CSC 225 Assignment 4

Dryden Bryson

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Question 1:

a) Selection Sort

Steps	Result of steps
(0) Initial Array	1 6 2 4 3 0 7 5
(1) Swap: $0 \leftrightarrow 1$	0 6 2 4 3 1 7 5
(2) Swap: $6 \leftrightarrow 1$	0 1 2 4 3 6 7 5
(3) No change since 2 is already in order	0 1 2 4 3 6 7 5
(4) Swap: $4 \leftrightarrow 3$	0 1 2 3 4 6 7 5
(5) No change since 4 is already in order	0 1 2 3 4 6 7 5
(6) Swap: $5 \leftrightarrow 6$	0 1 2 3 4 5 7 6
(7) Swap: $7 \leftrightarrow 6$	0 1 2 3 4 5 6 7

Array is Sorted (1)

(b) Bubble Sort

Steps	Result of steps
(0) Initial Array	1 6 2 4 3 0 7 5
(1) Swap $6 \leftrightarrow 2$, $6 \leftrightarrow 4$, $6 \leftrightarrow 3$, $6 \leftrightarrow 0$, $7 \leftrightarrow 5$	1 2 4 3 0 6 5 7
(2) Swap $4 \leftrightarrow 3$, $4 \leftrightarrow 0$, $6 \leftrightarrow 5$	1 2 3 0 4 5 6 7
(3) Swap $3 \leftrightarrow 0$	1 2 0 3 4 5 6 7
(4) Swap $2 \leftrightarrow 0$	1 0 2 3 4 5 6 7
(5) Swap $1 \leftrightarrow 0$	0 1 2 3 4 5 6 7
(6) Check Array is Sorted	0 1 2 3 4 5 6 7

Array is Sorted (2)

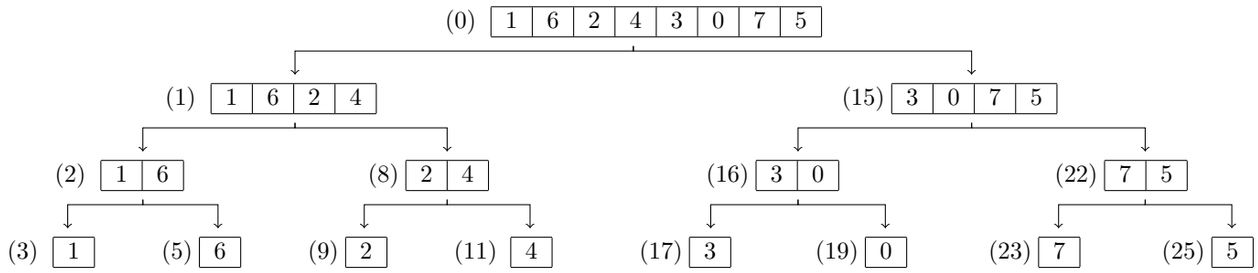
(c) Insertion Sort

Steps	Result of steps
(0) Initial Array	1 6 2 4 3 0 7 5
(1) No change since 6 is larger than one	1 6 2 4 3 0 7 5
(2) Insert 2 at index 1	1 2 6 4 3 0 7 5
(3) Insert 4 at index 2	1 2 4 6 3 0 7 5
(4) Insert 3 at index 2	1 2 3 4 6 0 7 5
(5) Insert 0 at index 0	0 1 2 3 4 6 7 5
(6) No change since 7 is larger than 6	0 1 2 3 4 6 7 5
(6) Insert 5 at index 5	0 1 2 3 4 5 6 7

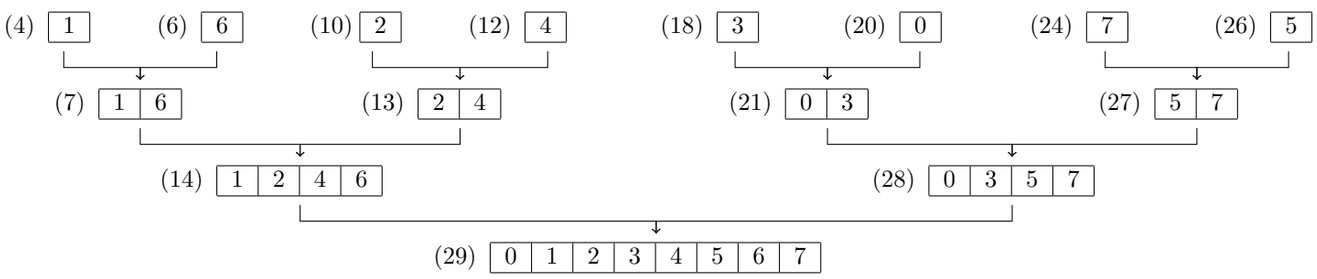
Array is Sorted (3)

d) Merge Sort

Divide:



Merge:



Question 2:

The invariant property we aim to show is true throughout the execution of the loop is as follows: Let k be the loop iteration counter starting at 1, then, the subarray of the array A from indexes 0 to $k-1$ is sorted.

1. **Initialization**

   During initialization $k = 1$, and thus the given sorted subarray is the subarray from 0 to $1 - 1 = 0$, which is a single element. An array containing a single element is always sorted; thus, the invariant holds throughout initialization.

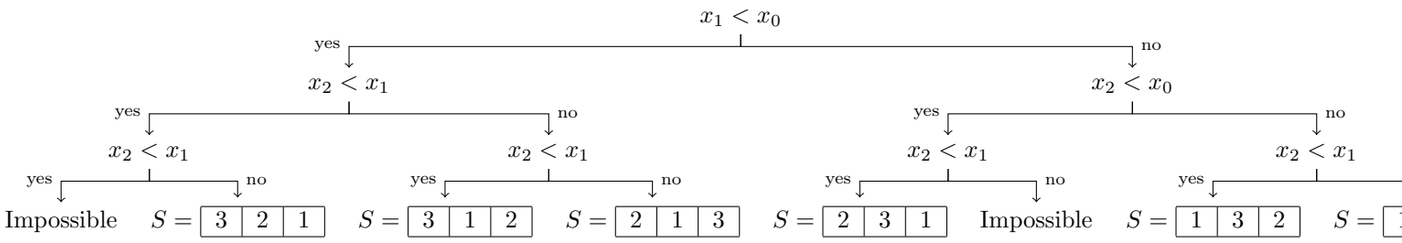
2. **Maintenance**

   We assume that the subarray $A[0..k-1]$ is sorted, and we assume that the inner loop is correct. We begin by storing the value at $A[k]$. Then, the inner loop shifts all elements of the subarray $A[0..k-1]$ that are greater than the stored value of $A[k]$ to the right once, with $A[k]$ getting overwritten on the first shift. This leaves a duplicate pair of values after the element that was found to be less than or equal to the stored value from $A[k]$. Then, $A[k]$ is inserted in the correct spot of the array after the element that was found to be less than or equal to $A[k]$. Thus, the subarray $A[0..k]$ is sorted, and the loop invariant holds during maintenance.

3. **Termination**

   Upon termination of the loop, $k = n-1$, where n is the length of the array A. Thus, the subarray $A[0..n-1]$ is sorted, and the loop invariant holds.

Question 3:


$x_1 < x_0$
yes
no
$x_2 < x_1$
$x_2 < x_0$
yes
no
yes
no
$x_2 < x_1$
$x_2 < x_1$
$x_2 < x_1$
$x_2 < x_1$
yes
no
yes
no
yes
no
yes
no
Impossible
$S =$ 3 2 1
$S =$ 3 1 2
$S =$ 2 1 3
$S =$ 2 3 1
Impossible
$S =$ 1 3 2
$S =$

Question 4:

We count the following primitive operations:

- 1 primary operation calling $\boxed{T.isInternal(v)}$
- c primary operations calling $\boxed{processNode(v)}$
- T(n-1) primary operations calling $\boxed{preorder(T, T.leftChild(v))}$
- 1 primary operation checking if $\boxed{T.isInternal(v)}$ when calling $\boxed{preorder(T, T.rightChild(v))}$

Here I have not counted method calls as operations as the question seems to specifiy only counting the operations of T.isInternal(v) and proccessNode(v), thus the recurrence equation is:

$$T(n) = T(n-1) + c + 2, \quad T(0) = 1$$

To find the closed form of this equation we will first compute:

- $T(n-1) = T(n-2) + c + 2$
- $T(n-2) = T(n-3) + c + 2$
- $T(n-3) = T(n-4) + c + 2$

Then perform the following substitutions:

$$\begin{aligned} T(n) &= T(n-1) + (c+2) \\ &= T(n-2) + (c+2) + (c+2) = T(n-2) + 2(c+2) \\ &= T(n-3) + (c+2) + (c+2) + (c+2) = T(n-3) + 3(c+2) \end{aligned}$$

We then have the following pattern:

$$T(n) = T(n-i) + i(c+2)$$

Solving for i we find that $i = n$ and thus our reccurence equation is as follows:

$$T(n) = 1 + n(c+2)$$

This worst case time complexity falls under the category of $O(n)$