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multiplicative filter

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Defines Gabriel Filter

Defines Multiplicative Filter

For any ring A, any set $S\subset A$ and any element $x\in A$, we use the notation

$$(S:x) := \{a \in A \ ax \in S\}$$

Let A be a commutative ring with unity, and let $\mathcal{I}(A)$ be the set of all ideals of A.

- A Multiplicative Filter of A is a filter \mathcal{F} on $\mathcal{I}(A)$ such that $I, J \in \mathcal{F} \Rightarrow IJ \in \mathcal{F}$.
- A Gabriel Filter of A is a filter \mathcal{F} on $\mathcal{I}(A)$ such that

$$[I \in \mathcal{F}, J \in \mathcal{I}(A) \text{ and } \forall x \in I, (J:x) \in \mathcal{F}] \Rightarrow J \in \mathcal{F}$$

Note that Gabriel Filters are also Multiplicative Filters.