another proof of cardinality of the rationals

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If we have a rational number p/q with p and q having no common factor, and each expressed in base 10 then we can view p/q as a base 11 integer, where the digits are $0, 1, 2, \ldots, 9$ and $/$. That is, slash ($/$) is a symbol for a digit. For example, the rational $3/2$ corresponds to the integer $3 \cdot 11^2 + 10 \cdot 11 + 2$. The rational $-3/2$ corresponds to the integer $-(3 \cdot 11^2 + 10 \cdot 11 + 2)$.

This gives a one-to-one map into the integers so the cardinality of the rationals is at most the cardinality of the integers. So the rationals are countable.