iterated forcing

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Defines	FS
Defines	CS
Defines	finite support
Defines	finite support iterated forcing
Defines	countable support
Defines	countable support iterated forcing
Defines	support iterated forcing

We can define an *iterated forcing* of length α by induction as follows:

Let $P_0 = \emptyset$.

Let $\hat{Q}_0$ be a forcing notion.

For $\beta \leq \alpha$, P_β is the set of all functions f such that $\operatorname{dom}(f) \subseteq \beta$ and for any $i \in \operatorname{dom}(f)$, $f(i)$ is a P_i-name for a member of $\hat{Q}_i$. Order P_β by the rule $f \leq g$ iff $\operatorname{dom}(g) \subseteq \operatorname{dom}(f)$ and for any $i \in \operatorname{dom}(f)$, $g \restriction i \Vdash f(i) \leq_{\hat{Q}_i} g(i)$. (Translated, this means that any generic subset including g restricted to i forces that $f(i)$, an element of $\hat{Q}_i$, be less than $g(i)$.)

For $\beta < \alpha$, $\hat{Q}_\beta$ is a forcing notion in P_β (so $\Vdash_{P_\beta} \hat{Q}_\beta$ `is a forcing notion`).

Then the sequence $\langle \hat{Q}_\beta \rangle_{\beta < \alpha}$ is an iterated forcing.

If P_β is restricted to finite functions that it is called a *finite support iterated forcing* (FS), if P_β is restricted to countable functions, it is called a *countable support iterated function* (CS), and in general if each function in each P_β has size less than κ then it is a $<\kappa$*-support iterated forcing*.

Typically we construct the sequence of $\hat{Q}_\beta$'s by induction, using a function F such that $F(\langle \hat{Q}_\beta \rangle_{\beta < \gamma}) = \hat{Q}_\gamma$.