proof of Tukey's lemma

Canonical name	ProofOfTukeysLemma
Date of creation	2013-03-22 13:54:58
Last modified on	2013-03-22 13:54:58
Owner	Koro (127)
Last modified by	Koro (127)
Numerical id	4
Author	Koro (127)
Entry type	Proof
Classification	msc 03E25

Let S be a set and F a set of subsets of S such that F is of finite character. By Zorn's lemma, it is enough to show that F is inductive. For that, it will be enough to show that if $(F_i)_{i \in I}$ is a family of elements of F which is totally ordered by inclusion, then the union U of the F_i is an element of F as well (since U is an upper bound on the family (F_i)). So, let K be a finite subset of U. Each element of U is in F_i for some $i \in I$. Since K is finite and the F_i are totally ordered by inclusion, there is some $j \in I$ such that all elements of K are in F_j. That is, $K \subset F_j$. Since F is of finite character, we get $K \in F$, QED.