# example of quantifier

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| Canonical name | ExampleOfQuantifier |
| Date of creation | 2013-05-23 19:14:17 |
| Last modified on | 2013-05-23 19:14:17 |
| Owner | hkkass (6035) |
| Last modified by | hkkass (6035) |
| Numerical id | 19 |
| Author | hkkass (6035) |
| Entry type | Example |
| Classification | msc 03B15 |
| Classification | msc 03B10 |

there are some examples and theorems about logical quantifiers in the Word Document below . you can download it:

http://www.freewebs.com/hkkass

or

http://www.hkkass.blogspot.com/

I include extracts of this Document below:

Definition: a property is something like $x > 0$ or $x = 0$ in which $x$ is a variable in some set. Such a formula is shown by $p(x)$, $q(x)$ ,etc. if x is fixed then $p(x)$ is a proposition, i.e. it is a true or a false sentence.

Example 1: let $p(x)$ be the property $0 < x$ where x is a real number. $p(1)$ is true and $p(0)$ is false.

Example 2: a property can have two or more variables. Let $p(x, y)$ be $x = y$. in this case $p(1, 1)$ is true but $p(0, 1)$ is false because 0 is not equal to 1.

Definition: let $p(x)$ be a property on the set $X$, i.e. $p(x)$ is a property and $x$ varies in the set $X$. a) The symbol $(\forall x \in X)(p(x))$ means for every $x$ in the set $X$ the proposition $p(x)$ is true. b) The symbol $(\exists x \in X)(p(x))$ means there is some $x$ in the set $X$ for which the proposition $p(x)$ is true. If $X = \emptyset$ , i.e. if the set $X$ is empty, $(\forall x \in X)(p(x))$ is defined to be true and $(\exists x \in X)(p(x))$ is defined to be false.

Example 1: $(\forall x \in \mathbb{R})(x = 0 \text{ or } x > 0 \text{ or } x < 0)$ is a true proposition.

Example 2: $(\exists x \in \mathbb{R})(x^2 + 1 = 0)$ is false, because no real number satisfies $x^2 + 10 = 0$.

Example 3: $(\forall x \in \mathbb{R})(x < y)$ is a property. $y$ varies in $\mathbb{R}$. As a result $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(x < y)$ is a proposition, i.e. it is a true or a false sentence. In fact $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(x < y)$ is false but $(\forall x \in \mathbb{R})(\forall y \in (x, \infty)(x < y)$ is true; here $(x, \infty)$ is the interval containing real numbers greater than $x$.

some theorems:

for proofs of the following theorems see the address above

Theorem 1: if $(\forall x \in A)(p(x))$ and $(\forall x \in A)(p(x) \to q(x))$ then $(\forall x \in A)(q(x))$.

Theorem 2: suppose $\{a\}$ is a singleton, i.e. a set with only one element. We have "$(\forall x \in \{a\})(p(x))$" is equivalent to $p(a)$.

Theorem 22: if $(\exists y \in B)(\forall x \in A)(r(x, y))$ then $(\forall x \in A)(\exists y \in B)(r(x, y))$.

here $r(x, y)$ is a property on $A \times B$.