axiom of power set

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The *axiom of power set* is an axiom of Zermelo-Fraenkel set theory which postulates that for any set X there exists a set $\mathcal{P}(X)$, called the *power set* of X, consisting of all subsets of X. In symbols, it reads:

$$\forall X \exists \mathcal{P}(X) \forall u (u \in \mathcal{P}(X) \leftrightarrow u \subseteq X).$$

In the above, $u \subseteq X$ is defined as $\forall z (z \in u \rightarrow z \in X)$. By the extensionality axiom, the set $\mathcal{P}(X)$ is unique.

The Power Set Axiom allows us to define the Cartesian product of two sets X and Y:

$$X \times Y = \{(x, y) : x \in X \wedge y \in Y\}.$$

The Cartesian product is a set since

$$X \times Y \subseteq \mathcal{P}(\mathcal{P}(X \cup Y)).$$

We may define the Cartesian product of any finite collection of sets recursively:

$$X_1 \times \cdots \times X_n = (X_1 \times \cdots \times X_{n-1}) \times X_n.$$