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primitive recursive number

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A special of computable numbers is so-called the *primitive recursive numbers*. Informally, these are numbers that can be measured by primitive recursive functions to an arbitrary degree of precision.

Definition. A non-negative real number r is said to be *primitive recursive* if there is a primitive recursive function $f:\mathbb{N}\to\mathbb{N}$ such that

$$f(n)=\begin{cases}[r] \text{ (the integer part of } r\text{)}, & \text{if } n=0,\\ n^{\text{th}} \text{ digit of } r \text{ when } r \text{ is expressed in its decimal representation}, & \text{if } n\neq 0.\end{cases}$$

A real number r is *primitive recursive* if $|r|$ is, and a complex number $x+yi$ is *primitive recursive* if both x and y are.

Clearly, any integer is primitive recursive. It is easy to see that all rational numbers are primitive recursive too, as the decimal representation of a rational number is periodic, so if

$$r=[r].\overline{a_1\cdots a_k},$$

we can define f so that

$$f(n)=\begin{cases}[r], & \text{if } n=0,\\ a_i & \text{if } n\neq 0 \text{ and } n\equiv i \pmod{k}.\end{cases}$$

Here, we assume that r is non-negative.

In addition, we can show that $\sqrt{n}$ is primitive recursive for any non-negative integer n.

Proof. Suppose $r=\sqrt{n}$. Write r in its decimal representation

$$r=n_0.n_1n_2\cdots n_k\cdots$$

Then $n_0=[\sqrt{n}]$. Multiply r by 10 to get its decimal representation

$$10r=n_0n_1.n_2\cdots n_k\cdots$$

Then $10n_0+n_1=[10r]=[\sqrt{100n}]$, so that $n_1=[\sqrt{100n}]-10n_0$ By induction, we see that

$$n_{k+1}=[\sqrt{100^{k+1}n}]-10(10^kn_0+10^{k-1}n_1+\cdots+n_k).$$

Define $f:\mathbb{N}^2\to\mathbb{N}$ by $f(n,m)=n_m$. Then $f(n,0)$ is primitive recursive. Next,

$$f(n,m)=[\sqrt{100^mn}]-10\sum_{i=0}^{m-1}10^{m-1-i}f(n,i)=h(n,m,\overline{f}(n,m)),$$

where

$$h(x,y,z) = [\sqrt{100^x y}] \dot{-} 10 \sum_{i=0}^{y \dot{-} 1} 10^{y \dot{-} s(i)} (z)_i$$

which is primitive recursive (all of the operations, including the bounded sum are primitive recursive). Since f is defined by course-of-values recursion via h, f is primitive recursive also. □

Remark. It can be shown that π is primitive recursive. A proof of this can be found in the link below.

References

[1] S. G. Simpson, `http://www.math.psu.edu/simpson/courses/math558/fom.pdf`Foundations of Mathematics. (2009).