# proof of Zermelo's postulate

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The following is a proof that the axiom of choice implies Zermelo's postulate.

*Proof.* Let $\mathcal{F}$ be a disjoint family of nonempty sets. Let $f\colon \mathcal{F} \to \bigcup \mathcal{F}$ be a choice function. Let $A, B \in \mathcal{F}$ with $A \neq B$. Since $\mathcal{F}$ is a disjoint family of sets, $A \cap B = \emptyset$. Since $f$ is a choice function, $f(A) \in A$ and $f(B) \in B$. Thus, $f(A) \notin B$. Hence, $f(A) \neq f(B)$. It follows that $f$ is injective.

Let $C = \left\{ f(B) \in \bigcup \mathcal{F} : B \in \mathcal{F} \right\}$. Then $C$ is a set.

Let $A \in \mathcal{F}$. Since $f$ is injective, $A \cap C = \{f(A)\}$. $\square$