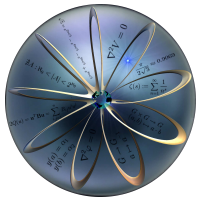

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rule of product

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If a process A can have altogether m different results and another process B altogether n different results, then the two processes can have altogether mn different combined results. Putting it to set-theoretical form,

$$\mathrm{card}(A \times B) \;=\; m \cdot n.$$

The *rule of product* is true also for the combination of several processes: If the processes A_i can have n_i possible results ($i = 1, 2, \ldots, k$), then their combined process has $n_1 n_2 \cdots n_k$ possible results. I.e.,

$$\mathrm{card}(A_1 \times A_2 \times \ldots \times A_k) \;=\; n_1 n_2 \cdots n_k.$$

Example. Arranging n elements, the first one may be chosen freely from all the n elements, the second from the remaining $n-1$ elements, the third from the remaining $n-2$, and so on, the penultimate one from two elements and the last one from the only remaining element; thus by the rule of product, there are in all

$$n(n-1)(n-2)\cdots 2 \cdot 1 \;=\; n!$$

different arrangements, i.e. permutations, as the result.