inverse image

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Let $f : A \longrightarrow B$ be a function, and let $U \subset B$ be a subset. The *inverse image* of U is the set $f^{-1}(U) \subset A$ consisting of all elements $a \in A$ such that $f(a) \in U$.

The inverse image commutes with all set operations: For any collection $\{U_i\}_{i \in I}$ of subsets of B, we have the following identities for

1. Unions:
$$f^{-1}\left(\bigcup_{i \in I} U_i\right) = \bigcup_{i \in I} f^{-1}(U_i)$$

2. Intersections:
$$f^{-1}\left(\bigcap_{i \in I} U_i\right) = \bigcap_{i \in I} f^{-1}(U_i)$$

and for any subsets U and V of B, we have identities for

3. Complements:
$$\left(f^{-1}(U)\right)^{\complement} = f^{-1}(U^{\complement})$$

4. Set differences:
$$f^{-1}(U \setminus V) = f^{-1}(U) \setminus f^{-1}(V)$$

5. Symmetric differences:
$$f^{-1}(U \triangle V) = f^{-1}(U) \triangle f^{-1}(V)$$

In addition, for $X \subset A$ and $Y \subset B$, the inverse image satisfies the miscellaneous identities

6. $(f|_X)^{-1}(Y) = X \cap f^{-1}(Y)$

7. $f\left(f^{-1}(Y)\right) = Y \cap f(A)$

8. $X \subset f^{-1}(f(X))$, with equality if f is injective.