example of strongly minimal

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Defines	language of rings

Let L_R be the language of rings. In other words L_R has two constant symbols $0, 1$, one unary symbol $-$, and two binary function symbols $+, \cdot$ satisfying the axioms (identities) of a ring. Let T be the L_R-theory that includes the field axioms and for each n the formula

$$\forall x_0, x_1, \ldots, x_n \exists y (\neg (\bigwedge_{1 \leq i \leq n} x_i = 0) \rightarrow \sum_{0 \leq i \leq n} x_i y^i = 0)$$

which expresses that every degree n polynomial which is non constant has a root. Then any model of T is an algebraically closed field.

One can show that this is a complete theory and has quantifier elimination (Tarski). Thus every B-definable subset of any $K \models T$ is definable by a quantifier free formula in $L_R(B)$ with one free variable y. A quantifier free formula is a Boolean combination of atomic formulas. Each of these is of the form $\sum_{i \leq n} b_i y^i = 0$ which defines a finite set. Thus every definable subset of K is a finite or cofinite set. Thus K and T are strongly minimal