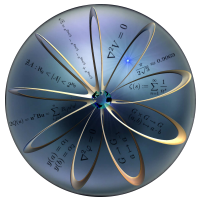

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arrows relation

Canonical name	ArrowsRelation
Date of creation	2013-03-22 17:48:54
Last modified on	2013-03-22 17:48:54
Owner	Henry (455)
Last modified by	Henry (455)
Numerical id	5
Author	Henry (455)
Entry type	Definition
Classification	msc 05A18
Classification	msc 03E05
Related topic	PartitionsLessThanCofinality
Related topic	ErdosRadoTheorem
Defines	homogeneous
Defines	arrows
Defines	homogeneous set
Defines	homogeneous subset

Let $[X]^\alpha = \{Y \subseteq X \mid |Y| = \alpha\}$, that is, the set of subsets of X of size α. Then given some cardinals κ, λ, α and β

$$\kappa \to (\lambda)^\alpha_\beta$$

states that for any set X of size κ and any function $f : [X]^\alpha \to \beta$, there is some $Y \subseteq X$ and some $\gamma \in \beta$ such that $|Y| = \lambda$ and for any $y \in [Y]^\alpha$, $f(y) = \gamma$.

In words, if f is a partition of $[X]^\alpha$ into β subsets then f is constant on a subset of size λ (a *homogeneous* subset).

As an example, the pigeonhole principle is the statement that if n is finite and $k < n$ then:

$$n \to 2^1_k$$

That is, if you try to partition n into fewer than n pieces then one piece has more than one element.

Observe that if

$$\kappa \to (\lambda)^\alpha_\beta$$

then the same statement holds if:

- κ is made larger (since the restriction of f to a set of size κ can be considered)
- λ is made smaller (since a subset of the homogeneous set will suffice)
- β is made smaller (since any partition into fewer than β pieces can be expanded by adding empty sets to the partition)
- α is made smaller (since a partition f of $[\kappa]^\gamma$ where $\gamma < \alpha$ can be extended to a partition f' of $[\kappa]^\alpha$ by $f'(X) = f(X_\gamma)$ where X_γ is the γ smallest elements of X)

$$\kappa \nrightarrow (\lambda)^\alpha_\beta$$

is used to state that the corresponding $\to$ relation is false.

References

- Jech, T. *Set Theory*, Springer-Verlag, 2003

- Just, W. and Weese, M. *Topics in Discovering Modern Set Theory, II*, American Mathematical Society, 1996