basic properties of a limit along a filter

Canonical name	BasicPropertiesOfALimitAlongAFilter
Date of creation	2013-03-22 15:32:23
Last modified on	2013-03-22 15:32:23
Owner	kompik (10588)
Last modified by	kompik (10588)
Numerical id	9
Author	kompik (10588)
Entry type	Theorem
Classification	msc 03E99
Classification	msc 40A05

Theorem 1. *Let $\mathcal{F}$ be a free filter (non-principal filter) and (x_n) be a real sequence.*

(i) *If* $\lim\limits_{n\to\infty} x_n = L$ *then* $\mathcal{F}\text{-}\lim x_n = L$.

(ii) *If* $\mathcal{F}\text{-}\lim x_n$ *exists, then* $\liminf x_n \leq \mathcal{F}\text{-}\lim x_n \leq \limsup x_n$.

(iii) *The $\mathcal{F}$-limits are unique.*

(iv) $\mathcal{F}\text{-}\lim(a.x_n + b.y_n) = a.\,\mathcal{F}\text{-}\lim x_n + b.\,\mathcal{F}\text{-}\lim y_n$ *(provided the $\mathcal{F}$-limits of (x_n) and (y_n) exist).*

(v) $\mathcal{F}\text{-}\lim(x_n.y_n) = \mathcal{F}\text{-}\lim x_n.\,\mathcal{F}\text{-}\lim y_n$ *(provided the $\mathcal{F}$-limits of (x_n) and (y_n) exist).*

(vi) *For every cluster point c of the sequence x_n there exists a free filter $\mathcal{F}$ such that $\mathcal{F}\text{-}\lim x_n = c$. On the other hand, if $\mathcal{F}\text{-}\lim x_n$ exists, it is a cluster point of the sequence (x_n).*