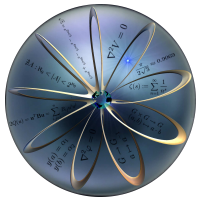

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axiom of extensionality

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If X and Y have the same elements, then $X = Y$.

The Axiom of Extensionality is one of the axioms of Zermelo-Fraenkel set theory. In symbols, it reads:

$$\forall u(u \in X \leftrightarrow u \in Y) \rightarrow X = Y.$$

Note that the converse,

$$X = Y \rightarrow \forall u(u \in X \leftrightarrow u \in Y)$$

is an axiom of the predicate calculus. Hence we have,

$$X = Y \leftrightarrow \forall u(u \in X \leftrightarrow u \in Y).$$

Therefore the Axiom of Extensionality expresses the most fundamental notion of a set: a set is determined by its elements.