operations on consequence operators

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Let L be a set and let $\mathcal{C}$ be the set of all consequence operators on S. Then we may define a binary relation $\leq \subset \mathcal{C} \times \mathcal{C}$ and binary operations $\wedge, \vee, \veebar \colon \mathcal{C} \times \mathcal{C} \to \mathcal{C}$ as follows:

Definition 1 *For $C_1, C_2 \in \mathcal{C}$, we have $C_1 \leq C_2$ when, for all $X \subseteq L$, we have $C_1(X) \subseteq C_2(X)$*

Definition 2 *For $C_1, C_2 \in \mathcal{C}$, we have $(C_1 \wedge C_2)(X) = C_1(X) \cap C_2(X)$ for all $X \subseteq L$.*

Definition 3 *For $C_1, C_2 \in \mathcal{C}$, we have $(C_1 \vee C_2)(X) = C_1(X) \cup C_2(X)$ for all $X \subseteq L$.*

Definition 4 *For $C_1, C_2 \in \mathcal{C}$, we have $(C_1 \veebar C_2)(X) = \cap\{Y \mid X \subseteq Y \subseteq L \wedge C_1(Y) = C_2(Y) = Y\}$ for all $X \subseteq L$.*