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Sheffer stroke

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Defines	Peirce arrow

In the late 19th century and early 20th century, Charles Sanders Peirce and H.M. Sheffer independently discovered that a single binary logical connective suffices to define all logical connectives (they are each functionally complete). Two such connectives are

- $\uparrow$: the *Sheffer stroke* (sometimes denoted by $|$) and
- $\downarrow$: the *Peirce arrow* (sometimes denoted by $\perp$).

The Sheffer stroke is defined by the truth table

P	Q	$P \uparrow Q$
F	F	T
F	T	T
T	F	T
T	T	F

Observe that $P \uparrow Q$ is true if and only if either P or Q is false. For this reason, the Sheffer stroke is sometimes called *alternative denial* or *NAND*.

The Peirce arrow is defined by the truth table

P	Q	$P \downarrow Q$
F	F	T
F	T	F
T	F	F
T	T	F

The proposition $P \downarrow Q$ is true if and only if both P and Q are false. For this reason, the Peirce arrow is sometimes called *joint denial* or *NOR*.

To show the sufficiency of the Sheffer stroke, all we have to do is define both $\neg$ and $\vee$ in terms of $\uparrow$. The proposition $P \uparrow P$ asserts that either P is false, or P is false; thus we can define $\neg$ by $\neg P := P \uparrow P$. We define $\vee$ by

$$P \vee Q := (P \uparrow P) \uparrow (Q \uparrow Q),$$

since this asserts that either $P \uparrow P$ is false (that is, that P is true) or that $Q \uparrow Q$ is false (that is, that Q is true).

We can show the sufficiency of the Peirce arrow in a similar way. Define

$$\neg P := P \downarrow P$$

and

$$P \vee Q := (P \downarrow Q) \downarrow (P \downarrow Q).$$

This expression asserts that $P \downarrow Q$ is false, that is, that it is false that both P and Q are false. By DeMorgan's law, this is equivalent to asserting that at least one of P and Q is true.

Remark. It can be shown that no binary connective, other than Sheffer stroke and Peirce arrow, is functionally complete.