composition of forcing notions

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Suppose P is a forcing notion in $\mathfrak{M}$ and $\hat{Q}$ is some P-name such that $\Vdash_P \hat{Q}$ `is a forcing notion`.

Then take a set of P-names Q such that given a P name $\tilde{Q}$ of Q, $\Vdash_P \tilde{Q} = \hat{Q}$ (that is, no matter which generic subset G of P we force with, the names in Q correspond precisely to the elements of $\hat{Q}[G]$). We can define

$$P * Q = \{\langle p, \hat{q}\rangle \mid p \in P, \hat{q} \in Q\}$$

We can define a partial order on $P * Q$ such that $\langle p_1, \hat{q}_1\rangle \leq \langle p_2, \hat{q}_2\rangle$ iff $p_1 \leq_P p_2$ and $p_1 \Vdash \hat{q}_1 \leq_{\hat{Q}} \hat{q}_2$. (A note on interpretation: q_1 and q_2 are P names; this requires only that $\hat{q}_1 \leq \hat{q}_2$ in generic subsets contain p_1, so in other generic subsets that fact could fail.)

Then $P * \hat{Q}$ is itself a forcing notion, and it can be shown that forcing by $P * \hat{Q}$ is equivalent to forcing first by P and then by $\hat{Q}[G]$.