A.1.7 W-types

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For W-types we introduce primitive constants c_{W} and c_{sup}. An expression of the form $c_{\mathsf{W}}(A, \lambda x.\, B)$ is written as $\mathsf{W}_{(x:A)} B$, and an expression of the form $c_{\mathsf{sup}}(x, u)$ is written as $\mathsf{sup}(x, u)$:

- if $A : \mathcal{U}_n$ and $B : A \to \mathcal{U}_n$, then $\mathsf{W}_{(x:A)} B(x) : \mathcal{U}_n$
- if moreover, $a : A$ and $g : B(a) \to \mathsf{W}_{(x:A)} B(x)$ then $\mathsf{sup}(a, g) : \mathsf{W}_{(x:A)} B(x)$.

Here also we can define functions by total recursion. If we have A and B as above and $C : \mathsf{W}_{(x:A)} B(x) \to \mathcal{U}_m$, then we can introduce a defined constant $f : \prod_{(z:\mathsf{W}_{(x:A)} B(x))} C(z)$ whenever we have

$$d : \prod_{(x:A)} \prod_{(u:B(x) \to \mathsf{W}_{(x:A)} B(x))} \big(\big(\prod_{(y:B(x))} C(u(y))\big) \to C(\mathsf{sup}(x, u))\big)$$

with the defining equation

$$f(\mathsf{sup}(x, u)) :\equiv d(x, u, f \circ u).$$