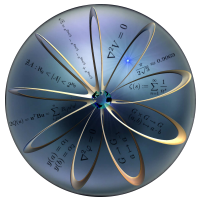

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arithmetic progression

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Arithmetic progression of length n, initial term a_1 and common difference d is the sequence $a_1, a_1 + d, a_1 + 2d, \ldots, a_1 + (n-1)d$.

The sum of terms of an arithmetic progression can be computed using Gauss's trick:

$$\begin{array}{rl} S = & (a_1 + 0) + (a_1 + d) + \cdots + (a_1 + (n-2)d) + (a_1 + (n-1)d) \\ +S = & (a_1 + (n-1)d) + (a_1 + (n-2)d) + \cdots + (a_1 + d) + (a_1 + 0) \\ \hline 2S = & (2a_1 + (n-1)d) + (2a_1 + (n-1)d) + \cdots + (2a_1 + (n-1)d) + (2a_1 + (n-1)d). \end{array}$$

We just add the sum with itself written backwards, and the sum of each of the columns equals to $(2a_1 + (n-1)d)$. The sum is then

$$S = \frac{(2a_1 + (n-1)d)n}{2}.$$