indexing set

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Synonym	index set
Defines	subscript
Defines	index
Defines	indices
Defines	indexed by
Defines	double indices
Defines	multiple indices

Let Λ and S be sets such that there exists a surjection $f\colon \Lambda \to S$. Then Λ is an *indexing set* for S. Also, S is *indexed by* Λ.

In such situations, the elements of S could be referenced by using the indexing set Λ, such as $f(\lambda)$ for some $\lambda \in \Lambda$. On the other hand, quite often, indexing sets are used without explicitly defining a surjective function. When this occurs, the elements of S are referenced by using *subscripts* (also called *indices*) which are elements of Λ, such as s_λ for some $\lambda \in \Lambda$. If, however, the surjection from Λ to S were called s, this notation would be quite to the function notation: $s(\lambda) = s_\lambda$.

Indexing sets are quite useful for describing sequences, nets, summations, products, unions, and intersections.

Multiple indices are possible. For example, consider the set $X = \{x_{aa}, x_{ab}, x_{ac}, x_{bb}, x_{bc}, x_{cc}\}$. Some people would consider the indexing set for X to be $\{aa, ab, ac, bb, bc, cc\}$. Others would consider the indexing set to be $\{a, b, c\} \times \{a, b, c\}$. (The double indices can be considered as ordered pairs.) Thus, in the case of multiple indices, it need not be the case that the underlying function f be a surjection. On the other hand, f must be a partial surjection. For example, if a set X is indexed by $A \times B$, the following must hold:

1. For every $x \in X$, there exist $i \in A$ and $j \in B$ such that $f(i, j) = x$;
2. For every $i \in A$, the map $f_i\colon B \to X$ defined by $f_i(j) = f(i, j)$ is a partial function;
3. For every $j \in B$, the map $f_j\colon A \to X$ defined by $f_j(i) = f(i, j)$ is a partial function.