Härtig's quantifier

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Härtig's quantifier is a quantifier which takes two variables and two formulas, written $Ixy\phi(x)\psi(y)$. It asserts that $|\{x \mid \phi(x)\}| = |\{y \mid \psi(y)\}|$. That is, the cardinality of the values of x which make ϕ is the same as the cardinality of the values which make $\psi(x)$ true. Viewed as a generalized quantifier, I is a $\langle 2 \rangle$ quantifier.

Closely related is the *Rescher quantifier*, which also takes two variables and two formulas, is written $Jxy\phi(x)\psi(y)$, and asserts that $|\{x \mid \phi(x)\}| \leq |\{y \mid \psi(y)|$. The Rescher quantifier is sometimes defined instead to be a similar but different quantifier, $Jx\phi(x) \leftrightarrow |\{x \mid \phi(x)\}| > |\{x \mid \neg\phi(x)\}|$. The first definition is a $\langle 2 \rangle$ quantifier while the second is a $\langle 1 \rangle$ quantifier.

Another similar quantifier is Chang's quantifier Q^C, a $\langle 1 \rangle$ quantifier defined by $Q^C_M = \{X \subseteq M \mid |X| = |M|\}$. That is, $Q^C x\phi(x)$ is true if the number of x satisfying ϕ has the same cardinality as the universe; for finite models this is the same as $\forall$, but for infinite ones it is not.