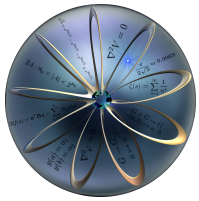

planetmath.org

Math for the people, by the people.

arithmetic-geometric-harmonic means inequality

Canonical name	ArithmeticgeometricharmonicMeansInequality
Date of creation	2013-03-22 11:42:32
Last modified on	2013-03-22 11:42:32
Owner	drini (3)
Last modified by	drini (3)
Numerical id	22
Author	drini (3)
Entry type	Theorem
Classification	msc 00A05
Classification	msc 20-XX
Classification	msc 26D15
Synonym	harmonic-geometric-arithmetic means inequality
Synonym	arithmetic-geometric means inequality
Synonym	AGM inequality
Synonym	AGMH inequality
Related topic	ArithmeticMean
Related topic	GeometricMean
Related topic	HarmonicMean
Related topic	GeneralMeansInequality
Related topic	WeightedPowerMean
Related topic	PowerMean
Related topic	RootMeanSquare3
Related topic	ProofOfGeneralMeansInequality
Related topic	JensensInequality
Related topic	DerivationOfHarmonicMeanAsTheLimitOfThePowerMean
Related topic	MinimalAndMaximalNumber
Related topic	ProofOfArithm

Let $x_1, x_2, \ldots, x_n$ be positive numbers. Then

$$\begin{aligned} \max\{x_1, x_2, \ldots, x_n\} &\geq \frac{x_1 + x_2 + \cdots + x_n}{n} \\ &\geq \sqrt[n]{x_1 x_2 \cdots x_n} \\ &\geq \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}} \\ &\geq \min\{x_1, x_2, \ldots, x_n\} \end{aligned}$$

The equality is obtained if and only if $x_1 = x_2 = \cdots = x_n$.

There are several generalizations to this inequality using power means and weighted power means.