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period of mapping

Canonical name	PeriodOfMapping
Date of creation	2013-03-22 13:48:53
Last modified on	2013-03-22 13:48:53
Owner	bwebste (988)
Last modified by	bwebste (988)
Numerical id	12
Author	bwebste (988)
Entry type	Definition
Classification	msc 03E20
Related topic	Retract
Related topic	Idempotency

**Definition** Suppose  $X$  is a set and  $f$  is a mapping  $f : X \rightarrow X$ . If  $f^n$  is the identity mapping on  $X$  for some  $n = 1, 2, \dots$ , then  $f$  is said to be a **mapping of period  $n$** . Here, the notation  $f^n$  means the  $n$ -fold composition  $f \circ \dots \circ f$ .

### 0.0.1 Examples

1. A mapping  $f$  is of period 1 if and only if  $f$  is the identity mapping.
2. Suppose  $V$  is a vector space. Then a linear involution  $L : V \rightarrow V$  is a mapping of period 2. For example, the reflection mapping  $x \mapsto -x$  is a mapping of period 2.
3. In the complex plane, the mapping  $z \mapsto e^{-2\pi i/n} z$  is a mapping of period  $n$  for  $n = 1, 2, \dots$ .
4. Let us consider the function space spanned by the trigonometric functions  $\sin$  and  $\cos$ . On this space, the derivative is a mapping of period 4.

### 0.0.2 Properties

1. Suppose  $X$  is a set. Then a mapping  $f : X \rightarrow X$  of period  $n$  is a bijection. <http://planetmath.org/MappingOfDegreeNIsASurjection>(proof.)
2. Suppose  $X$  is a topological space. Then a continuous mapping  $f : X \rightarrow X$  of period  $n$  is a homeomorphism.