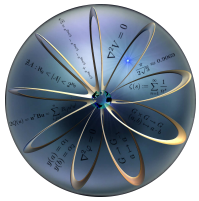

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characteristic function

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Synonym	indicator function
Related topic	SimpleFunction

Definition Suppose A is a subset of a set X. Then the function

$$\chi_A(x) = \begin{cases} 1, & \text{when } x \in A, \\ 0, & \text{when } x \in X \setminus A \end{cases}$$

is the *characteristic function* for A.

0.0.1 Properties

Suppose A, B are subsets of a set X.

1. For set intersections and set unions, we have

$$\begin{aligned} \chi_{A\cap B} &= \chi_A \chi_B, \\ \chi_{A\cup B} &= \chi_A + \chi_B - \chi_{A\cap B}, \\ \chi_{A\cap B} &= \min(\chi_A, \chi_B), \\ \chi_{A\cup B} &= \max(\chi_A, \chi_B). \end{aligned}$$

2. For the symmetric difference,

$$\chi_{A\triangle B} = \chi_A + \chi_B - 2\chi_{A\cap B}.$$

3. For the set complement,

$$\chi_{A^\complement} = 1 - \chi_A.$$

0.0.2 Remarks

A synonym for characteristic function is *indicator function* [**?**].

References

[1] G.B. Folland, *Real Analysis: Modern Techniques and Their Applications*, 2nd ed, John Wiley & Sons, Inc., 1999.