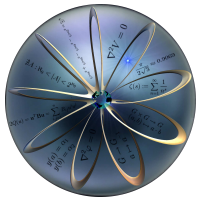

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circular reasoning

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Circular reasoning is an attempted proof of a statement that uses at least one of the following two things:

- the statement that is to be proven
- a fact that relies on the statement that is to be proven

Such proofs are not valid.

As an example, below is a faulty proof that the `http://planetmath.org/WellOrderingPrincip` ordering principle implies the axiom of choice. The step where circular reasoning is used is surrounded by brackets [ ].

Let C be a collection of nonempty sets. By the well-ordering principle, each $S \in C$ is well-ordered. [For each $S \in C$, let $<_S$ denote the well-ordering of S.] Let m_S denote the least member of each $S \in C$ with respect to $<_S$. Then a choice function $f\colon C \to \bigcup_{S \in C} S$ can be defined by $f(S) = m_S$.

The step surrounded by brackets is faulty because it actually uses the axiom of choice, which is what is to be proven. In the step, for each $S \in C$, an ordering is chosen. This cannot be done in general without appealing to the axiom of choice.