partitions less than cofinality

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If $\lambda < \mathrm{cf}(\kappa)$ then $\kappa \to (\kappa)^1_\lambda$.

This follows easily from the definition of cofinality. For any coloring $f : \kappa \to \lambda$ then define $g : \lambda \to \kappa + 1$ by $g(\alpha) = |f^{-1}(\alpha)|$. Then $\kappa = \sum_{\alpha < \lambda} g(\alpha)$, and by the normal rules of cardinal arithmetic $\sup_{\alpha < \lambda} g(\alpha) = \kappa$. Since $\lambda < \mathrm{cf}(\kappa)$, there must be some $\alpha < \lambda$ such that $g(\alpha) = \kappa$.