



Math for the people, by the people.

analytic hierarchy

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The *analytic hierarchy* is a hierarchy of either (depending on context) formulas or relations similar to the arithmetical hierarchy. It is essentially the second order equivalent. Like the arithmetical hierarchy, the relations in each level are exactly the relations defined by the formulas of that level.

The first level can be called Δ_0^1 , Δ_1^1 , Σ_0^1 , or Π_0^1 , and consists of the arithmetical formulas or relations.

A formula ϕ is Σ_n^1 if there is some arithmetical formula ψ such that:

$$\phi(\vec{k}) = \exists X_1 \forall X_2 \cdots Q X_n \psi(\vec{k}, \vec{X}_n)$$

where Q is either \forall or \exists , whichever maintains the pattern of alternating quantifiers, and each X_i

Similarly, a formula ϕ is Π_n^1 if there is some arithmetical formula ψ such that:

$$\phi(\vec{k}) = \forall X_1 \exists X_2 \cdots Q X_n \psi(\vec{k}, \vec{X}_n)$$

where Q is either \forall or \exists , whichever maintains the pattern of alternating quantifiers, and each X_i