Chu space

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Entry type	Definition
Classification	msc 03G99
Defines	perp
Defines	carrier
Defines	cocarrier
Defines	normal
Defines	normal Chu space
Defines	separable
Defines	extensional
Defines	biextensional
Defines	row
Defines	column

A *Chu space* over a set Σ is a triple $(\mathcal{A}, r, \mathcal{X})$ with $r : \mathcal{A} \times \mathcal{X} \to \Sigma$. $\mathcal{A}$ is called the *carrier* and $\mathcal{X}$ the *cocarrier*.

Although the definition is symmetrical, in practice asymmetric uses are common. In particular, often $\mathcal{X}$ is just taken to be a set of function from $\mathcal{A}$ to Σ, with $r(a, x) = x(a)$ (such a Chu space is called *normal* and is abbreviated $(\mathcal{A}, \mathcal{X})$).

We define the *perp* of a Chu space $\mathcal{C} = (\mathcal{A}, r, \mathcal{X})$ to be $\mathcal{C}^\perp = (\mathcal{X}, r^\smile, \mathcal{A})$ where $r^\smile(x, a) = r(a, x)$.

Define $\hat{r}$ and $\check{r}$ to be functions defining the *rows* and *columns* of $\mathcal{C}$ respectively, so that $\hat{r}(a) : \mathcal{X} \to \Sigma$ and $\check{r}(x) : \mathcal{A} \to \Sigma$ are given by $\hat{r}(a)(x) = \check{r}(x)(a) = r(a, x)$. Clearly the rows of $\mathcal{C}$ are the columns of $\mathcal{C}^\perp$.

Using these definitions, a Chu space can be represented using a matrix.

If $\hat{r}$ is injective then we call $\mathcal{C}$ *separable* and if $\check{r}$ is injective we call $\mathcal{C}$ *extensional*. A Chu space which is both separable and extensional is *biextensional*.