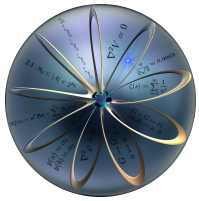

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a surjection between finite sets of the same cardinality is bijective

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Theorem. *Let A and B be finite sets of the same cardinality. If $f\colon A \to B$ is a surjection then f is a bijection.*

Proof. Let A and B be finite sets with $|A| = |B| = n$. Let $C = \{f^{-1}(\{b\}) \mid b \in B\}$. Then $\bigcup C \subseteq A$, so $|\bigcup C| \leq n$. Since f is a surjection, $|f^{-1}(\{b\})| \geq 1$ for each $b \in B$. The sets in C are pairwise disjoint because f is a function; therefore, $n \leq |\bigcup C|$ and

$$\left|\bigcup C\right| = \sum_{b \in B} |f^{-1}(\{b\})|.$$

In the last equation, n has been expressed as the sum of n positive integers; thus $|f^{-1}(\{b\})| = 1$ for each $b \in B$, so f is injective. □