well-defined

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A mathematical concept is *well-defined* (German *wohldefiniert*, French *bien défini*), if its contents is on the form or the alternative representative which is used for defining it.

For example, in defining the `http://planetmath.org/FractionPower`power x^r with x a positive real and r a rational number, we can freely choose the fraction form $\frac{m}{n}$ ($m \in \mathbb{Z},\; n \in \mathbb{Z}_+$) of r and take

$$x^r \;:=\; \sqrt[n]{x^m}$$

and be sure that the value of x^r does not depend on that choice (this is justified in the entry fraction power). So, the x^r is well-defined.

In many instances well-defined is a synonym for the formal definition of a function between sets. For example, the function $f(x) := x^2$ is a well-defined function from the real numbers to the real numbers because every input, x, is assigned to precisely one output, x^2. However, $f(x) := \pm\sqrt{x}$ is not well-defined in that one input x can be assigned any one of two possible outputs, $\sqrt{x}$ or $-\sqrt{x}$.

More subtle examples include expressions such as

$$f\left(\frac{a}{b}\right) \;:=\; a+b, \quad \frac{a}{b} \in \mathbb{Q}.$$

Certainly every input has an output, for instance, $f(1/2) = 3$. However, the expression is *not* well-defined since $1/2 = 2/4$ yet $f(1/2) = 3$ while $f(2/4) = 6$ and $3 \neq 6$.

One must question whether a function is well-defined whenever it is defined on a domain of equivalence classes in such a manner that each output is determined for a representative of each equivalence class. For example, the function $f(a/b) := a+b$ was defined using the representative a/b of the equivalence class of fractions equivalent to a/b.