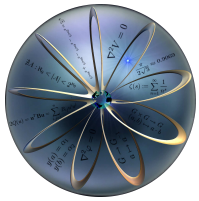

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Leibniz harmonic triangle

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The *Leibniz harmonic triangle* is a triangular arrangement of fractions in which the outermost diagonals consist of the reciprocals of the row numbers and each inner cell is the absolute value of the cell above minus the cell to the left. To put it algebraically, $L(r,1) = \frac{1}{n}$ (where r is the number of the row, starting from 1, and c is the column number, never more than r) and $L(r,c) = L(r-1,c-1) - L(r,c-1)$.

The first eight rows are:

$$
\begin{array}{ccccccccccccccc}
 & & & & & & 1 & & & & & & & & \\
 & & & & & \frac{1}{2} & & \frac{1}{2} & & & & & & & \\
 & & & & \frac{1}{3} & & \frac{1}{6} & & \frac{1}{3} & & & & & & \\
 & & & \frac{1}{4} & & \frac{1}{12} & & \frac{1}{12} & & \frac{1}{4} & & & & & \\
 & & \frac{1}{5} & & \frac{1}{20} & & \frac{1}{30} & & \frac{1}{20} & & \frac{1}{5} & & & & \\
 & \frac{1}{6} & & \frac{1}{30} & & \frac{1}{60} & & \frac{1}{60} & & \frac{1}{30} & & \frac{1}{6} & & & \\
\frac{1}{7} & & \frac{1}{42} & & \frac{1}{105} & & \frac{1}{140} & & \frac{1}{105} & & \frac{1}{42} & & \frac{1}{7} & & \\
\frac{1}{8} & & \frac{1}{56} & & \frac{1}{168} & & \frac{1}{280} & & \frac{1}{280} & & \frac{1}{168} & & \frac{1}{56} & & \frac{1}{8} \\
 & & \vdots & & & & \vdots & & & & \vdots & & & &
\end{array}
$$

The denominators are listed in A003506 of Sloane's OEIS, while the numerators, which are all 1s, are listed in A000012. The denominators of the second outermost diagonal are oblong numbers. The sum of the denominators in the nth row is $n2^{n-1}$.

Just as Pascal's triangle can be computed by using binomial coefficients, so can Leibniz's:

$$L(r,c) = \frac{1}{c\binom{r}{c}}$$

.

This triangle can be used to obtain examples for the `http://planetmath.org/ErdHosStrausCo` Straus conjecture when $4|n$.

References

[1] A. Ayoub, "The Harmonic Triangle and the Beta Function" *Math. Magazine* **60** 4 (1987): 223 - 225

[2] D. Darling, "Leibniz' harmonic triangle" in *The Universal Book of Mathematics: From Abracadabra To Zeno's paradoxes*. Hoboken, New Jersey: Wiley (2004)