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subgraph

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| Canonical name   | Subgraph            |
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| Entry type       | Definition          |
| Classification   | msc 05C99           |
| Related topic    | Graph               |
| Related topic    | Pseudograph         |
| Related topic    | Multigraph          |
| Defines          | induced             |
| Defines          | spanned by          |
| Defines          | spanning            |
| Defines          | spanning subgraph   |
| Defines          | induced subgraph    |

We say that  $G' = (V', E')$  is a *subgraph* of  $G = (V, E)$  if  $V' \subseteq V$  and  $E' \subseteq E$ . In this case we write  $G' \subseteq G$ .

If  $G'$  contains *all edges* of  $G$  that join two vertices in  $V'$  then  $G'$  is said to be the subgraph *induced* or *spanned by*  $V'$  and is denoted by  $G[V']$ . Thus, a subgraph  $G'$  of  $G$  is an induced subgraph if  $G' = G[V(G')]$ . If  $V' = V$ , then  $G'$  is said to be a *spanning* subgraph of  $G$ .

Often, new graphs are constructed from old ones by deleting or adding some vertices and edges. If  $W \subset V(G)$ , then  $G - W = G[V \setminus W]$  is the subgraph of  $G$  obtained by deleting the vertices in  $W$  *and all edges incident with them*. Similarly, if  $E' \subseteq E(G)$ , then  $G - E' = (V(G), E(G) \setminus E')$ . If  $W = w$  and  $E' = xy$ , then this notation is simplified to  $G - w$  and  $G - xy$ . Similarly, if  $x$  and  $y$  are nonadjacent vertices of  $G$ , then  $G + xy$  is obtained from  $G$  by joining  $x$  to  $y$ .

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