dependence relation

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Let X be a set. A (binary) relation $\prec$ between an element a of X and a subset S of X is called a *dependence relation*, written $a \prec S$, when the following conditions are satisfied:

1. if $a \in S$, then $a \prec S$;
2. if $a \prec S$, then there is a finite subset S_0 of S, such that $a \prec S_0$;
3. if T is a subset of X such that $b \in S$ implies $b \prec T$, then $a \prec S$ implies $a \prec T$;
4. if $a \prec S$ but $a \not\prec S - \{b\}$ for some $b \in S$, then $b \prec (S - \{b\}) \cup \{a\}$.

Given a *dependence relation* $\prec$ on X, a subset S of X is said to be *independent* if $a \not\prec S - \{a\}$ for all $a \in S$. If $S \subseteq T$, then S is said to *span* T if $t \prec S$ for every $t \in T$. S is said to be a *basis* of X if S is *independent* and S *spans* X.

Remark. If X is a non-empty set with a *dependence relation* $\prec$, then X always has a *basis* with respect to $\prec$. Furthermore, any two of X have the same cardinality.

Examples:

- Let V be a vector space over a field F. The relation $\prec$, defined by $\upsilon \prec S$ if υ is in the subspace S, is a dependence relatoin. This is equivalent to the definition of *http://planetmath.org/LinearIndependencelinear dependence*.
- Let K be a field extension of F. Define $\prec$ by $\alpha \prec S$ if α is algebraic over $F(S)$. Then $\prec$ is a dependence relation. This is equivalent to the definition of *algebraic dependence*.