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## Veblen function

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Defines strongly critical

Defines Feferman-Schutte ordinal

The *Veblen function* is used to obtain larger ordinal numbers than those provided by exponentiation. It builds on a hierarchy of closed and unbounded classes:

- Cr(0) is the additively indecomposable numbers,  $\mathbb{H}$
- Cr(Sn) = Cr(n)' the set of fixed points of the enumerating function of Cr(n)
- $Cr(\lambda) = \bigcap_{\alpha < \lambda} Cr(\alpha)$

The Veblen function  $\varphi_{\alpha}\beta$  is defined by setting  $\varphi_{\alpha}$  equal to the enumerating function of  $Cr(\alpha)$ .

We call a number  $\alpha$  strongly critical if  $\alpha \in Cr(\alpha)$ . The class of strongly critical ordinals is written SC, and the enumerating function is written  $f_{SC}(\alpha) = \Gamma_{\alpha}$ .

 $\Gamma_0$ , the first strongly critical ordinal, is also called the *Feferman-Schutte* ordinal.