# examples of ring of sets

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| Canonical name | ExamplesOfRingOfSets |
| Date of creation | 2013-03-22 15:47:52 |
| Last modified on | 2013-03-22 15:47:52 |
| Owner | rspuzio (6075) |
| Last modified by | rspuzio (6075) |
| Numerical id | 8 |
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| Entry type | Example |
| Classification | msc 03E20 |
| Classification | msc 28A05 |

Every field of sets is a ring of sets. Below are some examples of rings of sets that are not fields of sets.

1. Let $A$ be a non-empty set containing an element $a$. Let $\mathcal{R}$ be the family of subsets of $A$ containing $a$. Then $\mathcal{R}$ is a ring of sets, but not a field of sets, since $\{a\} \in \mathcal{R}$, but $A - \{a\} \notin \mathcal{R}$.

2. The collection of all open sets of a topological space is a ring of sets, which is in general not a field of sets, unless every open set is also closed. Likewise, the collection of all closed sets of a topological space is also a ring of sets.

3. A simple example of a ring of sets is the subset $\{\{a\}, \{a, b\}\}$ of $2^{\{a,b\}}$. That this is a ring of sets follows from the observations that $\{a\} \cap \{a, b\} = \{a\}$ and $\{a\} \cup \{a, b\} = \{a, b\}$. Note that it is not a field of sets because the complement of $\{a\}$, which is $\{b\}$, does not belong to the ring.

4. Another example involves an infinite set. Let $A$ be an infinite set. Let $\mathcal{R}$ be the collection of finite subsets of $A$. Since the union and the intersection of two finite set are finite sets, $\mathcal{R}$ is a ring of sets. However, it is not a field of sets, because the complement of a finite subset of $A$ is infinite, and thus not a member of $\mathcal{R}$.