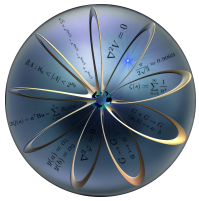

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proof of compactness theorem for first order logic

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The theorem states that if a set of sentences of a first-order language L is inconsistent, then some finite subset of it is inconsistent. Suppose $\Delta \subseteq L$ is inconsistent. Then by definition $\Delta \vdash \bot$, i.e. there is a formal proof of "false" using only assumptions from Δ. Formal proofs are finite objects, so let Γ collect all the formulas of Δ that are used in the proof.