irreflexive

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A binary relation $\mathcal{R}$ on a set A is said to be *irreflexive* (or *antireflexive*) if $\forall a \in A$, $\neg a\mathcal{R}a$. In other words, "no element is $\mathcal{R}$-related to itself."

For example, the relation $<$ ("less than") is an irreflexive relation on the set of natural numbers.

Note that "irreflexive" is not simply the negation of "`http://planetmath.org/Reflexive`reflexi ." Although it is impossible for a relation (on a nonempty set) to be both `http://planetmath.org/Reflexive`reflexive and irreflexive, there exist relations that are neither. For example, the relation $\{(a,a)\}$ on the two element set $\{a,b\}$ is neither reflexive nor irreflexive.

Here is an example of a non-reflexive, non-irreflexive relation "in nature." A subgroup in a group is said to be *self-normalizing* if it is equal to its own normalizer. For a group G, define a relation $\mathcal{R}$ on the set of all subgroups of G by declaring $H\mathcal{R}K$ if and only if H is the normalizer of K. Notice that every nontrivial group has a subgroup that is not self-normalizing; namely, the trivial subgroup $\{e\}$ consisting of only the identity. Thus, in any nontrivial group G, there is a subgroup H of G such that $\neg H\mathcal{R}H$. So the relation $\mathcal{R}$ is non-reflexive. Moreover, since the normalizer of a group G in G is G itself, we have $G\mathcal{R}G$. So $\mathcal{R}$ is non-irreflexive.