example of Gödel numbering

Canonical name	ExampleOfGodelNumbering
Date of creation	2013-03-22 12:58:28
Last modified on	2013-03-22 12:58:28
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Last modified by	Henry (455)
Numerical id	7
Author	Henry (455)
Entry type	Example
Classification	msc 03B10

We can define by recursion a function e from formulas of arithmetic to numbers, and the corresponding Gödel numbering as the inverse.

The symbols of the language of arithmetic are $=$, $\forall$, $\neg$, $\rightarrow$, 0, S, $<$, $+$, $\cdot$, the variables v_i for any integer i, and (and). (and) are only used to define the order of operations, and should be inferred where appropriate in the definition below.

We can define a function e by recursion as follows:

- $e(v_i) = \langle 0, i \rangle$
- $e(\phi = \psi) = \langle 1, e(\phi), e(\psi) \rangle$
- $e(\forall v_i \phi) = \langle 2, e(v_i), e(\phi) \rangle$
- $e(\neg \phi) = \langle 3, e(\phi) \rangle$
- $e(\phi \rightarrow \psi) = \langle 4, e(\phi), e(\psi) \rangle$
- $e(0) = \langle 5 \rangle$
- $e(S\phi) = \langle 6, e(\phi) \rangle$
- $e(\phi < \psi) = \langle 7, e(\phi), e(\psi) \rangle$
- $e(\phi + \psi) = \langle 8, e(\phi), e(\psi) \rangle$
- $e(\phi \cdot \psi) = \langle 9, e(\phi), e(\psi) \rangle$

Clearly e^{-1} is a Gödel numbering, with $\ulcorner \phi \urcorner = e(\phi)$.