axiom of union

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For any X there exists a set $Y = \bigcup X$.

The Axiom of Union is an axiom of Zermelo-Fraenkel set theory. In symbols, it reads

$$\forall X \exists Y \forall u (u \in Y \leftrightarrow \exists z (z \in X \wedge u \in z)).$$

Notice that this means that Y is the set of elements of all elements of X. More succinctly, the union of any set of sets is a set. By Extensionality, the set Y is unique. Y is called the *union* of X.

In particular, the Axiom of Union, along with the Axiom of Pairing allows us to define

$$X \cup Y = \bigcup \{X, Y\},$$

as well as the triple

$$\{a, b, c\} = \{a, b\} \cup \{c\}$$

and therefore the n-tuple

$$\{a_1, \ldots, a_n\} = \{a_1\} \cup \cdots \cup \{a_n\}$$