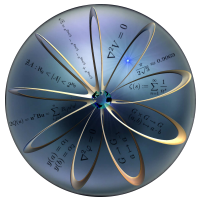

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subsequence

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Given a sequence $\{x_n\}_{n\in\mathbb{N}}$, any infinite subset of the sequence forms a subsequence. We formalize this as follows:

Definition. *If X is a set and $\{a_n\}_{n\in\mathbb{N}}$ is a sequence in X, then a* subsequence *of $\{a_n\}$ is a sequence of the form $\{a_{n_r}\}_{r\in\mathbb{N}}$ where $\{n_r\}_{r\in\mathbb{N}}$ is a strictly increasing sequence of natural numbers.*

Equivalently, $\{y_n\}_{n\in\mathbb{N}}$ is a subsequence of $\{x_n\}_{n\in\mathbb{N}}$ if

1. $\{y_n\}_{n\in\mathbb{N}}$ is a sequence of elements of X, and
2. there is a strictly increasing function $a : \mathbb{N} \to \mathbb{N}$ such that

$$y_n = x_{a(n)} \quad \text{for all } n \in \mathbb{N}.$$

Example. Let $X = \mathbb{R}$ and let $\{x_n\}$ be the sequence

$$\left\{\frac{1}{n}\right\}_{n\in\mathbb{N}} = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}.$$

Then, the sequence

$$\{y_n\}_{n\in\mathbb{N}} = \left\{\frac{1}{n^2}\right\}_{n\in\mathbb{N}} = \left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \dots\right\}$$

is a subsequence of $\{x_n\}$. The subsequence of natural numbers mentioned in the definition is $\{n^2\}_{n\in\mathbb{N}}$ and the function $a : \mathbb{N} \to \mathbb{N}$ mentioned above is $a(n) = n^2$.