I.C.A 11

Dryden Bryson

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Question 1:

In order to find the GCD with the euclidian algorithm we need to perform repeated division and taking the remainder, thus we perform:

$$\gcd(731, 578)$$
$$731 = 578 \times 1 + 153$$
$$578 = 153 \times 3 + 119$$
$$153 = 119 \times 1 + 34$$
$$119 = 34 \times 3 + 17$$
$$34 = 17 \times 2 + 0$$

Since we have that: $34 \div 17$ has a remainder of zero we conclude that:

$$\gcd(731, 578) = 17$$

Question 2:

We need to find integers x and y such that:

$$17 = 731x + 578y$$

We will use our work from the euclidian algorithm starting with the last division statement with a non-zero remainder. We have that:

$$119 = 34 \times 3 + 17 \quad \rightarrow \quad 17 = 119 - 3 \times 34$$

Substituting in from equation 3 to represent 17 in terms of 119 and 153 we have that:

$$\begin{aligned} 17 &= 119 - 3 \times 34 \\ &= 119 - 3(153 - 119) \\ &= 119 - 3 \times 153 + 3 \times 119 \\ &= 4 \times 119 - 3 \times 153 \end{aligned}$$

Now we substitute from equation 2 to represent 119 in terms of 578 and 153

$$\begin{aligned} 17 &= 4 \times 119 - 3 \times 153 \\ &= 4(578 - 3 \times 153) - 3 \times 153 \\ &= 4 \times 578 - 12 \times 153 - 3 \times 153 \\ &= 4 \times 578 - 15 \times 153 \end{aligned}$$

Now we substitute from the first question to represent 153 in terms of 731 and 578, we have that:

$$\begin{aligned} 17 &= 4 \times 578 - 15 \times 153 \\ &= 4 \times 578 - 15(731 - 578) \\ &= 4 \times 578 - 15 \times 731 + 15 \times 578 \\ &= 19 \times 578 - 15 \times 731 \end{aligned}$$

Now that 17 is represented in terms of 578 and 731 we have that:

$$17 = 731x + 578y$$
$$x = -15 \qquad y = 19$$

Question 3:

If we have that $\gcd(a, b) = 5$ this tells us that the only common prime factor between a and b is 5