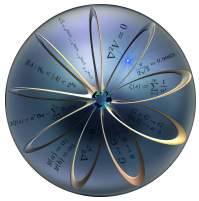

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proof that ω has the tree property

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Let T be a tree with finite levels and an infinite number of elements. Then consider the elements of T_0. T can be partitioned into the set of descendants of each of these elements, and since any finite partition of an infinite set has at least one infinite partition, some element x_0 in T_0 has an infinite number of descendants. The same procedure can be applied to the children of x_0 to give an element $x_1 \in T_1$ which has an infinite number of descendants, and then to the children of x_1, and so on. This gives a sequence $X = \langle x_0, x_1, \ldots \rangle$. The sequence is infinite since each element has an infinite number of descendants, and since x_{i+1} is always of child of x_i, X is a branch, and therefore an infinite branch of T.