type

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Defines	type
Defines	complete type
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Let L be a first order language. Let M be an `http://planetmath.org/node/3384`L-structure. Let $B \subseteq M$, and let $a \in M^n$. Then we define the *type of a over B* to be the set of L-formulas $\phi(x, \bar{b})$ with parameters $\bar{b}$ from B so that $M \models \phi(a, \bar{b})$. A collection of L-formulas is a *complete* n-type over B iff it is of the above form for some B, M and $a \in M^n$.

We call any consistent collection of formulas p in n variables with parameters from B a *partial* n-type over B. (See criterion for consistency of sets of formulas.)

Note that a complete n-type p over B is consistent so is in particular a partial type over B. Also p is maximal in the sense that for every formula $\psi(x, \bar{b})$ over B we have either $\psi(x, \bar{b}) \in p$ or $\neg\psi(x, \bar{b}) \in p$. In fact, for every collection of formulas p in n variables the following are equivalent:

- p is the type of some sequence of n elements a over B in some model $N \equiv M$
- p is a maximal consistent set of formulas.

For $n \in \omega$ we define $S_n(B)$ to be the set of complete n-types over B.

Some authors define a collection of formulas p to be a *n-type* iff p is a partial n-type. Others define p to be a *type* iff p is a complete n-type.

A type (resp. partial type/complete type) is any n-type (resp. partial type/complete type) for some $n \in \omega$.