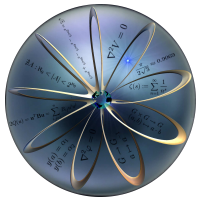

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structure homomorphism

Canonical name	StructureHomomorphism
Date of creation	2013-03-22 12:43:22
Last modified on	2013-03-22 12:43:22
Owner	almann (2526)
Last modified by	almann (2526)
Numerical id	14
Author	almann (2526)
Entry type	Definition
Classification	msc 03C07
Synonym	homomorphism
Synonym	morphism
Synonym	monomorphism
Synonym	epimorphism
Synonym	bimorphism
Synonym	embedding
Synonym	isomorphism
Synonym	endomorphism
Synonym	automorphism
Related topic	AxiomaticTheoryOfSupercategories
Defines	structure morphism
Defines	structure monomorphism
Defines	structure epimorphism
Defines	structure bimorphism
Defines	structure embedding
Defines	structure isomorphism
Defines	structure endomorphism
Defines	structure automorphism

Let Σ be a fixed signature, and $\mathfrak{A}$ and $\mathfrak{B}$ be two structures for Σ. The interesting functions from $\mathfrak{A}$ to $\mathfrak{B}$ are the ones that preserve the structure.

A function $f\colon \mathfrak{A} \to \mathfrak{B}$ is said to be a *homomorphism* (or simply *morphism*) if and only if:

1. For every constant symbol c of Σ, $f(c^{\mathfrak{A}}) = c^{\mathfrak{B}}$.
2. For every natural number n and every n-ary function symbol F of Σ,
$$f(F^{\mathfrak{A}}(a_1, ..., a_n)) = F^{\mathfrak{B}}(f(a_1), ..., f(a_n)).$$
3. For every natural number n and every n-ary relation symbol R of Σ,
$$R^{\mathfrak{A}}(a_1, \ldots, a_n) \Rightarrow R^{\mathfrak{B}}(f(a_1), \ldots, f(a_n)).$$

Homomorphisms with various additional properties have special names:

- An `http://planetmath.org/Injective`injective homomorphism is called a *monomorphism*.
- A surjective homomorphism is called an *epimorphism*.
- A bijective homomorphism is called a *bimorphism*.
- An injective homomorphism f is called an *embedding* if, for every natural number n and every n-ary relation symbol R of Σ,
$$R^{\mathfrak{B}}(f(a_1), \ldots, f(a_n)) \Rightarrow R^{\mathfrak{A}}(a_1, \ldots, a_n),$$
the converse of condition 3 above, holds.
- A surjective embedding is called an *isomorphism*.
- A homomorphism from a structure to itself (`http://planetmath.org/Eg`e.g., $f\colon \mathfrak{A} \to \mathfrak{A}$) is called an .
- An isomorphism from a structure to itself is called an *automorphism*.