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$$\mathcal{NJ}_p$$

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Synonym	NJp

$\mathcal{NJ}p$ is a natural deduction proof system for intuitionistic propositional logic. Its only axiom is $\alpha \Rightarrow \alpha$ for any atomic α . Its rules are:

$$\frac{\Gamma \Rightarrow \alpha}{\Gamma \Rightarrow \alpha \vee \beta} (\vee I) \quad \frac{\Gamma \Rightarrow \alpha \quad \Sigma, \alpha^0 \Rightarrow \phi \quad \Pi, \beta^0 \Rightarrow \phi}{[\Gamma, \Sigma, \Pi] \Rightarrow \phi} (\vee E)$$

The syntax α^0 indicates that the rule also holds if that formula is omitted.

$$\frac{\Gamma \Rightarrow \alpha \quad \Sigma \Rightarrow \beta}{[\Gamma, \Sigma] \Rightarrow \alpha \wedge \beta} (\wedge I) \quad \frac{\Gamma \Rightarrow \alpha \wedge \beta}{\Gamma \Rightarrow \alpha \quad \Gamma \Rightarrow \beta} (\wedge E)$$

$$\frac{\Gamma, \alpha \Rightarrow \beta}{\Gamma \Rightarrow \alpha \rightarrow \beta} (\rightarrow I) \quad \frac{\Gamma \Rightarrow \alpha \rightarrow \beta \quad \Sigma \Rightarrow \alpha}{[\Gamma, \Sigma] \Rightarrow \beta} (\rightarrow E)$$

$$\frac{\Gamma \Rightarrow \perp}{\Gamma \Rightarrow \alpha} (\perp_i), \quad \text{where } \alpha \text{ is atomic}$$