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superfluity of the third defining property for  
finite consequence operator

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In this entry, we demonstrate the claim made in section 1 of the <http://planetmath.org/node/> entry that the defining conditions for finitary consequence operator given there are redundant because one of them may be derived from the other two.

**Theorem.** *Let  $L$  be a set. Suppose that a mapping  $C: \mathcal{P}(L) \rightarrow \mathcal{P}(L)$  satisfies the following three properties:*

1. *For all  $X \subseteq L$ , it happens that  $X \subseteq C(X)$ .*
2.  *$C \circ C = C$*
3. *For all  $X \in L$ , it happens that  $C(X) = \bigcup_{Y \in \mathcal{F}(X)} C(Y)$ .*

*Then  $C$  also satisfies the following property: For all  $X, Y \subseteq L$ , if  $X \subseteq Y$ , then  $C(X) \subseteq C(Y)$ .*