac{3G_1^2 C b^2}{(C-1)} + \frac{9C^2 b^4}{4(C^2-1)} + 2c + \frac{4}{3}\right) \\
&+ \frac{1}{2}(\frac{G_1^2}{\mu} + \mu^2 + \frac{G_1^4}{4}),
\end{aligned}
$$

where $c = \frac{\mu+2}{2}\|\boldsymbol{\mathcal{W}}^0 - \boldsymbol{\mathcal{W}}^*\|_F^2$. Using Jensen's inequality, we have

$$
\mathbb{E}\left(\mathcal{L}(\bar{\boldsymbol{\mathcal{W}}}^T) - \mathcal{L}(\boldsymbol{\mathcal{W}}^*)\right) \leq \frac{1}{T}\sum_{t=0}^{T}\mathbb{E}\left(\mathcal{L}(\boldsymbol{\mathcal{W}}^t) - \mathcal{L}(\boldsymbol{\mathcal{W}}^*)\right),
$$

where $\bar{\boldsymbol{\mathcal{W}}}^T = \frac{1}{T}\sum_{t=1}^{T} = \boldsymbol{\mathcal{W}}^t$. Hence,

$$
\begin{aligned}
\mathbb{E}[\bar{\boldsymbol{\mathcal{W}}}^T - \mathcal{L}(\boldsymbol{\mathcal{W}}^*)] \leq& \frac{1}{2T}\left(4L + \frac{3G_1^2 U b^2}{(U-1)} + \frac{9U^2 b^4}{4(U^2-1)} + \frac{4}{3}\right) \\
&+ \frac{1}{2}(\frac{G_1^2}{L} + \mu^2 + \frac{G_1^4}{4}).
\end{aligned}
$$

As T converges to ∞, it converges to $\frac{1}{2}(\frac{G_1^2}{L} + \mu^2 + \frac{G_1^4}{4})$.

Q.E.D.

E. Proof of Corollary 1.

Proof. $M \geq \frac{n^d}{r^d+dnr} \geq \frac{n^d}{r^d+dnr^d}$. If $r \leq \left(\frac{n^d}{\phi(1+dn)}\right)^{1/d}$, $\frac{n^d}{r^d+dnr^d} \geq \phi$. Therefore, $M \geq \phi$. Q.E.D.

F. Proof of Theorem 3

Proof. For the uncompressed case, the computation is essentially matrix-matrix multiplication between matrices with size $n \times n^3$ and $n^3 \times m$ according to Eq. (1). So, the computation complexity is $\Theta(mn^4)$.

In the case of low-rank convolutional layer, there is an efficient way to implement the Eq. (11). Denoting $\text{im2col}(\boldsymbol{\mathcal{X}}) = \left[\text{vec}(\mathcal{P}^{(1)}), \text{vec}(\mathcal{P}^{(2)}), ..., \text{vec}(\mathcal{P}^{(m)})\right]$ where $\text{vec}(\cdot)$ is the vectorization operation and $\mathcal{P}^{(m)} \in \mathbb{R}^{n\times n\times n}$ is the patch of image to be convolved, then by applying the matrix equations in Kronecker product [2], for each $k = 1, 2, ..., m$ we have

$$(\boldsymbol{U}_2 \otimes \boldsymbol{U}_3 \otimes \boldsymbol{U}_4)^\intercal \cdot \text{vec}(\mathcal{P}^{(k)}) = (\boldsymbol{U}_3 \otimes \boldsymbol{U}_4)^\intercal (\mathcal{P}^{(k)}_{(1)})^\intercal \boldsymbol{U}_2. \tag{3}$$

Furthermore, applying the matrix equations again, we have

$$(\boldsymbol{U}_3 \otimes \boldsymbol{U}_4)^\intercal (\mathcal{P}^{(k)}_{(1)})^\intercal_{:,i} = \boldsymbol{U}_4^\intercal (\mathcal{P}^{(k)}_{i,:,:})^\intercal \boldsymbol{U}_3, \forall i = 1, 2, ..., n. \tag{4}$$

Hence, to compute $(\boldsymbol{U}_2 \otimes \boldsymbol{U}_3 \otimes \boldsymbol{U}_4)^\intercal \cdot \text{im2col}(\boldsymbol{\mathcal{X}})$, we need to compute m times Eq. (3) and mn times Eq. (4). The computation complexity is $\Theta\left(m\left(n(n^2r + nr^2) + nr^3\right)\right)$. Then, by performing matrix multiplication like $\boldsymbol{U}_1\left(\boldsymbol{\mathcal{G}}_{(1)}\left((\boldsymbol{U}_2 \otimes \boldsymbol{U}_3 \otimes \boldsymbol{U}_4)^\intercal \cdot \text{im2col}(\boldsymbol{\mathcal{X}})\right)\right)$, the total complexity is $\Theta\left(m(n^3r + n^2r^2 + nr^3 + r^4 + nr)\right)$. Hence, the speed-up ratio E is lower bounded by $\Omega\left(\frac{n^4}{r^4+n^3r+n^2r^2+nr^3+nr}\right)$. Q.E.D.

G. Proof of Corollary 2.

Proof. $E \geq \frac{n^4}{r^4+n^3r+n^2r^2+nr^3+nr} \geq \frac{n^4}{r^4(n^3+n^2+2n+1)}$. If $r \leq \left(\frac{n^4}{\tau(n^3+n^2+2n+1)}\right)^{\frac{1}{4}}$, then $\frac{n^4}{r^4(n^3+n^2+2n+1)} \geq \tau$. Therefore, $E \geq \tau$. Q.E.D.

References

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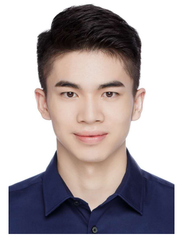
[2] T. G. Kolda and B. W. Bader, "Tensor decompositions and applications," SIAM review, vol. 51, no. 3, pp. 455–500, 2009.



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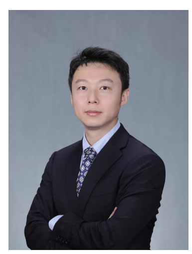

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