properties of functions

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Let $f\colon X \to Y$ be a function. Let $(A_i)_{i\in I}$ be a family of subsets of X, and let $(B_j)_{j\in J}$ be a family of subsets of Y, where I and J are non-empty index sets.

Then, it is easy to prove, directly from definitions, that the following hold:

- $f(\bigcup_{i\in I} A_i) = \bigcup_{i\in I} f(A_i)$ (i.e., the image of a union is the union of the images)
- $f(\bigcap_{i\in I} A_i) \subseteq \bigcap_{i\in I} f(A_i)$ (i.e., the image of an intersection is contained in the intersection of the images)
- $A \subseteq f^{-1}(f(A))$ for any $A \subseteq X$ (where $f^{-1}(f(A))$ is the inverse image of $f(A)$)
- $f(f^{-1}(B)) \subseteq B$ for any $B \subseteq Y$
- $f^{-1}(Y \setminus B) = X \setminus f^{-1}(B)$ for any $B \subseteq Y$
- $f^{-1}(\bigcup_{j\in J} B_j) = \bigcup_{j\in J} f^{-1}(B_j)$ (the inverse image of a union is the union of the inverse images)
- $f^{-1}(\bigcap_{j\in J} B_j) = \bigcap_{j\in J} f^{-1}(B_j)$ (the inverse image of an intersection is the intersection of the inverse images)
- $f(f^{-1}(B)) = B$ for every $B \subseteq Y$ if and only if f is surjective.

For more properties related specifically to inverse images, see the `http://planetmath.org/Inve` image entry.

Further, the following conditions are equivalent (for more, see the entry on injective functions):

- f is injective
- $f(S \cap T) = f(S) \cap f(T)$ for all $S, T \subseteq X$
- $f^{-1}(f(S)) = S$ for all $S \subseteq X$
- $f(S) \cap f(T) = \varnothing$ for all $S, T \subseteq X$ such that $S \cap T = \varnothing$
- $f(S \setminus T) = f(S) \setminus f(T)$ for all $S, T \subseteq X$