o-minimality

Canonical name	Ominimality
Date of creation	2013-03-22 13:23:01
Last modified on	2013-03-22 13:23:01
Owner	Timmy (1414)
Last modified by	Timmy (1414)
Numerical id	7
Author	Timmy (1414)
Entry type	Definition
Classification	msc 03C64
Classification	msc 14P10
Related topic	StronglyMinimal
Defines	o-minimal

Let M be an ordered structure. An interval in M is any subset of M that can be expressed in one of the following forms:

- $\{x : a < x < b\}$ for some a, b from M
- $\{x : x > a\}$ for some a from M
- $\{x : x < a\}$ for some a from M

Then we define M to be *o-minimal* iff every definable subset of M is a finite union of intervals and points. This is a property of the theory of M i.e. if $M \equiv N$ and M is o-minimal, then N is o-minimal. Note that M being o-minimal is equivalent to every definable subset of M being quantifier free definable in the language with just the ordering. Compare this with strong minimality.

The model theory of o-minimal structures is well understood, for an excellent account see Lou van den Dries, Tame topology and o-minimal structures, CUP 1998. In particular, although this condition is merely on definable subsets of M it gives very good information about definable subsets of M^n for $n \in \omega$.