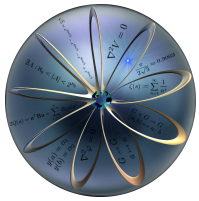

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Laplacian matrix of a graph

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Defines	Laplacian matrix

Let G be a finite graph with n vectices and let D be the `http://planetmath.org/IncidenceMatrixWithRespectToAnOrientation`incidence matrix of G with respect to some orientation. The *Laplacian matrix* of G is defined to be DD^T.

If we let A be the adjacency matrix of G then it can be shown that $DD^T = \Delta - A$, where $\Delta = \mathrm{diag}(\delta_1, \ldots, \delta_n)$ and δ_i is the degree of the vertex v_i. As a result, the Laplacian matrix is independent of what orientation is chosen for G.

The Laplacian matrix is usually denoted by $L(G)$. It is a positive semidefinite singular matrix, so that the smallest eigenvalue is 0.