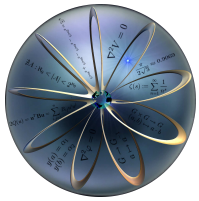

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Berry's paradox

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We begin with calling a positive integer *curious* if it can be defined in the English language using no more than 1234 words. Since there are finitely many English words, we see that there are only finitely many curious positive integers.

Define n_0 to be: *the least positive integer that is not curious.*

n_0 has just been described in $8 \leq 1234$ words, therefore, it is curious after all!

The paradox above is called Berry's Paradox. Berry's Paradox suggests the advantage of separating the language used to formulate mathematical statements or theory (the object language) from the language used to discuss those statements or the theory (the metalanguage).

Berry's Paradox can be avoided by the following reformulation:

1. fix the object language, called $\mathbf{E}^*$;
2. declare $\mathbf{E}^*$ to be different from our metalanguage, which is English here;
3. define a curious positive integer to be one which can be described in $\mathbf{E}^*$ using no more than 1234 words of the language;
4. define n_0 to be the least positive integer that is not curious.

In the reformulation, we have defined curious positive integers and n_0 in English, which is not $\mathbf{E}^*$. Thus, we have no basis to conclude that n_0 is curious, hence no contradiction arises.

Commonly, $\mathbf{E}^*$ is the first order logic. However, it is not often necessarily the case, and $\mathbf{E}^*$ above could have been English anyway. We only need to formally distinguish the statements formulating the mathematics from the statements discussing those formulations, i.e., declaring the two classes of statements to be disjunct, perhaps by italicizing the former. Nevertheless, such approach evidently involves more work and is understandably hard to follow.

References

[1] Schechter, E., *Handbook of Analysis and Its Foundations*, 1st ed., Academic Press, 1997.