implication

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An implication is a logical construction that essentially tells us if one condition is true, then another condition must be also true. Formally it is written

$$a \to b$$

or

$$a \Rightarrow b$$

which would be read "a implies b", or "a therefore b", or "if a, then b" (to name a few).

Implication is often confused for "if and only if", or the biconditional truth function ($\Leftrightarrow$). They are not, however, the same. The implication $a \to b$ is true even if only b is true. So the statement "pigs have wings, therefore it is raining today", is true if it is indeed raining, despite the fact that the first item is false.

In fact, any implication $a \to b$ is called *vacuously true* when a is false. By contrast, $a \Leftrightarrow b$ would be false if either a or b was by itself false ($a \Leftrightarrow b \equiv (a \wedge b) \vee (\neg a \wedge \neg b)$, or in terms of implication as $(a \to b) \wedge (b \to a)$).

It may be useful to remember that $a \to b$ only tells you that it *cannot* be the case that b is false while a is true; b must "follow" from a (and "false" does follow from "false"). Alternatively, $a \to b$ is in fact equivalent to

$$b \vee \neg a$$

The truth table for implication is therefore

a	b	$a \to b$
F	F	T
F	T	T
T	F	F
T	T	T