Conway's chained arrow notation

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Conway's chained arrow notation is a way of writing numbers even larger than those provided by the up arrow notation. We define $m \to n \to p = m^{(p+2)}n = m \underbrace{\uparrow \cdots \uparrow}_{p} n$ and $m \to n = m \to n \to 1 = m^n$. Longer chains are evaluated by

$$m \to \cdots \to n \to p \to 1 = m \to \cdots \to n \to p$$

$$m \to \cdots \to n \to 1 \to q = m \to \cdots \to n$$

and

$$m \to \cdots \to n \to p{+}1 \to q{+}1 = m \to \cdots \to n \to (m \to \cdots \to n \to p \to q{+}1) \to q$$

For example:

$$3 \to 3 \to 2 =$$
$$3 \to (3 \to 2 \to 2) \to 1 =$$
$$3 \to (3 \to 2 \to 2) =$$
$$3 \to (3 \to (3 \to 1 \to 2) \to 1) =$$
$$3 \to (3 \to 3 \to 1) =$$
$$3^{3^3} =$$
$$3^{27} = 7625597484987$$

A much larger example is:

$$3 \to 2 \to 4 \to 4 =$$
$$3 \to 2 \to (3 \to 2 \to 3 \to 4) \to 3 =$$
$$3 \to 2 \to (3 \to 2 \to (3 \to 2 \to 2 \to 4) \to 3) \to 3 =$$
$$3 \to 2 \to (3 \to 2 \to (3 \to 2 \to (3 \to 2 \to 1 \to 4) \to 3) \to 3) \to 3 =$$
$$3 \to 2 \to (3 \to 2 \to (3 \to 2 \to (3 \to 2) \to 3) \to 3) \to 3 =$$
$$3 \to 2 \to (3 \to 2 \to (3 \to 2 \to 9 \to 3) \to 3) \to 3$$

Clearly this is going to be a very large number. Note that, as large as it is, it is proceeding towards an eventual final evaluation, as evidenced by the fact that the final number in the chain is getting smaller.