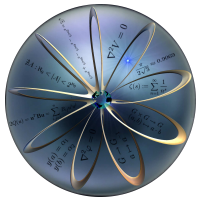

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cycle

Canonical name	Cycle1
Date of creation	2013-03-22 12:24:23
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Classification	msc 03-00
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Classification	msc 20F55
Related topic	Permutation
Related topic	SymmetricGroup
Related topic	Transposition
Related topic	Group
Related topic	Subgroup
Related topic	DihedralGroup
Related topic	CycleNotation
Related topic	PermutationNotation

Let S be a set. A *cycle* is a permutation (bijective function of a set onto itself) such that there exist distinct elements $a_1, a_2, \ldots, a_k$ of S such that

$$f(a_i) = a_{i+1} \qquad \text{and} \qquad f(a_k) = a_1$$

that is

$$\begin{aligned} f(a_1) &= a_2 \\ f(a_2) &= a_3 \\ &\vdots \\ f(a_k) &= a_1 \end{aligned}$$

and $f(x) = x$ for any other element of S.

This can also be pictured as

$$a_1 \mapsto a_2 \mapsto a_3 \mapsto \cdots \mapsto a_k \mapsto a_1$$

and

$$x \mapsto x$$

for any other element $x \in S$, where $\mapsto$ represents the action of f.

One of the basic results on symmetric groups says that any finite permutation can be expressed as product of disjoint cycles.