frequently in

Canonical name	FrequentlyIn
Date of creation	2013-03-22 17:14:23
Last modified on	2013-03-22 17:14:23
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	6
Author	CWoo (3771)
Entry type	Definition
Classification	msc 03E04
Synonym	clusters at
Defines	cluster point of a net

Recall that a net is a function x from a directed set D to a set X. The value of x at $i \in D$ is usually denoted by x_i. Let A be a subset of X. We say that a net x is *frequently in* A if for every $i \in D$, there is a $j \in D$ such that $i \leq j$ and $x_j \in A$.

Suppose a net x is frequently in $A \subseteq X$. Let $E := \{j \in D \mid x_j \in A\}$. Then E is a cofinal subset of D, for if $i \in D$, then by definition of A, there is $i \leq j \in D$ such that $x_j \in A$, and therefore $j \in E$.

The notion of "frequently in" is related to the notion of "eventually in" in the following sense: a net x is eventually in a set $A \subseteq X$ iff it is not frequently in $A^\complement$, its complement. Suppose x is eventually in A. There is $j \in D$ such that $x_k \in A$ for all $k \geq j$, or equivalently, $x_k \in A^\complement$ for no $k \geq j$. The converse is can be argued by tracing the previous statements backwards.

In a topological space X, a point $a \in X$ is said to be a *cluster point of a net* x (or, occasionally, x *clusters at* a) if x is frequently in every neighborhood of a. In this general definition, a limit point is always a cluster point. But a cluster point need not be a limit point. As an example, take the sequence $0, 2, 0, 4, 0, 6, 0, 8, \ldots, 0, 2n, 0, \ldots$ has 0 as a cluster point. But clearly 0 is not a limit point, as the sequence diverges in $\mathbb{R}$.