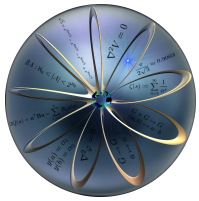

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empty product

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Related topic	EmptySum

The *empty product* of numbers is the borderline case of product, where the number of is empty. The most usual examples are the following.

- The http://planetmath.org/IntroducingOthPowerzeroth power of a non-zero number: a^0
- The factorial of 0: $0!$
- The http://planetmath.org/FundamentalTheoremOfArithmeticsprime factor presentation of unity, which has no prime factors

The value of the empty sum of numbers is equal to the additive identity number, 0. Similarly, the empty product of numbers is equal to the http://planetmath.org/Unitymul identity number, 1.

Note. When considering the complex numbers as pairs of real numbers one often identifies the pairs $(x, 0)$ and the reals x. In this sense one can think that the Cartesian product $\mathbb{R} \times \{0\}$ is equal to $\mathbb{R}$. This seems to the equation

$$\mathbb{R} \times \mathbb{R}^0 = \mathbb{R}^{1+0} = \mathbb{R}^1 = \mathbb{R},$$

although the http://planetmath.org/GeneralAssociativityassociativity of Cartesian product is nowhere stated. Nevertheless, it is sometimes natural to define that the Cartesian product of an empty collection of sets equals to a set with one element; so it may that e.g. $\mathbb{R}^0 = \{0\}$.

One can also consider empty products in categories. It follows directly from the definition that an object in a category is a http://planetmath.org/CategoricalDirectP of an empty family of objects in the category if and only if it is a terminal object of the category. Sets are a special case of this: in the category of sets the singletons are the terminal objects, so the empty product exists and is a singleton.