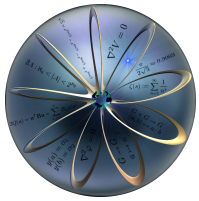

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sequent

Canonical name	Sequent
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Defines	contraction
Defines	premise
Defines	conclusion

A *sequent* represents a formal step in a proof. Typically it consists of two lists of formulas, one representing the premises and one the conclusions. A typical sequent might be:

$$\phi, \psi \Rightarrow \alpha, \beta$$

where ϕ and ψ are the premises and α and β are the conclusions.

This claims that, from premises ϕ and ψ either α or β must be true. Note that $\Rightarrow$ is not a symbol in the language, rather it is a symbol in the metalanguage used to discuss proofs. Also, notice the asymmetry: everything on the left must be true to conclude only one thing on the right. This does create a different kind of symmetry, since adding formulas to either side results in a weaker sequent, while removing them from either side gives a stronger one.

Some systems allow only one formula on the right.

Most proof systems provide ways to deduce one sequent from another. These rules are written with a list of sequents above and below a line. This rule indicates that if everything above the line is true, so is everything under the line. A typical rule is:

$$\frac{\Gamma \Rightarrow \Sigma}{\Gamma, \alpha \Rightarrow \Sigma \quad \alpha, \Gamma \Rightarrow \Sigma}$$

This indicates that if we can deduce Σ from Γ, we can also deduce it from Γ together with α.

Note that the capital Greek letters are usually used to denote a (possibly empty) list of formulas. $[\Gamma, \Sigma]$ is used to denote the *contraction* of Γ and Σ, that is, the list of those formulas appearing in either Γ or Σ but with no repeats.