superset

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Given two sets A and B, A is a *superset* of B if every element in B is also in A. We denote this relation as $A \supseteq B$. This is equivalent to saying that B is a subset of A, that is $A \supseteq B \Leftrightarrow B \subseteq A$.

Similar rules to those that hold for $\subseteq$ also hold for $\supseteq$. If $X \supseteq Y$ and $Y \supseteq X$, then $X = Y$. Every set is a superset of itself, and every set is a superset of the empty set.

We say A is a *proper superset* of B if $A \supseteq B$ and $A \neq B$. This relation is sometimes denoted by $A \supset B$, but $A \supset B$ is often used to mean the more general superset relation, so it should be made explicit when "proper superset" is intended, possibly by using $X \supsetneq Y$ or $X \underset{\neq}{\supset} Y$ (or $X \supsetneqq Y$ or $X \underset{\neq}{\supseteq} Y$).

One will occasionally see a collection C of subsets of some set X made into a partial order "by containment". Depending on context this can mean defining a partial order where $Y \leq Z$ means $Y \subseteq Z$, or it can mean defining the opposite partial order: $Y \leq Z$ means $Y \supseteq Z$. This is frequently used when applying Zorn's lemma.

One will also occasionally see a collection C of subsets of some set X made into a category, usually by defining a single abstract morphism $Y \to Z$ whenever $Y \subseteq Z$ (this being a special case of the general method of treating pre-orders as categories). This allows a concise definition of presheaves and sheaves, and it is generalized when defining a site.