maximal element

Canonical name	MaximalElement
Date of creation	2013-03-22 12:30:44
Last modified on	2013-03-22 12:30:44
Owner	akrowne (2)
Last modified by	akrowne (2)
Numerical id	9
Author	akrowne (2)
Entry type	Definition
Classification	msc 03E04
Defines	greatest element
Defines	least element
Defines	minimal element

Let $\leq$ be an ordering on a set S, and let $A \subseteq S$. Then, with respect to the ordering $\leq$,

- $a \in A$ is the *least* element of A if $a \leq x$, for all $x \in A$.
- $a \in A$ is a *minimal* element of A if there exists no $x \in A$ such that $x \leq a$ and $x \neq a$.
- $a \in A$ is the *greatest* element of A if $x \leq a$ for all $x \in A$.
- $a \in A$ is a *maximal* element of A if there exists no $x \in A$ such that $a \leq x$ and $x \neq a$.

Examples.

- The natural numbers $\mathbb{N}$ ordered by divisibility ($|$) have a least element, 1. The natural numbers greater than 1 ($\mathbb{N} \setminus \{1\}$) have no least element, but infinitely many minimal elements (the primes.) In neither case is there a greatest or maximal element.
- The negative integers ordered by the standard definition of $\leq$ have a maximal element which is also the greatest element, -1. They have no minimal or least element.
- The natural numbers $\mathbb{N}$ ordered by the standard $\leq$ have a least element, 1, which is also a minimal element. They have no greatest or maximal element.
- The rationals greater than zero with the standard ordering $\leq$ have no least element or minimal element, and no maximal or greatest element.