



Math for the people, by the people.

functions from empty set

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Sometimes, it is useful to consider functions whose domain is the empty set. Given a set, there exists exactly one function from the empty set to that set. The rationale for this comes from carefully examining the definition of function in this degenerate case. Recall that, in set theory, a function from a set D to a set R is a set of ordered pairs whose first element lies in D and whose second element lies in R such that every element of D appears as the first element of exactly one ordered pair. If we take D to be the empty set, we see that this definition is satisfied if we take our function to be set of no ordered pairs — since there are no elements in the empty set, it is technically correct to say that every element of the empty set appears as a first element of an ordered pair which is an element of the empty set!

This observation turns out to be more than just an exercise in logic, being useful in several contexts. Given a set S and a positive integer n , we may define S^n as the set of all functions from $\{1, \dots, n\}$ to S . If we choose $n = 0$, then S^0 consists of all maps from the empty set to S , hence consists of exactly one element — see the entry on empty products for a discussion of the usefulness of this convention. In category theory, it turns out that functions from the empty set are important because they make the empty set be an initial object in this category.