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Skolemization

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Defines	Skolem function
Defines	Skolem constant

*Skolemization* is a way of removing existential quantifiers from a formula. Variables bound by existential quantifiers which are not inside the scope of universal quantifiers can simply be replaced by constants:  $\exists x[x < 3]$  can be changed to  $c < 3$ , with  $c$  a suitable constant.

When the existential quantifier is inside a universal quantifier, the bound variable must be replaced by a *Skolem function* of the variables bound by universal quantifiers. Thus  $\forall x[x = 0 \vee \exists y[x = y + 1]]$  becomes  $\forall x[x = 0 \vee x = f(x) + 1]$ .

In general, the functions and constants symbols are new ones added to the language for the purpose of satisfying these formulas, and are often denoted by the formula they realize, for instance  $c_{\exists x\phi(x)}$ .

This is used in second order logic to move all existential quantifiers outside the scope of first order universal quantifiers. This can be done since second order quantifiers can quantify over functions. For instance  $\forall^1 x \forall^1 y \exists^1 z \phi(x, y, z)$  is equivalent to  $\exists^2 F \forall^1 x \forall^1 y \phi(x, y, F(x, y))$ .