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sentence

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Synonym	closed formula
Defines	open formula
Defines	universal closure
Defines	existential closure

A *sentence* is a formula with no free variables.

Simple examples include:

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$$\forall x \exists y [x < y]$$

•

$$\exists z [z + 7 - 43 = 0]$$

•

$$1 + 2 < 2 + 3$$

Note that the last sentence contains no variables.

A sentence is also called a *closed formula*. A formula that is not a sentence is called an *open formula*.

The following formula is open:

$$x + 2 = 3$$

**Remark.** In first-order logic, the main difference between a sentence and an open formula, semantically, is that a sentence has a definite truth value, whereas the truth value of an open formula may vary, depending on the interpretations of the free variables occurring in the formula. In the open formula above, if  $x$  were 1, then the formula is true. Otherwise, it is false.

Every open formula may be converted into a sentence by placing quantifiers in front of it. Given a formula  $\varphi$ , the *universal closure* of  $\varphi$  is the sentence

$$\forall x_1 \forall x_2 \cdots \forall x_n \varphi$$

where  $\{x_1, \dots, x_n\}$  is the set of all free variables occurring in  $\varphi$ .

The *existential closure* of a formula  $\varphi$  may be defined similarly.

For example, the universal closure of  $x + 2 = 3$  is

$$\forall x [x + 2 = 3],$$

and its existential closure is

$$\exists x [x + 2 = 3].$$

Note that the first sentence is false, while the second is true.