



planetmath.org

Math for the people, by the people.

definition of well ordered set, a variant

Canonical name	DefinitionOfWellOrderedSetAVariant
Date of creation	2013-03-22 18:04:47
Last modified on	2013-03-22 18:04:47
Owner	yesitis (13730)
Last modified by	yesitis (13730)
Numerical id	9
Author	yesitis (13730)
Entry type	Derivation
Classification	msc 03E25
Classification	msc 06A05
Related topic	SomethingRelatedToNaturalNumber
Related topic	NaturalNumbersAreWellOrdered

A well-ordered set is normally defined as a *totally* ordered set in which every nonempty subset has a least member, as the parent object does.

It is possible to define well-ordered sets as follows:

a well-ordered set X is a partially ordered set in which every nonempty subset of X has a least member.

To justify the alternative, we prove that every partially ordered set X in which every nonempty subset has a least member is total:

let $x \in X$ and $y \in X$, $x \neq y$. Now, $\{x, y\}$ has a least member, a fortiori, x, y are comparable. Hence, X is totally ordered.

The alternative has the benefit of being a stronger statement in the sense that

$$(\textit{partial order}) \implies (\textit{total order})$$

given that every nonempty subset has a least member.

References

- [1] Schechter, E., *Handbook of Analysis and Its Foundations*, 1st ed., Academic Press, 1997.
- [2] Jech, T., *Set Theory*, 3rd millennium ed., Springer, 2002.