proof of Nesbitt's inequality

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Owner	mathwizard (128)
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Starting from Nesbitt's inequality

$$\frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b} \geq \frac{3}{2}$$

we transform the left hand side:

$$\frac{a+b+c}{b+c} + \frac{a+b+c}{a+c} + \frac{a+b+c}{a+b} - 3 \geq \frac{3}{2}.$$

Now this can be transformed into:

$$((a+b) + (a+c) + (b+c)) \left(\frac{1}{a+b} + \frac{1}{a+c} + \frac{1}{b+c} \right) \geq 9.$$

Division by 3 and the right yields:

$$\frac{(a+b) + (a+c) + (b+c)}{3} \geq \frac{3}{\frac{1}{a+b} + \frac{1}{a+c} + \frac{1}{b+c}}.$$

Now on the left we have the arithmetic mean and on the right the harmonic mean, so this inequality is true.