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## proof of Cantor's theorem

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The proof of this theorem is fairly using the following construction, which is central to Cantor's diagonal argument.

Consider a function  $F: X \rightarrow \mathcal{P}(X)$  from a set  $X$  to its power set. Then we define the set  $Z \subseteq X$  as follows:

$$Z = \{x \in X \mid x \notin F(x)\}$$

Suppose that  $F$  is a bijection. Then there must exist an  $x \in X$  such that  $F(x) = Z$ . Then we have the following contradiction:

$$x \in Z \Leftrightarrow x \notin F(x) \Leftrightarrow x \notin Z$$

Hence,  $F$  cannot be a bijection between  $X$  and  $\mathcal{P}(X)$ .