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product of automata

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One way to manufacture an automaton out of existing automata is by taking products.

Products of Two Automata

Let $A_1 = (S_1, \Sigma_1, \delta_1, I_1, F_1)$ and $A_2 = (S_2, \Sigma_2, \delta_2, I_2, F_2)$ be two automata. We define the product A of A_1 and A_2, written $A_1 \times A_2$, as the quituple

$$(S, \Sigma, \delta, I, F) := (S_1 \times S_2, \Sigma_1 \times \Sigma_2, \delta_1 \times \delta_2, I_1 \times I_2, F_1 \times F_2),$$

where δ is a function from $S \times \Sigma$ to $P(S_1) \times P(S_2) \subseteq P(S)$, given by

$$\delta((s_1, s_2), (\alpha_1, \alpha_2)) := \delta_1(s_1, \alpha_1) \times \delta_2(s_2, \alpha_2).$$

Since S, Σ, I, F are non-empty, A is an automaton. The automaton A can be thought of as a machine that runs automata A_1 and A_2 simultaneously. A pair (α_1, α_2) of symbols being fed into A at start state $(q_1, q_2) \in I$ is the same as A_1 reading α_1 at state q_1 and A_2 reading α_2 at state q_2. The set of all possible next states for the configuration $((s_1, s_2), (\alpha_1, \alpha_2))$ in A is the same as the set of all possible combinations (t_1, t_2), where t_1 is a next state for the configuration (s_1, α_1) in A_1 and t_2 is a next state for the configuration (s_2, α_2) in A_2.

If A_1 and A_2 are FSA, so is A. In addition, if both A_1 and A_2 are deterministic, so is A, because

$$\delta((s_1, s_2), (\alpha_1, \alpha_2)) = (\delta_1(s_1, \alpha_1), \delta_2(s_2, \alpha_2)),$$

and I is a singleton.

As usual, δ can be extended to read words over Σ, and it is easy to see that

$$\delta((s_1, s_2), (a_1, a_2)) = \delta_1(s_1, a_1) \times \delta_2(s_2, a_2),$$

where a_1 and a_2 are words over Σ_1 and Σ_2 respectively. A word (a_1, a_2) is accepted by A iff a_1 is accepted by A_1 and a_2 is accepted by A_2.

Intersection of Two Automata

Again, we assume A_1 and A_2 are automata specified above. Now, suppose $\Sigma_1 = \Sigma_2 = \Delta$. Then Δ can be identified as the diagonal in $\Sigma = \Sigma_1 \times \Sigma_2 = \Delta^2$. We are then led to an automaton

$$A_1 \cap A_2 := (S, \Delta, \delta, I, F),$$

where S, I, and F are defined previously, and δ is given by

$$\delta((s_1, s_2), \alpha) = \delta_1(s_1, \alpha) \times \delta_2(s_2, \alpha).$$

Suppose in addition that Δ is finite. From the discussion in the previous section, it is evident that the language accepted by $A_1 \cap A_2$ is the same as the intersection of the language accepted by A_1 and the language accepted by A_2:

$$L(A_1 \cap A_2) = L(A_1) \cap L(A_2).$$