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A.1.6 Natural numbers

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The type of natural numbers is obtained by introducing primitive constants $\mathbb{N}$, 0, and succ with the following rules:

- $\mathbb{N} : \mathcal{U}_0$,

- $0 : \mathbb{N}$,

- $\mathsf{succ} : \mathbb{N} \to \mathbb{N}$.

Furthermore, we can define functions by primitive recursion. If we have $C : \mathbb{N} \to \mathcal{U}_k$ we can introduce a defined constant $f : \prod_{(x:\mathbb{N})} C(x)$ whenever we have

$$d : C(0)$$
$$e : \prod_{(x:\mathbb{N})} (C(x) \to C(\mathsf{succ}(x)))$$

with the defining equations

$$f(0) \equiv d \qquad \text{and} \qquad f(\mathsf{succ}(x)) \equiv e(x, f(x)).$$