another proof of pigeonhole principle

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By induction on n. It is harmless to let $n = m + 1$, since 0 lacks proper subsets. Suppose that $f : n \to n$ is injective.

To begin, note that $m \in f[n]$. Otherwise, $f[m] \subseteq m$, so that by the induction hypothesis, $f[m] = m$. Then $f[n] = f[m]$, since $f[n] \subseteq m$. Therefore, for some $k < m$, $f(k) = f(m)$.

Let $g : f[n] \to f[n]$ transpose m and $f(m)$. Then $h|_m : m \to m$ is injective, where $h = g \circ f$. By the induction hypothesis, $h|_m[m] = m$. Therefore:

$$\begin{aligned} f[n] &= g \circ h[n] \\ &= h[n] \\ &= m \cup \{m\} \\ &= m + 1 \\ &= n. \end{aligned}$$