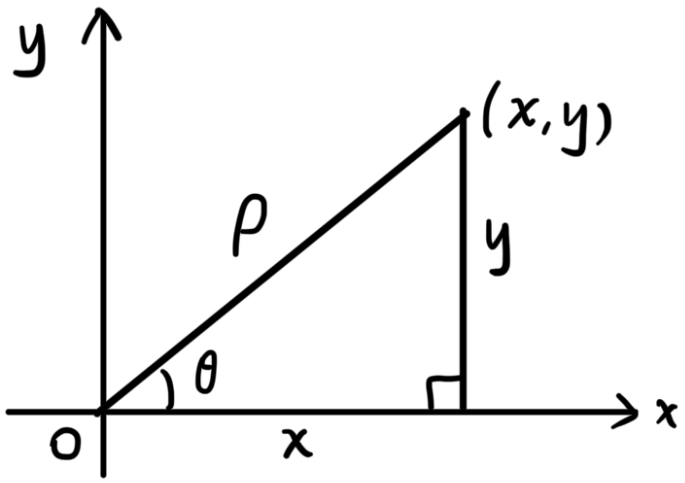
极坐标（定积分）

θ：鸡脚（极角）

ρ：鸡精（极径）



$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$$

$$x^2+y^2=\rho^2$$

$$\tan\theta = \frac{y}{x}$$

$$(x-1)^2+y^2=1$$

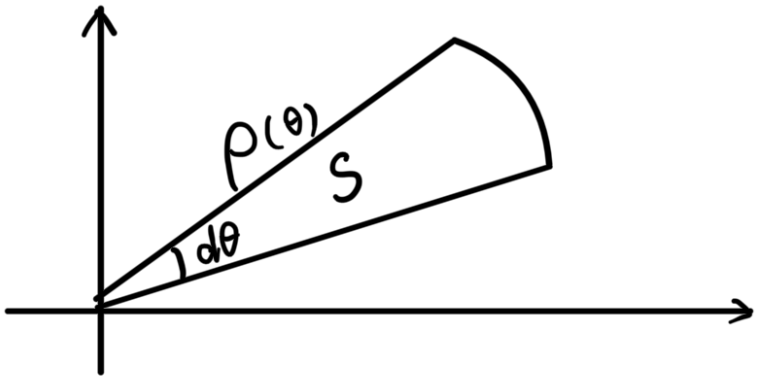
$$\Rightarrow x^2-2x+y^2=0$$

$$\Rightarrow x^2+y^2=2x$$

$$\Rightarrow \rho^2=2\rho\cos\theta$$

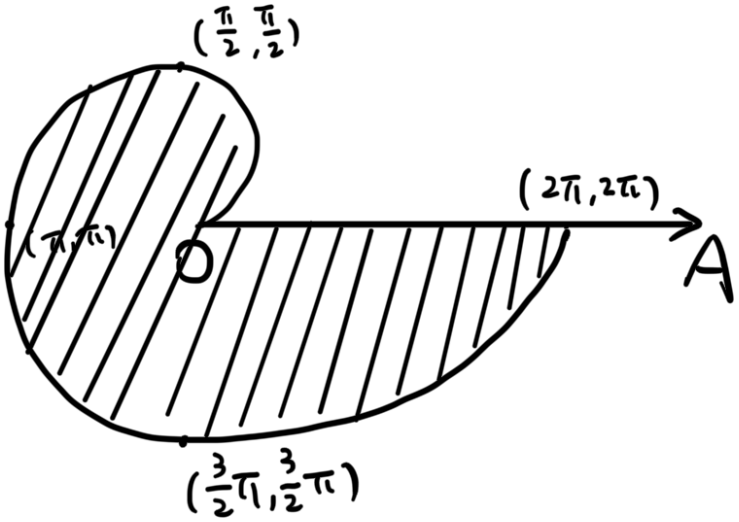
$$\Rightarrow \rho=2\cos\theta$$

极坐标下求面积



$$S=\int_a^\beta \frac{1}{2}d\theta\cdot\rho^2(\theta)$$

阿基米德螺线



$$\rho=a\theta\ (a>0)\quad \theta:0\to 2\pi$$

$$S=\int_0^{2\pi}\frac{1}{2}a^2\theta^2 d\theta$$

$$=\frac{1}{2}a^2\int_0^{2\pi}\theta^2 d\theta$$

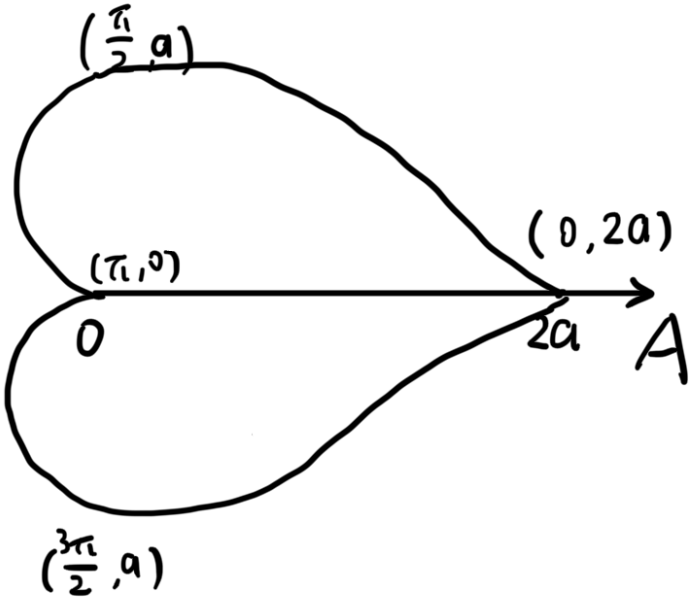
$$= \frac{1}{2}a^2 \left[\frac{8}{3}\theta^3\right]_0$$

$$= \frac{1}{2}a^2 \times \frac{8}{3}\pi^3$$

$$= \frac{4}{3}\pi^3 a^2$$

心形线

$$\rho = a(1+\cos\theta)\ (a>0)$$



$$S = 2\int_0^{\pi} \frac{1}{2}[\rho(\theta)]^2 d\theta$$

$$= a^2\int_0^{\pi} (1+\cos\theta)^2 d\theta$$

$$= a^2\int_0^{\pi} (1+2\cos\theta+\cos^2\theta)\, d\theta$$

$$= [a^2\theta]_0^{\pi} + [2a^2\sin\theta]_0^{\pi}$$

$$+ \frac{a^2}{4}\int_0^{\pi} (1+\cos 2\theta)\, d(2\theta)$$

$$= a^2\pi + 0 + [\frac{1}{2}a^2\theta]_0^{\pi}$$

$$+ \left[\frac{1}{4}\sin 2\theta\right]_0^{\pi}$$

$$= a^2\pi + \frac{1}{2}a^2\pi$$

$$= \frac{3}{2}a^2\pi$$