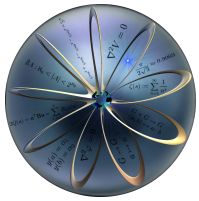

criterion for a near-linear space being a linear space

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Theorem

Suppose $\mathscr{S}$ is near-linear space with v points and b lines, and s_i is the number of points in the ith line, for $i = 1, \ldots, b$. Then

$$\sum_{i=1}^{b} s_i(s_i - 1) \leq v(v-1),$$

and equality holds if and only if $\mathscr{S}$ is a linear space.

Proof Let N be the number of ordered pairs of points that are joined by a line. Clearly N can be no more than $v(v-1)$, and $N = v(v-1)$ if and only if every pair of points are joined by a line. Since two points in a near-linear space are on at most one line, we can label each pair by the line to which the two points belong to. We thus have a partition of the N pairs into b groups, and each group is associated with a distinct line. The group corresponding to the line consisting of s_i points contributes $s_i(s_i - 1)$ to the total sum. Therefore

$$\sum_{i=1}^{b} s_i(s_i - 1) = N \leq v(v-1).$$