Zermelo-Fraenkel axioms

Canonical name	ZermeloFraenkelAxioms
Date of creation	2013-03-22 11:47:51
Last modified on	2013-03-22 11:47:51
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	20
Author	mathcam (2727)
Entry type	Axiom
Classification	msc 03E99
Synonym	Zermelo-Fraenkel set theory
Synonym	ZFC
Synonym	ZF
Related topic	AxiomOfChoice
Related topic	RussellsParadox
Related topic	VonNeumannOrdinal
Related topic	Axiom
Related topic	ContinuumHypothesis
Related topic	GeneralizedContinuumHypothesis
Related topic	SetTheory
Related topic	VonNeumannBernausGodelSetTheory
Related topic	Set

Ernst Zermelo and Abraham Fraenkel proposed the following axioms as a for what is now called Zermelo-Fraenkel set theory, or ZF. If this set of axioms are accepted along with the Axiom of Choice, it is often denoted ZFC.

- *Equality of sets*: If X and Y are sets, and $x \in X$ iff $x \in Y$, then $X = Y$.
- *Pair set*: If X and Y are sets, then there is a set Z containing only X and Y.
- `http://planetmath.org/Union`*Union over a set*: If X is a set, then there exists a set that contains every element of each $x \in X$.
- : If X is a set, then there exists a set $\mathcal{P}(x)$ with the property that $Y \in \mathcal{P}(x)$ iff any element $y \in Y$ is also in X.
- *Replacement axiom*: Let $F(x, y)$ be some formula. If, for all x, there is exactly one y such that $F(x, y)$ is true, then for any set A there exists a set B with the property that $b \in B$ iff there exists some $a \in A$ such that $F(a, b)$ is true.
- : Let $F(x)$ be some formula. If there is some x that makes $F(x)$ true, then there is a set Y such that $F(Y)$ is true, but for no $y \in Y$ is $F(y)$ true.
- *Existence of an infinite set*: There exists a non-empty set X with the property that, for any $x \in X$, there is some $y \in X$ such that $x \subseteq y$ but $x \neq y$.
- : If X is a set and P is a condition on sets, there exists a set Y whose members are precisely the members of X satisfying P. (This axiom is also occasionally referred to as the).