transfinite induction

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Suppose $\Phi(\alpha)$ is a property defined for every ordinal α, the principle of *transfinite induction* states that in the case where for every α, if the fact that $\Phi(\beta)$ is true for every $\beta < \alpha$ implies that $\Phi(\alpha)$ is true, then $\Phi(\alpha)$ is true for every ordinal α. Formally :

$$\forall\alpha(\forall\beta(\beta < \alpha \Rightarrow \Phi(\beta)) \Rightarrow \Phi(\alpha)) \Rightarrow \forall\alpha(\Phi(\alpha))$$

The principle of transfinite induction is very similar to the principle of finite induction, except that it is stated in terms of the whole class of the ordinals.