principle of inclusion-exclusion

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The *principle of inclusion-exclusion* provides a way of methodically counting the union of possibly non-disjoint sets.

Let $C = \{A_1, A_2, \dots A_N\}$ be a finite collection of finite sets. Let I_k represent the set of k-fold intersections of members of C (e.g., I_2 contains all possible intersections of two sets chosen from C).

Then

$$\left|\bigcup_{i=1}^{N} A_i\right| = \sum_{j=1}^{N}\left((-1)^{(j+1)}\sum_{S\in I_j}|S|\right)$$

For example:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - (|A \cap B| + |A \cap C| + |B \cap C|) + |A \cap B \cap C|$$

The principle of inclusion-exclusion, combined with de Morgan's laws, can be used to count the intersection of sets as well. Let A be some universal set such that $A_k \subseteq A$ for each k, and let $\overline{A_k}$ represent the complement of A_k with respect to A. Then we have

$$\left|\bigcap_{i=1}^{N} A_i\right| = \left|\overline{\bigcup_{i=1}^{N} \overline{A_i}}\right|$$

thereby turning the problem of finding an intersection into the problem of finding a union.