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additively indecomposable

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Defines	epsilon number
Defines	epsilon zero

An ordinal α is called *additively indecomposable* if it is not 0 and for any $\beta, \gamma < \alpha$, we have $\beta + \gamma < \alpha$. The set of additively indecomposable ordinals is denoted $\mathbb{H}$.

Obviously $1 \in \mathbb{H}$, since $0 + 0 < 1$. No finite ordinal other than 1 is in $\mathbb{H}$. Also, $\omega \in \mathbb{H}$, since the sum of two finite ordinals is still finite. More generally, every infinite cardinal is in $\mathbb{H}$.

$\mathbb{H}$ is closed and unbounded, so the enumerating function of $\mathbb{H}$ is normal. In fact, $f_{\mathbb{H}}(\alpha) = \omega^\alpha$.

The derivative $f'_{\mathbb{H}}(\alpha)$ is written ϵ_α. Ordinals of this form (that is, fixed points of $f_{\mathbb{H}}$) are called *epsilon numbers*. The number $\epsilon_0 = \omega^{\omega^{\omega^{\cdot^{\cdot^{\cdot}}}}}$ is therefore the first fixed point of the series $\omega, \omega^\omega, \omega^{\omega^\omega}, \ldots$