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simple interest

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Suppose a bank account is opened at time 0 and M_0 is deposited into the account. A *simple interest* is interest with the following characteristics:

1. it is earned at subsequent time periods $t, 2t, \dots$, where t is the length of the initial time interval (1 for 1 month, 12 for 1 year, etc...)
2. the interest earned at the end of each time period is the same regardless of the time period

The following table illustrates the structure of the simple interest.

time period at	principal	interest	interest accrued
0	M_0	0	0
t	M_0	i	i
$2t$	M_0	i	$2i$
$3t$	M_0	i	$3i$
\vdots	\vdots	\vdots	\vdots
nt	M_0	i	ni

The “total” interest $i(nt)$ earned (accrued) at the end of time nt is ni . If the account is closed and the money withdrawn at the end of nt , and the total amount of money received is

$$M(nt) = M_0 + ni.$$

The interest rate associated with the simple interest as presented above between two time periods, say at and bt , is given by

$$r(at, bt) = \frac{1}{M_0} \frac{i(bt) - i(at)}{bt - at} = \frac{i}{M_0 t},$$

which does not depend on the choice of a and b . In other words, the original principal M_0 , the amount of interest i , and the length of the initial time interval t are enough to determine the interest rate.

Remark.

- The expression for the effective interest rate for simple interest is a bit more complicated:

$$\text{eff. } r(at, bt) = \frac{1}{M(at)} \frac{i(bt) - i(at)}{bt - at} = \frac{1}{M_0 + ai} \frac{i}{t},$$

which decreases with increasing a . Imagine as a becomes very large, the increase in interest has practically no impact on the “accumulated” principal $M(at)$.

- More generally, we say that an interest is *simple* if its interest rate r is constant with respect to time t . Solving

$$r = \frac{1}{M_0} \frac{i(t) - i(0)}{t - 0}$$

for $i(t)$, we get $i(t) = M_0 r t$, or that the accrued interest is a linear function of t . It grows directly proportionally with respect to time.