不定积分草稿3

分部积分法 ☆.

$$(uv)' = u'v + uv'$$

$$\Rightarrow uv' = (uv)' - u'v$$

$$\Rightarrow \int uv'dx = uv - \int u'vdx$$

$$\Rightarrow \int udv = uv - \int vdu$$

$$\int udv = uv - \int vdu$$

$$\int xe^x dx = \int xde^x$$

$$= xe^x - \int e^x dx$$

$$= xe^x - e^x + C$$

$$= (x-1)e^x + C$$

$$\int x\cos x\,dx$$

$$=\int x\,d\sin x$$

$$=x\sin x-\int \sin x\,dx$$

$$=x\sin x+\cos x+C$$

$$\int \sin x\,e^x\,dx$$

$$=\int \sin x\,de^x$$

$$=\sin x\,e^x-\int e^x\,d\sin x$$

$$=\sin x\,e^x-\int e^x\cos x\,dx$$

$$=\sin x\,e^x-\int \cos x\,de^x$$

$$=\sin x\,e^x-\cos x\,e^x-\int e^x\sin x\,dx$$

$\Rightarrow 2$原式$=\sin x\,e^x-\cos x\,e^x$

$\Rightarrow$原式$=\dfrac{\sin x\,e^x-\cos x\,e^x}{2}$

导数$=\frac{1}{2}(\cos x\,e^x+\sin x\,e^x+\sin x\,e^x-\cos x\,e^x)$

$=\sin x\,e^x$

$$\int x^2e^x\,dx$$

$$=\int x^2\,de^x$$

$$= \int x^2 de^x$$

$$= x^2e^x - \int e^x dx^2$$

$$= x^2e^x - \int 2x\, de^x$$

$$= x^2e^x - 2xe^x + \int e^x d2x$$

$$= (x^2-2x)e^x + 2e^x + C$$

$$= (x^2-2x+2)e^x + C$$

$$\int \ln x\, dx$$

$$= x\ln x - \int x\, d\ln x$$

$$= x\ln x - \int 1 dx$$

$$= x\ln x - x + C$$

$$\int x\ln x\, dx$$

$$= \frac{1}{2}\int \ln x\, dx^2$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{2}\int x^2 d\ln x$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$$

$$\int x\ln x\, dx$$

$$= \int x\, d(x\ln x - x)$$

$= x(x\ln x - x) - \int (x\ln x - x)dx$

$= x^2\ln x - x^2 + \frac{1}{2}x^2 - \int x\ln x dx$

$\Rightarrow 2y = x^2\ln x - \frac{1}{2}x^2 + C$

$\Rightarrow y = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$

$\int \arccos x dx$

$= x\arccos x - \int x d\arccos x$

$= x\arccos x + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}} dx^2$

$= x\arccos x - \frac{1}{2}\int (1-x^2)^{-\frac{1}{2}} d(1-x^2)$

$= x\arccos x - (1-x^2)^{\frac{1}{2}} + C$

$\int x\arctan x dx$

$= \frac{1}{2}\int \arctan x dx^2$

$= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int x^2 \cdot \frac{1}{1+x^2} dx$

$\int x^2 d\arctan x$

$$\frac{1+x^2-1}{1+x^2}$$

$$= \int x^2 \cdot \frac{1}{1+x^2}\, dx$$

$$= \frac{1}{2}x^2 \arctan x - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right) dx$$

$$= \frac{1}{2}x^2 \arctan x - \frac{1}{2}x + \frac{1}{2}\arctan x + c$$

$$= \frac{1}{2}(x^2+1)\arctan x - \frac{1}{2}x + C$$

$$\int e^x \sin x\, dx$$

$$= \int \sin x\, de^x$$

$$= \sin x e^x - \int e^x \cos x\, dx$$

$$= \sin x e^x - \int \cos x\, de^x$$

$$= \sin x e^x - \cos x e^x - \int e^x \sin x\, dx$$

$$\Rightarrow 2y = e^x(\sin x - \cos x) + C$$

$$\Rightarrow y = \frac{1}{2}e^x(\sin x - \cos x) + C$$

$$\int \sec^3 x\, dx$$

$$= \int \sec x \, d\tan x$$

$$= \sec x \tan x - \int \tan x \, d\sec x$$

$$= \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1) dx$$

$$= \sec x \tan x - \int (\sec^3 x - \sec x) \, dx$$

$$= \sec x \tan x + \ln|\sec x + \tan x| - \int \sec^3 x \, dx$$

$$\Rightarrow 2y = \sec x \tan x + \ln|\sec x + \tan x|$$

$$\Rightarrow y = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|)$$

$$\int e^{\sqrt{x}} dx$$

令 $t = \sqrt{x}$，则 $x = t^2$

$$\int e^{\sqrt{x}} dx = \int e^t \, dt^2$$

$$= 2\int t \, de^t$$

$$= 2\int t\, de^{t}$$

$$= 2\left(t \cdot e^{t} - \int e^{t} dt\right)$$

$$= 2\left(t \cdot e^{t} - e^{t}\right) + C$$

$$= 2(t-1)e^{t} + C$$

$$= 2(\sqrt{x}-1)e^{\sqrt{x}} + C$$