partial order with chain condition does not collapse cardinals

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If P is a partial order which satisfies the κ chain condition and G is a generic subset of P then for any $\kappa < \lambda \in \mathfrak{M}$, λ is also a cardinal in $\mathfrak{M}[G]$, and if $\mathrm{cf}(\alpha) = \lambda$ in $\mathfrak{M}$ then also $\mathrm{cf}(\alpha) = \lambda$ in $\mathfrak{M}[G]$.

This theorem is the simplest way to control a notion of forcing, since it means that a notion of forcing does not have an effect above a certain point. Given that any P satisfies the $|P|^+$ chain condition, this means that most forcings leaves all of $\mathfrak{M}$ above a certain point alone. (Although it is possible to get around this limit by forcing with a proper class.)