planetmath.org

Math for the people, by the people.

k-connected graph

Canonical name	kconnectedGraph
Date of creation	2013-03-22 13:10:58
Last modified on	2013-03-22 13:10:58
Owner	marijke (8873)
Last modified by	marijke (8873)
Numerical id	7
Author	marijke (8873)
Entry type	Definition
Classification	msc 05C40
Related topic	ConnectedGraph
Related topic	ClosedPath
Defines	k-connected k-vertex-connected k-edge-connected connectivity

Connectivity of graphs, when it isn't specified which flavor is intended, usually refers to *vertex connectivity*, unless it is clear from the context that it refers to *edge connectivity*.

The (**vertex**) **connectivity** $\kappa(G)$ is the minimum number of vertices (aka nodes) you have to remove to *either* make the graph no longer connected, *or* reduce it to a single vertex (node). G is said to be k**-(vertex)-connected** for any $k \leqslant \kappa(G) \in \mathbb{N}$. Note that "removing a vertex" in graph theory also involves removing all the edges incident to that vertex.

The **edge connectivity** $\kappa'(G)$ of a graph G is more straightforward, it is just the minimum number of edges you have to remove to make the graph no longer connected. G is said to be k**-edge-connected** for any $k \leqslant \kappa'(G) \in \mathbb{N}$. And note "removing an edge" is simply that; it does not entail removing any vertices.

- If $\kappa'(G) = 0$ also $\kappa(G) = 0$ and vice versa; such graphs are called disconnected and consist of several connected components. If κ and κ' are nonzero (the graph is 1-connected) it is a connected graph (a single connected component)

- If $\kappa'(G) = 1$ the graph contains at least one bridge. If $\kappa'(G) > 1$ (the graph is 2-edge-connected) every edge is part of a cycle (circuit, closed path).

  If $\kappa(G) = 1$ the graph contains at least one cutvertex. If $\kappa(G) > 1$ (the graph is 2-vertex-connected) there is, for any pair of vertices, a cycle they both lie on.

- For the complete graphs we have $\kappa(K_n) = \kappa'(K_n) = n - 1$.

- For any graph G we have $\kappa(G) \leqslant \kappa'(G) \leqslant \delta(G)$ where the latter is the minimum valency in G (the valency of a vertex is the number of edges sprouting from it).

Everything on this page applies equally well to multigraphs and pseudographs.

For directed graphs there are `http://planetmath.org/ConnectedGraph`two notions of connectivity ("weak" if the underlying graph is connected, "strong" if you can get *from* everywhere *to* everywhere).

There are now `http://planetmath.org/ExamplesOfKConnectedGraphs`pictures to go with this entry.