sum and product and quotient of functions

Canonical name	SumAndProductAndQuotientOfFunctions
Date of creation	2013-03-22 17:44:24
Last modified on	2013-03-22 17:44:24
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	12
Author	pahio (2872)
Entry type	Definition
Classification	msc 03E20
Related topic	DirectSumOfEvenoddFunctionsExample
Related topic	LimitRulesOfFunctions
Related topic	PolynomialFunction
Related topic	ProofOfLimitRuleOfProduct
Related topic	ContinuousDerivativeImpliesBoundedVariation
Related topic	PropertiesOfRiemannStieltjesIntegral
Related topic	InfimumAndSupremumOfSumAndProduct
Related topic	PropertiesOfVectorValuedFunctio
Defines	sum of functions
Defines	product of functions
Defines	quotient of functions
Defines	scalar-multiplied function

Let A be a set and M a left R-module. If $f\colon A\to M$ and $g\colon A\to M$, then one may define the *sum of functions* f and g as the following function $f{+}g\colon A\to M$:

$$(f{+}g)(x)\ :=\ f(x){+}g(x)\quad \forall x\in A$$

If r is any element of the ring R, then the *scalar-multiplied function* $rf: A\to M$ is defined as

$$(rf)(x)\ :=\ r{\cdot}f(x)\quad \forall x\in A.$$

Let A again be a set and K a field or a skew field. If $f: A\to K$ and $g: A\to K$, then one can define the *product of functions* f and g as the function $fg\colon A\to K$ as follows:

$$(fg)(x)\ :=\ f(x){\cdot}g(x)\quad \forall x\in A$$

The *quotient of functions* f and g is the function $\dfrac{f}{g}\colon \{a\in A\colon\ g(a)\neq 0\}\to K$ defined as

$$\frac{f}{g}(x)\ :=\ \frac{f(x)}{g(x)}\quad \forall x\in A\smallsetminus\{a\in A\colon\ g(a)=0\}.$$

In particular, the incremental quotient of functions $\frac{f(y)-f(x)}{y-x}$, as y tends to x, gave rise to the important concept of derivative. As another example, we can with a conscience say that the `http://planetmath.org/TrigonometricFunction`tangent function is the quotient of the `http://planetmath.org/TrigonometricFunction`sine and the cosine functions.