

COMSM1302

Overview of Computer Architecture

Lecture 4

Simple devices



In this lecture

Foundations

- Data representation, logic, Boolean algebra.

Building blocks

- Transistors, transistor based logic, **simple devices**, storage.

Modules

- Memory, simple controllers, FSMs, processors and execution.

Programming

- Machine code, assembly, high-level languages, compilers.

Wrap-up

- Operating systems, energy aware computing.



🔥 Today, we learn to add!

- And also...
 - Subtract
 - Select 1 signal from many
 - Distribute 1 signal to many
- The circuits shown hereafter will be drawn in Logisim.
 - You can download it from:
<http://sourceforge.net/projects/circuit/>✗
 - Install Logisim on your own computer and practice.

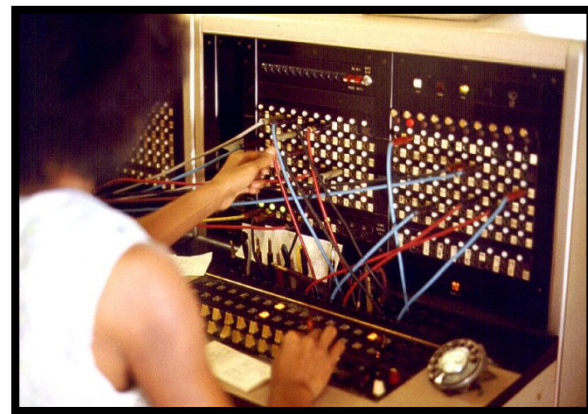
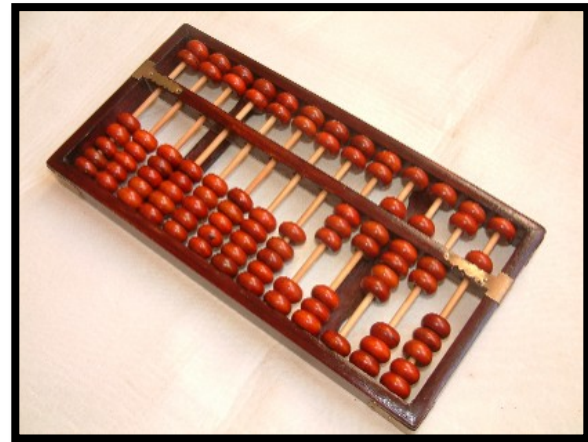


Photo by Joseph A. Carr, 1975

🔥 COMSM1302 NAND board kit

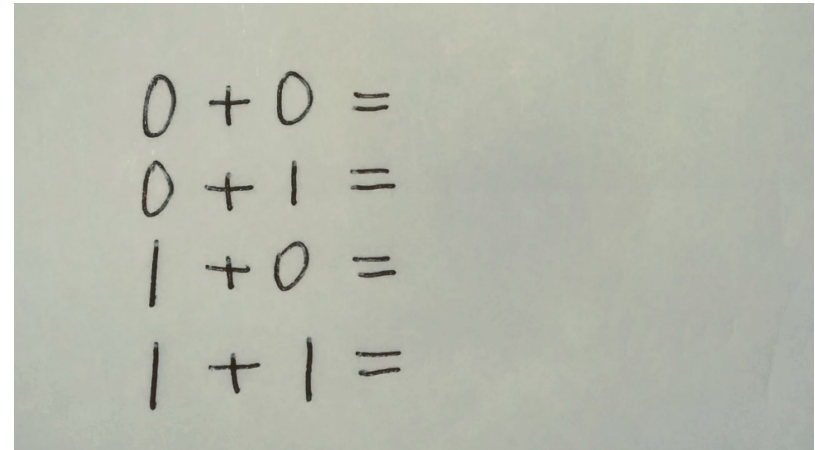


- Your task in the NAND labs will involve building some of the circuits that will be introduced today.
- **You should always design with Logisim BEFORE you start building with the NAND board kit.**



🔥 The simplest of binary addition

- How to add two, single-digit binary numbers?
 - Each digit is either 0 or 1
 - What are the possible results?

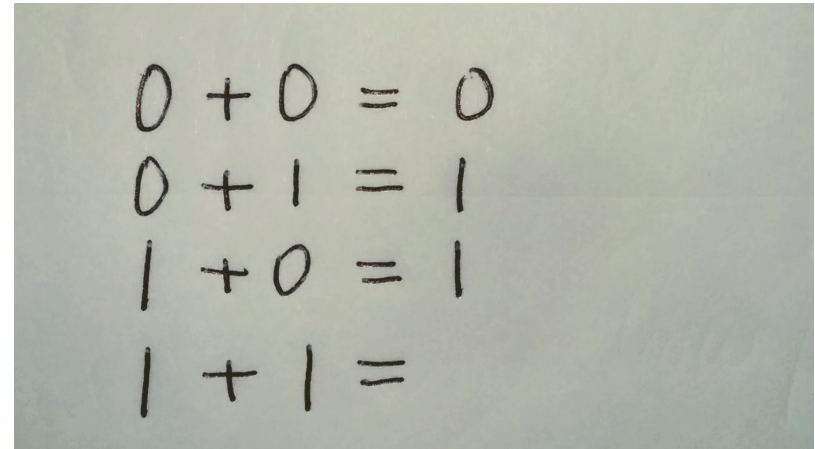


A photograph of a piece of paper with four handwritten binary addition equations:

$$\begin{array}{l} 0 + 0 = \\ 0 + 1 = \\ 1 + 0 = \\ 1 + 1 = \end{array}$$

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 - Each digit is either 0 or 1
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$$\begin{array}{l} 0 + 0 = \underline{0} \\ 0 + 1 = \underline{1} \\ 1 + 0 = \underline{1} \\ 1 + 1 = \cancel{2} \quad 10 \checkmark \end{array}$$

🔥 The simplest of binary addition

- How to add two, single-digit binary numbers?
 - Each digit is either 0 or 1
 - There are three possible results
 - 0, 1, 2 ←
 - 0b00, 0b01, 0b10 ←
 - Two inputs
 - A, B
 - Output
 - Sum (S)

A	B	S
0	0	0
0	1	1
1	0	1
1	1	<u>10</u>

🔥 The simplest of binary addition

- How to add two, single-digit binary numbers?
 - Each digit is either 0 or 1
 - There are three possible results
 - 0, 1, 2
 - 0b00, 0b01, 0b10
 - Two inputs
 - A, B
 - Two outputs
 - Sum (S)
 - Carry (C)

Handwritten binary addition examples:

0	+	0	=	00
0	+	1	=	01
1	+	0	=	01
1	+	1	=	<u>1</u> 0

🔥 The simplest of binary addition

- How to add two, single-digit binary numbers?
 - Each digit is either 0 or 1
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 - 0b00, 0b01, 0b10
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A	B	C	S
0	0	0	0
0	1	0	1
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1	1	1	0

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 - 0, 1, 2
 - 0b00, 0b01, 0b10
 - Two inputs
 - A, B
 - Two outputs
 - Carry (C)
 - Sum (S)

A	B	2^1	2^0
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

With some Boolean algebra

- Remember our truth tables for the connectives in Boolean algebra, e.g. AND, OR, XOR, etc?
- Which operations can be used to help us generate S and C?

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

🔥 With some Boolean algebra

- Remember our truth tables for the connectives in Boolean algebra, e.g. AND, OR, XOR, etc?
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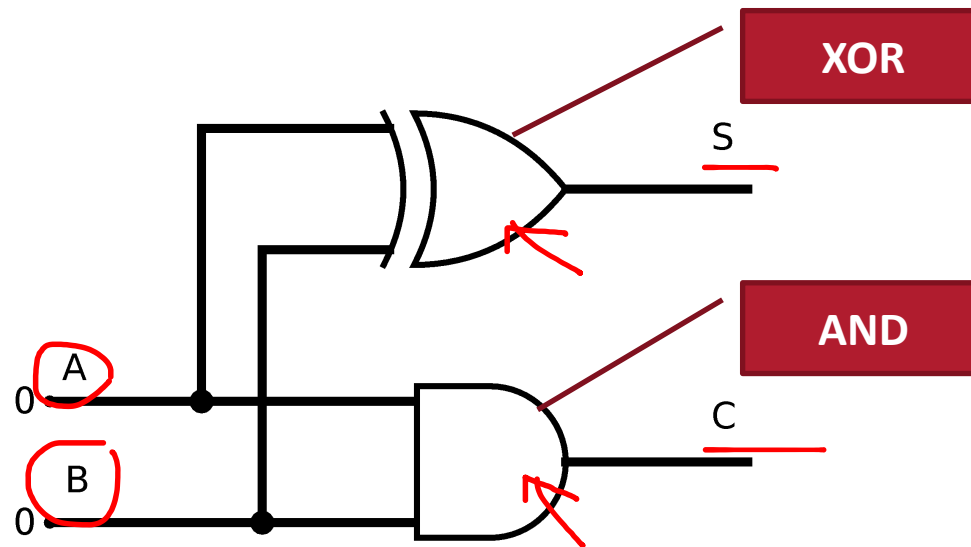
A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$A \oplus B$	$A \rightarrow B$	$A \equiv B$
0	0	1	0	0	0	1	1
0	1	1	0	1	1	1	0
1	0	0	0	1	1	0	0
1	1	0	1	1	0	1	1

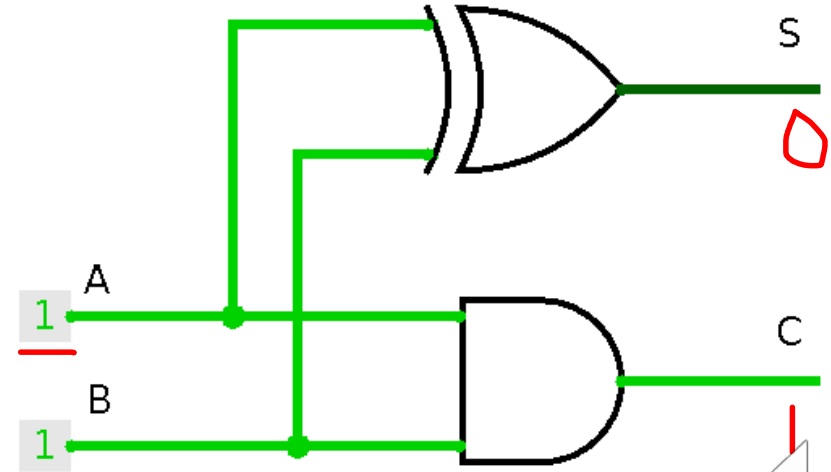
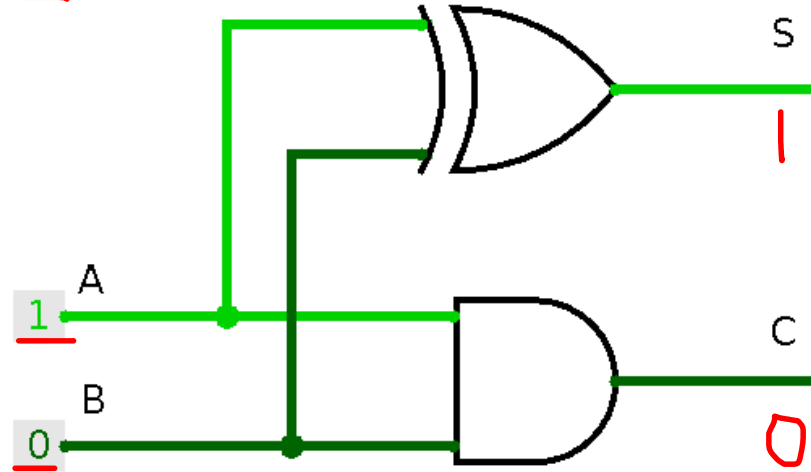
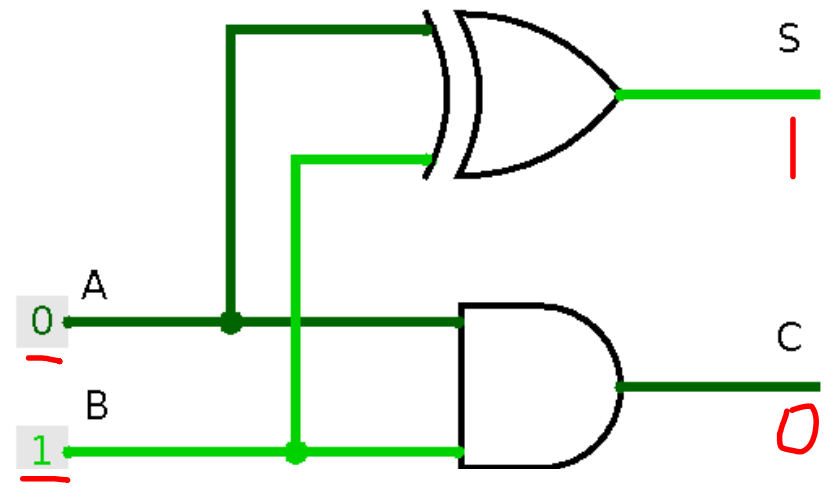
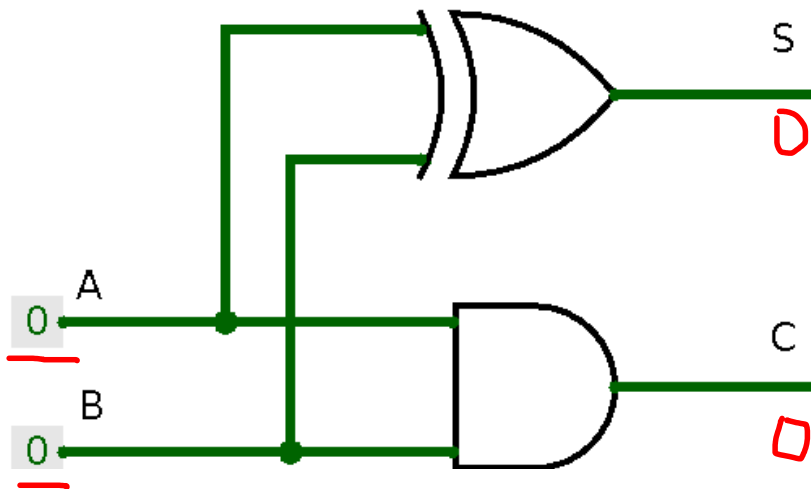


🔥 The half-adder

- $S = A \oplus B$
- $C = A \wedge B$ add two bits together, no carry in
- Now let's build it with logic gates.



🔥 The half-adder in action



🔥 Now what?

- We can add two bits together.
 - By generating a sum and a carry bit.
- How do we add multiple bits together?

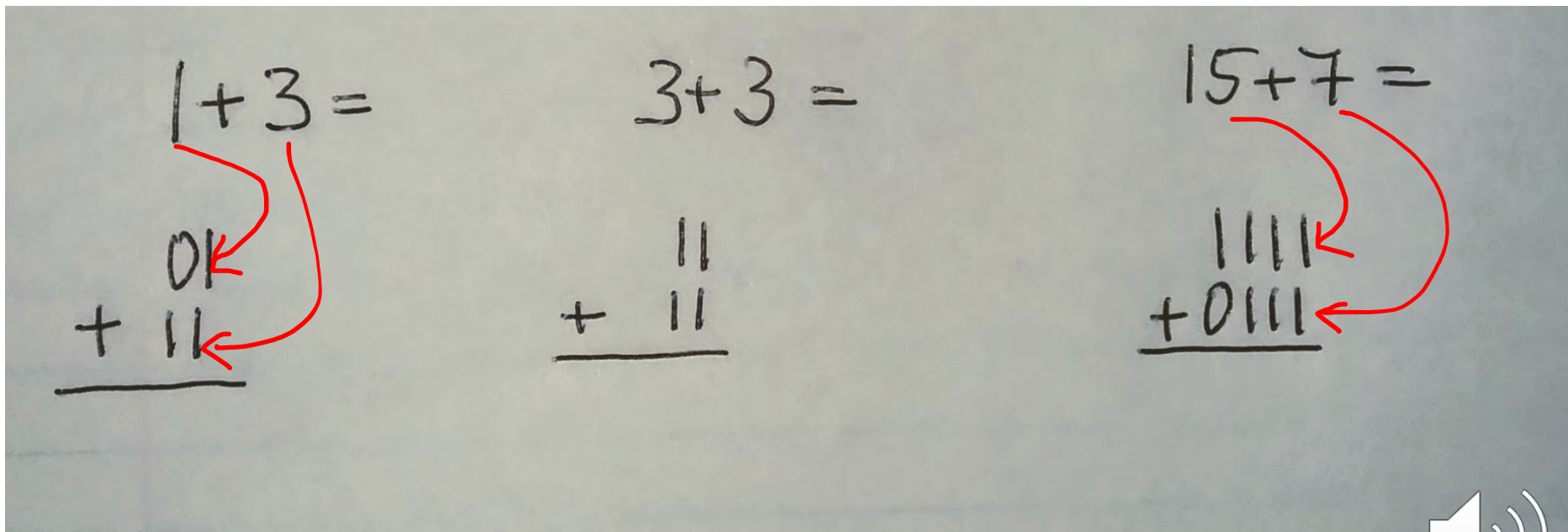
$$1 + 3 =$$

$$3 + 3 =$$

$$15 + 7 =$$

🔥 Now what?

- We can add two bits together.
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🔥 Now what?

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 - By generating a sum and a carry bit.
- How do we add multiple bits together?

The image shows three handwritten binary addition examples on a light background. Each example consists of a decimal equation at the top and a corresponding binary addition at the bottom. Red annotations highlight the carry process.

- Example 1:** $1 + 3 = 4$. The binary addition is $01 + 11 = 100$. Red circles are drawn around the two 1s in the first column, with an arrow pointing to them labeled "carry-in". The result 100 is underlined.
- Example 2:** $3 + 3 = 6$. The binary addition is $11 + 11 = 110$. Red circles are drawn around the two 1s in the first column, with an arrow pointing to them labeled "carry". The result 110 is underlined.
- Example 3:** $15 + 7 = 22$. The binary addition is $1111 + 0111 = 10110$. Red circles are drawn around the four 1s in the first column, with an arrow pointing to them labeled "carry". The result 10110 is underlined.

🔥 Now what?

- We can add two bits together
 - By generating a sum and a carry
- How do we add multiple

For each addition (except for the first) we need to account for two input bits and a **carry_in**, and we produce a **sum** and a **carry_out**.

Handwritten binary addition examples illustrating carry propagation:

- $1 + 3 = 4$:
$$\begin{array}{r} 01 \\ + 11 \\ \hline 100 \end{array} \equiv 4$$

A red arrow labeled "carry-in" points to the first column (LSB) where 1 + 1 = 0 with a carry-out of 1.
- $3 + 3 = 6$:
$$\begin{array}{r} 11 \\ + 11 \\ \hline 110 \end{array} \equiv 6$$

A red arrow labeled "carry" points to the second column where 1 + 1 + 1 (carry-in) = 0 with a carry-out of 1.
- $5 + 7 = 12$:
$$\begin{array}{r} 1111 \\ + 1011 \\ \hline 10110 \end{array} \equiv 22$$

A red arrow labeled "carry" points to the third column where 1 + 1 + 1 (carry-in) = 0 with a carry-out of 1.

The full-adder

<u>C_in</u>	<u>A</u>	<u>B</u>	<u>C_out</u>	<u>S</u>
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

From the half-adder:

- $S = A \oplus B$
- $C_{out} = A \wedge B$

(Note that the above covers only the top-half of this table!)

The full-adder



<u>C_in</u>	<u>A</u>	<u>B</u>	C_out	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



The full-adder

For multiple arguments, XOR is defined to be true iff an odd number of its arguments is true, and false otherwise.

C_in	A	B	C_out	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

- $S = \underline{A \oplus B} \oplus \underline{C_{in}}$
- $C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \vee B))$
- Also valid:
 $C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \oplus B))$
– Why?

The full-adder

C_in	A	B	C_out	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

- $S = A \oplus B \oplus C_{in}$

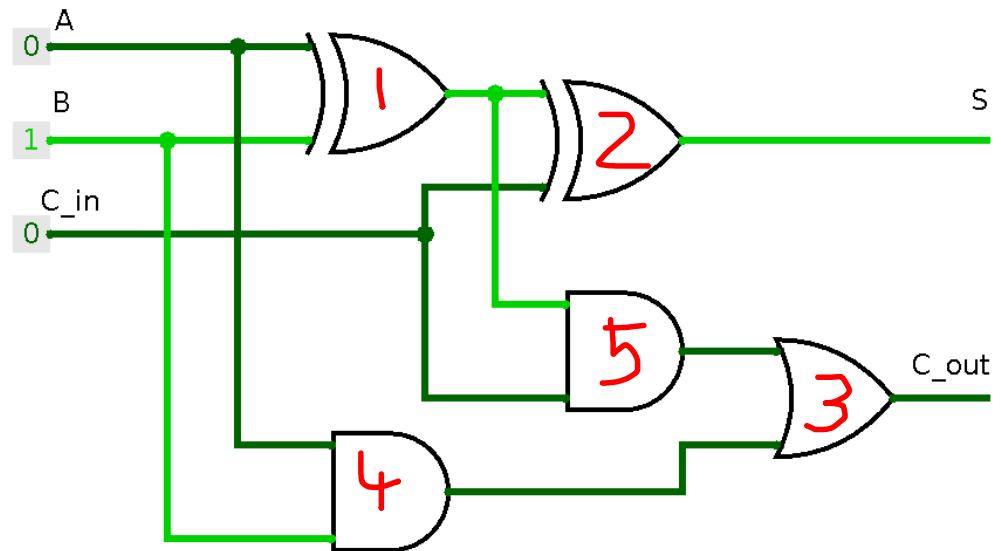
- $C_{out} =$
 $(A \wedge B) \vee (C_{in} \wedge (A \vee B))$

- Also valid:

$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \oplus B))$
– Why?

🔥 The full-adder

- 8 different combinations of input
- Try them yourself!

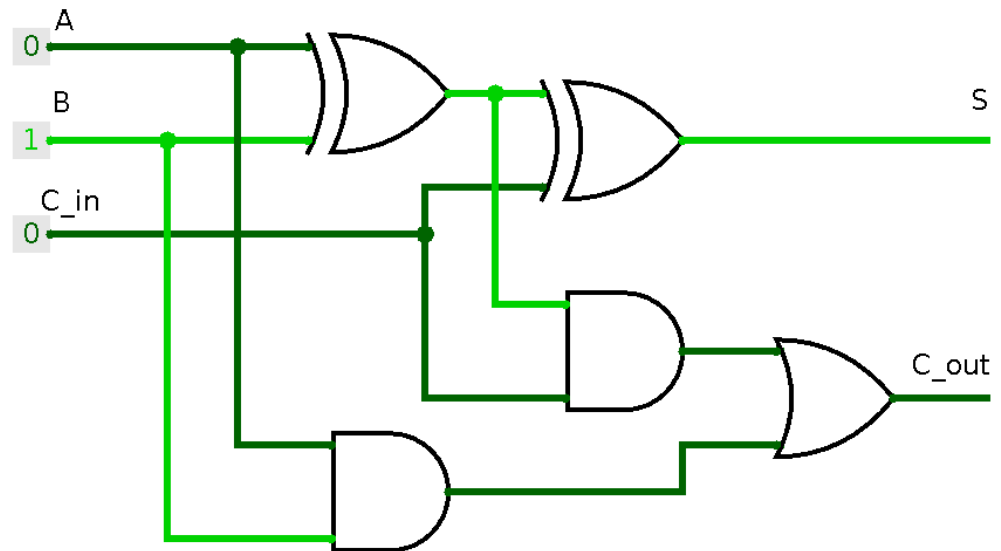


- S = A \oplus B \oplus C_in
- C_out = (A \wedge B) \vee (C_in \wedge (A \oplus B))



🔥 The full-adder

- 8 different combinations of input
- Try them yourself!



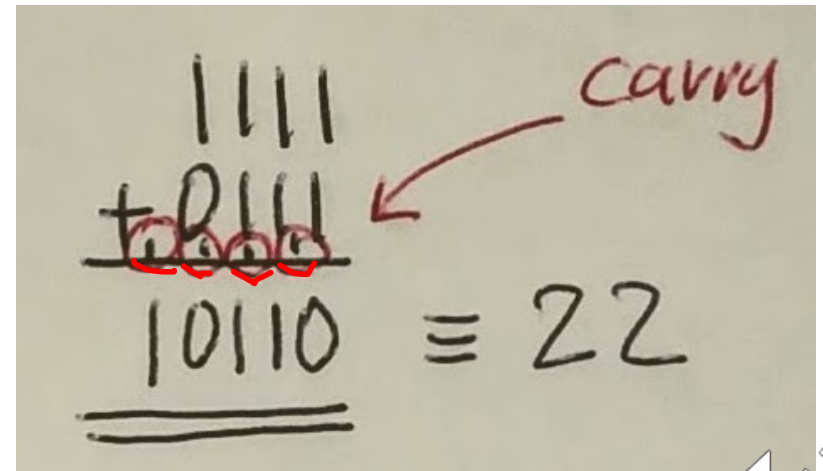
- To build this with the NAND kits you need to:
 - use Boolean algebra to obtain a design that is based purely on NAND gates,
 - implement this in Logisim to gain confidence your design is correct, then
 - *(and only then)* transfer to NAND boards and test.

Now what?

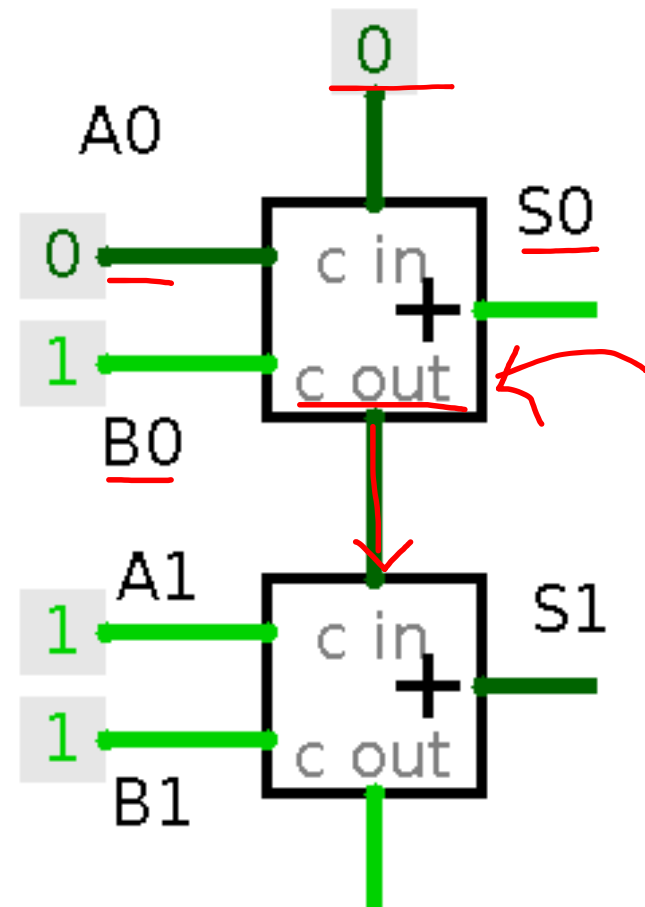
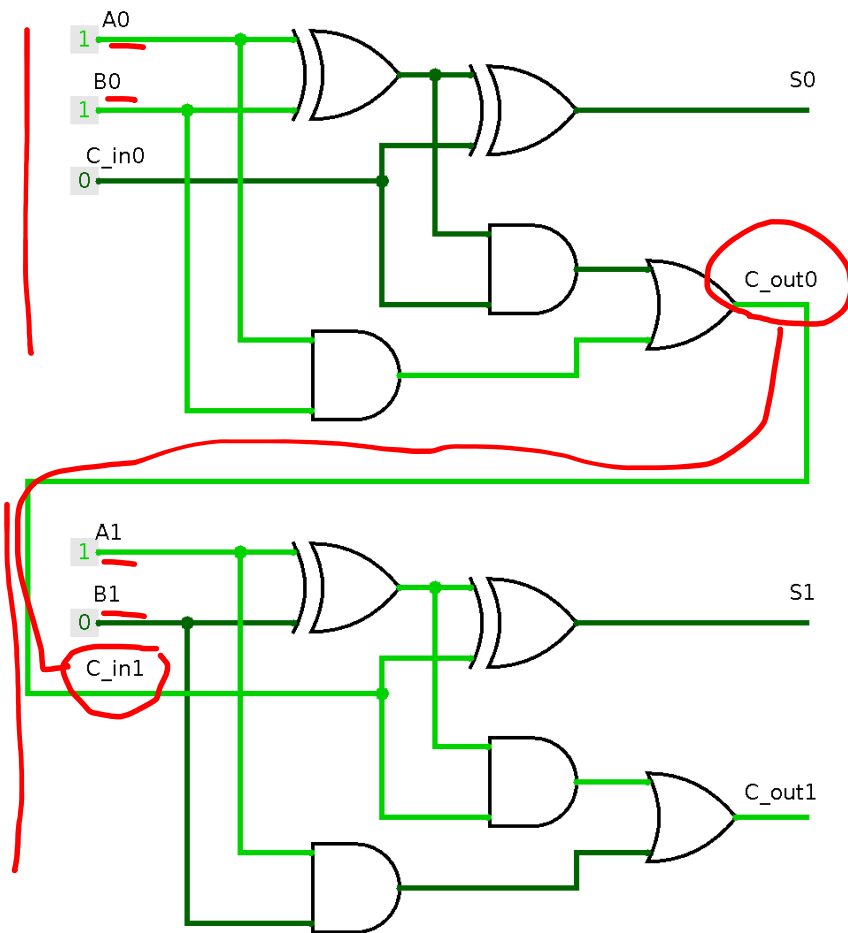
- We can add two bits together.
 - By generating a sum and a carry bit.
- We can add three bits together
 - By accommodating a carry-in as well as our regular two inputs.
- How do we add multiple bits together?

🔥 Now what?

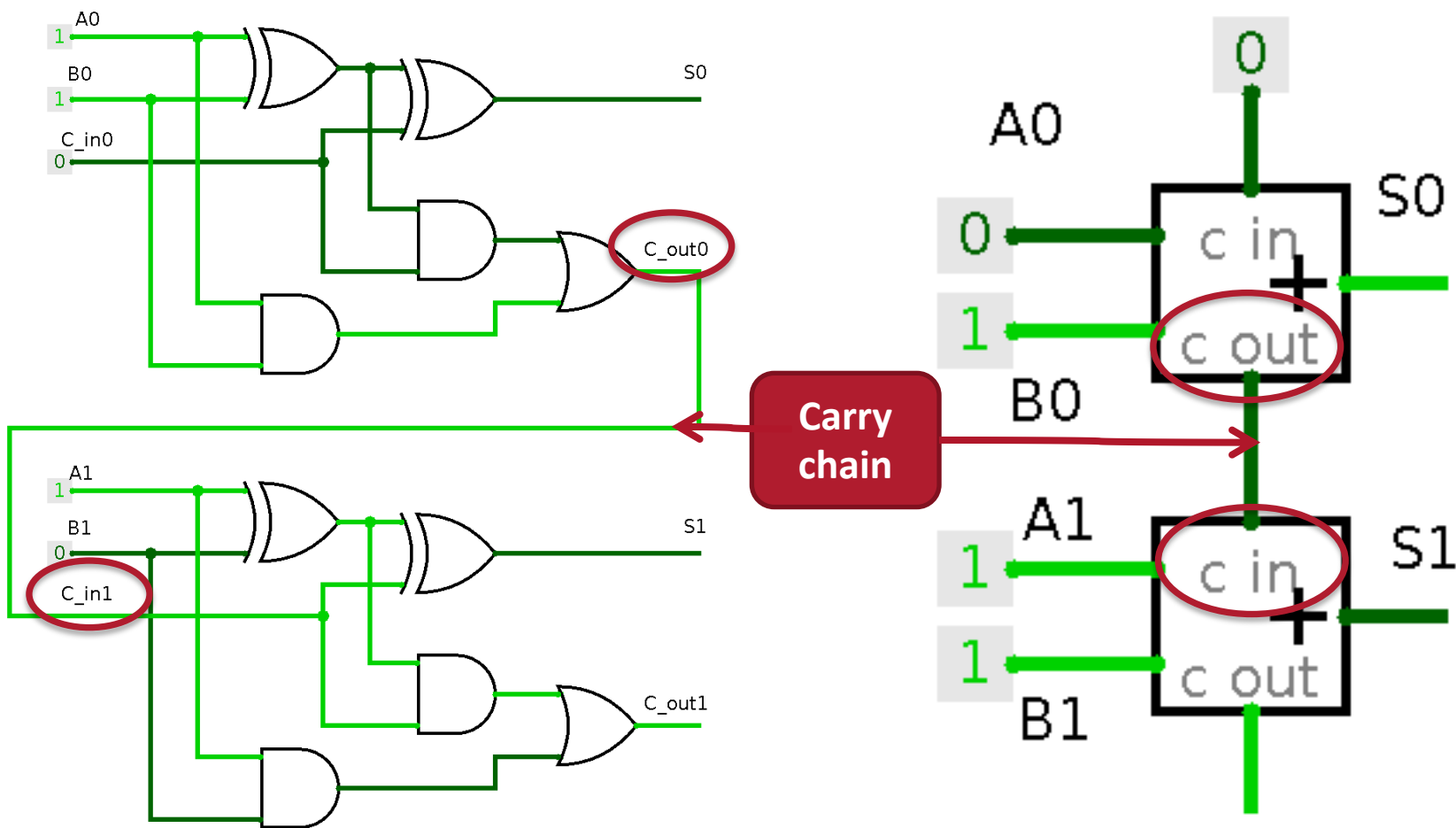
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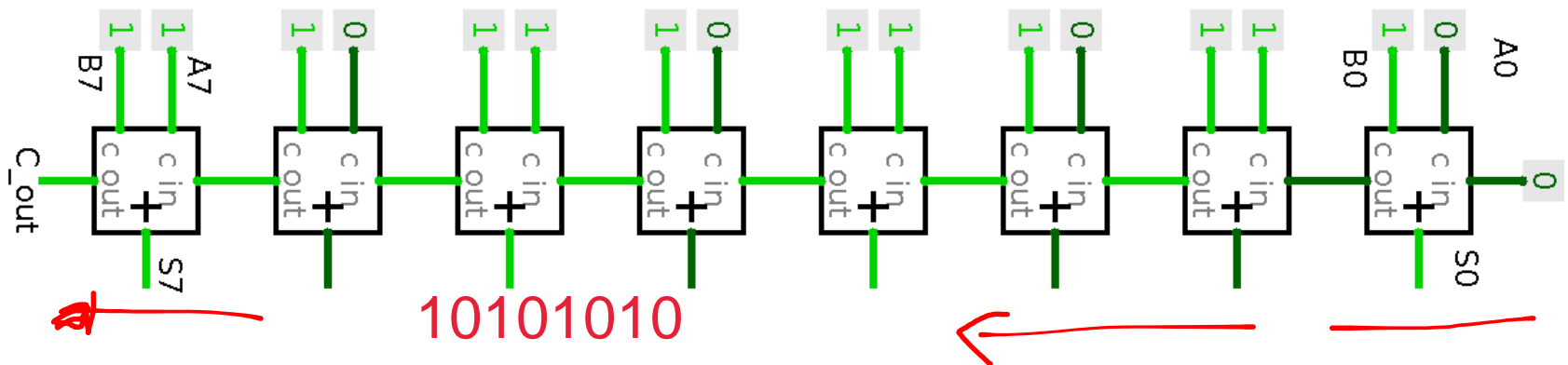
🔥 Chaining full-adders



🔥 Chaining full-adders



8-bit adder, ripple-carry adder



- Named **ripple-carry** because a carry signal generated at the LSB (Least Significant Bit... bit 0) of the device can affect the result on any/all more significant bits.
- Does this have any implications?

Building blocks

- To recap what we've done:
 - Used Boolean algebra to identify our **building blocks**.
 - Built a unit capable of adding two bits.
 - **Half-adder**
 - Extended it to handle carry-in.
 - **Full-adder**
 - Chained them together to make an adder of **arbitrary size**.
 - **Ripple-carry adder**
- Now we can add anything!
 - Modern processors typically have adders between 8 and 64 bits.
 - **Why?**



Subtraction

- Subtraction is easy if we think of it as **adding one number to a negative number**.
- So let's represent this subtraction:

$$\underline{1 - 2} = -1$$

- As:

$$\underline{1 + (-2)} = -1$$

- How to negate a number?

Subtraction

- Subtraction is easy if we think of it as **adding one number to a negative number**.
- So let's represent this subtraction:
$$1 - 2 = -1$$
- As:
$$1 + -2 = -1$$
- How to negate a number?
 - We use 2s complement!



🔥 Reminder: 2s complement



$$Y = -x_{N-1} \cdot 2^{N-1} + \sum_{i=0}^{N-2} x_i \cdot 2^i$$

-128	64	32	16	8	4	2	1	Base 10
0	0	0	0	1	0	1	0	10
0	0	0	0	0	0	0	1	1
1	1	1	1	1	1	1	1	-1
1	1	1	1	1	1	1	0	-2
1	0	0	0	1	0	1	0	-118



Calculating the 2s complement

To calculate the 2's complement of an integer:

1. invert the binary equivalent of the number by changing all of the ones to zeroes and all of the zeroes to ones (also called 1's complement), then
2. add one.

- How do we represent -6?

🔥 Calculating the 2s complement

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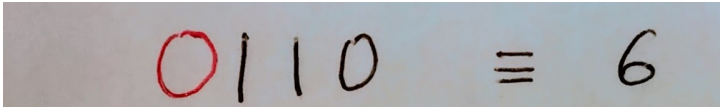
A photograph of a piece of paper with the binary number 110 written on it, followed by an equals sign and the decimal number 6. The paper is placed on a dark surface, and there is a red underline on the surface to the left of the paper.

$$110 \equiv 6$$

🔥 Calculating the 2s complement

To calculate the 2's complement of an integer:

1. invert the binary equivalent of the number by changing all of the ones to zeroes and all of the zeroes to ones (also called 1's complement), then
2. add one.



A photograph of a whiteboard showing the binary number 0110 written in red marker, followed by an equals sign and the decimal number 6.

$$0110 \equiv 6$$

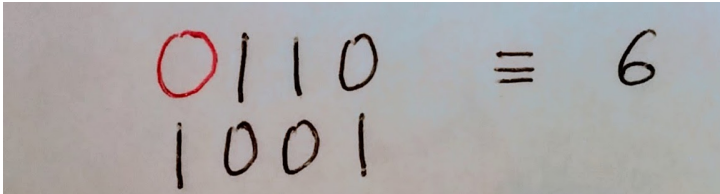
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2. add one.

- How do we represent -6?



Handwritten binary representation of 6 and its 2's complement. The number 6 is represented as 0110, with the leading 0 circled in red. Below it, the 2's complement representation 1001 is shown. To the right of the binary numbers is an equals sign followed by the decimal number 6.

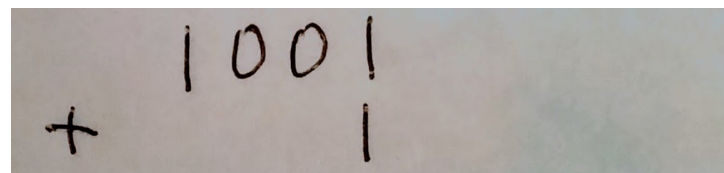
$$\begin{array}{r} 0110 \\ 1001 \end{array} \equiv 6$$

🔥 Calculating the 2s complement

To calculate the 2's complement of an integer:

1. invert the binary equivalent of the number by changing all of the ones to zeroes and all of the zeroes to ones (also called 1's complement), then
2. add one.

- How do we represent -6?



Handwritten binary addition showing 1001 plus 1. The result is 1010, which is the 2's complement of 6.

$$\begin{array}{r} 1001 \\ + 1 \\ \hline 1010 \end{array}$$

🔥 Calculating the 2s complement

To calculate the 2's complement of an integer:

1. invert the binary equivalent of the number by changing all of the ones to zeroes and all of the zeroes to ones (also called 1's complement), then
2. add one.

- How do we represent -6?

$$\begin{array}{r} 1001 \\ + \quad 1 \\ \hline 1010 \end{array} \equiv -6$$

1010

↑ ↑ ↑ ↑

-8 4 2 1

🔥 Subtraction with 2s complement

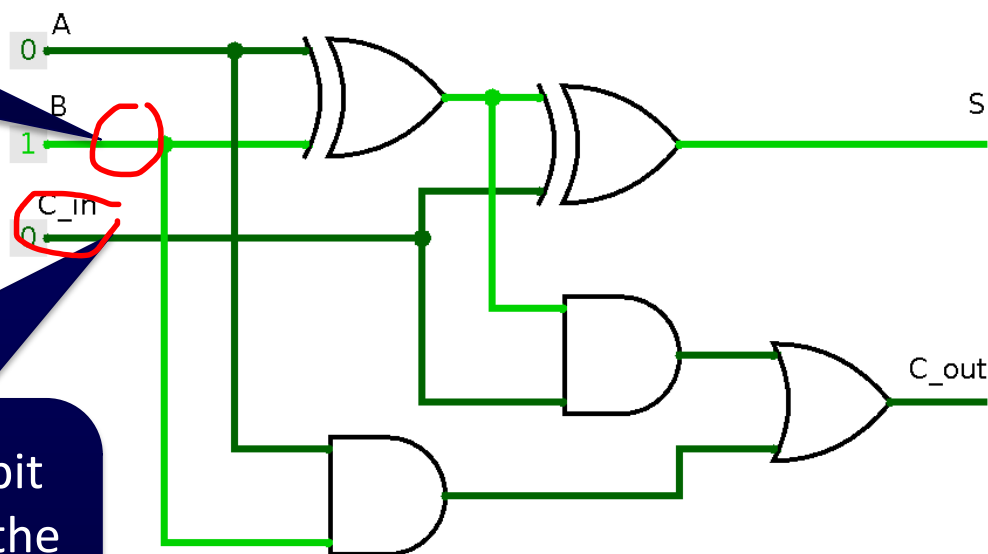
- $A - B = A + (-B) = A + (\text{Not}(B) + 1)$
- We already have all the building blocks we need to implement this!
 - NOT gates to flip bits
 - An unused C_in at the beginning of our adder's carry chain to provide the extra 1.

🔥 Subtraction with 2s complement

- $A - B = A + (-B) = A + (\text{Not}(B) + 1)$
- We already have all the building blocks we need to implement this!
 - NOT gates to flip bits
 - An unused C_in at the beginning of our adder's carry chain to provide the extra 1.

Add a NOT gate here to obtain the 1s complement of B, i.e. NOT(B)

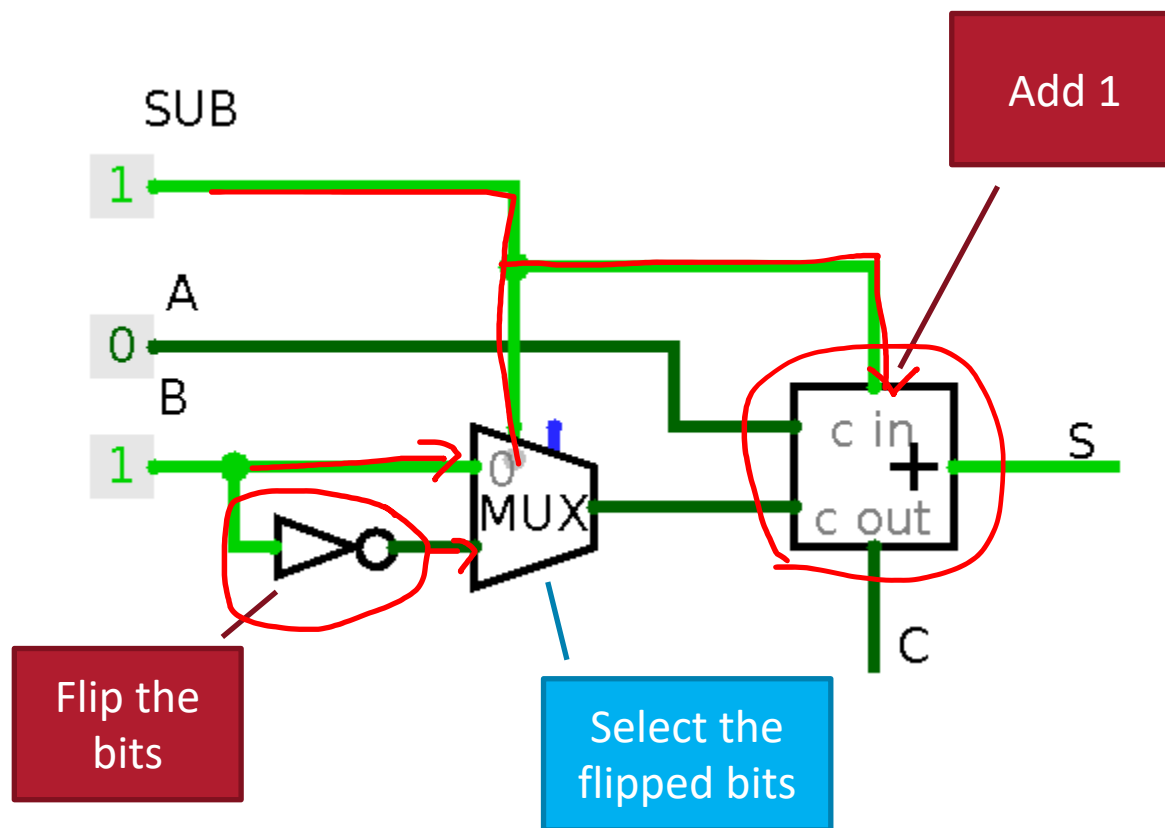
Set the C_in (at the lowest bit position) to 1; this provides the “extra” 1 we need to obtain the 2s complement of B.



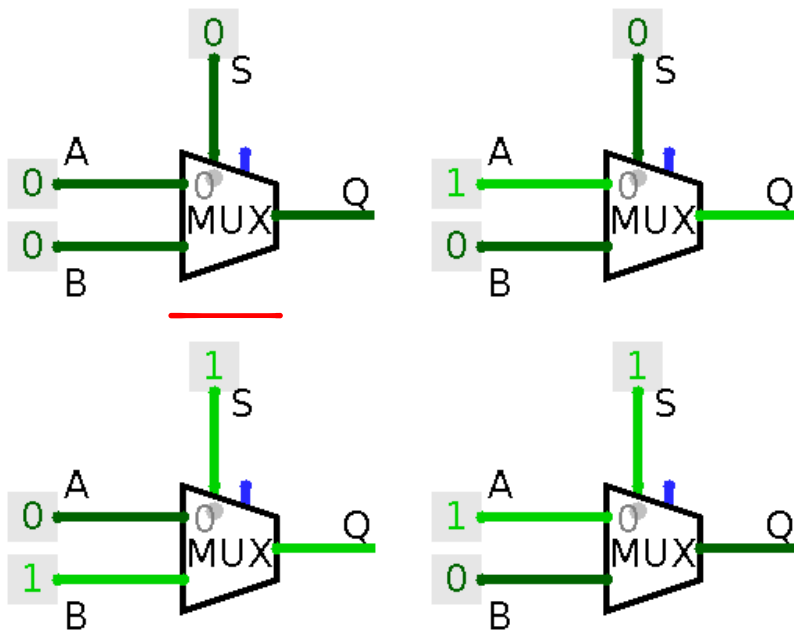
The adder-subtractor?

- We can build an adder **or** a subtractor.
- They are **very similar**.
- Can we build one unit that does **both**?
- Almost...

🔥 Adder-subtractor



🔥 Selecting a signal



<u>S (Select)</u>	<u>A</u>	<u>B</u>	<u>Q</u>
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Selecting a signal

- $\underline{Q} = (\underline{A} \wedge \underline{\neg S}) \vee (\underline{B} \wedge \underline{S})$

- Consider S = 0

- Consider S = 1

S	A	B	Q
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

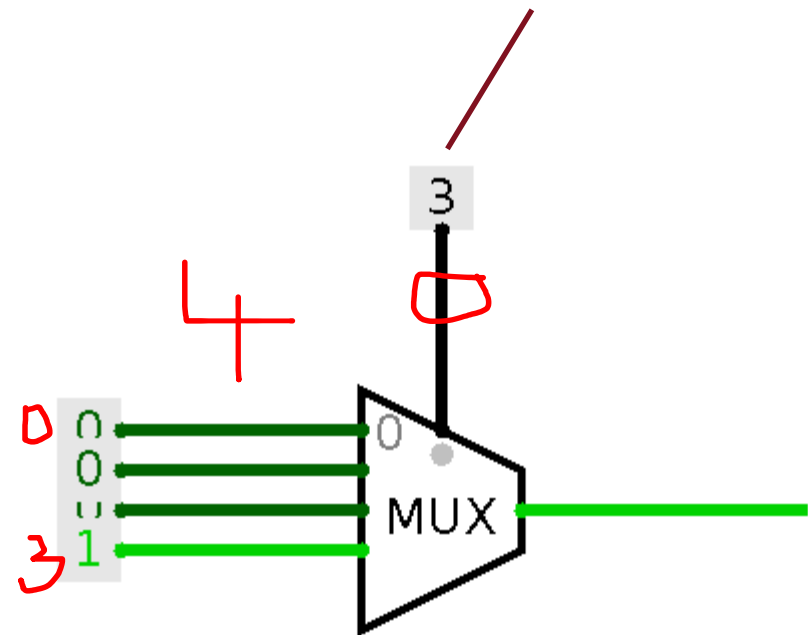
Selecting a signal

- $Q = (A \wedge \underline{\neg S}) \vee (B \wedge \underline{S})$
- Consider $S = 0$
 - $Q = (A \wedge \underline{1}) \vee (B \wedge \underline{0})$
 - $Q = A \vee 0$
 - $Q = A$
- Consider $S = 1$
 - $Q = (A \wedge 0) \vee (B \wedge 1)$
 - $Q = 0 \vee B$
 - $Q = B$

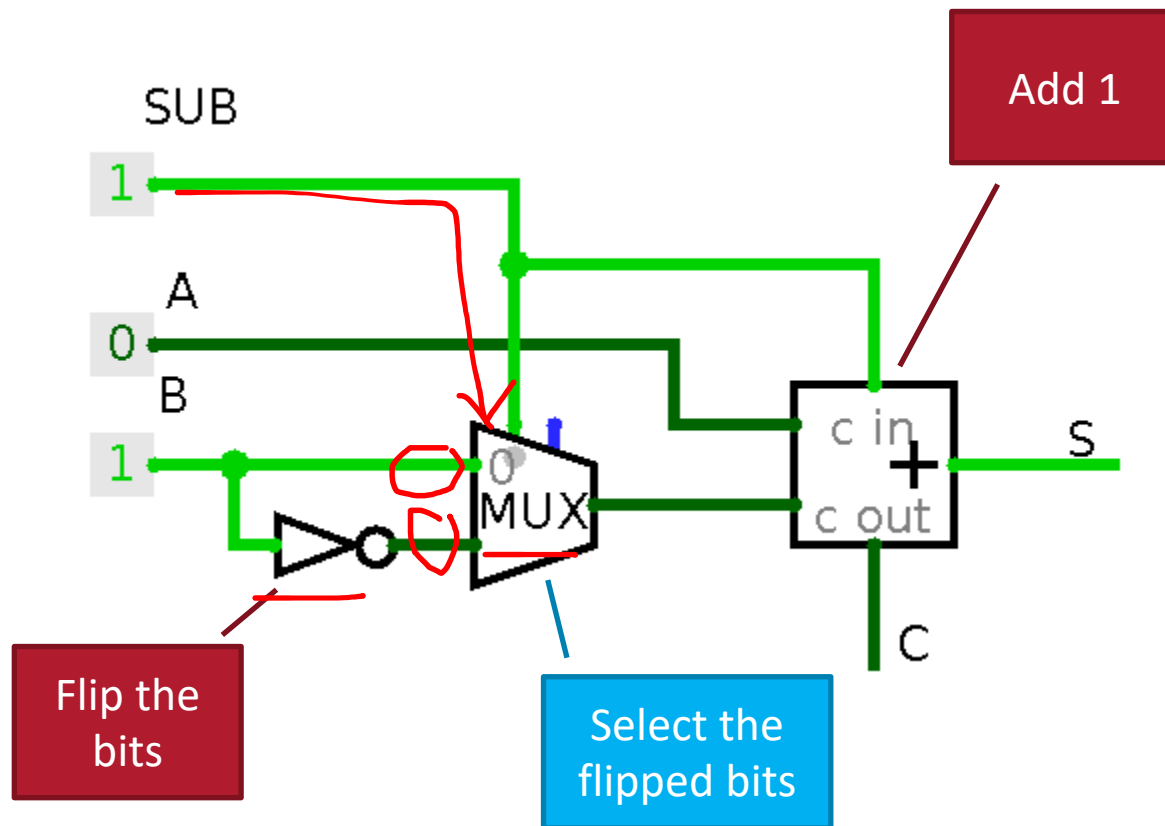
S	A	B	Q
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

🔥 The multiplexer

- S selects which input to propagate to the output.
- 2-to-1 multiplexer
 - 1 select bit
- **N**-to-1 multiplexers are also possible
 - $\log_2(N)$ select bits needed
 - Why?

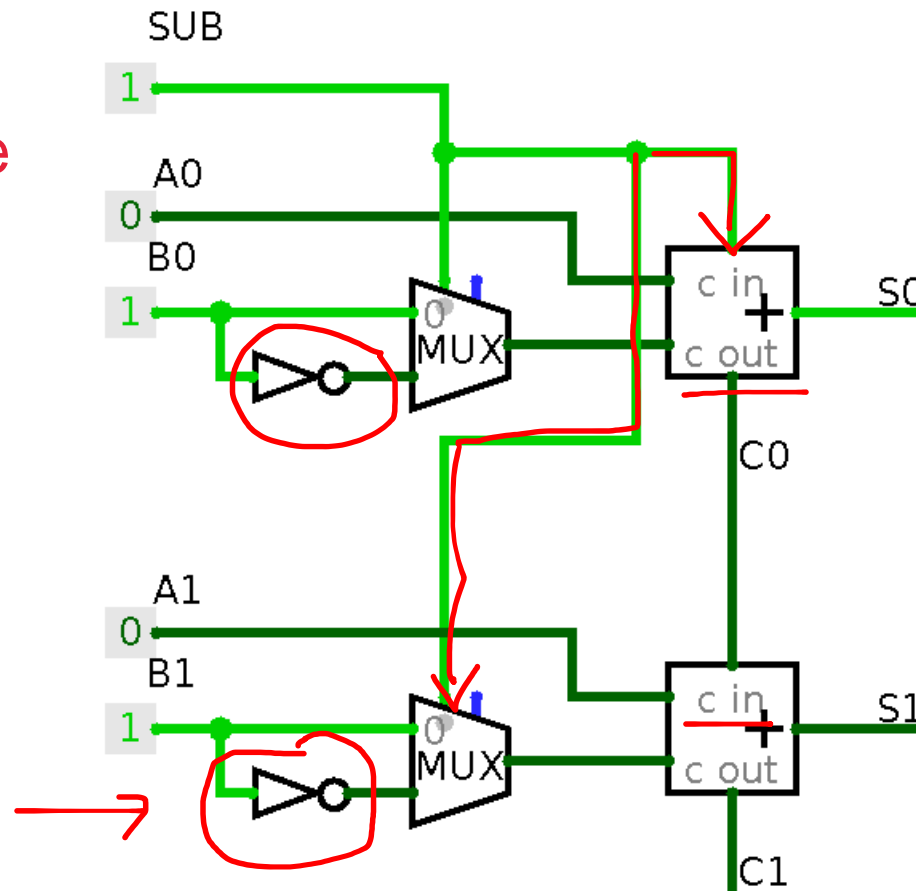


🔥 Adder-subtractor



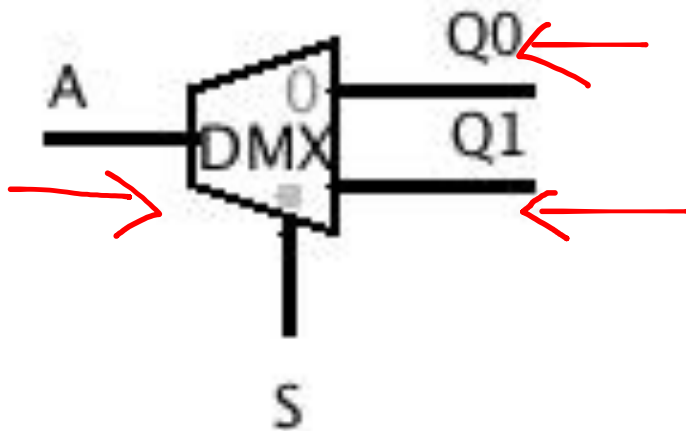
🔥 Ripple-carry adder-subtractor

Only a one is needed at the beginning



Multi bits: like 8-bit 64-bit
Attention: If we want to add $a + b + c$, calculate the first item, then add the result and c

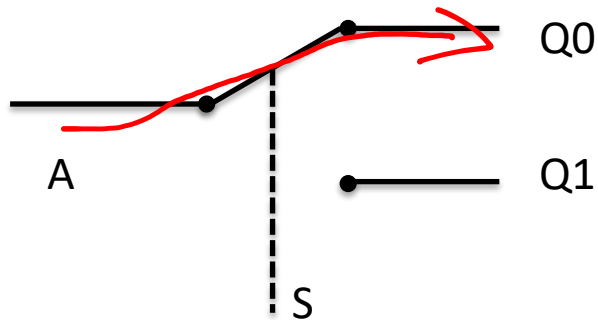
Demultiplexer



- 1-to-2 demultiplexer
 - Choose **which of 2 wires** to **propagate** the input signal onto.
 - $Q0 = A \wedge \neg S$
 - $Q1 = A \wedge S$
- Instead of choosing which signal to select, we choose **where to send** a signal to, i.e. which output carries the input.



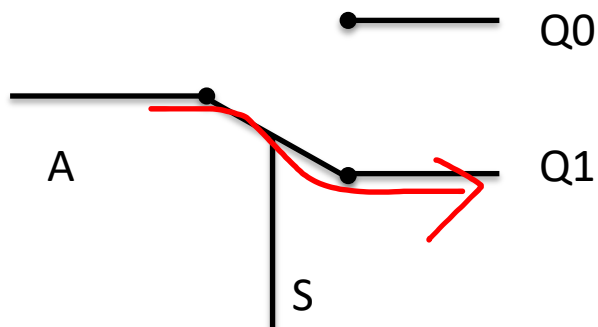
Demultiplexer



The demultiplexer acts like a switch.

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Demultiplexer



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 - **$Q1 = A \wedge S$**
- Instead of choosing which signal to select, we choose **where to send** a signal to, i.e. which output carries the input.

Demultiplexer



S	A	<u>Q0</u>	<u>Q1</u>
0	0	0	0
0	1	1	0
1	0	0	0
1	1	0	1

- 1-to-2 demultiplexer
 - Choose **which of 2 wires** to **propagate** the input signal onto.
 - $Q0 = A \wedge \neg S$
 - $Q1 = A \wedge S$
- Instead of choosing which signal to select, we choose **where to send** a signal to, i.e. which output carries the input.



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Summary

- Used Boolean algebra to build **four simple devices**:
 - Demux, Mux, Adder, Subtractor.
- Combined Mux, NOT gate and adder to build **adder-subtractor**.
- We can do **basic arithmetic** with a **bunch of NAND gates**!

Imagine if we could **store** the results of that arithmetic, somehow...



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Building blocks

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Modules

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Programming

- Machine code, assembly, high-level languages, compilers.

Wrap-up

- Operating systems, energy aware computing.



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