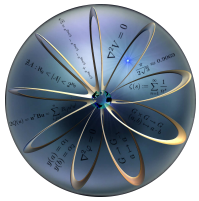

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Hausdorff's maximum principle

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Theorem Let X be a partially ordered set. Then there exists a maximal totally ordered subset of X.

The Hausdorff's maximum principle is one of the many theorems equivalent to the `http://planetmath.org/AxiomOfChoice`axiom of choice. The below proof uses Zorn's lemma, which is also equivalent to the .

Proof. Let S be the set of all totally ordered subsets of X. S is not empty, since the empty set is an element of S. Partial order S by inclusion. Let τ be a chain (of elements) in S. Being each totally ordered, the union of all these elements of τ is again a totally ordered subset of X, and hence an element of S, as is easily verified. This shows that S, ordered by inclusion, is inductive. The result now follows from Zorn's lemma. □