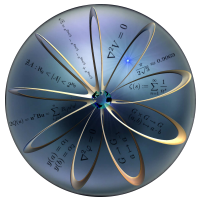

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preservation and reflection

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In mathematics, the word "preserve" usually means the "preservation of properties". Loosely speaking, whenever a mathematical construct A has some property P, after A is somehow "transformed" into A', the transformed object A' also has property P. The constructs usually refer to sets and the transformations typically are functions or something similar.

Here is a simple example, let f be a function from a set A to B. Let A be a finite set. Let P be the property of a set being finite. Then f preserves P, since $f(A)$ is finite. Note that we are not saying that B is finite. We are merely saying that the portion of B that is the *image* of A (the transformed portion) is finite.

Here is another example. The property of being connected in a topological space is preserved under a continuous function. Here, the constructs are topological spaces, and the transformation is a continuous function. In other words, if $f : X \to Y$ is a continuous function from X to Y. If X is connected, so is $f(X) \subseteq Y$.

Many more examples can be found in abstract algebra. Group homomorphisms, for example, preserve commutativity, as well as the property of being finitely generated.

The word "reflect" is the dual notion of "preserve". It means that if the transformed object has property P, then the original object also has property P. This usage is rarely found outside of category theory, and is almost exclusively reserved for functors. For example, a faithful functor reflects isomorphism: if F is a faithful functor from $\mathcal{C}$ to $\mathcal{D}$, and the object $F(A)$ is isomorphic to the object $F(B)$ in $\mathcal{D}$, then A is isomorphic to B in $\mathcal{C}$.