不定积分草稿1

$$\int \sqrt{x}\,(x^2-5)\,dx$$

$$= \int \left(x^{\frac{5}{2}} - 5x^{\frac{1}{2}}\right) dx$$

$$= \frac{2}{7}x^{\frac{7}{2}} - \frac{10}{3}x^{\frac{3}{2}} + C$$

$$\int \frac{(x-1)^3}{x^2}\,dx$$

$$= \int \frac{(x-1)(x^2-2x+1)}{x^2}\,dx$$

$$= \int \frac{x^3-3x^2+3x-1}{x^2}\,dx$$

$$= \int \left(x - 3 + 3x^{-1} - x^{-2}\right) dx$$

$$= \frac{1}{2}x^2 - 3x + 3\ln|x| + x^{-1} + C$$

$$\int \left(e^x - 3\cos x\right) dx$$

$$= e^x - 3\sin x + C$$

$$\int 2^x e^x\,dx$$

$$\int 2^x e^x \, dx$$

$$= \int (2e)^x \, dx$$

$$= \frac{(2e)^x}{\ln(2e)} + C$$

$$= \frac{2^x e^x}{1+\ln 2} + C$$

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

$$\int \tan^2 x \, dx$$

$$= \int (\sec^2 x - 1) \, dx$$

$$= \tan x - x + C$$

$$\int \sin^2 \frac{x}{2} \, dx$$

$$\cos x = 1 - 2\sin^2 \frac{x}{2}$$

$$\Rightarrow \cancel{2}\sin^2 \frac{x}{2} = \frac{1-\cos x}{2}$$

$$= \int \left(\frac{1}{2} - \frac{1}{2}\cos x\right) dx$$

$$= \frac{1}{2}x - \frac{1}{2}\sin x + C$$

$$\int \frac{1}{\sin^2 \frac{x}{2} \cos^2 \frac{x}{2}} \, dx$$

$$= \int \frac{4}{\sin^2 x} \, dx$$

$$= -4\cot x + C$$

$$\int \frac{2x^4+x^2+3}{x^2+1}\,dx$$

$$= \int \frac{2(x^4+x^2)-(x^2+1)+4}{x^2+1}\,dx$$

$$= \int \left(2x^2-1+\frac{4}{x^2+1}\right)dx$$

$$= \frac{2}{3}x^3 - x + 4\arctan x + C$$

$$x^3-4x^2+3x+2$$

$$x^2+x+1 \,\big)\, x^5-3x^4+x^2-6x+1$$

$$x^5+x^4+x^3$$

$$-4x^4-x^3+x^2-6x+1$$

$$-4x^4-4x^3-4x^2$$

$$3x^3+5x^2-6x+1$$

$$3x^3+3x^2+3x$$

$$2x^2-9x+1$$

$$2x^2+2x+2$$

$$-11x-1$$

凑微分法：

$$\int 2\cos 2x \, dx$$
$$= \int \cos 2x \, d2x$$
$$= \sin(2x) + C$$

$$\int \frac{1}{3+2x} dx$$
$$= \frac{1}{2} \int \frac{d(3+2x)}{3+2x}$$
$$= \frac{1}{2} \ln|2x+3| + C$$

$$\int f(\varphi(x)) \, d\varphi(x) = F(\varphi(x)) + C$$

$$\int 2\cos 2x \, dx$$
$$= \int \cos 2x \, d2x$$
$$= \sin(2x) + C$$

$$\int \frac{1}{2x+3} dx$$

$$= \frac{1}{2}\int \frac{1}{2x+3} \cdot 2dx$$

$$= \frac{1}{2}\int \frac{1}{2x+3}\, d(2x+3)$$

$$= \frac{1}{2} \ln|2x+3| + C$$

$$\int 2xe^{x^2}dx$$

$$= \int e^{x^2}\, d(x^2)$$

$$= e^{x^2} + C$$

$$\int x\sqrt{1-x^2}\, dx$$

$$= -\frac{1}{2}\int \sqrt{1-x^2}\, d(1-x^2)$$

$$= -\frac{1}{2} \cdot \frac{(1-x^2)^{\frac{3}{2}}}{\frac{3}{2}}$$

$$= -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x^2}{(x+2)^3}dx$$

令 $x+2=u$

$\Rightarrow x=u-2$

$$= \int \frac{(u-2)^2}{}\, du$$

$$\int \frac{\quad}{u^{3}}\,du$$

$$= \int \frac{u^2-4u+4}{u^3}\,du$$

$$= \int (u^{-1} - 4u^{-2} + 4u^{-3})\,du$$

$$= \ln|u| - \frac{4u^{-1}}{-1} + \frac{4u^{-2}}{-2} + C$$

$$= \ln|u| + 4u^{-1} - 2u^{-2} + C$$

$$= \ln|x+2| + 4(x+2)^{-1} - 2(x+2)^{-2} + C$$

$$\int \frac{1}{a^2+x^2}\,dx \quad (a \neq 0)$$

$$= \frac{1}{a} \int \frac{1}{1+(\frac{x}{a})^2}\,d(\frac{x}{a})$$

$$= \frac{1}{a} \arctan(\frac{x}{a}) + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}}$$

$$= \int \frac{dx}{a\sqrt{1-(\frac{x}{a})^2}}$$

$$= \int \frac{1}{\sqrt{1-(\frac{x}{a})^2}}\,d(\frac{x}{a})$$

$$= \arcsin\left(\frac{}{a}\right) + C$$

$$\int \frac{1}{x^2-a^2}\,dx$$

$$= \int \frac{1}{(x+a)(x-a)}\,dx$$

$$= \int \left(\frac{1}{x-a} - \frac{1}{x+a}\right) \times \frac{1}{2a}\,dx$$

$$= \frac{1}{2a}\left[\int \frac{1}{x-a}\,d(x-a) - \int \frac{1}{x+a}\,d(x+a)\right]$$

$$= \frac{1}{2a}\left[\ln|x-a| - \ln|x+a|\right] + C$$

$$= \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{dx}{x(1+2\ln x)} = \int \frac{d\ln x}{1+2\ln x}$$

$$= \frac{1}{2}\int \frac{1}{1+2\ln x}\,d(1+2\ln x)$$

$$= \frac{1}{2}\ln|1+2\ln x| + C$$

$$\int \frac{e^{3\sqrt{x}}}{\sqrt{x}} dx$$

$$= \int e^{3x^{\frac{1}{2}}} \cdot x^{-\frac{1}{2}} dx$$

$$= 2\int e^{3x^{\frac{1}{2}}} \cdot d(x^{\frac{1}{2}})$$

$$= \frac{2}{3} \int e^{3x^{\frac{1}{2}}} d(3x^{\frac{1}{2}})$$

$$= \frac{2}{3} e^{3x^{\frac{1}{2}}} + C$$

$$\int \sin^3 x \, dx$$

$$= \int (1-\cos^2 x) \sin x \, dx$$

$$= -\int (1-\cos^2 x) \, d(\cos x)$$

$$= -\cos x + \frac{1}{3}\cos^3 x + C$$

$$\int \sin^2 x \cos^5 x \, dx$$

$$= \int \sin^2 x \cos^4 x \, d \sin x$$

$$= \int \sin^2 x \, (1 - \sin^2 x)^2 \, d \sin x$$

$$= \int \sin^2 x (1 - 2\sin^2 x + \sin^4 x) \, d \sin x$$

$$= \int (\sin^6 x - 2\sin^4 x + \sin^2 x) \, d \sin x$$

$$= \frac{1}{7}\sin^7 x - \frac{2}{5}\sin^5 x + \frac{1}{3}\sin^3 x + C$$

$$\int \tan x \, dx = -\int \frac{1}{\cos x} \, d \cos x$$

$$= -\ln|\cos x| + C$$

$\ln|\cos x| + C$

$$\int \cot x \, dx = \int \frac{1}{\sin x} \, d \sin x$$

$$= \ln|\sin x| + C$$

$$\int \cos^2 x \, dx$$

$\cos 2x = 2\cos^2 x - 1$

$\Rightarrow \cos^2 x = \dfrac{\cos 2x + 1}{2}$

$$= \int \frac{\cos 2x + 1}{2} \, dx$$

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$

$$= \int \frac{\cos 2x + 1}{4} \, d(2x)$$

$$= \frac{1}{4} \sin 2x + \frac{1}{2} x + C$$

$$\int \sin^2 x \cos^4 x \, dx$$

$$= \int \frac{1-\cos 2x}{2} \cdot \left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int \cos^2 x \, dx$$

$$= \int \frac{1+\cos 2x}{2} dx$$

$$= \frac{1}{4} \int (1+\cos 2x) \, d2x$$

$$= \frac{1}{2} + \frac{1}{4}\sin 2x + C$$

$$\int \sin^2 x \cos^4 x \, dx$$

$$= \int \frac{1-\cos 2x}{2} \cdot \left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{8} \int (1-\cos 2x)(1+\cos 2x)^2 dx$$

$$= \frac{1}{8} \int (1+\cos 2x - \cos^2 2x - \cos^3 2x) \, dx$$

$$\frac{1}{2}x$$

$$= \frac{1}{8}\left(x + \cancel{\frac{1}{2}\sin 2x} - \frac{1}{2}x - \frac{1}{8}\sin 4x \cancel{- \frac{1}{2}\sin 2x} + \frac{1}{3}\sin^3 2x\right) + C$$

$$= \frac{1}{48}\sin^3 2x + \frac{1}{16}x - \frac{1}{64}\sin 4x + C$$

$$\int \cos^3 2x\, dx$$

$$= \frac{1}{2}\int \cos^3 2x\, d2x$$

$$= \frac{1}{2}\int \cos^2 2x\, d\sin 2x$$

$$= \frac{1}{2}\int (1-\sin^2 2x)\, d\sin 2x$$

$$= \frac{1}{2}\left(\sin 2x - \frac{1}{3}\sin^3 2x\right)+C$$

$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$$\int \csc x\, dx = \int \frac{1}{\sin x}\, dx$$

$$= \frac{1}{2}\int \frac{1}{\sin\frac{x}{2}\cos\frac{x}{2}}\, dx$$

$\tan x = \frac{\sin x}{\cos x}$

$\Rightarrow \sin x = \tan x \cdot \cos x$

$$= \frac{1}{2}\int \frac{1}{\tan\frac{x}{2}\cos^2\frac{x}{2}}\, dx$$

$$= \int \frac{1}{\tan\frac{x}{2}}\, d\left(\tan\frac{x}{2}\right)$$

$$= \ln\left|\tan\frac{x}{2}\right|+C$$

$$\int \sec^6 x\, dx$$

$$= \int (\sec^4 x)\cdot \sec^2 x \, dx$$

$$= \int \sec^4 x \, d\tan x$$

$$= \int (1+\tan^2 x)^2 \, d\tan x$$

$$= \int (\tan^4 x + 2\tan^2 x + 1) \, d\tan x$$

$$= \frac{1}{5}\tan^5 x + \frac{2}{3}\tan^3 x + \tan x + C$$

$$\int \csc x \, dx = \int \frac{1}{\sin x} \, dx$$

$$= \int \frac{1}{\sin\frac{x}{2}\cos\frac{x}{2}} \, d\frac{x}{2}$$

$$= \int \frac{1}{\tan\frac{x}{2}\cos^2\frac{x}{2}} \, d\frac{x}{2}$$

$$= \int \frac{1}{\quad} \, d\tan\frac{x}{2}$$

$$\int \quad \tan\frac{x}{2}$$

$$= \ln\left|\tan\frac{x}{2}\right| + C$$

$$\int \cos 3x \cos 2x \, dx$$

$$= \frac{1}{2}\int (\cos x + \cos 5x)\, dx$$

$$= \frac{1}{2}\sin x + \frac{1}{10}\sin 5x + C$$

$$\int \cos x \, dx = \sin x$$

$$\int \cos^2 x \, dx \neq \sin^2 x$$

$$\int \cos^2 x \, dx$$

$$= \int \frac{1+\cos 2x}{2}\, dx$$

$$= \frac{1}{4}\int (1+\cos 2x)\, d2x$$

$$= \frac{1}{4}(2x + \sin 2x) + C$$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\Rightarrow 2\cos^2 x = 1 + \cos 2x$$

$$\Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\sin 2x = 1 - 2\sin^2 x$$

$$\Rightarrow 2\sin^2 x = 1 - \sin 2x$$

$$\Rightarrow \sin^2 x = \frac{1-\sin 2x}{2}$$