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random graph (infinite)

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Suppose we have some method M of generating sequences of letters from $\{p, q\}$ so that at each generation the probability of obtaining p is x, a real number strictly between 0 and 1.

Let $\{a_i : i < \omega\}$ be a set of vertices. For each $i < \omega$, $i \geq 1$ we construct a graph G_i on the vertices $a_1, \ldots, a_i$ recursively.

- G_1 is the unique graph on one vertex.
- For $i > 1$ we must describe for any $j < k \leq i$ when a_j and a_k are joined.
  - If $k < i$ then join a_j and a_k in G_i iff a_j and a_k are joined in G_{i-1}
  - If $k = i$ then generate a letter $l(j, k)$ with M. Join a_j to a_k iff $l(j, k) = p$.

Now let Γ be the graph on $\{a_i : i < \omega\}$ so that for any $n, m < \omega$, a_n is joined to a_m in Γ iff it is in some G_i.

Then we call Γ a *random graph.* Consider the following property which we shall call *f-saturation*:

Given any finite disjoint U and V, subsets of $\{a_i : i < \omega\}$ there is some $a_n \in \{a_i : i < \omega\} \setminus (U \cup V)$ so that a_n is joined to every point of U and no points in V.

Proposition 1 *A random graph has f-saturation with probability* 1.

Proof: Let $b_1, b_2, \ldots, b_n, \ldots$ be an enumeration of $\{a_i : i < \omega\} \setminus (U \cup V)$. We say that b_i is *correctly joined* to (U, V) iff it is joined to all the members of U and non of the members of V. Then the probability that b_i is not correctly joined is $(1 - x^{|U|}(1 - x)^{|V|})$ which is some real number y strictly between 0 and 1. The probability that none of the first m are correctly joined is y^m and the probability that none of the b_is are correctly joined is $\lim_{n\to\infty} y^n = 0$. Thus one of the b_is is correctly joined.

Proposition 2 *Any two countable graphs with f-saturation are isomorphic.*

Proof: This is via a back and forth argument. The property of f-saturation is exactly what is needed.

Thus although the system of generation of a random graph looked as though it could deliver many potentially different graphs, this is not the case. Thus we talk about *the* random graph.

The random graph can also be constructed as a Fraisse limit of all finite graphs, and in many other ways. It is homogeneous and universal for the class of all countable graphs.

The theorem that almost every two infinite random graphs are isomorphic was first proved in [**?**].

References

[1] Paul Erdős and Alfréd Rényi. Asymmetric graphs. *Acta Math. Acad. Sci. Hung.*, 14:295–315, 1963.