example of well-founded induction

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This proof of the fundamental theorem of arithmetic (every natural number has a prime factorization) affords an example of proof by well-founded induction over a well-founded relation that is not a linear order.

First note that the division relation is obviously well-founded. The $|$-minimal elements are the prime numbers. We detail the two steps of the proof :

1. If n is prime, then n is its own factorization into primes, so the assertion is true for the $|$-minimal elements.

2. If n is not prime, then n has a non-trivial factorization (by definition of not being prime), i.e. $n = m\ell$, where $m, n \neq 1$. By induction, m and ℓ have prime factorizations, the product of which is a prime factorization of n.