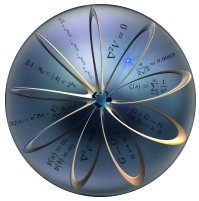

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limit cardinal

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A *limit cardinal* is a cardinal κ such that $\lambda^+ < \kappa$ for every cardinal $\lambda < \kappa$. Here λ^+ denotes the cardinal successor of λ. If $2^\lambda < \kappa$ for every cardinal $\lambda < \kappa$, then κ is called a *strong limit cardinal*.

Every strong limit cardinal is a limit cardinal, because $\lambda^+ \leq 2^\lambda$ holds for every cardinal λ. Under GCH, every limit cardinal is a strong limit cardinal because in this case $\lambda^+ = 2^\lambda$ for every infinite cardinal λ.

The three smallest limit cardinals are 0, $\aleph_0$ and $\aleph_\omega$. Note that some authors do not count 0, or sometimes even $\aleph_0$, as a limit cardinal. An infinite cardinal $\aleph_\alpha$ is a limit cardinal if and only if α is either 0 or a limit ordinal.