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sum of odd numbers

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The sum of the first n positive odd integers can be calculated by using the well-known of the arithmetic progression, that the sum of its is equal to the arithmetic mean of the first and the last , multiplied by the number of the :

$$\underbrace{1 + 3 + 5 + 7 + 9 + \cdots + (2n-1)}_n = n \cdot \frac{1+(2n-1)}{2} = n^2$$

Thus, the sum of the first n odd numbers is n^2 (this result has been proved first time in 1575 by Francesco Maurolico).

Below, the odd numbers have been set to form a triangle, each n^{th} row containing the next n consecutive odd numbers. The arithmetic mean on the row is n^2 and the sum of its numbers is $n \cdot n^2 = n^3$.

$$\begin{array}{cccccccc} & & & & 1 & & & \\ & & & & 3 & & 5 & \\ & & & 7 & & 9 & & 11 \\ & & 13 & & 15 & & 17 & & 19 \\ & 21 & & 23 & & 25 & & 27 & & 29 \\ 31 & & 33 & & 35 & & 37 & & 39 & & 41 \\ & \vdots & & & \vdots & & & & \vdots & & \end{array}$$