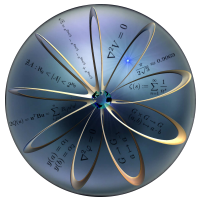

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weakly compact cardinal

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Defines	weakly compact cardinal
Defines	weak compactness theorem

Weakly compact cardinals are (large) infinite cardinals which have a property related to the syntactic compactness theorem for first order logic. Specifically, for any infinite cardinal κ, consider the language $L_{\kappa,\kappa}$.

This language is identical to first logic except that:

- infinite conjunctions and disjunctions of fewer than κ formulas are allowed
- infinite strings of fewer than κ quantifiers are allowed

The weak compactness theorem for $L_{\kappa,\kappa}$ states that if Δ is a set of sentences of $L_{\kappa,\kappa}$ such that $|\Delta| = \kappa$ and any $\theta \subset \Delta$ with $|\theta| < \kappa$ is consistent then Δ is consistent.

A cardinal is weakly compact if the weak compactness theorem holds for $L_{\kappa,\kappa}$.