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substructure

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Let Σ be a fixed signature, and $\mathfrak{A}$ and $\mathfrak{B}$ structures for Σ. We say $\mathfrak{A}$ is a *substructure* of $\mathfrak{B}$, denoted $\mathfrak{A} \subseteq \mathfrak{B}$, if for all $x \in \mathfrak{A}$ we have $x \in \mathfrak{B}$, and the inclusion map $i \colon \mathfrak{A} \to \mathfrak{B} : x \mapsto x$ is an embedding.

When $\mathfrak{A}$ is a substructure of $\mathfrak{B}$, we also say that $\mathfrak{B}$ is an *extension* of $\mathfrak{A}$.

A *submodel* $\mathfrak{A}$ of a model $\mathfrak{B}$ of a (first-order) language $\mathcal{L}$ if $\mathfrak{A}$ is a model of $\mathcal{L}$ and $\mathfrak{A}$ is a substructure of $\mathfrak{B}$.