inverse function

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Definition Suppose $f : X \to Y$ is a function between sets X and Y, and suppose $f^{-1} : Y \to X$ is a mapping that satisfies

$$\begin{aligned} f^{-1} \circ f &= \mathrm{id}_X, \\ f \circ f^{-1} &= \mathrm{id}_Y, \end{aligned}$$

where id_A denotes the identity function on the set A. Then f^{-1} is called the *inverse of* f, or the *inverse function of* f. If f has an inverse near a point $x \in X$, then f is *invertible near* x. (That is, if there is a set U containing x such that the restriction of f to U is invertible, then f is invertible near x.) If f is invertible near all $x \in X$, then f is *invertible*.

Properties

1. When an inverse function exists, it is unique.
2. The inverse function and the inverse image of a set coincide in the following sense. Suppose $f^{-1}(A)$ is the inverse image of a set $A \subset Y$ under a function $f : X \to Y$. If f is a bijection, then $f^{-1}(y) = f^{-1}(\{y\})$.
3. The inverse function of a function $f : X \to Y$ exists if and only if f is a bijection, that is, f is an injection and a surjection.
4. A linear mapping between vector spaces is invertible if and only if the determinant of the mapping is nonzero.
5. For differentiable functions between Euclidean spaces, the inverse function theorem gives a necessary and sufficient condition for the inverse to exist. This can be generalized to maps between Banach spaces which are differentiable in the sense of Frechet.

Remarks

When f is a linear mapping (for instance, a matrix), the term *non-singular* is also used as a synonym for invertible.