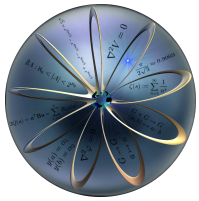

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Church integer

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A *Church integer* is a representation of integers as functions, invented by Alonzo Church. An integer N is represented as a higher-order function, which applies a given function to a given expression N times.

For example, in the programming language Haskell, a function that returns a particular Church integer might be

$$\begin{aligned}
&\text{church}\,0 = \;\; fx \to x \\
&\text{church}\,n = c \\
&\qquad\qquad \text{where} : cfx = c'f(fx) \\
&\qquad\qquad\qquad\qquad \text{where} : c' = \text{church}(n-1)
\end{aligned}$$

The transformation from a Church integer to an integer might be

```
unchurch n = n (+1) 0
```

Thus we can generate the integers–the `(+1)` function would be applied to an initial value of 0 n times, yielding the ordinary integer n.