filtration

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Let $M = (W, R, V)$ be a Kripke model for a modal logic L. Let Δ be a set of wff's. Define a binary relation $\sim_\Delta$ on W:

$$w \sim_\Delta u \qquad \text{iff} \qquad \models_w A \text{ iff } \models_u A \text{ for any } A \in \Delta.$$

Then $\sim_\Delta$ is an equivalence relation on W. Let W' be the set of equivalence classes of $\sim_\Delta$ on W. It is easy to see that if Δ is finite, so is W'. Next, let

$$V'(p) := \{[w] \in W' \mid w \in V(p)\}.$$

Then V' is a well-defined function. We call a binary relation R' on W' a *filtration* of R if

- wRu implies $[w]R'[u]$
- $[w]R'[u]$ implies that for any wff A with $\Box A \in \Delta$, if $\models_w \Box A$, then $\models_u A$.

The triple $M' := (W', R', V')$ is called a *filtration* of the model M.

Proposition 1. *(Filtration Lemma) Let Δ be a set of wff's closed under the formation of subformulas: any subformula of any formula in Δ is again in Δ. Then*

$$M' \models_{[w]} A \qquad \textit{iff} \qquad M \models_w A.$$