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recursively axiomatizable theory

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Defines	finitely axiomatizable

Let T be a first order theory. A subset $\Delta \subseteq T$ is a set of **axioms** for T if and only if T is the set of all consequences of the formulas in Δ. In other words, $\varphi \in T$ if and only if φ is provable using only assumptions from Δ.

Definition. A theory T is said to be **finitely axiomatizable** if and only if there is a finite set of axioms for T; it is said to be **recursively axiomatizable** if and only if it has a recursive set of axioms.

For example, group theory is finitely axiomatizable (it has only three axioms), and Peano arithmetic is recursivaly axiomatizable : there is clearly an algorithm that can decide if a formula of the language of the natural numbers is an axiom.

Theorem. recursively axiomatizable theories are decidable.

As an example of the use of this theorem, consider the theory of algebraically closed fields of characteristic p for any number p prime or 0. It is complete, and the set of axioms is obviously recursive, and so it is decidable.