generalized quantifier

Canonical name	GeneralizedQuantifier
Date of creation	2013-03-22 12:59:57
Last modified on	2013-03-22 12:59:57
Owner	Henry (455)
Last modified by	Henry (455)
Numerical id	5
Author	Henry (455)
Entry type	Definition
Classification	msc 03C80
Classification	msc 03B15
Classification	msc 03B10
Related topic	quantifier
Related topic	Quantifier
Defines	monadic
Defines	polyadic

Generalized quantifiers are an abstract way of defining quantifiers.

The underlying principle is that formulas quantified by a generalized quantifier are true if the set of elements satisfying those formulas belong in some relation associated with the quantifier.

Every generalized quantifier has an arity, which is the number of formulas it takes as arguments, and a type, which for an n-ary quantifier is a tuple of length n. The tuple represents the number of quantified variables for each argument.

The most common quantifiers are those of type $\langle 1 \rangle$, including $\forall$ and $\exists$. If Q is a quantifier of type $\langle 1 \rangle$, M is the universe of a model, and Q_M is the relation associated with Q in that model, then $Qx\phi(x) \leftrightarrow \{x \in M \mid \phi(x)\} \in Q_M$.

So $\forall_M = \{M\}$, since the quantified formula is only true when all elements satisfy it. On the other hand $\exists_M = P(M) - \{\emptyset\}$.

In general, the *monadic* quantifiers are those of type $\langle 1, \ldots, 1 \rangle$ and if Q is an n-ary monadic quantifier then $Q_M \subseteq P(M)^n$. Härtig's quantifier, for instance, is $\langle 1, 1 \rangle$, and $I_M = \{\langle X, Y \rangle \mid X, Y \subseteq M \wedge |X| = |Y|\}$.

A quantifier Q is *polyadic* if it is of type $\langle n_1, \ldots, n_n \rangle$ where each $n_i \in \mathbb{N}$. Then:

$$Q_M \subseteq \prod_i P(M^{n_i})$$

These can get quite elaborate; $Wxy\phi(x, y)$ is a $\langle 2 \rangle$ quantifier where $X \in W_M \leftrightarrow X$ `is a well-ordering`. That is, it is true if the set of pairs making ϕ true is a well-ordering.

1