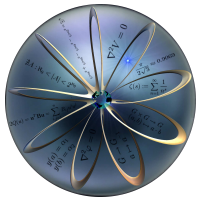

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quotient structure

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Let Σ be a fixed signature, $\mathfrak{A}$ a structure for Σ, and $\sim$ a congruence on $\mathfrak{A}$. The *quotient structure* of $\mathfrak{A}$ by $\sim$, denoted $\mathfrak{A}/\sim$, is defined as follows:

1. The universe of $\mathfrak{A}/\sim$ is the set $\{[\![a]\!] \mid a \in \mathfrak{A}\}$.
2. For each constant symbol c of Σ, $c^{\mathfrak{A}/\sim} = [\![c^{\mathfrak{A}}]\!]$.
3. For every natural number n and every n-ary function symbol F of Σ,
$$F^{\mathfrak{A}/\sim}([\![a_1]\!], \ldots [\![a_n]\!]) = [\![F^{\mathfrak{A}}(a_1, \ldots a_n)]\!].$$
4. For every natural number n and every n-ary relation symbol R of Σ, $R^{\mathfrak{A}/\sim}([\![a_1]\!], \ldots, [\![a_n]\!])$ if and only if for some $a_i' \sim a_i$ we have $R^{\mathfrak{A}}(a_1', \ldots, a_n')$.