simple function

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In measure theory, a *simple function* is a function that is a finite linear combination

$$h = \sum_{k=1}^{n} c_k \chi_{A_k}$$

of characteristic functions, where the c_k are real coefficients and every A_k is a measurable set with respect to a fixed measure space.

If the measure space is $\mathbb{R}$ and each A_k is an interval, then the function is called a *step function*. Thus, every step function is a simple function.

Simple functions are used in analysis to interpolate between characteristic functions and measurable functions. In other words, characteristic functions are easy to integrate:

$$\int_E \chi_A \, dx = |A|,$$

while simple functions are not much harder to integrate:

$$\int_E \sum_{k=1}^{n} c_k \chi_{A_k} \, dx = \sum_{k=1}^{n} c_k |A_k|.$$

To integrate a measurable function, one approximates it from below by simple functions. Thus, simple functions can be used to define the Lebesgue integral over a subset of the measure space.

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