向量代数与空间解析几何(1)

一、向量及其线性运算

① 加法（△，□ 法则）

$\vec{a}+\vec{b}=\vec{b}+\vec{a}$

$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$

② 减法

$\vec{b}-\vec{a}=\vec{b}+(-\vec{a})$

$\vec{b}-\vec{a}$ 为 $\vec{a}$ 的终点到 $\vec{b}$ 的终点、所引的向量

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$

③ 数乘

$\lambda>0$，方向相同；$\lambda<0$，方向相反

$|\lambda\vec{a}|=|\lambda||\vec{a}|$

$\lambda(\mu\vec{a})=\mu(\lambda\vec{a})=\lambda\mu\vec{a}$

$(\lambda+\mu)\vec{a}=\lambda\vec{a}+\mu\vec{a}$

$\lambda(\vec{a}+\vec{b})=\lambda\vec{a}+\lambda\vec{b}$

例1

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$\vec{AC}=\vec{a}+\vec{b}$

$\vec{MA}=-\frac{1}{2}\vec{AC}=-\frac{1}{2}(\vec{a}+\vec{b})$

$\vec{MC}=\frac{1}{2}\vec{AC}=\frac{1}{2}(\vec{a}+\vec{b})$

$\vec{MB}=\frac{1}{2}\vec{DB}=\frac{1}{2}(\vec{DA}+\vec{AB})$

$=\frac{1}{2}(-\vec{b}+\vec{a})$

$\vec{MD}=-\vec{MB}=\frac{1}{2}(\vec{b}-\vec{a})$

例2. $\begin{cases} 5\vec{x}-3\vec{y}=\vec{a} & ① \\ 3\vec{x}-2\vec{y}=\vec{b} & ② \end{cases}$

其中 $\vec{a}=(2,1,2)$, $\vec{b}=(-1,1,-2)$

①×2 − ②×3

$\Rightarrow \vec{x}=2\vec{a}-3\vec{b}=(7,-1,10)$

代入②得: $\vec{y}=(11,-2,16)$

例3. $A(x_1,y_1,z_1)$

$B(x_2,y_2,z_2)$

求M, 使得 $\vec{AM}=\lambda\vec{MB}$.

$\vec{AM}=\vec{OM}-\vec{OA}$

$MB = OB - OM$

又 $\because \overrightarrow{AM} = \lambda \overrightarrow{MB}$

$\therefore \overrightarrow{OM} - \overrightarrow{OA} = \lambda(\overrightarrow{OB} - \overrightarrow{OM})$

$\Rightarrow (1+\lambda)\overrightarrow{OM} = \overrightarrow{OA} + \lambda\overrightarrow{OB}$

$$\Rightarrow \overrightarrow{OM} = \frac{\overrightarrow{OA} + \lambda\overrightarrow{OB}}{1+\lambda}$$

$$= \left(\frac{x_1+\lambda x_2}{1+\lambda}, \frac{y_1+\lambda y_2}{1+\lambda}, \frac{z_1+\lambda z_2}{1+\lambda}\right)$$

$$\Rightarrow M\left(\frac{x_1+\lambda x_2}{1+\lambda}, \frac{y_1+\lambda y_2}{1+\lambda}, \frac{z_1+\lambda z_2}{1+\lambda}\right)$$

$A(a,b,c)$, $B(d,e,f)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (d-a, e-b, f-c)$

$$|\overrightarrow{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

例4.

$M_1(4,3,1)$, $M_2(7,1,2)$, $M_3(5,2,3)$

求证等腰△

$|\overrightarrow{M_1M_2}| = \sqrt{3^2+2^2+1^2} = \sqrt{14}$

$|\overrightarrow{M_2M_3}| = \sqrt{2^2+1^2+1^2} = \sqrt{6}$

$|\overrightarrow{M_1M_3}| = \sqrt{1^2+1^2+2^2} = \sqrt{6}$

$\therefore |\overrightarrow{M_2M_3}| = |\overrightarrow{M_1M_3}|$

$\therefore \triangle M_1M_2M_3$ 为等腰三角形

例5.

设 $M(0,0,z)$

由题：$|\overrightarrow{MA}| = |\overrightarrow{MB}|$

$\Rightarrow 4^2+1^2+(7-z)^2 = 3^2+5^2+(-2-z)^2$

$\Rightarrow 66-14z+\cancel{z^2} = 38+4z+\cancel{z^2}$

$\Rightarrow z = \frac{14}{9}$

$\therefore M(0,0,\frac{14}{9})$

例6. $A(4,0,5), B(7,1,3)$

求 $\vec{e}_{\overrightarrow{AB}}$

$\overrightarrow{AB} = (3,1,-2)$

$|\overrightarrow{AB}| = \sqrt{14}$

$\therefore \vec{e}_{\overrightarrow{AB}} = \frac{1}{\sqrt{14}}(3,1,-2)$

方向余弦：

$$\cos\alpha = \frac{x}{|\vec{r}|}$$

$$\cos\beta = \frac{y}{|\vec{r}|}$$

$$\cos r = \frac{z}{|\vec{r}|}$$

$$(\cos\alpha, \cos\beta, \cos r)$$

$$= \frac{1}{|\vec{r}|}(x, y, z) = \frac{\vec{r}}{|\vec{r}|} = \vec{e}_{\vec{r}}$$

例17. $M_1(2, 2, \sqrt{2})$

$M_2(1, 3, 0)$

求 $\overrightarrow{M_1M_2}$ 的模、方向余弦、方向角.

$$\overrightarrow{M_1M_2} = (-1, 1, -\sqrt{2})$$

$$|\overrightarrow{M_1M_2}| = \sqrt{1+1+2} = 2$$

$$\cos\alpha = -\frac{1}{2}$$

$$\cos\beta = \frac{1}{2}$$

$$\cos r = -\frac{\sqrt{2}}{2}$$

$$\alpha = \frac{2\pi}{3}, \quad \beta = \frac{\pi}{3}, \quad r = \frac{3\pi}{4}$$

(1) $Prj_u \vec{a} = |\vec{a}| \cos\varphi$

(2) $Prj_u (\vec{a} + \vec{b}) = Prj_u \vec{a} + Prj_u \vec{b}$

(3) $Prj_u (\lambda \vec{a}) = \lambda\, Prj_u \vec{a}$

$\ldots J_a(\lambda a) = \lambda \ldots J_{a}$