COMSM1302
Overview of Computer Architecture

Lecture 13
Advanced Math Operations

In the previous lecture

- General introduction to ARM architecture.
- Conditional code flags and conditional execution.
- Data processing instructions
  - Arithmetic and logical operations.
  - Comparisons (no results - just set condition codes)
  - Data movement between registers.
- Branching instructions.

In this lecture

Multiplication and Division → Shifting operations → Fixed Point Math → Square Root

- At the end of this lecture:
  - Solve problems that require multiplication and division.
  - Use shifting operations to do efficient calculations.
  - Use fixed point math to do accurate calculations.

Advance Math Operations

Multiplication and Division → Shifting operations → Fixed Point Math → Square Root

Multiplication Instructions

- Multiply
  - MUL{cond}{S} Rd, Rm, Rs
  - Rd = Rm * Rs

32 32 32*32

Usually, we need a 64 bits register to store the multiplication of two 33 bits value. Rd only stores the 32 LSB.

- Multiply Accumulate
  - Does addition for free
  - MLA{cond}{S} Rd, Rm, Rs, Rn
  - Rd = (Rm * Rs) + Rn

Division Instructions

- Unsigned integer division
  - UDIV{cond} Rd, Rm, Rs ; Rd = Rm / Rs

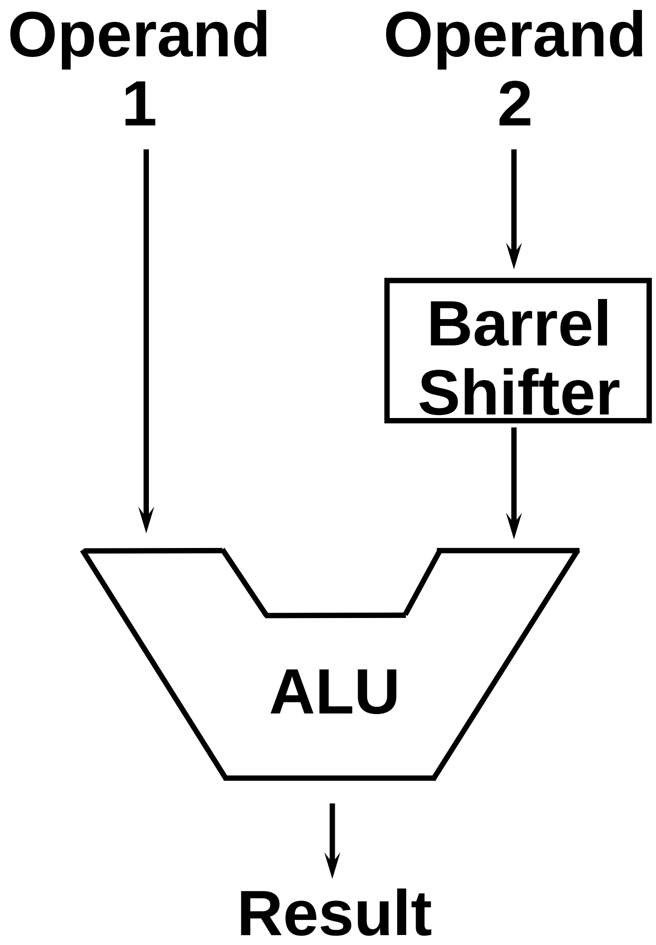
9/2 = 4

- Signed integer division
  - SDIV{cond} Rd, Rm, Rs ; Rd = Rm / Rs

Advance Math Operations

Multiplication and Division → **Shifting operations** → Fixed Point Math → Square Root

The Barrel Shifter



very quick, no need for a clock phase, can be done during any data processing instruction.

Barrel Shifter – Logical Left Shift

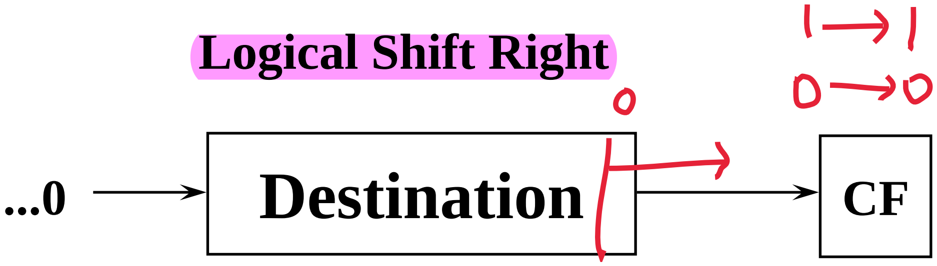
- Shifts left by the specified amount (multiplies by powers of two) e.g.
  - LSL{cond}{S} Rd, Rm, Rs
  - LSL{cond}{S} Rd, Rm, #sh
  - `MOV r1,r0,LSL #1`
  - `LSL r1,r0,#1`

$r_1 = 2 * r_0$

Logical Shift Left (LSL)

Barrel Shifter – Logical Right Shift

- Shifts right by the specified amount (divides by powers of two) e.g.   ÷2
  - LSR{cond}{S} Rd, Rm, Rs
  - LSR{cond}{S} Rd, Rm, #sh
  - `MOV r1,r0,LSR #1`
  - `LSR r1,r0,#1`



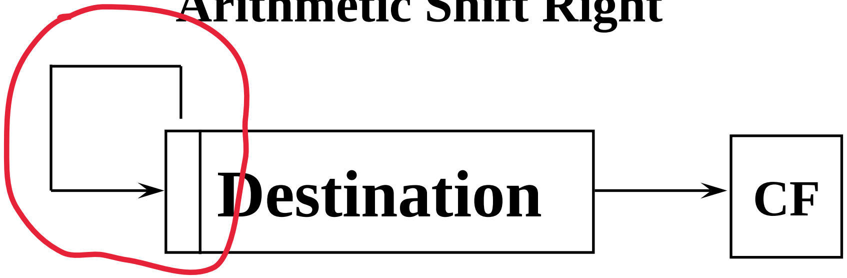
Barrel Shifter – Arithmetic Right Shift

- Shifts right (divides by powers of two) and preserves the sign bit, for 2's complement operations.
  - ASR{cond}{S} Rd, Rm, Rs
  - ASR{cond}{S} Rd, Rm, #sh
  - `MOV r1,r0,ASR #1`
  - `ASR r1,r0,#1`

$r_0 = -6$

$r_1 = -3$

Arithmetic Shift Right



Sign bit shifted in

Barrel Shifter – Shift Operations

- Logical Shift Left: LSL
- Logical Shift Right: LSR
- Arithmetic Shift Right: ASR

- Why do not we have Arithmetic Shift Left (ARL) ?

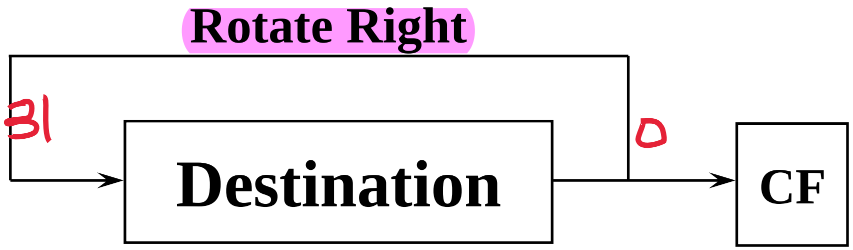
  Because there will be a carryout in the left

Barrel Shifter - Rotations

Rotate Right (ROR)

- Similar to an ASR but the bits wrap around as they leave the LSB and appear as the MSB.
  - ROR{cond}{S} Rd, Rm, Rs
  - ROR{cond}{S} Rd, Rm, #sh
  - `MOV r1,r0,ROR #1`
  - `ROR r1,r0,#1`

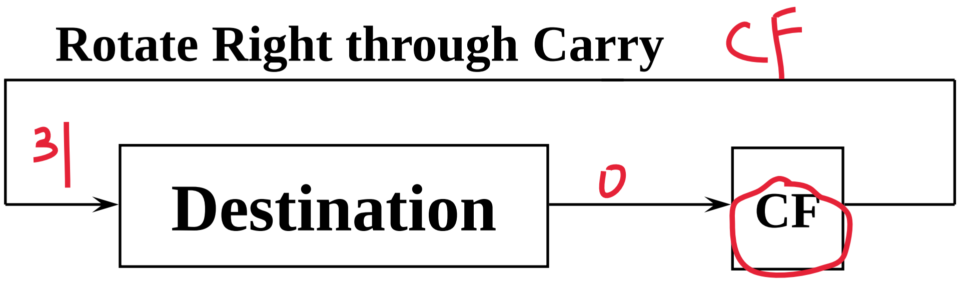
No ROS, because it can be achieved by ROR



Barrel Shifter - Rotations

Rotate Right Extended (RRX) by one bit.

- This operation uses the CPSR C flag as a 33rd bit.
  - RRX{cond}{S} Rd, Rm
  - `MOV r1,r0,RRX`
  - `RRX r1,r0`



Using a Shifted Register

- Multiplications by a constant equal to a ((power of 2) ± 1) can be done in one cycle.

- Example: r0 = r1 * 5
  = r1 + (r1 * 4)

  ADD r0, r1, r1, LSL #2

Using a Shifted Register -2

- Example: r2 = r3 * 105
  = r3 * 15 * 7
  = r3 * (16 - 1) * (8 - 1)

```
RSB r2, r3, r3, LSL #4    ; r2 = r3 * 15
RSB r2, r2, r2, LSL #3    ; r2 = r2 * 7
```

Advance Math Operations

Multiplication and Division → Shifting operations → **Fixed Point Math** → Square Root

Fixed Point Math

- We will use first 8 bits for the fraction and the last 24 bits for the integer part. 24 8
- What is r0 value that represent the value 1.5

r0

0 1 1 0

23 7

Fixed point

Choose the location of the point carefully, considering

- What **range** do you need?
  - from *<smallest number>* to *<largest number>*
- What **precision** do you need?
  - *What is the required distance between successive numbers?*

2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	
8	4	2	1	0.5	0.25	0.125	0.0625	**Base 10**
0	0	0	0	1	0	1	1	**0.6875**
1	0	0	0	1	0	0	0	**8.5**

Multiplication with Fixed Point Math

- Multiplication of two fixed point registers cause lose in integer precision.
  - @24.8 * @24.8 = @16.16

The fraction part bits will grow to 16 bits

if we want to formalize the result to 24.8, lsr #8(right, not left !!!!!!!!!!)

$2 \times 1.5 = 3$

$0010\underbrace{0}_{8} \times 00011000\underbrace{0}_{4} \rightarrow \underbrace{0}_{12}00110\underbrace{0}_{16}$

When calculating the mul, do not calculate integer and fraction seperately

$$
\begin{array}{r}
0010\,\underline{0} \\
\times\, 00011\,\underline{0} \\
\hline
0011\,\underline{0}
\end{array}
$$

Division with Fixed Point Math

- Division of two fixed point registers cause loss in fractional precision.
  - @16.16 / @24.8 = @16.8

3/2 = 1.5

0x300 ÷ 0x200

↓

0x30000 ÷ 0x200

↓

0x180

Advance Math Operations

Multiplication and Division → Shifting operations → Fixed Point Math → **Square Root**

Square Root Algorithm -1/3

- Newton-Raphson's Method (Numerical).
- $y_{n+1} = \left(y_n + \frac{w}{y_n}\right) * {}^{1}/_{2}$

n	w	y_n	$\frac{w}{y_n}$	$\left(y_n + \frac{w}{y_n}\right)$	$\left(y_n + \frac{w}{y_n}\right) * {}^{1}/_{2}$
0	4	4	1	5	2.5
1	4				
2	4				

Square Root Algorithm -2/3

- Newton-Raphson's Method (Numerical).
- $y_{n+1} = \left(y_n + \frac{w}{y_n}\right) * {}^{1}/_{2}$

n	w	y_n	$\frac{w}{y_n}$	$\left(y_n + \frac{w}{y_n}\right)$	$\left(y_n + \frac{w}{y_n}\right) * {}^{1}/_{2}$
0	4	4	1	5	2.5
1	4	2.5	1.6	4.1	2.05
2	4				

Square Root Algorithm -3/3

- Newton-Raphson's Method (Numerical).
- $y_{n+1} = \left(y_n + \frac{w}{y_n}\right) * {}^{1}/_{2}$

n	w	y_n	$\frac{w}{y_n}$	$\left(y_n + \frac{w}{y_n}\right)$	$\left(y_n + \frac{w}{y_n}\right) * {}^{1}/_{2}$
0	4	4	1	5	2.5
1	4	2.5	1.6	4.1	2.05
2	4	2.05	1.951	4.001	2.0006

Square Root Code -1/4

n	w	y_n	$\frac{w}{y_n}$	$\left(y_n + \frac{w}{y_n}\right)$	$\left(y_n + \frac{w}{y_n}\right) * {}^{1}/_{2}$
0	4	4	1	5	2.5
1	4	2.5	1.6	4.1	2.05
2	4	2.05	1.951	4.001	2.0006

Square Root Code -2/4

```
MOV r0,#4
MOV r1,r0
_loop:
UDIV r2,r0,r1
ADD r3,r1,r2
MOV r4,r3, lsr #1
MOV r1,r4
B _loop
```

infinite loop

	r_0	r_1	r_2	r_3	r_4
n	w	y_n	$\frac{w}{y_n}$	$\left(y_n + \frac{w}{y_n}\right)$	$\left(y_n + \frac{w}{y_n}\right) * {}^{1}/_{2}$
0	4	4	1	5	2.5
1	4	2.5	1.6	4.1	2.05
2	4	2.05	1.951	4.001	2.0006

Square Root Code -3/4

```
MOV r0,#4
MOV r1,r0
_loop:
UDIV r2,r0,r1
ADD r3,r1,r2
MOV r4,r3, lsr #1
SUB r5,r1,r4
CMP r5,0.001
BLT _end
MOV r1,r4
B _loop
_end: b _end
```

Consider precision
When yn - final result < 0.001, program will end.

	r_0	r_1	r_2	r_3	r_4
n	w	y_n	$\frac{w}{y_n}$	$\left(y_n + \frac{w}{y_n}\right)$	$\left(y_n + \frac{w}{y_n}\right) * {}^1/_2$
0	4	4	1	5	2.5
1	4	2.5	1.6	4.1	2.05
2	4	2.05	1.951	4.001	2.0006

Square Root Code -4/4

```
MOV r0,#4
MOV r1,r0
MOV r0,r0,lsl #16 @16.16
MOV r1,r1,lsl #8 @24.8
_loop:
UDIV r2,r0,r1 @16.8
ADD r3,r1,r2 @24.8
MOV r4,r3, lsr #1 @24.8
SUB r5,r1,r4 @24.8
CMP r5,#1 @24.8
BLT _end
MOV r1,r4 @24.8
B _loop
_end: b _end
```

n	w	y_n	$\frac{w}{y_n}$	$\left(y_n + \frac{w}{y_n}\right)$	$\left(y_n + \frac{w}{y_n}\right) * {}^1/_2$
0	4	4	1	5	2.5
1	4	2.5	1.6	4.1	2.05
2	4	2.05	1.951	4.001	2.0006

Summary

1. Multiplication and division instructions.
2. Shifting operations.
3. Fixed point math.
4. Example: Newton and Raphson's method