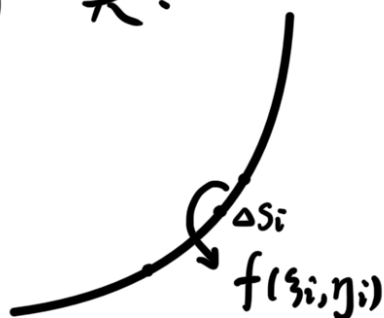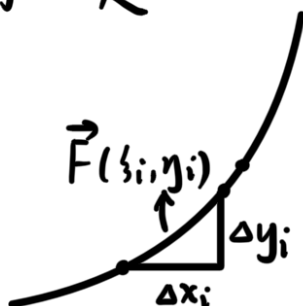曲线积分与曲面积分 (3)

两类曲线积分之间的联系
第一类：

$$\Delta m \approx f(\xi_i, \eta_i) \Delta s_i$$

$$\Rightarrow m \approx \sum_{i=1}^{n} f(\xi_i, \eta_i) \Delta s_i$$

$$f \Delta s$$

$$\Rightarrow f \sqrt{\Delta x^2 + \Delta y^2}$$

第二类：

$$\Delta W_i \approx \vec{F}(\xi_i, \eta_i) \cdot \overrightarrow{M_{i-1} M_i}$$

$$\Delta W_i \approx P(\xi_i, \eta_i) \Delta x_i + Q(\xi_i, \eta_i) \Delta y_i$$

$$\left(P \frac{\Delta x}{\sqrt{\Delta x^2 + \Delta y^2}} + Q \frac{\Delta y}{\sqrt{\Delta x^2 + \Delta y^2}} \right) \sqrt{\Delta x^2 + \Delta y^2}$$

$$\Rightarrow (P \cos\alpha + Q \cos\beta) \sqrt{\Delta x^2 + \Delta y^2}$$

$$\Rightarrow f \sqrt{\Delta x^2 + \Delta y^2}$$

第一类：

$$\int_L f(x,y) ds = \int_\alpha^\beta f(\varphi(t), \psi(t)) \sqrt{\varphi'^2(t) + \psi'^2(t)}\, dt$$

第二类：

$$\int_L P(x,y)dx + Q(x,y)dy$$

$$= \int_\alpha^\beta \left(P(\varphi(t),\psi(t))\varphi'(t) + Q(\varphi(t),\psi(t))\psi'(t) \right)dt$$

$$= \int_\alpha^\beta \left(P(\varphi(t),\psi(t))\frac{\varphi'(t)}{\sqrt{\varphi'^2(t)+\psi'^2(t)}} + Q(\varphi(t),\psi(t))\frac{\psi'(t)}{\sqrt{\varphi'^2(t)+\psi'^2(t)}} \right) \sqrt{\varphi'^2(t)+\psi'^2(t)}\; dt$$

$$= \int_\alpha^\beta \left(P(\varphi(t),\psi(t))\cos\alpha + Q(\varphi(t),\psi(t))\cos\beta \right)\sqrt{\varphi'^2(t)+\psi'^2(t)}\,dt$$

$$= \int_L \left[P(x,y)\cos\alpha + Q(x,y)\cos\beta \right]dS$$