曲率：

$$\Delta s = 2\cancel{\pi} r \times \frac{\Delta\alpha}{\cancel{2\pi}} = \Delta\alpha \cdot r$$

$$k = \frac{\Delta\alpha}{\Delta s} = \frac{\Delta\alpha}{\Delta\alpha \cdot r} = \frac{1}{r}$$

$$k = \frac{\Delta\alpha}{\Delta s}$$

$$y = f(x)$$
$$y' = \tan\alpha$$
$$y'' = \sec^2\alpha \cdot \frac{d\alpha}{dx}$$

$$\Rightarrow \frac{d\alpha}{dx} = \frac{y''}{\sec^2\alpha} = \frac{y''}{1+\tan^2\alpha}$$
$$= \frac{y''}{1+(y')^2}$$

$$\Rightarrow d\alpha = \frac{y'' \cdot dx}{1+(y')^2}$$

$$ds = \sqrt{1+(y')^2}\,dx$$

$$\therefore k = \left|\frac{d\alpha}{[illegible]}\right| = \left|\frac{\frac{y''\cancel{dx}}{1+(y')^2}}{[illegible]}\right| = \frac{|y''|}{[illegible]},$$

$\therefore K = |ds| = \left| \sqrt{1+(y')^2}\,\cancel{dx} \right| = [1+(y')^2]^{\frac{3}{2}}$

$$\left(\frac{\Delta s}{\Delta x}\right)^2 = \left(\frac{MM'}{\Delta x}\right)^2$$

$$= \left(\frac{MM'}{|MM'|}\right)^2 \cdot \left(\frac{|MM'|}{\Delta x}\right)^2$$

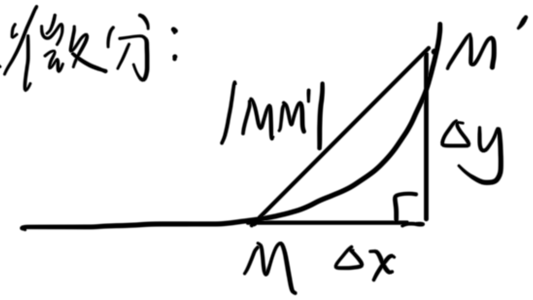
$$= \left(\frac{MM'}{|MM'|}\right)^2 \cdot \frac{\Delta x^2 + \Delta y^2}{\Delta x^2}$$

$$= \left(\frac{MM'}{|MM'|}\right)^2 \cdot [1 + (y')^2]$$

$$\therefore \frac{ds}{dx} = \sqrt{1+y'^2}$$

$$\Rightarrow ds = \sqrt{1+y'^2}\,dx$$

弧微分：



$$\frac{\Delta s}{\Delta x} = \frac{MM'}{\Delta x}$$

当 $M \to M'$ 时，$\dfrac{MM'}{|MM'|} = 1$

$\simeq 1$

$ds \sim MM' \quad |MM'|$

$$\therefore \frac{\quad}{dx} = \frac{\quad}{|MM'|} \cdot \frac{\quad}{\Delta x}$$

$$= \frac{\sqrt{\Delta x^2 + \Delta y^2}}{\Delta x}$$

$$= \sqrt{\frac{\Delta x^2 + \Delta y^2}{\Delta x^2}}$$

$$= \sqrt{1 + y'^2}$$

$$\Rightarrow ds = \sqrt{1+y'^2} \cdot dx$$

$$y' = \tan\alpha$$

$$y'' = \sec^2\alpha \cdot \frac{d\alpha}{dx}$$

$$\Rightarrow d\alpha = \frac{y''}{\sec^2\alpha} \cdot dx$$

$$= \frac{y''}{1+\tan^2\alpha} \cdot dx$$

$$= \frac{y''}{1+y'^2} \cdot dx$$

$$\therefore k = \left| \frac{d\alpha}{ds} \right|$$

$$= \left| \frac{y''}{1+y'^2} \cdot \cancel{dx} \right|$$

$$\left| \frac{\quad}{\sqrt{1+y'^2}\cdot \cancel{dx}} \right|$$

$$= \left| \frac{y''}{(1+y'^2)^{\frac{3}{2}}} \right|$$

$$\tan\alpha(t) = \frac{y'(t)}{x'(t)}$$

$$\Rightarrow \alpha(t) = \arctan\frac{y'(t)}{x'(t)}$$

$$\Rightarrow \alpha'(t) = \frac{1}{1+\left(\frac{y'}{x'}\right)^2}\cdot\frac{x'y''-x''y'}{(x')^2}$$

$$又 \because s'(t) = (x'^2+y'^2)^{\frac{1}{2}}$$

$$\therefore \bar{K} = \left|\frac{\alpha'(t)}{s'(t)}\right|$$

$$= \frac{\dfrac{x'y''-x''y'}{(x')^2\cdot\left[1+\left(\frac{y'}{x'}\right)^2\right]}}{(x'^2+y'^2)^{\frac{1}{2}}}$$

$$= \frac{x'y''-x''y'}{(x'^2+y'^2)^{\frac{3}{2}}}$$

$$y = x^{-1}$$

$$y' = -x^{-2}$$
$$y'' = 2x^{-3}$$

$$\therefore K = \frac{|y''|}{(1+y'^2)^{\frac{3}{2}}} = \frac{2}{\sqrt{2^3}}$$
$$= \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$y = ax^2 + bx + c$$
$$y' = 2ax + b$$
$$y'' = 2a$$

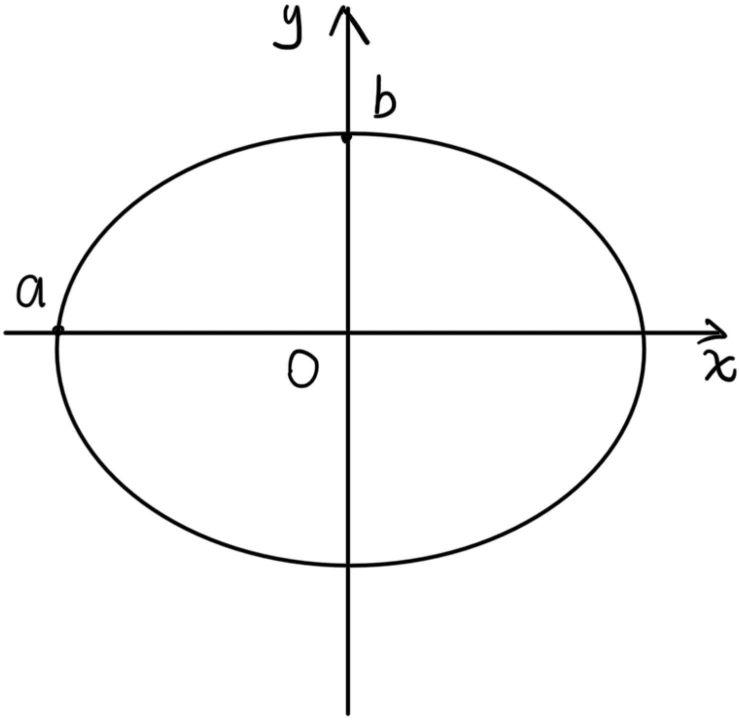
$$K = \frac{|y''|}{(1+y'^2)^{\frac{3}{2}}}$$
$$= \frac{|2a|}{[1+(2ax+b)^2]^{\frac{3}{2}}}$$

要使 K 最大，

只要使 $2ax+b=0$

$$\Rightarrow x = -\frac{b}{2a}$$

（对称轴）



$$x = a \cdot \cos t$$

$$y = b \cdot \sin t$$

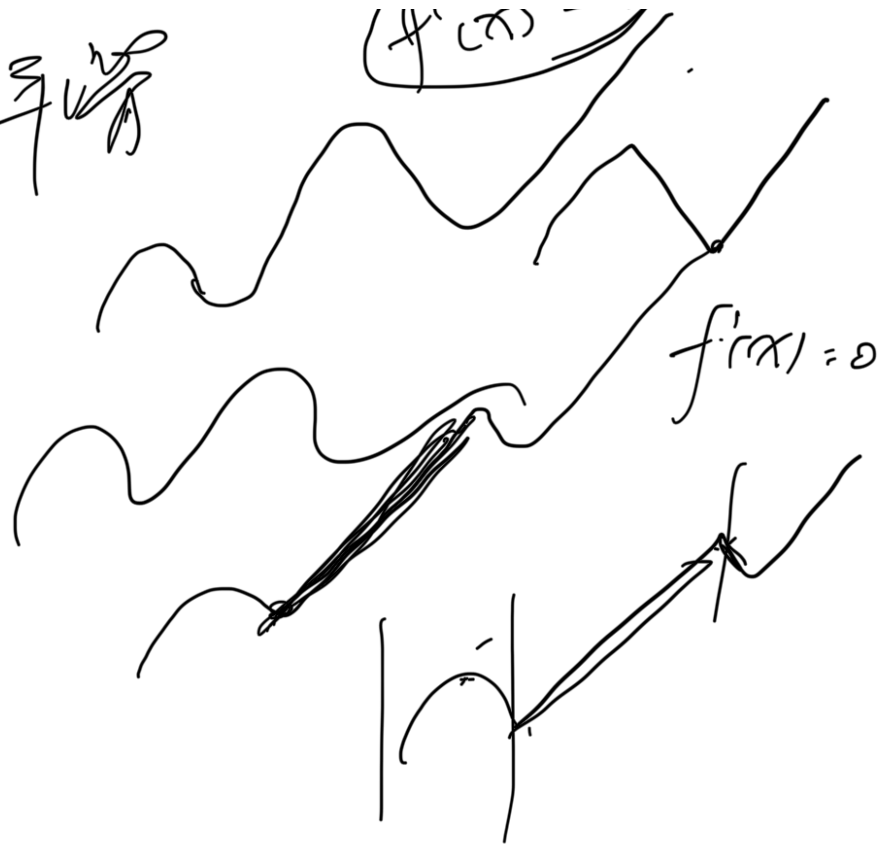
$$K = \frac{|x'y'' - x''y'|}{(x'^2 + y'^2)^{\frac{3}{2}}}$$

$$= \frac{a\sin t \cdot b\sin t + a\cos t \cdot b\cos t}{(a^2\sin^2 t + b^2\cos^2 t)^{\frac{3}{2}}}$$

$$= \frac{ab(\sin^2 t + \cos^2 t)}{(a^2\sin^2 t + b^2 - b^2\sin^2 t)^{\frac{3}{2}}}$$

$$= \frac{ab}{[(a^2 - b^2)\sin^2 t + b^2]^{\frac{3}{2}}}$$

平滑



$(x_1, 0) \cup (x_1, x_0)$ 极大

$x_0 \to 0$

x_2 极小