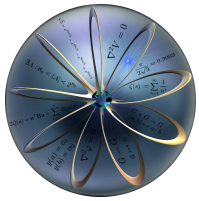

planetmath.org

Math for the people, by the people.

continued fraction of pi

Canonical name	ContinuedFractionOfPi
Date of creation	2013-03-22 16:35:57
Last modified on	2013-03-22 16:35:57
Owner	PrimeFan (13766)
Last modified by	PrimeFan (13766)
Numerical id	5
Author	PrimeFan (13766)
Entry type	Example
Classification	msc 01A40
Related topic	ContinuedFraction

The simple continued fraction for π begins

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cdots}}}$$

and continues with the numerators 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1, 84, 2, 1, 1, 15, 3, 13, 1, 4, 2, 6, 6, 99, 1, 2, 2, 6, 3, 5, 1, 1, 6, 8, 1, 7, 1, 2, 3, 7, 1, 2, 1, 1, 12, ... listed in A001203 of Sloane's OEIS.

Here are a few more terms: 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1, 84, 2, 1, 1, 15, 3, 13, 1, 4, 2, 6, 6, 99, 1, 2, 2, 6, 3, 5, 1, 1, 6, 8, 1, 7, 1, 2, 3, 7, 1, 2, 1, 1, 12, 1, 1, 1, 3, 1, 1, 8, 1, 1, 2, 1, 6, 1, 1, 5, 2, 2, 3, 1, 2, 4, 4, 16, 1, 161, 45, 1, 22, 1, 2, 2, 1, 4, 1, 2, 24, 1, 2, 1, 3, 1, 2, 1, 1, 10, 2, 5, 4, 1, 2, 2, 8, 1, 5, 2, 2, 26, 1, 4, 1, 1, 8, 2, 42, 2, 1, 7, 3, 3, 1, 1, 7, 2, 4, 9, 7, 2, 3, 1, 57, 1, 18, 1, 9, 19, 1, 2, 18, 1, 3, 7, 30, 1, 1, 1, 3, 3, 3, 1, 2, 8, 1, 1, 2, 1, 15, 1, 2, 13, 1, 2, 1, 4, 1, 12, 1, 1, 3, 3, 28, 1, 10, 3, 2, 20, 1, 1, 1, 1, 4, 1, 1, 1, 5, 3, 2, 1, 6, 1, 4, 1, 120, 2, 1, 1, 3, 1, 23, 1, 15, 1, 3, 7, 1, 16, 1, 2, 1, 21, 2, 1, 1, 2, 9, 1, 6, 4, 127, 14, 5, 1, 3, 13, 7, 9, 1, 1, 1, 1, 1, 5, 4, 1, 1, 3, 1, 1, 29, 3, 1, 1, 2, 2, 1, 3, 1, 1, 1, 3, 1, 1, 10, 3, 1, 3, 1, 2, 1, 12, 1, 4, 1, 1, 1, 1, 7, 1, 1, 2, 1, 11, 3, 1, 7, 1, 4, 1, 48, 16, 1, 4, 5, 2, 1, 1, 4, 3, 1, 2, 3, 1, 2, 2, 1, 2, 5, 20, 1, 1, 5, 4, 1, 436, 8, 1, 2, 2, 1, 1, 1, 1, 1, 5, 1, 2, 1, 3, 6, 11, 4, 3, 1, 1, 1, 2, 5, 4, 6, 9, 1, 5, 1, 5, 15, 1, 11, 24, 4, 4, 5, 2, 1, 4, 1, 6, 1, 1, 1, 4, 3, 2, 2, 1, 1, 2, 1, 58, 5, 1, 2, 1, 2, 1, 1, 2, 2, 7, 1, 15, 1, 4, 8, 1, 1, 4, 2, 1, 1, 1, 3, 1, 1, 1, 2, 1, 1, 1, 1, 1, 9, 1, 4, 3, 15, 1, 2, 1, 13, 1, 1, 1, 3, 24, 1, 2, 4, 10, 5, 12, 3, 3, 21, 1, 2, 1, 34, 1, 1, 1, 4, 15, 1, 4, 44, 1, 4, 20776, 1, 1, 1, 1, 1, 1, 1, 23, 1, 7, 2, 1, 94, 55, 1, 1, 2, 1, 1, 3, 1, 1, 32, 5, 1, 14, 1, 1, 1, 1, 1, 3, 50, 2, 16, 5, 1, 2, 1, 4, 6, 3, 1, 3, 3, 1, 2, 2, 2, 5, 2, 2, 2, 28, 1, 1, 13, 1, 5, 43, 1, 4, 3, 5, 3, 1, 4, 1, 1, 2, 2, 1, 1, 19, 2, 7, 1, 72, 3, 1, 2, 3, 7, 11, 1, 2, 1, 1, 2, 2, 1, 1, 2, 1, 1, 1, 1, 1, 33, 7, 19, 1, 19, 3, 1, 4, 1, 1, 1, 1, 2, 3, 1, 3, 2, 2, 2, 2, 4, 1, 1, 1, 4, 2, 3, 1, 1, 1, 1, 11, 1, 1, 2, 1, 2, 1, 2, 2, 1, 7, 2, 27, 1, 1, 6, 2, 1, 9, 6, 26, 1, 1, 3, 2, 1, 1, 1, 1, 1, 15, 1, 36, 4, 2, 2, 1, 22, 2, 1, 106, 2, 2, 1, 3, 1, 12, 10, 7, 1, 2, 1, 1, 1, 1, 8, 2, 4, 5, 3, 2, 1, 4, 23, 1, 18, 2, 10, 3, 1, 6, 6, 13, 8, 6, 2, 2, 2, 2, 1, 1, 1, 3, 1, 7, 17, 1, 1, 1, 2, 5, 5, 1, 1, 2, 11, 1, 6, 1, 6, 1, 29, 4, 29, 3, 5, 3, 1, 141, 1, 2, 7, 7, 2, 2, 7, 1, 1, 7, 1, 7, 1, 2, 4, 1, 1, 1, 30, 1, 12, 4, 18, 10, 2, 8, 1, 2, 2, 2, 4, 13, 1, 5, 4, 1, 6, 1, 1, 11, 2, 4, 2, 1, 1, 3, 3, 12, 1, 1, 39, 5, 1, 1, 16, 125, 1, 4, 1, 2, 1, 19, 1, 4, 1, 1, 2, 1, 4, 1, 10, 1, 4, 2, 1, 1, 1, 5, 10, 4, 14, 1, 13, 41, 1, 4, 1, 8, 1, 1, 2, 1, 3, 1, 6, 1, 3, 2, 2, 2, 1, 4, 1, 14, 1, 2, 8, 1, 8, 3, 3, 3, 1, 37, 4, 2, 4, 1, 3, 4, 25, 4, 27, 2, 7, 1, 1, 2, 6, 1, 1, 1, 12, 1, 2, 2, 2, 13, 12, 1, 3, 1, 6, 1, 1, 33, 1, 5, 3, 1, 5, 15, 8, 8, 47, 1, 3, 2, 12, 2, 12, 1, 12, 1, 2,

5, 3, 1, 1, 1, 1, 2, 3, 5, 4, 2, 1, 1, 5, 1, 9, 14, 1, 1, 3, 2, 1, 9, 3, 22, 13, 1, 1, 3, 20, 1, 1, 61, 1, 376, 2, 107, 1, 10, 3, 2, 2, 31, 1, 2, 10, 2, 2, 62, 2, 2, 7, 4, 5, 6, 1, 1, 1, 1, 2, 8, 2, 73, 3, 5, 42, 1, 3, 2, 1, 1, 59, 6, 1, 1, 1, 5, 1, 6, 1, 2, 6, 1, 1, 1, 1, 3, 2, 1, 3, 1, 8, 1, 4, 2, 5, 4, 7, 1, 4, 2, 2, 6, 1, 1, 2, 2, 1, 1, 1, 1, 1, 2, 1, 2, 2, 5, 1, 2, 1, 1, 10, 1, 6, 1, 129, 1, 4, 65, 2, 4, 4, 3, 2, 3, 1, 1, 5, 1, 1, 1, 1, 1, 2, 2, 1, 2, 1, 1, 2, 2, 1, 2, 3, 1, 2, 1, 2, 4, 2, 1, 2, 27, 6, 2, 1, 193, 1, 3, 9, 1, 3, 35, 2, 1, 8, 1, 1, 1, 1, 9, 3, 56, 1, 6, 6, 2, 8, 1, 8, 1, 2, 3, 6, 3, 1, 3, 1, 1, 1, 2, 13, 1, 1, 1, 1, 13, 2, 1, 3, 1, 3, 15, 2, 1, 1, 2, 4, 1, 4, 5, 2, 2, 1, 2, 1, 6, 1, 4, 12, 1, 1, 1, 1, 13, 1, 3, 4, 1, 1, 1, 2, 9, 1, 7, 1, 1, 1, 1, 4, 1, 3, 4, 1, 1, 4, 3, 1, 39, 2, 1, 1, 1, 1, 1, 4, 7, 2, 2, 2, 1, 1, 1, 1, 2, 114, 12, 4, 1, 3, 2, 1, 19, 1, 1, 2, 1, 1, 3, 4, 1, 60, 3, 72, 2, 1, 1, 1, 50, 1, 1, 1, 1, 3, 1, 1, 2, 2, 1, 4, 1, 7, 3, 1, 2, 1, 5, 1, 1, 1, 2, 6, 2, 21, 2, 6, 1, 6, 1, 1, 2, 1, 7, 1, 8, 1, 1, 5, 4, 1, 1, 1, 1, 1, 1, 1, 1, 4, 1, 11, 2, 4, 10, 2, 1, 1, 13, 1, 1, 7, 15, 1, 1, 1, 2, 3, 15, 8, 8, 2, 1, 13, 3, 5, 1, 2, 1, 6, 1, 10, 123, 3, 1, 4, 59, 4, 156, 88, 1, 5, 4, 1, 3, 1, 4, 2, 9, 1, 7, 4, 2, 1, 2, 3, 2, 1, 2, 11, 1, 13, 7, 7, 1, 63, 37, 12, 86, 1, 1, 1, 1, 2, 2, 4, 2, 18, 1, 1, 1, 41, 2, 1, 1, 12, 1, 2, 1, 1, 2, 10, 1, 1, 1, 5, 1, 1, 3, 1, 7, 5, 1, 9, 1, 2, 2, 7, 1, 1, 5, 2, 1, 3, 3, 5, 2, 1, 11, 3, 1, 3, 2, 1, 1, 2, 1, 14, 5, 2, 2, 1, 1, 1, 1, 3, 1, 3, 3, 2, 2, 1, 3, 2, 1, 2, 1, 4, 1, 14, 1, 1, 58, 7, 1, 2, 1, 1, 5, 1, 2, 1, 5, 18, 1, 4, 3, 1, 1, 1, 4, 1, 1, 2, 5, 1, 148, 1, 9, 2, 1, 2, 1, 5, 4, 93, 1, 1, 2, 4, 1, 2, 73, 1, 1, 3, 1, 1, 1, 1, 2, 1, 34, 1, 5, 6, 1, 2, 1, 3, 4, 1, 16, 28, 17, 2, 5, 5, 26, ...

Most computer algebra systems provide relatively simple commands for obtaining as many terms as one wants. In Mathematica, for example, one can issue the command `ContinuedFraction[Pi, 1000]`, replacing 1000 with whichever number one prefers. In this particular case, it only takes Mathematica a couple of seconds to give a thousand terms.