difference set

Canonical name	DifferenceSet
Date of creation	2013-03-22 16:50:04
Last modified on	2013-03-22 16:50:04
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	9
Author	CWoo (3771)
Entry type	Definition
Classification	msc 05B10
Defines	non-trivial difference set
Defines	planar difference set

Definition. Let A be a finite abelian group of order n. A subset D of A is said to be a *difference set* (in A) if there is a positive integer m such that every non-zero element of A can be expressed as the difference of elements of D in exactly m ways.

If D has d elements, then we have the equation

$$m(n-1) = d(d-1).$$

In the equation, we are counting the number of pairs of distinct elements of D. On the left hand side, we are counting it by noting that there are $m(n-1)$ pairs of elements of D such that their difference is non-zero. On the right hand side, we first count the number of elements in D^2, which is d^2, then subtracted by d, since there are d pairs of $(x, y) \in D^2$ such that $x = y$.

A difference set with parameters n, m, d defined above is also called a (n, d, m)-difference set. A difference set is said to be *non-trivial* if $1 < d < n-1$. A difference set is said to be *planar* if $m = 1$.

Difference sets versus square designs. Recall that a square design is a τ-(ν, κ, λ)-http://planetmath.org/Designdesign where $\tau = 2$ and the number ν of points is the same as the number b of blocks. In a general design, b is related to the other numbers by the equation

$$b\binom{\kappa}{\tau} = \lambda\binom{\nu}{\tau}.$$

So in a square design, the equation reduces to $b\kappa(\kappa-1) = \lambda\nu(\nu-1)$, or

$$\lambda(\nu-1) = \kappa(\kappa-1),$$

which is identical to the equation above for the difference set. A square design with parameters λ, ν, κ is called a square (ν, κ, λ)-design.

One can show that a subset D of an abelian group A is an (n, d, m)-difference set iff it is a square (n, d, m)-design where A is the set of points and $\{D + a \mid a \in A\}$ is the set of blocks.