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point-free geometry

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Point-free geometry is based on the idea that in geometry it is not necessary to assume as a primitive the notion of point. Instead, we can start from the notion of a "region" in the space. The points are defined by suitable *abstraction processes*, i.e. order-reversing sequences of regions. Firstly, such a question was analyzed by A. N. Whitehead in the books "An Inquiry Concerning the Principles of Natural Knowledge" and "The concept of Nature". In these books the "inclusion relation" is the only primitive ("mereology" is the correct collocation). Successively in "Process and Reality" Whitehead proposed a new approach in which the "connection relation" and the one of "oval" is considered which are topological and affine in nature, respectively. Notice that Whitehead's analysis was philosophical but that several authors translated this analysis into systems of axioms for a formal mathematical treatment (for further information see [1]). Successively several approaches to point-free geometry were proposed metrical in nature. The primitives are the inclusion and either the diameter ([4] and [5]) or the diameter and the minimum distance between regions [2]. A totally different approach to point-free geometry is proposed in a very interesting book by H. J. Schmidt. We quote also the deep and extensive investigation of several authors in point-free topology [3].

References

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