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discontinuity of characteristic function

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Theorem. For a subset A of $\mathbb{R}^n$, the set of the http://planetmath.org/Continuousdiscontinu points of the characteristic function χ_A is the http://planetmath.org/BoundaryFrontierboundar of A.

Proof. Let a be a discontinuity point of χ_A. Then any http://planetmath.org/Neighborhood of a contains the points b and c such that $\chi_A(b) = 1$ and $\chi_A(c) = 0$. Thus $b \in A$ and $c \notin A$, whence a is a boundary point of A.

If, on the contrary, a is a boundary point of A and $U(a)$ an arbitrary neighborhood of a, it follows that $U(a)$ contains both points belonging to A and points not belonging to A. So we have in $U(a)$ the points b and c such that $\chi_A(b) = 1$ and $\chi_A(c) = 0$. This means that χ_A cannot be continuous at the point a (N.B. that one does not need to know the value $\chi_A(a)$).