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von Neumann ordinal

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The is a method of defining ordinals in set theory.

The von Neumann ordinal α is defined to be the well-ordered set containing the von Neumann ordinals which precede α . The set of finite von Neumann ordinals is known as the von Neumann integers. Every well-ordered set is isomorphic to a von Neumann ordinal.

They can be constructed by transfinite recursion as follows:

- The empty set is 0.
- Given any ordinal α , the ordinal $\alpha + 1$ (the *successor* of α) is defined to be $\alpha \cup \{\alpha\}$.
- Given a set A of ordinals, $\bigcup_{a \in A} a$ is an ordinal.

If an ordinal is the successor of another ordinal, it is an *successor ordinal*. If an ordinal is neither 0 nor a successor ordinal then it is a *limit ordinal*. The first limit ordinal is named ω .

The class of ordinals is denoted **On**.

The von Neumann ordinals have the convenient property that if $a < b$ then $a \in b$ and $a \subset b$.