example of tree (set theoretic)

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The set $\mathbb{Z}^+$ is a tree with $<_T=<$. This isn't a very interesting tree, since it simply consists of a line of nodes. However note that the height is ω even though no particular node has that height.

A more interesting tree using $\mathbb{Z}^+$ defines $m <_T n$ if $i^a = m$ and $i^b = n$ for some $i, a, b \in \mathbb{Z}^+ \cup \{0\}$. Then 1 is the root, and all numbers which are not powers of another number are in T_1. Then all squares (which are not also fourth powers) for T_2, and so on.

To illustrate the concept of a cofinal branch, observe that for any limit ordinal κ we can construct a κ-tree which has no cofinal branches. We let $T = \{(\alpha, \beta) | \alpha < \beta < \kappa\}$ and $(\alpha_1, \beta_1) <_T (\alpha_2, \beta_2) \leftrightarrow \alpha_1 < \alpha_2 \wedge \beta_1 = \beta_2$. The tree then has κ disjoint branches, each consisting of the set $\{(\alpha, \beta) | \alpha < \beta\}$ for some $\beta < \kappa$. No branch is cofinal, since each branch is capped at β elements, but for any $\gamma < \kappa$, there is a branch of height $\gamma + 1$. Hence the supremum of the heights is κ.