modus tollens

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The law of *modus tollens* is the inference rule which allows one to conclude $\neg P$ from $P \Rightarrow Q$ and $\neg Q$. The name "modus tollens" refers to the fact that this rule allows one to take away the conclusion of a conditional statement and conclude the negation of the condition. As an example of this rule, we may cite the following:

> If the postman is at the door, the doorbell will ring twice
> The bell is not ringing.
> ———————————————————————————
> The postman is not at the door.

The validity of this rule may be established by means of the following truth table:

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg Q$
F	F	T	T	T
F	T	T	T	F
T	F	F	F	T
T	T	T	F	F

This rule can be used to justify the popular technique of proof by contradiction. In this technique, one assumes a hypothesis P and then derives a conclusion Q. This is tantamount to showing that $P \Rightarrow Q$. Next one demonstrates $\neg Q$. Applying modus tollens, one then concludes $\neg P$.