antisymmetric

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A relation $\mathcal{R}$ on A is *antisymmetric* iff $\forall x, y \in A$, $(x\mathcal{R}y \wedge y\mathcal{R}x) \rightarrow (x = y)$. For a finite set A with n elements, the number of possible antisymmetric relations is $2^n 3^{\frac{n^2-n}{2}}$ out of the 2^{n^2} total possible relations.

Antisymmetric is not the same thing as "not symmetric", as it is possible to have both at the same time. However, a relation $\mathcal{R}$ that is both antisymmetric and symmetric has the condition that $x\mathcal{R}y \Rightarrow x = y$. There are only 2^n such possible relations on A.

An example of an antisymmetric relation on $A = \{\circ, \times, \star\}$ would be $\mathcal{R} = \{(\star, \star), (\times, \circ), (\circ, \star), (\star, \times)\}$. One relation that isn't antisymmetric is $\mathcal{R} = \{(\times, \circ), (\star, \circ), (\circ, \star)\}$ because we have both $\star\mathcal{R}\circ$ and $\circ\mathcal{R}\star$, but $\circ \neq \star$