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kernel of a homomorphism is a congruence

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Let Σ be a fixed signature, and \mathfrak{A} and \mathfrak{B} two structures for Σ . If $f: \mathfrak{A} \rightarrow \mathfrak{B}$ is a homomorphism, then $\ker(f)$ is a congruence on \mathfrak{A} .

Proof. If F is an n -ary function symbol of Σ , and $f(a_i) = f(a'_i)$, then

$$\begin{aligned} f(F^{\mathfrak{A}}(a_1, \dots, a_n)) &= F^{\mathfrak{B}}(f(a_1), \dots, f(a_n)) \\ &= F^{\mathfrak{B}}(f(a'_1), \dots, f(a'_n)) \\ &= f(F^{\mathfrak{A}}(a'_1, \dots, a'_n)). \end{aligned} \quad \square$$