$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$

Canonical name	abcdIfAndOnlyIfAcAndBd
Date of creation	2013-03-22 16:13:19
Last modified on	2013-03-22 16:13:19
Owner	Wkbj79 (1863)
Last modified by	Wkbj79 (1863)
Numerical id	9
Author	Wkbj79 (1863)
Entry type	Proof
Classification	msc 03-00

Following is a proof that the ordered pairs (a,b) and (c,d) are equal if and only if $a=c$ and $b=d$.

Proof. If $a=c$ and $b=d$, then $(a,b)=\{\{a\},\{a,b\}\}=\{\{c\},\{c,d\}\}=(c,d)$.

Assume that $(a,b)=(c,d)$ and $a=b$. Then $\{\{c\},\{c,d\}\}=(c,d)=(a,b)=\{\{a\},\{a,b\}\}=\{\{a\},\{a,a\}\}=\{\{a\},\{a\}\}=\{\{a\}\}$. Thus, $\{c,d\}\in\{\{a\}\}$. Therefore, $\{c,d\}=\{a\}$. Hence, $a=c$ and $a=d$. Since it was also assumed that $a=b$, it follows that $a=c$ and $b=d$.

Finally, assume that $(a,b)=(c,d)$ and $a\neq b$. Then $\{a\}\neq\{a,b\}$. Note that $\{\{a\},\{a,b\}\}=(a,b)=(c,d)=\{\{c\},\{c,d\}\}$. Thus, $\{c\}\in\{\{a\},\{a,b\}\}$. It cannot be the case that $\{c\}=\{a,b\}$ (lest $a=c=b$). Thus, $\{c\}=\{a\}$. Therefore, $a=c$. Hence, $\{\{a\},\{a,b\}\}=\{\{c\},\{c,d\}\}=\{\{a\},\{a,d\}\}$. Note that $\{a,b\}\in\{\{a\},\{a,d\}\}$. Since $\{a\}\neq\{a,b\}$, it must be the case that $\{a,b\}=\{a,d\}$. Thus, $b\in\{a,d\}$. Since $a\neq b$, it must be the case that $b=d$. It follows that $a=c$ and $b=d$. $\square$