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axiom of infinity

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There exists an infinite set.

The Axiom of Infinity is an axiom of Zermelo-Fraenkel set theory. At first glance, this axiom seems to be ill-defined. How are we to know what constitutes an infinite set when we have not yet defined the notion of a finite set? However, once we have a theory of ordinal numbers in hand, the axiom makes sense.

Meanwhile, we can give a definition of finiteness that does not rely upon the concept of number. We do this by introducing the notion of an inductive set. A set S is said to be inductive if $\emptyset \in S$ and for every $x \in S$, $x \cup \{x\} \in S$. We may then state the Axiom of Infinity as follows:

There exists an inductive set.

In symbols:

$$\exists S[\emptyset \in S \wedge (\forall x \in S)[x \cup \{x\} \in S]]$$

We shall then be able to prove that the following conditions are equivalent:

1. There exists an inductive set.

2. There exists an infinite set.

3. The least nonzero limit ordinal, ω, is a set.