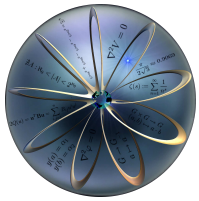

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taxicab numbers

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The number 1729 has a reputation of its own. The reason is the famous exchange between http://www-groups.dcs.st-and.ac.uk/ history/Mathematicians/Hardy.html H. Hardy, a famous British mathematician (1877-1947), and http://www-groups.dcs.st-and.ac.u Ramanujan , one of India's greatest mathematical geniuses (1887-1920):

> In 1917, during one visit to Ramanujan in a hospital (he was ill for much of his last three years), Hardy mentioned that the number of the taxi cab that had brought him was 1729, which, as numbers go, Hardy thought was "rather a dull number". At this, Ramanujan perked up, and said "No, it is a very interesting number; it is the smallest number expressible as a sum of two cubes in two different ways."

Indeed:

$$1729 = 1 + 12^3 = 9^3 + 10^3.$$

Moreover, there are other reasons why 1729 is far from dull. 1729 is the third Carmichael number. Even more strange, beginning at the 1729th decimal digit of the transcental number e, the next ten successive digits of e are 0719425863. This is the first appearance of all ten digits in a row without repititions.

More generally, the smallest natural number which can be expressed as the sum of n positive cubes is called the nth taxicab number. The first taxicab numbers are:

$2 = 1^3+1^3$, $1729 = 1^3+12^3 = 9^3+10^3$, $87539319 = 167^3+436^3 = 228^3+423^3 = 255^3+414^3$

followed by 6963472309248 (found by E. Rosenstiel, J.A. Dardis, and C.R. Rosenstiel in 1991) and 48988659276962496 (found by David Wilson on November 21st, 1997).