Math 122. I.C.A 10

Dryden Bryson

October 30, 2024

Question 1:

First we find the prime factorization of !12:

$$!12 = 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$$

Then we get the prime factorization of each non-prime number in the factorization:

$$!12 = (3 \times 2^2) \times 11 \times (5 \times 2) \times (3^2) \times (2^3) \times 7 \times (3 \times 2) \times 5 \times (2^2) \times 3 \times 2$$

Then collecting the terms:

$$!12 = 11 \times 7 \times 5 \times 3^5 \times 2^{10}$$

Now we can square both sides and simplify using the distributive propery:

$$\begin{aligned}(!12)^2 &= (11 \times 7 \times 5 \times 3^5 \times 2^{10})^2 \\ &= 11^2 \times 7^2 \times 5^2 \times 3^{5\times 2} \times 2^{10\times 2} \\ &= 11^2 \times 7^2 \times 5^2 \times 3^{10} \times 2^{20}\end{aligned}$$

Question 2:

Since $a|b$ and $b|c^2$ there exists integers k and m such that $a \times k = b$ and $b \times m = c^2$. We have that

$$\begin{aligned}c^2 &= b \times m \\ \left(c^2\right)^2 &= (b \times m)^2 \\ c^4 &= b^2 \times m^2\end{aligned}$$

Let us know substitute b for $b = a \times k$:

$$\begin{aligned}c^4 &= b^2 \times m^2 \\ c^4 &= (b \times b) \times m^2 \\ &= ((a \times k) \times b) \times m^2 \\ &= (a \times b)(k \times m^2)\end{aligned}$$

Thus we have that $c^4 = (a \times b) \times n$ for $n = (k \times m^2)$ thus:

$$ab|c^4 \quad \text{or} \quad c^4 = (ab)n$$

Since $n = (k \times m^2)$ is an integer we conclude that $ab|c^4$. $\square$