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example of definable type

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Consider $(\mathbf{Q}, <)$ as a structure in a language with one binary relation, which we interpret as the order. This is a universal, $\aleph_0$-categorical structure (see example of universal structure).

The theory of $(\mathbf{Q}, <)$ has quantifier elimination, and so is o-minimal. Thus a type over the set $\mathbf{Q}$ is determined by the quantifier free formulas over $\mathbf{Q}$, which in turn are determined by the atomic formulas over $\mathbf{Q}$. An atomic formula in one variable over B is of the form $x < b$ or $x > b$ or $x = b$ for some $b \in B$. Thus each 1-type over $\mathbf{Q}$ determines a Dedekind cut over $\mathbf{Q}$, and conversely a Dedekind cut determines a complete type over $\mathbf{Q}$. Let $D(p) := \{a \in \mathbf{Q} : x > a \in p\}$.

Thus there are two classes of type over $\mathbf{Q}$.

1. Ones where $D(p)$ is of the form $(-\infty, a)$ or $(-\infty, a]$ for some $a \in \mathbf{Q}$. It is clear that these are definable from the above discussion.

2. Ones where $D(p)$ has no supremum in $\mathbf{Q}$. These are clearly not definable by o-minimality of $\mathbf{Q}$.