# power set

| | |
|---|---|
| Canonical name | PowerSet |
| Date of creation | 2013-03-22 11:43:46 |
| Last modified on | 2013-03-22 11:43:46 |
| Owner | matte (1858) |
| Last modified by | matte (1858) |
| Numerical id | 23 |
| Author | matte (1858) |
| Entry type | Definition |
| Classification | msc 03E99 |
| Classification | msc 03E10 |
| Classification | msc 37-01 |
| Synonym | powerset |
| Related topic | PowerObject |
| Related topic | ProofOfGeneralAssociativity |
| Defines | finite power set |
| Defines | finite powerset |

**Definition** If $X$ is a set, then the *power set of* $X$, denoted by $\mathcal{P}(X)$, is the set whose elements are the subsets of $X$.

### Properties

1. If $X$ is finite, then $|\mathcal{P}(X)| = 2^{|X|}$.
2. The above property also holds when $X$ is not finite. For a set $X$, let $|X|$ be the cardinality of $X$. Then $|\mathcal{P}(X)| = 2^{|X|} = |2^X|$, where $2^X$ is the set of all functions from $X$ to $\{0, 1\}$.
3. For an arbitrary set $X$, Cantor's theorem states: a) there is no bijection between $X$ and $\mathcal{P}(X)$, and b) the cardinality of $\mathcal{P}(X)$ is greater than the cardinality of $X$.

### Example

Suppose $S = \{a, b\}$. Then $\mathcal{P}(S) = \{\emptyset, \{a\}, \{b\}, S\}$. In particular, $|\mathcal{P}(S)| = 2^{|S|} = 4$.

### Related definition

If $X$ is a set, then the *finite power set of* $X$, denoted by $\mathcal{F}(X)$, is the set whose elements are the **finite** subsets of $X$.

### Remark

Due to the canonical correspondence between elements of $\mathcal{P}(X)$ and elements of $2^X$, the power set is sometimes also denoted by $2^X$.