proof of the well-founded induction principle

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This proof is very similar to the proof of the transfinite induction theorem. Suppose Φ is defined for a well-founded set (S, R), and suppose Φ is not true for every $a \in S$. Assume further that Φ satisfies requirements 1 and 2 of the statement. Since R is a well-founded relation, the set $\{a \in S : \neg\Phi(a)\}$ has an R minimal element r. This element is either an R minimal element of S itself, in which case condition 1 is violated, or it has R predessors. In this case, we have by minimality $\Phi(s)$ for every s such that sRr, and by condition 2, $\Phi(r)$ is true, contradiction.