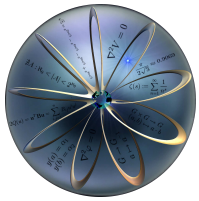

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proof of Bondy and Chvátal theorem

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Proof. The sufficiency of the condition is obvious and we shall prove the necessity by contradiction.

Assume that $G + uv$ is Hamiltonian but G is not. Then $G + uv$ has a Hamiltonian cycle containing the edge uv. Thus there exists a path $P = (x_1, \ldots, x_n)$ in G from $x_1 = u$ to $x_n = v$ meeting all the vertices of G. If x_i is adjacent to x_1 $(2 \leq i \leq n)$ then x_{i-1} is not adjacent to x_n, for otherwise $(x_1, x_i, x_{i+1}, \ldots, x_n, x_{i-1}, x_{i-2}, \ldots, x_1)$ is a Hamiltonian cycle of G. Thus $d(x_n) \leq (n-1) - d(x_1)$, that is $d(u) + d(v) \leq n - 1$, a contradiction $\square$