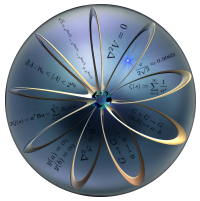

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axiomatizable class

Canonical name	AxiomatizableClass
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Entry type	Definition
Classification	msc 03C52
Synonym	axiomatisable class
Synonym	finitely axiomatizable
Synonym	finitely axiomatisable
Synonym	EC
Synonym	EC_Δ
Related topic	Supercategories3
Related topic	AxiomaticAndCategoricalFoundationsOfMathematicsII2
Defines	elementary class

Let L be a first order language and T a theory in L. Recall that a model M is an L-structure such that M satisfies every sentence in T. We say that the structure M is a model of T. Let us write $\mathrm{Mod}(T)$ the class of all L-structures that are models of T.

Definition. A class K of L-structures is said to be *axiomatizable* if there is a theory T such that $K = \mathrm{Mod}(T)$. Furthermore, K is a *finitely axiomatizable* or *elemenary class* if T is finite.

For example, the class of groups is elementary (and hence axiomatizable), because the set of group axioms is finite. However, the class of infinite groups is axiomatizable but not elementary. Similarly, the class of R-modules is elementary iff R is finite. The class of locally finite groups is an example of a non-axiomatizable class.

Remarks.

- K is an elementary class iff there is a sentence φ such that $K = \mathrm{Mod}(\{\varphi\})$, for sentences $\varphi_1, \ldots, \varphi_n$ can be combined to form $\varphi_1 \wedge \cdots \wedge \varphi_n$, which is also a sentence since it has no free variables.

- A class is axiomatizable iff it is an intersection of elementary classes. As such elementary class is sometimes abbreviated EC, and axiomatizable class EC_Δ, where Δ means is another symbol for intersection.

- A caution to the reader: some authors call an elementary class an axiomatizable class that is defined here.