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**Hall’s marriage theorem**

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Let  $S = \{S_1, S_2, \dots, S_n\}$  be a finite collection of finite sets. There exists a system of distinct representatives of  $S$  if and only if the following condition holds for any  $T \subseteq S$ :

$$|\cup T| \geq |T|$$

As a corollary, if this condition fails to hold anywhere, then no SDR exists.

This is known as Hall's marriage theorem. The name arises from a particular application of this theorem. Suppose we have a finite set of single men/women, and, for each man/woman, a finite collection of women/men to whom this person is attracted. An SDR for this collection would be a way each man/woman could be (theoretically) married happily. Hence, Hall's marriage theorem can be used to determine if this is possible.

An application of this theorem to graph theory gives that if  $G(V_1, V_2, E)$  is a bipartite graph, then  $G$  has a complete matching that saturates every vertex of  $V_1$  if and only if  $|S| \leq |N(S)|$  for every subset  $S \subset V_1$ .