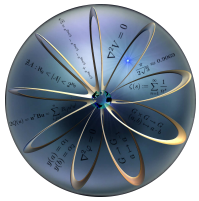

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consequence operator is determined by its fixed points

Canonical name	ConsequenceOperatorIsDeterminedByItsFixedPoints
Date of creation	2013-03-22 16:29:48
Last modified on	2013-03-22 16:29:48
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	6
Author	rspuzio (6075)
Entry type	Theorem
Classification	msc 03G10
Classification	msc 03B22
Classification	msc 03G25

Theorem 1 *Suppose that C_1 and C_2 are consequence operators on a set L and that, for every $X \subseteq L$, it happens that $C_1(X) = X$ if and only if $C_2(X) = X$. Then $C_1 = C_2$.*

Theorem 2 *Suppose that C is a consequence operators on a set L. Define $K = \{X \subseteq L \mid C(X) = X\}$. Then, for every $X \in L$, there exists a $Y \in K$ such that $X \subseteq Y$ and, for every $Z \in K$ such that $X \subseteq Z$, one has $Y \subseteq Z$.*

Theorem 3 *Given a set L, suppose that K is a subset of L such that, for every $X \in L$, there exists a $Y \in K$ such that $X \subseteq Y$ and, for every $Z \in K$ such that $X \subseteq Z$, one has $Y \subseteq Z$. Then there exists a consequence operator $C \colon \mathcal{P}(L) \to \mathcal{P}(L)$ such that $C(X) = X$ if and only if $X \in K$.*