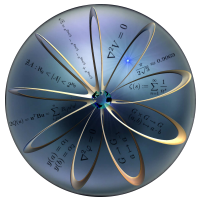

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kernel

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Let Σ be a fixed signature, and $\mathfrak{A}$ and $\mathfrak{B}$ be two structures for Σ. Given a homomorphism $f\colon \mathfrak{A} \to \mathfrak{B}$, the *kernel* of f is the relation $\ker(f)$ on A defined by

$$\langle a, a' \rangle \in \ker(f) \Leftrightarrow f(a) = f(a').$$

So defined, the kernel of f is a congruence on $\mathfrak{A}$. If Σ has a constant symbol 0, then the kernel of f is often defined to be the preimage of $0^{\mathfrak{B}}$ under f. Under this definition, if $\{0^{\mathfrak{B}}\}$ is a substructure of $\mathfrak{B}$, then the kernel of f is a substructure of $\mathfrak{A}$.