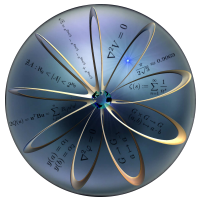

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direct image

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Let $f\colon A \longrightarrow B$ be a function, and let $U \subset A$ be a subset. The *direct image* of U is the set $f(U) \subset B$ consisting of all elements of B which equal $f(u)$ for some $u \in U$.

Direct images satisfy the following properties:

1. Unions: For any collection $\{U_i\}_{i\in I}$ of subsets of A,
$$f\left(\bigcup_{i\in I} U_i\right) = \bigcup_{i\in I} f(U_i).$$
2. Intersections: For any collection $\{U_i\}_{i\in I}$ of subsets of A,
$$f\left(\bigcap_{i\in I} U_i\right) \subset \bigcap_{i\in I} f(U_i).$$
3. Set difference: For any $U, V \subset A$,
$$f(V \setminus U) \supset f(V) \setminus f(U).$$
In particular, the complement of U satisfies $f(U^{\complement}) \supset f(A) \setminus f(U)$.
4. Subsets: If $U \subset V \subset A$, then $f(U) \subset f(V) \subset B$.
5. Inverse image of a direct image: For any $U \subset A$,
$$f^{-1}(f(U)) \supset U$$
with equality if f is injective.
6. Direct image of an inverse image: For any $V \subset B$,
$$f(f^{-1}(V)) \subset V$$
with equality if f is surjective.