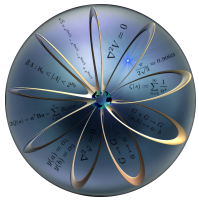

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PA

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Related topic	PeanoArithmetic

(PA) is the restriction of Peano's axioms to a first order theory of . The only change is that the induction axiom is replaced by induction restricted to arithmetic formulas:

$$\phi(0) \wedge \forall x(\phi(x) \rightarrow \phi(x')) \rightarrow \forall x \phi(x))\text{where } \phi \text{ is arithmetical}$$

Note that this replaces the single, second-order, axiom of induction with a countably infinite schema of axioms.

Appropriate axioms defining $+$, $\cdot$, and $<$ are included. A full list of the axioms of PA looks like this (although the exact list of axioms varies somewhat from source to source):

- $\forall x(x' \neq 0)$ (0 is the first number)
- $\forall x, y(x' = y' \rightarrow x = y)$ (the successor function is one-to-one)
- $\forall x(x + 0 = x)$ (0 is the additive identity)
- $\forall x, y(x + y' = (x + y)')$ (addition is the repeated application of the successor function)
- $\forall x(x \cdot 0 = 0)$
- $\forall x, y(x \cdot (y') = x \cdot y + x)$ (multiplication is repeated addition)
- $\forall x(\neg(x < 0))$ (0 is the smallest number)
- $\forall x, y(x < y' \leftrightarrow x < y \vee x = y)$
- $\phi(0) \wedge \forall x(\phi(x) \rightarrow \phi(x')) \rightarrow \forall x \phi(x))$where ϕ is arithmetical