



Poincaré formula

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Let K be finite oriented simplicial complex of dimension n . Then

$$\chi(K) = \sum_{p=0}^n (-1)^p R_p(K),$$

where $\chi(K)$ is the Euler characteristic of K , and $R_p(K)$ is the p -th Betti number of K .

This formula also works when K is any finite CW complex. The Poincaré formula is also known as the Euler-Poincaré formula, for it is a generalization of the Euler formula for polyhedra.

If K is a compact connected orientable surface with no boundary and with genus h , then $\chi(K) = 2 - 2h$. If K is non-orientable instead, then $\chi(K) = 2 - h$.