well-ordering principle implies axiom of choice

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Theorem. *The well-ordering principle implies the axiom of choice.*

Proof. Let C be a collection of nonempty sets. Then $\bigcup\limits_{S \in C} S$ is a set. By the well-ordering principle, $\bigcup\limits_{S \in C} S$ is well-ordered under some relation $<$. Since each S is a nonempty subset of $\bigcup\limits_{S \in C} S$, each S has a least member m_S with respect to the relation $<$.

Define $f \colon C \to \bigcup\limits_{S \in C} S$ by $f(S) = m_S$. Then f is a choice function. Hence, the axiom of choice holds. $\square$