division

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Entry type	Definition
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Related topic	InverseFormingInProportionToGroupOperation
Related topic	DivisionInGroup
Related topic	ConjugationMnemonic
Related topic	Difference2
Related topic	UniquenessOfDivisionAlgorithmInEuclideanDomain
Defines	quotient
Defines	ratio
Defines	fundamental property of quotient
Defines	reduction

Division is the operation which assigns to every two numbers (or more generally, elements of a field) a and b their quotient or ratio, provided that the latter, b, is distinct from zero.

The *quotient* (or *ratio*) $\frac{a}{b}$ of a and b may be defined as such a number (or element of the field) x that $b \cdot x = a$. Thus,

$$b \cdot \frac{a}{b} = a,$$

which is the "fundamental property of quotient".

The quotient of the numbers a and b ($\neq 0$) is a uniquely determined number, since if one had

$$\frac{a}{b} = x \neq y = \frac{a}{b},$$

then we could write

$$b(x-y) = bx - by = a - a = 0$$

from which the supposition $b \neq 0$ would imply $x - y = 0$, i.e. $x = y$.

The explicit general expression for $\frac{a}{b}$ is

$$\frac{a}{b} = b^{-1} \cdot a$$

where b^{-1} is the inverse number (the multiplicative inverse) of a, because

$$b(b^{-1}a) = (bb^{-1})a = 1a = a.$$

- For positive numbers the quotient may be obtained by performing the division algorithm with a and b. If $a > b > 0$, then $\frac{a}{b}$ indicates how many times b fits in a.

- The quotient of a and b does not change if both numbers (elements) are multiplied (or divided, which is called *reduction*) by any $k \neq 0$:

$$\frac{ka}{kb} = (kb)^{-1}(ka) = b^{-1}k^{-1}ka = b^{-1}a = \frac{a}{b}$$

So we have the method for getting the quotient of complex numbers,

$$\frac{a}{b} = \frac{\bar{b}a}{\bar{b}b},$$

where $\bar{b}$ is the complex conjugate of b, and the quotient of `http://planetmath.org/SquareRo`
root polynomials, e.g.

$$\frac{1}{5+2\sqrt{2}} = \frac{5-2\sqrt{2}}{(5-2\sqrt{2})(5+2\sqrt{2})} = \frac{5-2\sqrt{2}}{25-8} = \frac{5-2\sqrt{2}}{17};$$

in the first case one aspires after a real and in the second case after a rational denominator.

- The division is neither associative nor commutative, but it is right distributive over addition:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$