(closed) walk / trek / trail / path

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Defines	closed trek
Defines	closed trail
Defines	closed path
Defines	closed chain
Defines	open walk
Defines	open trek
Defines	open trail
Defines	open path
Defines	open chain
Defines	s-arc
Defines	s-cycle
Defines	elementary cycle
Defines	s-circuit

Graph theory terminology is notoriously variable so the following definitions should be used with caution. In books, most authors define their usage at the beginning.

In a graph, multigraph or even pseudograph G,

- a **walk** of length s is formed by a sequence of s edges such that any two successive edges in the sequence share a vertex (aka node). The walk is also considered to include all the vertices (nodes) incident to those edges, making it a subgraph.

  In the case of a simple graph (i.e. not a multigraph) it is also possible to define the walk uniquely by the vertices it visits: a **walk** of length s is then a sequence of vertices ν_0, ν_1, $\ldots$ ν_s such that an edge $\nu_i\nu_{i+1}$ exists for all $0 \leqslant i < s$. Again the walk is considered to contain those edges as well as the vertices.

- A **trek** is a walk that does not backtrack, i.e. no two successive edges are the same.

  For simple graphs this also implies $\nu_i \neq \nu_{i+2}$ for all $0 \leqslant i \leqslant s-2$.

- A **trail** is a walk where all edges are distinct, and

- a **path** is one where all vertices are distinct.

The walk, etc. is said to **run from** ν_0 **to** ν_s, to **run between** them, to **connect** them etc. The term *trek* was introduced by Cameron [**?**] who notes the lexicographic mnemonic

$$\textit{paths} \subset \textit{trails} \subset \textit{treks} \subset \textit{walks}$$

The other terms are fairly widespread, cf. [**?**], but **beware:** many authors call walks **paths**, and some then call paths **chains**. And when edges are called **arcs**, a trek of length s sometimes goes by the name s**-arc**.

Note that for the purpose of defining connectivity any of these types of wanderings can be used; if a walk exists between vertices μ and ν then there also exists a path between them. And here we must allow $s = 0$ to make "are connected by a path" an equivalence relation on vertices (in order to define connected components as its equivalence classes).

- A **closed walk** aka **circuit** of length $s \neq 0$ is a walk where $\nu_0 = \nu_s$,

- a **closed trek** is a trek that's closed in the same way, and
- a **closed trail** likewise;
- a **closed path** aka (**elementary**) **cycle** is like a path (except that we only demand that ν_i for $0 \leqslant i < s$ are distinct) and again closed (ν_s again coincides with ν_0).

Beware: cycles are often called circuits [**?**]; the distinction between circuits and cycles here follows Wilson [**?**]. These closed wanderings are often called after their length: s-**circuits**, s-**cycles**.

The case $s = 0$ is excluded from these definitions; 1-cycles are loops so imply a pseudograph; 2-cycles are double edges implying multigraphs; so 3 is the minimum cycle length in a proper graph.

Note also that in trivalent aka cubic graphs a closed trail is automatically a closed path: it is impossible to visit a vertex (in via edge a, out via edge b say) and visit it again (in via c, out via d) without also revisiting an edge, because there are only three edges at each vertex.

- An **open walk**, **open trek**, **open trail** is one that isn't closed.
- An **open path** (sometimes **open chain**) is just a path as defined above (because a closed path isn't actually a path). Still, the term is useful when you want to emphasise the contrast with a closed path.

References

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