computable real function

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Entry type	Definition
Classification	msc 03F60
Defines	sequentially computable
Defines	effectively uniformly continuous
Defines	effective uniform continuity

A function $f\colon \mathbb{R} \to \mathbb{R}$ is *sequentially computable* if, for every computable sequence $\{x_i\}_{i=1}^{\infty}$ of real numbers, the sequence $\{f(x_i)\}_{i=1}^{\infty}$ is also computable.

A function $f\colon \mathbb{R} \to \mathbb{R}$ is *effectively uniformly continuous* if there exists a recursive function $d\colon \mathbb{N} \to \mathbb{N}$ such that, if

$$|x - y| < \frac{1}{d(n)}$$

then

$$|f(x) - f(y)| < \frac{1}{n}$$

A real function is *computable* if it is both sequentially computable and effectively uniformly continuous.

It is not hard to generalize these definitions to functions of more than one variable or functions only defined on a subset of $\mathbb{R}^n$. The generalizations of the latter two definitions are so obvious that they need not be restated. A suitable generalization of the first definition is:

Let D be a subset of $\mathbb{R}^n$. A function $f\colon D \to \mathbb{R}$ is *sequentially computable* if, for every n-tuplet $(\{x_{i\,1}\}_{i=1}^{\infty}, \ldots \{x_{i\,n}\}_{i=1}^{\infty})$ of computable sequences of real numbers such that

$$(\forall i) \quad (x_{i\,1}, \ldots x_{i\,n}) \in D \qquad ,$$

the sequence $\{f(x_i)\}_{i=1}^{\infty}$ is also computable.