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Canonical name	clubsuit
Date of creation	2013-03-22 12:53:52
Last modified on	2013-03-22 12:53:52
Owner	Henry (455)
Last modified by	Henry (455)
Numerical id	4
Author	Henry (455)
Entry type	Definition
Classification	msc 03E65
Synonym	clubsuit
Related topic	Diamond
Related topic	DiamondIsEquivalentToClubsuitAndContinuumHypothesis
Related topic	ProofOfDiamondIsEquivalentToClubsuitAndContinuumHypothesis
Related topic	CombinatorialPrinciple

$\clubsuit_S$ is a combinatoric principle weaker than $\Diamond_S$. It states that, for S stationary in κ, there is a sequence $\langle A_\alpha \rangle_{\alpha \in S}$ such that $A_\alpha \subseteq \alpha$ and $\sup(A_\alpha) = \alpha$ and with the property that for each unbounded subset $T \subseteq \kappa$ there is some $A_\alpha \subseteq X$.

Any sequence satisfying $\Diamond_S$ can be adjusted so that $\sup(A_\alpha) = \alpha$, so this is indeed a weakened form of $\Diamond_S$.

Any such sequence actually contains a stationary set of α such that $A_\alpha \subseteq T$ for each T: given any club C and any unbounded T, construct a κ sequence, C^* and T^*, from the elements of each, such that the α-th member of C^* is greater than the α-th member of T^*, which is in turn greater than any earlier member of C^*. Since both sets are unbounded, this construction is possible, and T^* is a subset of T still unbounded in κ. So there is some α such that $A_\alpha \subseteq T^*$, and since $\sup(A_\alpha) = \alpha$, α is also the limit of a subsequence of C^* and therefore an element of C.