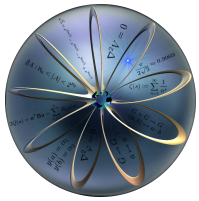

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negation

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In logics and mathematics, *negation* (from Latin *negare* 'to deny') is the unary operation "$\neg$" which swaps the truth value of any operand to the truth value. So, if the statement P is true then its negated statement $\neg P$ is false, and vice versa.

Note 1. The negated statement $\neg P$ (by Heyting) has been denoted also with $-P$ (Peano), $\sim P$ (Russell), $\overline{P}$ (Hilbert) and NP (by the Polish notation).

Note 2. $\neg P$ may be expressed by implication as

$$P \to \lightning$$

where $\lightning$ means any contradictory statement.

Note 3. The negation of logical or and logical and give the results

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q, \qquad \neg(P \wedge Q) \equiv \neg P \vee \neg Q.$$

Analogical results concern the quantifier statements:

$$\neg(\exists x)P(x) \equiv (\forall x)\neg P(x), \qquad \neg(\forall x)P(x) \equiv (\exists x)\neg P(x).$$

These all are known as de Morgan's laws.

Note 4. Many mathematical relation statements, expressed with such special relation symbols as $=$, $\subseteq$, $\in$, $\cong$, $\parallel$, $\mid$, are negated by using in the symbol an additional cross line: $\neq$, $\not\subseteq$, $\notin$, $\ncong$, $\nparallel$, $\nmid$.