logically equivalent

Canonical name	LogicallyEquivalent
Date of creation	2013-03-22 13:17:00
Last modified on	2013-03-22 13:17:00
Owner	sleske (997)
Last modified by	sleske (997)
Numerical id	10
Author	sleske (997)
Entry type	Definition
Classification	msc 03B05
Synonym	tautologically equivalent
Synonym	semantically equivalent
Synonym	tautological equivalence
Synonym	semantical equivalence
Synonym	tautological consequence
Synonym	semantical consequence
Related topic	Biconditional
Defines	logical equivalence
Defines	logical consequence

Two formulas A and B are said to be *logically equivalent* (typically shortened to *equivalent*) when A is true if and only if B is true (that is, A implies B and B implies A):

$$\models A \leftrightarrow B.$$

This is sometimes abbreviated as $A \Leftrightarrow B$.

For example, for any integer z, the statement "z is positive" is equivalent to "z is not negative and $z \neq 0$".

More generally, one says that a formula A is a logical consequence of a set Γ of formulas, written

$$\Gamma \models A$$

if whenever every formula in Γ is true, so is A. If Γ is a singleton consisting of formula B, we also write

$$B \models A.$$

Using this, one sees that

$$\models A \leftrightarrow B \qquad \text{iff} \qquad A \models B \text{ and } B \models A.$$

To see this: if $\models A \leftrightarrow B$, then $A \to B$ and $B \to A$ are both true, which means that if A is true so is B and that if B is true so is A, or $A \models B$ and $B \models A$. The argument can be reversed.

Remark. Some authors call the above notion semantical equivalence or tautological equivalence, rather than logical equivalence. In their view, logical equivalence is a syntactic notion: A and B are logically equivalent whenever A is deducible from B and B is deducible from A in some deductive system.