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line segment

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Defines	open line segment
Defines	closed line segment

**Definition** Suppose  $V$  is a vector space over  $\mathbb{R}$  or  $\mathbb{C}$ , and  $L$  is a subset of  $V$ . Then  $L$  is a *line segment* if  $L$  can be parametrized as

$$L = \{a + tb \mid t \in [0, 1]\}$$

for some  $a, b$  in  $V$  with  $b \neq 0$ .

Sometimes one needs to distinguish between open and <http://planetmath.org/Closed> closed line segments. Then one defines a *closed line segment* as above, and an *open line segment* as a subset  $L$  that can be parametrized as

$$L = \{a + tb \mid t \in (0, 1)\}$$

for some  $a, b$  in  $V$  with  $b \neq 0$ .

If  $x$  and  $y$  are two vectors in  $V$  and  $x \neq y$ , then we denote by  $[x, y]$  the set connecting  $x$  and  $y$ . This is,  $\{\alpha x + (1 - \alpha)y \mid 0 \leq \alpha \leq 1\}$ . One can easily check that  $[x, y]$  is a closed line segment.

### Remarks

- An alternative, equivalent, definition is as follows: A (closed) line segment is a convex hull of two distinct points.
- A line segment is connected, non-empty set.
- If  $V$  is a topological vector space, then a closed line segment is a closed set in  $V$ . However, an open line segment is an open set in  $V$  if and only if  $V$  is one-dimensional.
- More generally than above, the concept of a line segment can be defined in an ordered geometry.