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# axiom of pairing

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| Canonical name | AxiomOfPairing |
| Date of creation | 2013-03-22 13:42:43 |
| Last modified on | 2013-03-22 13:42:43 |
| Owner | Sabean (2546) |
| Last modified by | Sabean (2546) |
| Numerical id | 7 |
| Author | Sabean (2546) |
| Entry type | Axiom |
| Classification | msc 03E30 |
| Synonym | pairing |

For any $a$ and $b$ there exists a set $\{a, b\}$ that contains exactly $a$ and $b$.

The Axiom of Pairing is one of the axioms of Zermelo-Fraenkel set theory. In symbols, it reads:

$$\forall a \forall b \exists c \forall x (x \in c \leftrightarrow x = a \vee x = b).$$

Using the Axiom of Extensionality, we see that the set $c$ is unique, so it makes sense to define the pair

$$\{a, b\} = \text{ the unique } c \text{ such that } \forall x (x \in c \leftrightarrow x = a \vee x = b).$$

Using the Axiom of Pairing, we may define, for any set $a$, the singleton

$$\{a\} = \{a, a\}.$$

We may also define, for any set $a$ and $b$, the ordered pair

$$(a, b) = \{\{a\}, \{a, b\}\}.$$

Note that this definition satisfies the condition

$$(a, b) = (c, d) \text{ iff } a = c \text{ and } b = d.$$

We may define the ordered $n$-tuple recursively

$$(a_1, \ldots, a_n) = ((a_1, \ldots, a_{n-1}), a_n).$$