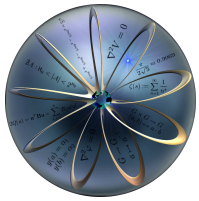

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proof of homomorphic image of a Σ-structure is a Σ-structure

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We need to show that $\mathrm{im}(f)$ is closed under functions. For every constant symbol c of Σ, $c^{\mathfrak{B}} = f(c^{\mathfrak{A}})$. Hence $c^{\mathfrak{B}} \in \mathrm{im}(f)$. Also, if $b_1, \ldots, b_n \in \mathrm{im}(f)$ and F is an n-ary function symbol of Σ, then for some $a_1, \ldots, a_n \in \mathfrak{A}$ we have

$$F^{\mathfrak{B}}(b_1, \ldots, b_n) = F^{\mathfrak{B}}(f(a_1), \ldots, f(a_n)) = f(F^{\mathfrak{A}}(a_1, \ldots, a_n)).$$

Hence $F^{\mathfrak{B}}(b_1, \ldots, b_n) \in \mathrm{im}(f)$.