proof of Fodor's lemma

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If we let $f^{-1} : \kappa \to P(S)$ be the inverse of f restricted to S then Fodor's lemma is equivalent to the claim that for any function such that $\alpha \in f(\kappa) \to \alpha > \kappa$ there is some $\alpha \in S$ such that $f^{-1}(\alpha)$ is stationary.

Then if Fodor's lemma is false, for every $\alpha \in S$ there is some club set C_α such that $C_\alpha \cap f^{-1}(\alpha) = \emptyset$. Let $C = \Delta_{\alpha<\kappa} C_\alpha$. The club sets are closed under diagonal intersection, so C is also club and therefore there is some $\alpha \in S \cap C$. Then $\alpha \in C_\beta$ for each $\beta < \alpha$, and so there can be no $\beta < \alpha$ such that $\alpha \in f^{-1}(\beta)$, so $f(\alpha) \geq \alpha$, a contradiction.