inverse function theorem

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Let $\mathbf{f}$ be a continuously differentiable, vector-valued function mapping the open set $E \subset \mathbb{R}^n$ to $\mathbb{R}^n$ and let $S = \mathbf{f}(E)$. If, for some point $\mathbf{a} \in E$, the Jacobian, $|J_{\mathbf{f}}(\mathbf{a})|$, is non-zero, then there is a uniquely defined function $\mathbf{g}$ and two open sets $X \subset E$ and $Y \subset S$ such that

1. $\mathbf{a} \in X$, $\mathbf{f}(\mathbf{a}) \in Y$;
2. $Y = \mathbf{f}(X)$;
3. $\mathbf{f} : X \to Y$ is one-one;
4. $\mathbf{g}$ is continuously differentiable on Y and $\mathbf{g}(\mathbf{f}(\mathbf{x})) = \mathbf{x}$ for all $\mathbf{x} \in X$.

0.0.1 Simplest case

When $n = 1$, this theorem becomes: Let f be a continuously differentiable, real-valued function defined on the open interval I. If for some point $a \in I$, $f'(a) \neq 0$, then there is a neighbourhood $[\alpha, \beta]$ of a in which f is strictly monotonic. Then $y \to f^{-1}(y)$ is a continuously differentiable, strictly monotonic function from $[f(\alpha), f(\beta)]$ to $[\alpha, \beta]$. If f is increasing (or decreasing) on $[\alpha, \beta]$, then so is f^{-1} on $[f(\alpha), f(\beta)]$.

0.0.2 Note

The inverse function theorem is a special case of the implicit function theorem where the dimension of each variable is the same.