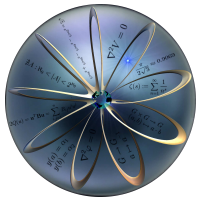

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## beth numbers

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The *beth numbers* are infinite cardinal numbers defined in a similar manner to the aleph numbers, as described below. They are written $\beth_\alpha$, where $\beth$ is beth, the second letter of the Hebrew alphabet, and $\alpha$ is an ordinal number.

We define $\beth_0$ to be the first infinite cardinal (that is, $\aleph_0$). For each ordinal $\alpha$, we define $\beth_{\alpha+1} = 2^{\beth_\alpha}$. For each limit ordinal $\delta$, we define $\beth_\delta = \bigcup_{\alpha\in\delta} \beth_\alpha$.

Note that $\beth_1$ is the cardinality of the continuum.

For any ordinal $\alpha$ the inequality $\aleph_\alpha \leqslant \beth_\alpha$ holds. The Generalized Continuum Hypothesis is equivalent to the assertion that $\aleph_\alpha = \beth_\alpha$ for every ordinal $\alpha$.

For every limit ordinal $\delta$, the cardinal $\beth_\delta$ is a strong limit cardinal. Every uncountable strong limit cardinal arises in this way.