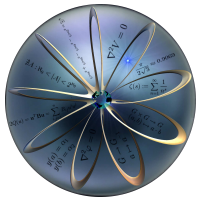

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Boolean valued model

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Defines	Boolean-valued model

A traditional model of a language makes every formula of that language either true or false. A *Boolean valued model* is a generalization in which formulas take on any value in a Boolean algebra.

Specifically, a Boolean valued model of a signature Σ over the language $\mathcal{L}$ is a set $\mathcal{A}$ together with a Boolean algebra $\mathcal{B}$. Then the objects of the model are the functions $\mathcal{A}^{\mathcal{B}} = \mathcal{B} \to \mathcal{A}$.

For any formula ϕ, we can assign a value $\|\phi\|$ from the Boolean algebra. For example, if $\mathcal{L}$ is the language of first order logic, a typical recursive definition of $\|\phi\|$ might look something like this:

- $\|f = g\| = \bigvee_{f(b)=g(b)} b$
- $\|\neg\phi\| = \|\phi\|'$
- $\|\phi \vee \psi\| = \|\phi\| \vee \|\psi\|$
- $\|\exists x \phi(x)\| = \bigvee_{f \in \mathcal{A}^{\mathcal{B}}} \|\phi(f)\|$