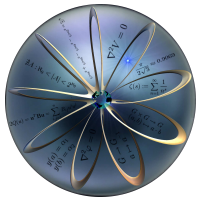

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aleph numbers

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The *aleph numbers* are infinite cardinal numbers defined by transfinite recursion, as described below. They are written $\aleph_\alpha$, where $\aleph$ is aleph, the first letter of the Hebrew alphabet, and α is an ordinal number. Sometimes we write ω_α instead of $\aleph_\alpha$, usually to emphasise that it is an ordinal.

To start the transfinite recursion, we define $\aleph_0$ to be the first infinite ordinal. This is the cardinality of countably infinite sets, such as $\mathbb{N}$ and $\mathbb{Q}$. For each ordinal α, the cardinal number $\aleph_{\alpha+1}$ is defined to be the least ordinal of cardinality greater than $\aleph_\alpha$. For each limit ordinal δ, we define $\aleph_\delta = \bigcup_{\alpha\in\delta} \aleph_\alpha$.

As a consequence of the `http://planetmath.org/ZermelosWellOrderingTheorem`Well-Ordering Principle, every infinite set is equinumerous with an aleph number. Every infinite cardinal is therefore an aleph. More precisely, for every infinite cardinal κ there is exactly one ordinal α such that $\kappa = \aleph_\alpha$.