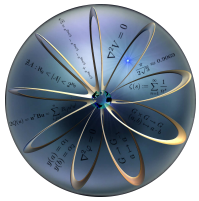

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proof that Hadamard matrix has order 1 or 2 or 4n

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Let m be the order of a Hadamard matrix. The matrix [1] shows that order 1 is possible, and the entry has a 2×2 Hadamard matrix , so assume $m > 2$.

We can assume that the first row of the matrix is all 1's by multiplying selected columns by -1. Then permute columns as needed to arrive at a matrix whose first three rows have the following form, where P denotes a submatrix of one row and all 1's and N denotes a submatrix of one row and all -1's.

$$\begin{bmatrix} \overbrace{P}^{x} & \overbrace{P}^{y} & \overbrace{P}^{z} & \overbrace{P}^{w} \\ P & P & N & N \\ P & N & P & N \end{bmatrix}$$

Since the rows are orthogonal and there are m columns we have

$$\begin{cases} x+y+z+w &= m \\ x+y-z-w &= 0 \\ x-y+z-w &= 0 \\ x-y-z+w &= 0. \end{cases}$$

Adding the 4 equations together we get

$$4x = m.$$

so that m must be divisible by 4.