# permutation

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A *permutation* of a finite set $\{a_1, a_2, \ldots, a_n\}$ is an arrangement of its elements. For example, if $S = \{A, B, C\}$ then $ABC$, $CAB$ , $CBA$ are three different permutations of $S$.

The number of permutations of a set with $n$ elements is $n!$ (see the rule of product).

A permutation can also be seen as a bijective function of a set into itself. For example, the permutation $ABC \mapsto CAB$ could be seen a function $f : \{A, B, C\} \to \{A, B, C\}$ that assigns:

$$f(A) = C, \qquad f(B) = A, \qquad f(C) = B.$$

In fact, every bijection of a set into itself gives a permutation, and any permutation gives rise to a bijective function.

Therefore, we can say that there are $n!$ bijective functions from a set with $n$ elements into itself.

Using the function approach, it can be proved that any permutation can be expressed as a composition of disjoint cycles and also as composition of (not necessarily disjoint) transpositions.

Moreover, if $\sigma = \tau_1\tau_2 \cdots \tau_m = \rho_1\rho_2 \cdots \rho_n$ are two factorization of a permutation $\sigma$ into transpositions, then $m$ and $n$ must be both even or both odd. So we can label permutations as *even* or *odd* depending on the number of transpositions for any decomposition.

Permutations (as functions) form in general a non-abelian group with function composition as binary operation called *symmetric group of order* $n$. The subset of even permutations becomes a subgroup called the alternating group of order $n$.