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forcing relation

Canonical name	ForcingRelation
Date of creation	2013-03-22 12:53:28
Last modified on	2013-03-22 12:53:28
Owner	Henry (455)
Last modified by	Henry (455)
Numerical id	5
Author	Henry (455)
Entry type	Definition
Classification	msc 03E35
Classification	msc 03E40
Related topic	Forcing
Defines	forcing relation
Defines	forces

If  $\mathfrak{M}$  is a transitive model of set theory and  $P$  is a partial order then we can define a *forcing relation*:

$$p \Vdash_P \phi(\tau_1, \dots, \tau_n)$$

( $p$  forces  $\phi(\tau_1, \dots, \tau_n)$ )

for any  $p \in P$ , where  $\tau_1, \dots, \tau_n$  are  $P$ - names.

Specifically, the relation holds if for every generic filter  $G$  over  $P$  which contains  $p$ ,

$$\mathfrak{M}[G] \models \phi(\tau_1[G], \dots, \tau_n[G])$$

That is,  $p$  forces  $\phi$  if every  $\phi$  of  $\mathfrak{M}$  by a generic filter over  $P$  containing  $p$  makes  $\phi$  true.

If  $p \Vdash_P \phi$  holds for every  $p \in P$  then we can write  $\Vdash_P \phi$  to mean that for any generic  $G \subseteq P$ ,  $\mathfrak{M}[G] \models \phi$ .