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von Neumann integer

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A *von Neumann* is not an integer, but instead a construction of a natural number using some basic set notation. The von Neumann integers are defined inductively. The von Neumann integer zero is defined to be the empty set, \emptyset , and there are no smaller von Neumann integers. The von Neumann integer N is then the set of all von Neumann integers less than N . The set of von Neumann integers is the set of all finite <http://planetmath.org/VonNeumannOrdinal> von Neumann ordinals.

This form of construction from very basic notions of sets is applicable to various forms of set theory (for instance, Zermelo-Fraenkel set theory). While this construction suffices to define the set of natural numbers, a little more work must be done to define the set of all <http://planetmath.org/Integer> integers.

Examples

$$\begin{aligned}
 0 &= \emptyset \\
 1 &= \{0\} = \{\emptyset\} \\
 2 &= \{0, 1\} = \{\emptyset, \{\emptyset\}\} \\
 3 &= \{0, 1, 2\} = \{\emptyset, \{\emptyset\}, \{\{\emptyset, \{\emptyset\}\}\}\} \\
 &\vdots \\
 N &= \{0, 1, \dots, N-1\}
 \end{aligned}$$