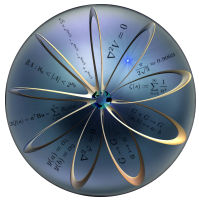

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axiom of foundation

Canonical name	AxiomOfFoundation
Date of creation	2013-03-22 13:04:31
Last modified on	2013-03-22 13:04:31
Owner	Henry (455)
Last modified by	Henry (455)
Numerical id	10
Author	Henry (455)
Entry type	Definition
Classification	msc 03C99
Synonym	foundation
Synonym	regularity
Synonym	axiom of regularity
Defines	artinian
Defines	artinian set
Defines	artinian sets

The *axiom of foundation* (also called the *axiom of regularity*) is an axiom of ZF set theory prohibiting circular sets and sets with infinite levels of containment. Intuitively, it that every set can be built up from the empty set. There are several equivalent formulations, for instance:

For any nonempty set X there is some $y \in X$ such that $y \cap X = \emptyset$.

For any set X, there is no function f from ω to the transitive closure of X such that for every n, $f(n+1) \in f(n)$.

For any formula ϕ, if there is any set x such that $\phi(x)$ then there is some X such that $\phi(X)$ but there is no $y \in X$ such that $\phi(y)$.

Sets which satisfy this axiom are called *artinian*. It is known that, if ZF without this axiom is consistent, then this axiom does not add any inconsistencies.

One important consequence of this property is that no set can contain itself. For instance, if there were a set X such that $X \in X$ then we could define a function $f(n) = X$ for all n, which would then have the property that $f(n+1) \in f(n)$ for all n.