special elements in a relation algebra

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Entry type	Definition
Classification	msc 03G15
Defines	function element
Defines	injective element
Defines	surjective element
Defines	reflexive element
Defines	symmetric element
Defines	transitive element
Defines	equivalence element
Defines	domain element
Defines	range element
Defines	ideal element
Defines	rectangle
Defines	square
Defines	antisymmetric element
Defines	subidentity

Let A be a relation algebra with operators $(\vee, \wedge, \ ;, ', ^{-}, 0, 1, i)$ of type $(2,2,2,1,1,0,0,0)$. Then $a \in A$ is called a

- *function element* if $e^{-} \ ; e \leq i$,
- *injective element* if it is a function element such that $e \ ; e^{-} \leq i$,
- *surjective element* if $e^{-} \ ; e = i$,
- *reflexive element* if $i \leq a$,
- *symmetric element* if $a^{-} \leq a$,
- *transitive element* if $a \ ; a \leq a$,
- *subidentity* if $a \leq i$,
- *antisymmetric element* if $a \wedge a^{-}$ is a subidentity,
- *equivalence element* if it is symmetric and transitive (not necessarily reflexive!),
- *domain element* if $a \ ; 1 = a$,
- *range element* if $1 \ ; a = a$,
- *ideal element* if $1 \ ; a \ ; 1 = a$,
- *rectangle* if $a = b \ ; 1 \ ; c$ for some $b, c \in A$, and
- *square* if it is a rectangle where $b = c$ (using the notations above).

These special elements are so named because they are the names of the corresponding binary relations on a set. The following table shows the correspondence.

element in relation algebra A	binary relation on set S
function element	function (on S)
injective element	injection
surjective element	surjection
reflexive element	reflexive relation
symmetric element	symmetric relation
transitive element	transitive relation
subidentity	$I_T := \{(x, x) \mid x \in T\}$ where $T \subseteq S$
antisymmetric element	antisymmetric relation
equivalence element	symmetric reflexive relation (not an equivalence relation!)
domain element	$\mathrm{dom}(R) \times S$ where $R \subseteq S^2$
range element	$S \times \mathrm{ran}(R)$ where $R \subseteq S^2$
ideal element	
rectangle	$U \times V \subseteq S^2$
square	U^2, where $U \subseteq S$

References

[1] S. R. Givant, *The Structure of Relation Algebras Generated by Relativizations*, American Mathematical Society (1994).