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additive

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Synonym	additivity
Defines	countable additivity
Defines	countably additive
Defines	σ-additive
Defines	sigma-additive

Let ϕ be some positive-valued set function defined on an algebra of sets $\mathcal{A}$. We say that ϕ is *additive* if, whenever A and B are disjoint sets in $\mathcal{A}$, we have

$$\phi(A \cup B) = \phi(A) + \phi(B).$$

Given any sequence $\langle A_i \rangle$ of disjoint sets in A and whose union is also in A, if we have

$$\phi\left(\bigcup A_i\right) = \sum \phi(A_i)$$

we say that ϕ is *countably additive* or σ*-additive*.

Useful properties of an additive set function ϕ include the following:

1. $\phi(\emptyset) = 0$.
2. If $A \subseteq B$, then $\phi(A) \leq \phi(B)$.
3. If $A \subseteq B$, then $\phi(B \setminus A) = \phi(B) - \phi(A)$.
4. Given A and B, $\phi(A \cup B) + \phi(A \cap B) = \phi(A) + \phi(B)$.