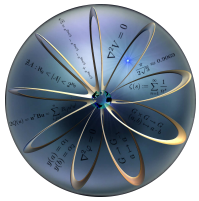

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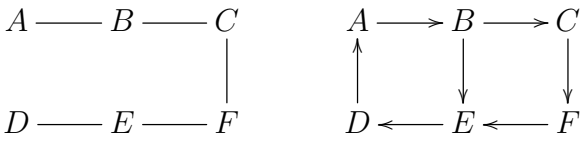
connected graph

Canonical name	ConnectedGraph
Date of creation	2013-03-22 12:30:34
Last modified on	2013-03-22 12:30:34
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	7
Author	rspuzio (6075)
Entry type	Definition
Classification	msc 05C40
Synonym	connected
Synonym	strongly connected
Synonym	component
Related topic	Graph
Related topic	Bridge
Related topic	Cutvertex
Related topic	LocallyConnected
Related topic	KConnectedGraph
Related topic	GraphTopology
Related topic	ClosedPath
Related topic	ConnectedPoset
Related topic	DepthFirstSearch2
Related topic	DepthFirstSearch
Related topic	VectorValuedFunction2
Defines	strongly connected graph
Defines	connected components
Defines	strongly connected components

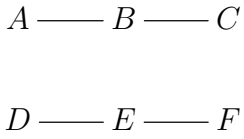
A *connected graph* is a graph such that there exists a path between all pairs of vertices. If the graph is a directed graph, and there exists a path from each vertex to every other vertex, then it is a *strongly connected graph.*

A *connected component* is a maximal (under inclusion) subset of vertices of any graph and any edges between them that forms a connected graph. Similarly, a *strongly connected component* is a maximal (under inclusion) subset of vertices of any digraph and any edges between them that forms a strongly connected graph. Any graph or digraph is a union of connected or strongly connected components, plus some edges to join the components together. Thus any graph can be decomposed into its connected or strongly connected components. For instance, Tarjan's algorithm can decompose any digraph into its strongly connected components.

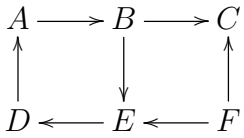
For example, the following graph and digraph are connected and strongly connected, respectively.



On the other hand, the following graph is *not* connected, and consists of the union of two connected components.



The following digraph is *not* strongly connected, because there is no way to reach F from other vertices, and there is no vertex reachable from C.



The three strongly connected components of this graph are