inference rule

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In logic, an *inference rule* is a rule whereby one may correctly draw a conclusion from one or more premises. For example, the law of the contrapositive allows one to conclude a statement of the form

$$\neg Q \Rightarrow \neg P$$

from a premise of the form

$$P \Rightarrow Q.$$

Here, 'P' and 'Q' are propositional variables, which can stand for arbitrary propositions. A popular way to indicate applications of rules of inference is to list the premises above a line and write the conclusions below the line. For instance, we might indicate the law of the contrapositive thus:

$$\frac{P \Rightarrow Q}{\neg Q \Rightarrow \neg P}$$

A typical application of the law of contrapositive would be to conclude "If my clothes are dry, then it is not raining", from "If it rains, then my clothes will be wet." which could be expressed as follows using the notation described above:

$$\frac{\text{If it rains, then my clothes will be wet.}}{\text{If my clothes are dry, then it is not raining.}}$$

(In this instance, P is "It is raining" and Q is "My clothes are dry".

An important feature of rules of inference is that they are purely formal, which means that all that matters is the form of the expression; meaning is not a consideration in applying a rule of inference. Thus, the following are equally valid applications of the rule of the contrapositive:

$$\frac{\text{If the jabberwocky is mimsy, then the toves blithe.}}{\text{If the toves are not blithing, then the jabberwocky is not mimsy.}}$$

$$\frac{\text{If my cat has a tail, then my cat is a dog.}}{\text{If my cat is not a dog, then my cat does not have a tail.}}$$

In the first example, the statements are nonsense and in the second example, the statements are false, but this doesn't matter — both examples constitute valid apllications of the rule of the contrapositive. Of course, in order to draw valid conclusions, we need to start with valid premises, but the point of these examples is clarify the distinction between valid statements and valid applications of rules of inference.