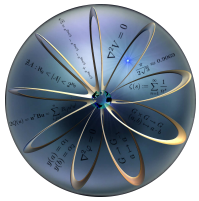

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proof of Vaught's test

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Let φ be an L-sentence, and let $\mathcal{A}$ be the unique model of S of cardinality κ. Suppose $\mathcal{A} \vDash \varphi$. Then if $\mathcal{B}$ is any model of S then by the http://planetmath.org/UpwardLowenheimSkolemTheoremupward and downward Lowenheim-Skolem theorems, there is a model $\mathcal{C}$ of S which is elementarily equivalent to $\mathcal{B}$ such that $|\mathcal{C}| = \kappa$. Then $\mathcal{C}$ is isomorphic to $\mathcal{A}$, and so $\mathcal{C} \vDash \varphi$, and $\mathcal{B} \vDash \varphi$. So $\mathcal{B} \vDash \varphi$ for all models $\mathcal{B}$ of S, so $S \vDash \varphi$.

Similarly, if $\mathcal{A} \vDash \neg\varphi$ then $S \vDash \neg\varphi$. So S is http://planetmath.org/Complete6complete.$\square$