limit along a filter

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Definition 1. *Let $\mathcal{F}$ be a filter on $\mathbb{N}$ and (x_n) be a sequence in a metric space (X,d). We say that L is the* $\mathcal{F}$-limit *of (x_n) if*

$$A(\varepsilon)=\{n\in\mathbb{N}: d(x_n,L)<\varepsilon\}\in\mathcal{F}$$

for every $\varepsilon>0$.

The name *along $\mathcal{F}$* is used as well.

In the usual definition of limit one requires all sets $A(\varepsilon)$ to be cofinite - i.e. they have to be large. In the definition of $\mathcal{F}$-limit we simply choose which sets are considered to be large - namely the sets from the filter $\mathcal{F}$.

Remarks

This notion shouldn't be confused with the notion of `http://planetmath.org/filter`limit of a filter defined in general topology.

Let us note that the same notion is defined by some authors using the dual notion of ideal instead of filter and, of course, all results can be reformulated using ideals as well. For this approach see e.g. [**?**].

Examples

Limit along the Fréchet filter, which consist of complements of finite sets, is the usual limit of a sequence.

Limit of the sequence (x_n) along the principal filter $\mathcal{F}_k=\{A\subseteq\mathbb{N}; k\in A\}$ is x_k.

If we put $\mathcal{F}=\{\mathbb{N}\setminus A : d(A)=0\}$, where d denotes the asymptotic density, then it can be shown that $\mathcal{F}$ is a filter. In this case $\mathcal{F}$-convergence is known as statistical convergence.

References

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