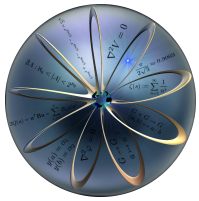

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joint embedding property

Canonical name	JointEmbeddingProperty
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Entry type	Definition
Classification	msc 03C52
Synonym	JEP
Synonym	SJEP
Related topic	ultrauniversal
Related topic	amalgamationproperty
Related topic	UltraUniversal
Related topic	FactorEmbeddable
Defines	joint embedding property
Defines	strong joint embedding property

Let K be a class of models (structures) of a given signature. We say that K has the *joint embedding property* (abbreviated *JEP*) iff for any models A and B in K there exists a model C in K such that both A and B are embeddable in C. [**?**, **?**]

0.0.1 Examples

Examples include [**?**]:

- The class of all groups.
- The class of all monoids.
- The class of all non-trivial Boolean algebras.

As is the case with the above examples, classes having the joint embedding property often satisfy an even stronger condition - for every indexed family of models in the class there is a model in the class into which each member of the family can be embedded. This is known as the *strong joint embedding property* (abbreviated *SJEP*). [**?**]

In general any factor embeddable class closed under products will have the strong joint embedding property. [**?**]

0.0.2 Characterizations

Elementary classes with the joint embedding property may be characterized syntactically and semantically:

Let T be a first order theory in a language L and let K be the class of models of T then:

1. K has the joint embedding property iff for all universal sentences ϕ and ψ in L, $T \vdash \phi \vee \psi$ implies either $T \vdash \phi$ or $T \vdash \psi$. [**?**]
2. If T is consistent, then K has the joint embedding property iff T has an ultra-universal model. [**?**]

References

[1] Abraham Robinson: *Forcing in model theory*, Actes du Congrès International des Mathématiciens (Nice, 1970) Gauthier-Villars, Paris, 1971, pp. 245-250

[2] Colin Naturman, Henry Rose: *Ultra-universal models*, Quaestiones Mathematicae, 15(2), 1992, 189-195

[3] Colin Naturman: *Interior Algebras and Topology*, Ph.D. thesis, University of Cape Town Department of Mathematics, 1991