subsets of countable sets are countable

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The definition of countable sets would not serve us well if it did not conform with our intuition about countable sets. So let us prove that countability is in a sense hereditary.

Theorem 1. *Every subset of a countable set is itself countable.*

Proof. Let $B \subseteq A$ and A countable with $f : A \to K$, $K \subseteq \mathbb{N}$ a bijective function as in the definition of countable sets.

Let us consider $f|_B$, the function f restricted to B, i.e. $f|_B : B \to f(B)$. Then $f|_B$ is trivially onto, but also one-to-one (f was one-to-one!). So we have a bijective function from B onto $f(B) \subseteq K \subseteq \mathbb{N}$, which the proof. $\square$