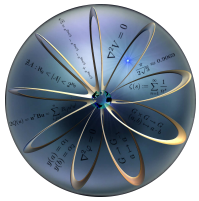

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structure

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Defines	structure
Defines	interpretation

Let τ be a signature. A *τ-structure* $\mathcal{A}$ comprises of a set A, called the (or *underlying set* or) of $\mathcal{A}$, and an *interpretation* of the symbols of τ as follows:

- for each constant symbol $c \in \tau$, an element $c^A \in A$;
- for each n-ary function symbol $f \in \tau$, a function (or operation) $f^A : A^n \to A$;
- for each n-ary relation symbol $R \in \tau$, a n-ary relation R^A on A.

Some authors require that A be non-empty.

If $\mathcal{A}$ is a structure, then the *cardinality* (or *power*) of $\mathcal{A}$, $|\mathcal{A}|$, is the cardinality of its A.

Examples of structures abound in mathematics. Here are some of them:

1. A set is a structure, with no constants, no functions, and no relations on it.
2. A partially ordered set is a structure, with one binary relation call partial order defined on the underlying set.
3. A group is a structure, with one binary operation called multiplication, one unary operation called inverse, and one constant called the multiplicative identity.
4. A vector space is a structure, with one binary operation called addition, unary operations called scalar multiplications, one for each element of the underlying set, and one constant 0, the additive identity.
5. A partially ordered group is a structure like a group, but with the addition of a partial order on the underlying set.

If τ contains only relation symbols, then a τ-structure is called a relational structure. If τ contains only function symbols, then a τ-structure is called an algebraic structure. In the examples above, 2 is a relation structure, while $3, 4$ are algebraic structures.