choice function

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A *choice function* on a set S is a function f with domain S such that $f(x) \in x$ for all $x \in S$.

A choice function on S simply picks one element from each member of S. So in order for S to have a choice function, every member of S must be a nonempty set. The http://planetmath.org/AxiomOfChoiceAxiom of Choice (AC) states that every set of nonempty sets does have a choice function.

Without AC the situation is more complicated, but we can still show that some sets have a choice function. Here are some examples:

- If S is a finite set of nonempty sets, then we can construct a choice function on S by picking one element from each member of S. This requires only finitely many choices, so we don't need to use AC.

- If every member of S is a well-ordered nonempty set, then we can pick the least element of each member of S. In this case we may be making infinitely many choices, but we have a rule for making the choices, so AC is not needed. The distinction between "well-ordered" and "well-orderable" is important here: if the members of S were merely well-orderable, we would first have to choose a well-ordering of each member, and this might require infinitely many arbitrary choices, and therefore AC.

- If every member of S is a nonempty set, and the union $\cup S$ is well-orderable, then we can choose a well-ordering for this union, and this induces a well-ordering on every member of S, so we can now proceed as in the previous example. In this case we were able to well-order every member of S by making just one choice, so AC wasn't needed. (This example shows that the Well-Ordering Principle, which states that every set is well-orderable, implies AC. The converse is also true, but less trivial — http://planetmath.org/ProofOfZermelosWellOrderingTheoremsee the proof.)