product map

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Notation: If $\{X_i\}_{i\in I}$ is a collection of sets (indexed by I) then $\prod_{i\in I} X_i$ denotes the generalized Cartesian product of $\{X_i\}_{1\in I}$.

Let $\{A_i\}_{i\in I}$ and $\{B_i\}_{i\in I}$ be collections of sets indexed by the same set I and $f_i : A_i \longrightarrow B_i$ a collection of functions.

The **product map** is the function

$$\prod_{i\in I} f_i : \prod_{i\in I} A_i \longrightarrow \prod_{i\in I} B_i$$
$$\Big(\prod_{i\in I} f_i\Big)(a_i)_{i\in I} := (f_i(a_i))_{i\in I}$$

0.1 Properties:

- If $f_i : A_i \longrightarrow B_i$ and $g_i : B_i \longrightarrow C_i$ are collections of functions then

$$\prod_{i\in I} g_i \circ \prod_{i\in I} f_i = \prod_{i\in I} g_i \circ f_i$$

- $\prod_{i\in I} f_i$ is injective if and only if each f_i is injective.

- $\prod_{i\in I} f_i$ is surjective if and only if each f_i is surjective.

- Suppose $\{A_i\}_{i\in I}$ and $\{B_i\}_{i\in I}$ are topological spaces. Then $\prod_{i\in I} f_i$ is `http://planetmath.org/Continuous`continuous (in the product topology) if and only if each f_i is continuous.

- Suppose $\{A_i\}_{i\in I}$ and $\{B_i\}_{i\in I}$ are groups, or rings or algebras. Then $\prod_{i\in I} f_i$ is a group (ring or) homomorphism if and only if each f_i is a group (ring or) homomorphism.