

高等数学上册补充(1)

1. 已知 $f(\ln x) = \frac{1-x}{1+x}$,

求 $f'(x)$

解: 令 $\ln x = t$

$$\Rightarrow x = e^t$$

$$\begin{aligned}\therefore f(t) &= \frac{1-e^t}{1+e^t} = \frac{-e^t-1+2}{1+e^t} \\ &= -1 + 2 \frac{1}{1+e^t}\end{aligned}$$

$$\therefore f'(t) = \frac{-2e^t}{(1+e^t)^2}$$

$$\therefore f'(x) = \frac{-2e^x}{(1+e^x)^2}$$

2. 设 $x = g(y)$ 是 $y = x^3 + 1$ 的反函数, 求 $g'(2)$.

$y = x^3 + 1$ 的反函数为 $x = y^{\frac{1}{3}} + 1$

$$\Rightarrow y = (x-1)^{\frac{1}{3}}$$

$$\therefore g(y) = (y-1)^{\frac{1}{3}}$$

$$\Rightarrow g'(y) = \frac{1}{3} (y-1)^{-\frac{2}{3}}$$

$$\therefore g'(2) = \frac{1}{3}$$

另法: $g(f(x)) = x$ 是 $f(x) = x^3 + 1$
的反函数.

$$g(x^3 + 1) = x$$

$$\text{令 } t = x^3 + 1 \Rightarrow x = (t - 1)^{\frac{1}{3}}$$

$$\Rightarrow g(t) = (t - 1)^{\frac{1}{3}}$$