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Dynkin system

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Let Ω be a set, and $\mathcal{P}(\Omega)$ be the power set of Ω . A *Dynkin system* on Ω is a set $\mathcal{D} \subset \mathcal{P}(\Omega)$ such that

1. $\Omega \in \mathcal{D}$
2. $A, B \in \mathcal{D}$ and $A \subset B \Rightarrow B \setminus A \in \mathcal{D}$
3. $A_n \in \mathcal{D}$, $A_n \subset A_{n+1}$, $n \geq 1 \Rightarrow \bigcup_{k=1}^{\infty} A_k \in \mathcal{D}$.

Let $F \subset \mathcal{P}(\Omega)$, and consider

$$\Gamma = \{X : X \subset \mathcal{P}(\Omega) \text{ is a Dynkin system and } F \subset X\}. \quad (1)$$

We define the intersection of all the Dynkin systems containing F as

$$\mathcal{D}(F) := \bigcap_{X \in \Gamma} X \quad (2)$$

One can easily verify that $\mathcal{D}(F)$ is itself a Dynkin system and that it contains F . We call $\mathcal{D}(F)$ the *Dynkin system generated by F* . It is the “smallest” Dynkin system containing F .

A Dynkin system which is also <http://planetmath.org/PiSystem>-system is a <http://planetmath.org/SigmaAlgebra>-algebra.