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Pascal’s rule proof

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We need to show

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$$

Let us begin by writing the left-hand side as

$$\frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-(k-1))!}$$

Getting a common denominator and simplifying, we have

$$\begin{aligned} \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-k+1)!} &= \frac{(n-k+1)n!}{(n-k+1)k!(n-k)!} + \frac{kn!}{k(k-1)!(n-k+1)!} \\ &= \frac{(n-k+1)n! + kn!}{k!(n-k+1)!} \\ &= \frac{(n+1)n!}{k!((n+1)-k)!} \\ &= \frac{(n+1)!}{k!((n+1)-k)!} \\ &= \binom{n+1}{k} \end{aligned}$$