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level sets of smooth functions on manifolds

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Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be smooth. Further suppose that the gradient of f differs from zero at every point of a level set. Then it follows from the implicit function theorem that that level set is a smooth hypersurface. Furthermore, at any point of the level set, the gradient of the function at that point is orthogonal to the level set.

One can generalize this observation to manifolds. Suppose that M is a smooth manifold and that $f : M \rightarrow \mathbb{R}$ is smooth. Further suppose that the gradient of f differs from zero at every point of a level set. Then it follows from the implicit function theorem that that level set is a smooth hypersurface. If one chooses a Riemannian metric on the manifold, the gradient of the function at that point will be orthogonal to the level set.