CSC 225 SPRING 2025 ALGORITHMS AND DATA STRUCTURES I ASSIGNMENT 1 UNIVERSITY OF VICTORIA

- 1. [4 marks] I have seven different programming textbooks on my bookshelf, three C++ and four Java. In how many ways can I arrange the books,
 - a) if there are no restrictions?
 - b) if the languages should alternate?
 - c) if all the C++ books must be next to each other?
 - d) if all the C++ books must be next to each other and all the Java books must be next to each other?
- 2. [4 marks] Suppose that you draw a five card hand (i.e. order doesn't matter) from a standard deck of 52.
 - a) How many ways can you draw exactly 3 spades?
 - b) How many ways can you draw at most 3 spades?
 - c) How many ways can you draw 2 spades and 3 hearts?
 - d) How many ways can you draw exactly 1 spade, 2 diamonds and 2 kings, where there may be overlap (that is, 1 king may be a spade, and/or 1 king may be a diamond, etc.)?
- 3. [4 marks] Show that if n is a positive integer and n > 2, then

$$\binom{n}{2} + \binom{n-1}{2}$$

is a <u>perfect square</u> (i.e. its square root is an integer.)

- 4. [4 marks] Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 20$, where
 - a) $x_i \ge 0$, $1 \le i \le 4$
 - b) $x_1, x_2 \ge 2, x_3, x_4 \ge 1$
 - c) $x_i \ge -1, 1 \le i \le 4$
 - d) $x_i \ge 0$, $1 \le i \le 3$, and $2 \le x_4 \le 7$
- 5. [4 marks] Let S be a set of five distinct positive integers, the maximum of which is at most 9. Consider all the subsets A of S where $1 \le |A| \le 3$. Let s_A denote the sum of the elements in subset A. Prove that there must be at least two such subsets of S, say A and B, such that $s_A = s_B$.