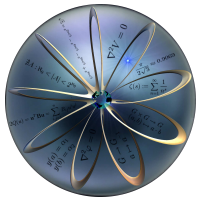

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criterion for a set to be transitive

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Theorem. *A set X is transitive if and only if its power set $\mathcal{P}(X)$ is transitive.*

Proof. First assume X is transitive. Let $A \in B \in \mathcal{P}(X)$. Since $B \in \mathcal{P}(X)$, $B \subseteq X$. Thus, $A \in X$. Since X is transitive, $A \subseteq X$. Hence, $A \in \mathcal{P}(X)$. It follows that $\mathcal{P}(X)$ is transitive.

Conversely, assume $\mathcal{P}(X)$ is transitive. Let $a \in X$. Then $\{a\} \in \mathcal{P}(X)$. Since $\mathcal{P}(X)$ is transitive, $\{a\} \subseteq \mathcal{P}(X)$. Thus, $a \in \mathcal{P}(X)$. Hence, $a \subseteq X$. It follows that X is transitive. □