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proof of fixed points of normal functions

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Owner	Henry (455)
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Author	Henry (455)
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Suppose f is a κ-normal function and consider any $\alpha < \kappa$ and define a sequence by $\alpha_0 = \alpha$ and $\alpha_{n+1} = f(\alpha_n)$. Let $\alpha_\omega = \sup_{n<\omega} \alpha_n$. Then, since f is continuous,

$$f(\alpha_\omega) = \sup_{n<\omega} f(\alpha_n) = \sup_{n<\omega} \alpha_{n+1} = \alpha_\omega$$

So $\mathrm{Fix}(f)$ is unbounded.

Suppose N is a set of fixed points of f with $|N| < \kappa$. Then

$$f(\sup N) = \sup_{\alpha \in N} f(\alpha) = \sup_{\alpha \in N} \alpha = \sup N$$

so $\sup N$ is also a fixed point of f, and therefore $\mathrm{Fix}(f)$ is closed.