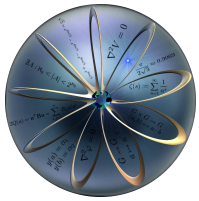

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first order logic

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Synonym	FO

A logic is *first order* if it has exactly one type. Usually the term refers specifically to the logic with connectives $\neg$, $\vee$, $\wedge$, $\rightarrow$, and $\leftrightarrow$ and the quantifiers $\forall$ and $\exists$, all given the usual semantics:

- $\neg\phi$ is true iff ϕ is not true
- $\phi \vee \psi$ is true if either ϕ is true or ψ is true
- $\forall x \phi(x)$ is true iff ϕ_x^t is true for every object t (where ϕ_x^t is the result of replacing every unbound occurrence of x in ϕ with t)
- $\phi \wedge \psi$ is the same as $\neg(\neg\phi \vee \neg\psi)$
- $\phi \rightarrow \psi$ is the same as $(\neg\phi) \vee \psi$
- $\phi \leftrightarrow \psi$ is the same as $(\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$
- $\exists x \phi(x)$ is the same as $\neg\forall x \neg\phi(x)$

However languages with slightly different quantifiers and connectives are sometimes still called first order as long as there is only one type.