proof of theorems in additively indecomposable

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- $\mathbb{H}$ is closed.

  Let $\{\alpha_i \mid i < \kappa\}$ be some increasing sequence of elements of $\mathbb{H}$ and let $\alpha = \sup\{\alpha_i \mid i < \kappa\}$. Then for any $x, y < \alpha$, it must be that $x < \alpha_i$ and $y < \alpha_j$ for some $i, j < \kappa$. But then $x + y < \alpha_{\max\{i,j\}} < \alpha$.

- $\mathbb{H}$ is unbounded.

  Consider any α, and define a sequence by $\alpha_0 = S\alpha$ and $\alpha_{n+1} = \alpha_n + \alpha_n$. Let $\alpha_\omega = \sup_{n<\omega} \alpha_n$ be the limit of this sequence. If $x, y < \alpha_\omega$ then it must be that $x < \alpha_i$ and $y < \alpha_j$ for some $i, j < \omega$, and therefore $x + y < \alpha_{\max\{i,j\}+1}$. Note that α_ω is, in fact, the next element of $\mathbb{H}$, since every element in the sequence is clearly additively decomposable.

- $f_{\mathbb{H}}(\alpha) = \omega^\alpha$.

  Since 0 is not in $\mathbb{H}$, we have $f_{\mathbb{H}}(0) = 1$.

  For any $\alpha + 1$, we have $f_{\mathbb{H}}(\alpha + 1)$ is the least additively indecomposable number greater than $f_{\mathbb{H}}(\alpha)$. Let $\alpha_0 = Sf_{\mathbb{H}}(\alpha)$ and $\alpha_{n+1} = \alpha_n + \alpha_n = \alpha_n \cdot 2$. Then $f_{\mathbb{H}}(\alpha + 1) = \sup_{n<\omega} \alpha_n = \sup_{n<\omega} S\alpha \cdot 2 \cdots 2 = f_{\mathbb{H}}(\alpha) \cdot \omega$. The limit case is trivial since $\mathbb{H}$ is closed and unbounded, so $f_{\mathbb{H}}$ is continuous.