# example of polyadic algebra with equality

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Recall that given a triple $(A, I, X)$ where $A$ is a Boolean algebra, $I$ and $X \neq \varnothing$ are sets. we can construct a polyadic algebra $(B, I, \exists, S)$ called the functional polyadic algebra for $(A, I, X)$. In this entry, we will construct an example of a polyadic algebra with equality called the *functional polyadic algebra with equality* from $(B, I, \exists, S)$.

We start with a simpler structure. Let $B$ be an arbitrary Boolean algebra, $I$ and $X \neq \varnothing$ are sets. Let $Y = X^I$, the set of all $I$-indexed $X$-valued sequences, and $Z = B^Y$, the set of all functions from $Y$ to $B$. Call the function $e : I \times I \to Z$ the *functional equality associated with* $(B, I, X)$, if for each $i, j \in I$, $e(i, j)$ is the function defined by

$$e(i,j)(x) := \begin{cases} 1 & \text{if } x_i = x_j, \\ 0 & \text{otherwise.} \end{cases}$$

The quadruple $(B, I, X, e)$ is called a *functional equality algebra*.

Now, $B$ will have the additional structure of being a polyadic algebra. Start with a Boolean algebra $A$, and let $I$ and $X$ be defined as in the last paragraph. Then, as stated above in the first paragraph, and illustrated in `http://planetmath.org/ExampleOfPolyadicAlgebra`here, $(B, I, \exists, S)$ is a polyadic algebra (called the functional polyadic algebra for $(A, I, X)$). Using the $B$ just constructed, the quadruple $(B, I, X, e)$ is a functional equality algebra, and is called the *functional polyadic algebra with equality* for $(A, I, X)$.

It is not hard to show that $e$ is an equality predicate on $C = (B, I, \exists, S)$, and as a result $(C, e)$ is a polyadic algebra with equality.

# References


[1] P. Halmos, *Algebraic Logic*, Chelsea Publishing Co. New York (1962).