# metalanguage

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A remedy for Berry's Paradox and related paradoxes is to separate the language used to formulate a particular mathematical theory from the language used for its discourse.

The language used to formulate a mathematical theory is called the *object language* to contrast it from the *metalanguage* used for the discourse.

The most widely used object language is the first-order logic. The metalanguage could be English or other natural languages plus mathematical symbols such as $\Rightarrow$.

Examples

1. The object language speaks of $(\neg A_n)$, but we speak of $\langle (, \neg, A_n, ) \rangle$ in the metalanguage. [Recall that a formula is some finite sequence of the symbols. Cf. First Order Logic or Propositional Logic.]

2. In induction proofs, one might encounter "the first symbol in the formula $\varphi$ is (;" we know that the first symbol is indeed ( and not $\langle$ because $\langle$ is a symbol in our metalanguage. Similarly, "the third symbol is $A_n$" and not , because , is a symbol in our metalanguage.

3. $\vdash$ and $\models$ are members of the metalanguage, *not* of object language.

4. Parallel with the notion of metalanguage is metatheorem. "$\Gamma \vdash (\varphi \to \psi)$ if $\Gamma \cup \{\varphi\} \vdash \psi, \Gamma \subseteq \mathcal{L}_0, \varphi, \psi \in \mathcal{L}_0$" is a metatheorem.

5. *Examples from Set Theory.* Let "Con" denote consistency. Then Con(ZF) and Con(ZF+AC+GCH) are metamathematical statements; they are statements in the metalanguage.

# References

[1] Schechter, E., *Handbook of Analysis and Its Foundations*, 1st ed., Academic Press, 1997.