model

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Defines	model

Let τ be a signature and φ be a sentence over τ. A http://planetmath.org/Structurestructure $\mathcal{M}$ for τ is called a *model* of φ if

$$\mathcal{M} \models \varphi,$$

where $\models$ is the satisfaction relation. When $\mathcal{M} \models \varphi$, we says that φ *satisfies* $\mathcal{M}$, or that $\mathcal{M}$ is *satisfied by* φ.

More generally, we say that a τ-structure $\mathcal{M}$ is a *model* of a theory T over τ, if $\mathcal{M} \models \varphi$ for every $\varphi \in T$. When $\mathcal{M}$ is a model of T, we say that T *satisfies* $\mathcal{M}$, or that $\mathcal{M}$ is satisfied by T, and is written

$$\mathcal{M} \models T.$$

Example. Let $\tau = \{\cdot\}$, where $\cdot$ is a binary operation symbol. Let x, y, z be variables and

$$T = \{\forall x \forall y \forall z \, ((x \cdot y) \cdot z = x \cdot (y \cdot z))\}.$$

Then it is easy to see that any model of T is a semigroup, and vice versa.

Next, let $\tau' = \tau \cup \{e\}$, where e is a constant symbol, and

$$T' = T \cup \{\forall x (x \cdot e = x), \forall x \exists y (x \cdot y = e)\}.$$

Then G is a model of T' iff G is a group. Clearly any group is a model of T'. To see the converse, let G be a model of T' and let $1 \in G$ be the interpretation of $e \in \tau'$ and $\cdot : G \times G \to G$ be the interpretation of $\cdot \in \tau'$. Let us write xy for the product $x \cdot y$. For any $x \in G$, let $y \in G$ such that $xy = 1$ and $z \in G$ such that $yz = 1$. Then $1z = (xy)z = x(yz) = x1 = x$, so that $1x = 1(1z) = (1 \cdot 1)z = 1z = x$. This shows that 1 is the identity of G with respect to $\cdot$. In particular, $x = 1z = z$, which implies $1 = yz = yx$, or that y is a inverse of x with respect to $\cdot$.

Remark. Let T be a theory. A class of τ-structures is said to be *axiomatized by* T if it is the class of all models of T. T is said to be the *set of axioms* for this class. This class is necessarily unique, and is denoted by $\operatorname{Mod}(T)$. When T consists of a single sentence φ, we write $\operatorname{Mod}(\varphi)$.