flow

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Defines	maximum flow
Defines	source
Defines	sink
Defines	Kirchoff's law

On a digraph, define a *sink* to be a vertex with out-degree zero and a *source* to be a vertex with in-degree zero. Let G be a digraph with non-negative weights and with exactly one sink and exactly one source. A *flow* on G is an assignment $f : E(G) \to \mathbb{R}$ of values to each edge of G satisfying certain rules:

1. For any edge e, we must have $0 \leq f(e) \leq W(e)$ (where $W(e)$ is the weight of e).

2. For any vertex v, excluding the source and the sink, let E_{in} be the set of edges incident to v and let E_{out} be the set of edges incident from v. Then we must have

$$\sum_{e \in E_{in}} f(e) = \sum_{e \in E_{out}} f(e).$$

Let E_{source} be the edges incident from the source, and let E_{sink} be the set of edges incident to the sink. If f is a flow, then

$$\sum_{e \in E_{sink}} f(e) = \sum_{e \in E_{source}} f(e)\ .$$

We will refer to this quantity as the amount of flow.

Note that a flow given by $f(e) = 0$ trivially satisfies these conditions. We are typically more interested in *maximum flows*, where the amount of flow is maximized for a particular graph.

We may interpret a flow as a means of transmitting something through a network. Suppose we think of the edges in a graph as pipes, with the weights corresponding with the capacities of the pipes; we are pouring water into the system through the source and draining it through the sink. Then the first rule requires that we do not pump more water through a pipe than is possible, and the second rule requires that any water entering a junction of pipes must leave. Under this interpretation, the maximum amount of flow corresponds to the maximum amount of water we could pump through this network.

Instead of water in pipes, one may think of electric charge in a network of conductors. Rule (2) above is one of Kirchoff's two laws for such networks; the other says that the sum of the voltage drops around any circuit is zero.