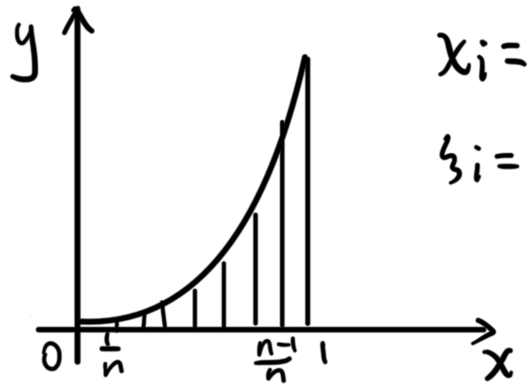
定积分草稿（一）

$$\int_0^1 x^2 dx$$

$$= \frac{1}{3} \times 1^3 = \frac{1}{3} ?$$



$x_i = \frac{i}{n}$

$\xi_i = \frac{i}{n}$

$$\therefore 原式 = \sum_{i=1}^{n} f(\xi_i) \Delta x_i$$

$$= \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \cdot \frac{1}{n}$$

$$= \frac{1}{n^3} \sum_{i=1}^{n} i^2$$

$$= \frac{1}{n^3} \cdot \frac{1}{6} n(n+1)(2n+1)$$

$$= \frac{n(n+1)(2n+1)}{6n^3}$$

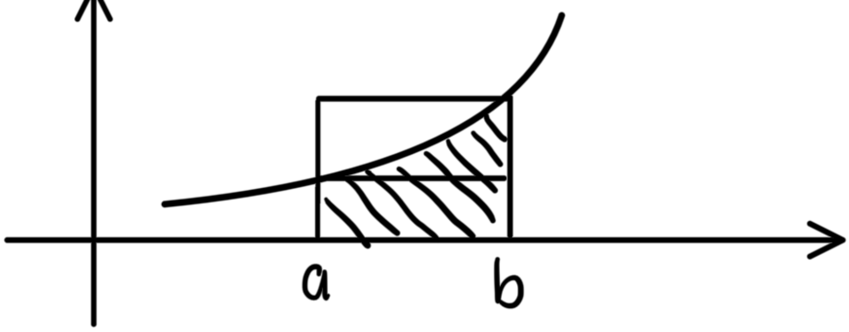
$$\lim_{n \to 0} \frac{n(n+1)(2n+1)}{6n^3} = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \int_0^1 x^2 dx = \frac{1}{3}$$

$$\int_a^b f(x)\,dx$$
$$= \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\lim_{\lambda \to 0} \sum_{i=1}^{n} f(\xi_i)\,\Delta x$$

$$\left| \int_a^b f(x)\,dx \right| \leq \int_a^b |f(x)|\,dx$$



定积分中值定理

$$\int_a^b f(x)\,dx = f(\xi)(b-a)$$
$$(a \leq \xi \leq b)$$

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$
$$\Rightarrow m \leq \frac{1}{b-a} \int_a^b f(x)\,dx \leq M$$

$$f(\xi) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

微积分基本公式

积分上限函数

$$\Phi(x) = \int_a^x f(t)\,dt$$

$$\Phi'(x) = \frac{d}{dx}\int_a^x f(t)\,dt$$

$$= f(x) \quad (a \le x \le b)$$

$\Phi(x) = \int_a^x f(t)\,dt$ 是 $f(x)$ 的一个原函数

$$\int_a^{\varphi(x)} f(t)\,dt = f(\varphi(x)) \cdot \varphi'(x)$$

$$\left[\int_{\psi(x)}^{\varphi(x)} f(t)\,dt\right]'$$

$$= f(\varphi(x))\varphi'(x)$$

$$- f(\psi(x))\psi'(x)$$

$\left[\int^{x^3} \right.$ ... $]'$

$$\left[\int_{x^2} \sin t\,dt\right]$$

$$= f(x^3)\cdot 3x^2 - f(x^2)\cdot 2x$$

$$= \sin(x^3)\cdot 3x^2 - \sin(x^2)\cdot 2x$$

☆. 牛顿-莱布尼茨公式

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

证： $\Phi(x) = \int_a^x f(t)\,dt$

$F(x) - \Phi(x) = C$

$F(a) = C$ ($\Phi(a) = 0$)

$\therefore \int_a^x f(t)\,dt = F(x) - F(a)$

$\Rightarrow \int_a^b f(t)\,dt = F(b) - F(a)$

证明完毕！

$$\int_0^1 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$$

$$\int_{-1}^{\sqrt{3}} \frac{1}{1+x^2}\,dx$$

$$= \left[\arctan x\right]_{-1}^{\sqrt{3}}$$

$$= \frac{\pi}{3} + \frac{\pi}{4} = \frac{7}{12}\pi$$

$$\int_{-2}^{-1} \frac{1}{x}\,dx$$

$$= \left[\ln|x| \right]_{-2}^{-1}$$

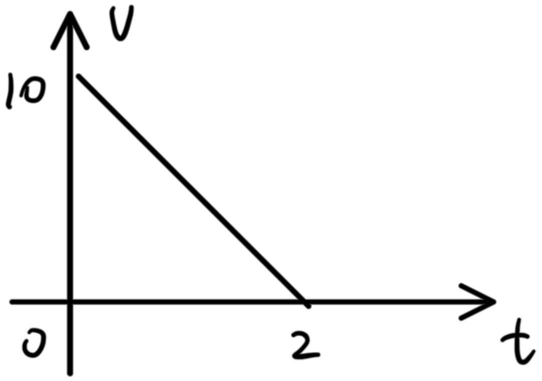
$$= 0 - \ln 2$$

$$= -\ln 2$$

$$\int_{0}^{\pi} \sin x\,dx$$

$$= \left[-\cos x \right]_{0}^{\pi}$$

$$= 1 - (-1) = 2$$



$36 km/h = 10 m/s$

$$V(t) = v_0 + at = 10 - 5t$$

令 $10 - 5t = 0 \Rightarrow t = 2$

$$s = \int_{0}^{2} (10 - 5t)\,dt$$

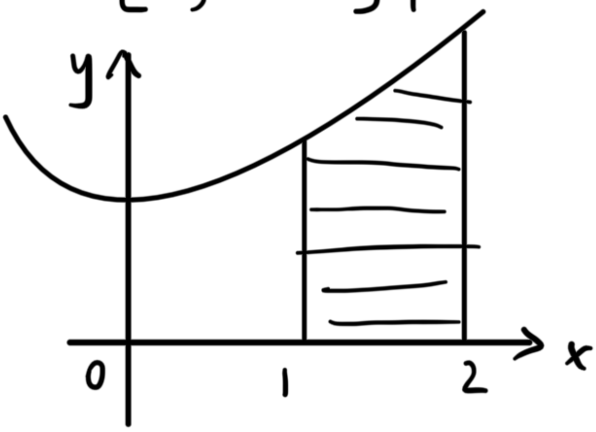
$$= \left[10t - \frac{5}{2}t^2 \right]^{2}$$

$= [10t \quad 2t]_0$

$= 10(m)$

$$\int_1^2 (x^2+1)\,dx$$

$$= \left[\frac{1}{3}x^3+x\right]_1^2$$



$$\left[\frac{1}{3}x^3+x\right]_1^2$$

$$= \frac{8}{3}+2-\frac{1}{3}-1$$

$$= \frac{10}{3}$$

(f(x)在[0,+∞)内连续且>0)

$$F(x) = \frac{\int_0^x t f(t)\,dt}{\int_0^x f(t)\,dt}$$

$$F'(x) = \frac{x f(x)\int_0^x f(t)\,dt - f(x)\int_0^x t f(t)\,dt}{\left[\int_0^x f(t)\,dt\right]^2}$$

$$= \frac{f(x)\left[\int_0^x x f(t)\,dt - \int_0^x t f(t)\,dt\right]}{\left[\int_0^x f(t)\,dt\right]^2}$$

$$= \frac{f(x)\int_0^x (x-t) f(t)\,dt}{\left[\int_0^x f(t)\,dt\right]^2}$$

$\because$ $f(t) > 0$, $x - t > 0$

$\therefore$ $(x-t) f(t) > 0$

定积分中值定理

$$\int_a^b f(x)\,dx = f(\xi)(b-a)$$

$$\therefore \int_0^x \underbrace{(x-t) f(t)}_{>0}\,dt > 0$$

$\therefore$ $F'(x) > 0$

$\therefore$ $F(x)$ 在 $(0, +\infty)$ 上 $\uparrow$

$$\lim_{x \to 0} \frac{\int_{\cos x}^{1} e^{-t^2}\,dt}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{-\cos^2 x} \cdot \sin x}{2x}$$

$\left[\int^{\varphi(x)} \quad \right]'$

$$\left[\int_{\psi(x)}^{\varphi(x)} f(t)dt\right]$$

$$= f(\varphi(x)\varphi'(x) - f(\psi(x))\psi'(x)$$

$$= \lim_{x\to 0} \frac{e^{-\cos^2 x}}{2}$$

$$= \frac{e^{-1}}{2} = \frac{1}{2e}$$