measurable and real-valued measurable cardinals

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Owner	yesitis (13730)
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Author	yesitis (13730)
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Let κ be an uncountable cardinal. Then

1. κ is *measurable* if there exists a nonprincipal κ-complete ultrafilter U on κ;

2. κ is *real-valued measurable* if there exists a nontrivial κ-additive measure μ on κ.

If κ is measurable, then it is real-valued measurable. This is so because the ultrafilter U and its dual ideal I induce a two-valued measure μ on κ where every member of U is mapped to 1 and every member of I is mapped to 0. Since U is κ-complete, I is also κ-complete. It can then be proved that if I_μ–the ideal of those sets whose measures are 0–is κ-complete, then I_μ is κ-additive.

On the converse side, if κ is not real-valued measurable, then $\kappa \leq 2^{\aleph_0}$. It can be shown that if κ is real-valued measurable, then it is regular; a further result is that κ is weakly inaccessible. Inaccessible cardinals are in some sense "large."