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fuzzy subset

Canonical name	FuzzySubset
Date of creation	2013-03-22 16:34:54
Last modified on	2013-03-22 16:34:54
Owner	ggerla (15808)
Last modified by	ggerla (15808)
Numerical id	11
Author	ggerla (15808)
Entry type	Definition
Classification	msc 03E72
Classification	msc 03G20
Synonym	fuzzy set
Synonym	L-subset
Related topic	Logic
Related topic	FuzzyLogic2

Fuzzy set theory is based on the idea that vague notions as "big", "near", "hold" can be modelled by "fuzzy subsets". The idea of a fuzzy subset T of a set S is the following: each element $x \in S$, there is a number $p \in [0,1]$ such that p_x is the "probability" that x is in T.

To formally define a fuzzy set, let us first recall a well-known fact about subsets: a subset T of a set S corresponds uniquely to the characteristic function $c_T : S \to \{0,1\}$, such that $c_T(x) = 1$ iff $x \in T$. So if one were to replace $\{0,1\}$ with the the closed unit interval $[0,1]$, one obtains a fuzzy subset:

> A *fuzzy subset* of a set S is a map $s : S \to [0,1]$ from S into the interval $[0,1]$.

More precisely, the interval $[0,1]$ is considered as a complete lattice with an involution $1-x$. We call *fuzzy subset of* S any element of the direct power $[0,1]^S$. Whereas there are $2^{|S|}$ subsets of S, there are $\aleph_1^{|S|}$ fuzzy subsets of S.

The join and meet operations in the complete lattice $[0,1]^S$ are named *union* and *intersection*, respectively. The operation induced by the involution is called *complement*. This means that if s and t are two fuzzy subsets, then the fuzzy subsets $s \cup t, s \cap t, -s$, are defined by the equations

$$(s \cup t)(x) = \max\{s(x), t(x)\} \;\; ; \;\; (s \cap t)(x) = \min\{s(x), t(x)\} \;\; ; \;\; -s(x) = 1 - s(x).$$

It is also possible to consider any lattice L instead of $[0,1]$. In such a case we call *L-subset of* S any element of the direct power L^S and the union and the intersection are defined by setting

$$(s \cup t)(x) = s(x) \vee t(x) \;\; ; \;\; (s \cap t)(x) = s(x) \wedge t(x)$$

where $\vee$ and $\wedge$ denote the join and the meet operations in L, respectively. In the case an order reversing function $\neg : L \to L$ is defined in L, the *complement* $-s$ of s is defined by setting

$$-s(x) = \neg s(x).$$

Fuzzy set theory is devoted mainly to applications. The main success is perhaps *fuzzy control*.

References

[1] Cignoli R., D Ottaviano I. M. L. and Mundici D.,*Algebraic Foundations of Many-Valued Reasoning*. Kluwer, Dordrecht, (1999).

[2] Elkan C., *The Paradoxical Success of Fuzzy Logic*. (November 1993). Available from `http://www.cse.ucsd.edu/users/elkan/`http://www.cse.ucsd.edu/users/elkan/ Elkan's home page.

[3] Gerla G., *Fuzzy logic: Mathematical tools for approximate reasoning*, Kluwer Academic Publishers, Dordrecht, (2001).

[4] Goguen J., The logic of inexact concepts, *Synthese*, vol. 19 (1968/69)

[5] Gottwald S., *A treatise on many-valued logics*, Research Studies Press, Baldock (2000).

[6] Hjek P., *Metamathematics of fuzzy logic*. Kluwer (1998).

[7] Klir G. , UTE H. St.Clair and Bo Yuan, *Fuzzy Set Theory Foundations and Applications*, (1997).

[8] Zimmermann H., *Fuzzy Set Theory and its Applications* (2001), ISBN 0-7923-7435-5.

[9] Zadeh L.A., Fuzzy Sets, *Information and Control*, 8 (1965) 338-353.

[10] Zadeh L. A., The concept of a linguistic variable and its application to approximate reasoning I, II, III, *Information Sciences*, vol. 8, 9(1975), pp. 199-275, 301-357, 43-80.