# model theory

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*Model theory* is a branch of mathematical logic which deals with the general problem of classifying mathematical structures. While every branch of mathematics has its own classifying problem (such as the classification of http://planetmath.org/ExamplesOfFiniteSimpleGroups finite simple groups or the classification of smooth manifolds), the approach of model theory is unique in attempting to classify structures in general by the sentences which are true of those structures. Like most classification problems, this problem is almost certainly unsolvable when stated in full generality. However, many special cases are worthy of study.

The basic theoretical notions in model theory are the structure and the theory. A *http://planetmath.org/StructuresAndSatisfaction structure*, sometimes called an $L$-structure, is a set with associated http://planetmath.org/Signature symbols representing constants, relations, and functions. A *theory* is a collection of sentences in a formal language. We say that a structure $\mathfrak{K}$ *satisfies* a sentence $\varphi$ provided that the sentence is true no matter how its variables are interpreted in $\mathfrak{K}$. If $\mathfrak{K}$ satisfies all sentences of a theory $T$, then we say that $\mathfrak{K}$ is a *model* or $T$, or $\mathfrak{K}$ *models* $T$, and write $\mathfrak{K} \models T$. This allows us to talk about the class of models of a particular theory or the theory of a particular structure.

Basic constructions in model theory include elementary extensions, ultrapowers and ultraproducts, and the elimination of imaginaries. Some of the basic results of model theory include the http://planetmath.org/DownwardLowenheimSkolemTheorem Skolem theorem (in downward and http://planetmath.org/UpwardLowenheimSkolemTheorem upward forms), the http://planetmath.org/CompactnessTheoremForFirstOrderLogic compactness theorem for first-order logic, and Łoś's theorem. Important concepts include Morley rank, o-minimality, and quantifier elimination.

Early pioneers of model theory include Löwenheim, Skolem, Gödel, Tarski, and Maltsev, and the work of Henkin, Robinson, and (again) Tarski helped distinguish model theory from the rest of mathematical logic. Ax and Kochen used model theory to prove a result on Diophantine problems over function fields, and Hrushovski used model theory to prove the Mordell–Lang conjecture for function fields.

# References

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