Pascal's triangle is symmetrical along its central column

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Owner	PrimeFan (13766)
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Author	PrimeFan (13766)
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As a consequence of Pascal's rule, we see that Pascal's triangle is symmetrical along its central column (the column containing the central binomial coefficients). Expressing individual values in Pascal's triangle T as $T(n, k)$, with n and k being integers obeying the relation $-1 < k \leq n$, this means that each $T(n, k) = T(n, n - k)$.

Since Pascal's triangle is essentially a table in which to look up binomial coefficients,

$$T(n, k) = \binom{n}{k}.$$

From Pascal's rule it follows that $T(n, k) = T(n - 1, k - 1) + T(n - 1, k)$.

Obviously $T(0, k) = 1$ because there is only one way to choose no items from a collection of k items; likewise, $T(k, k) = 1$ because there is only one way to choose k items from a collection of k items. Therefore, the leftmost and rightmost column of Pascal's triangle are filled with 1's. Almost as obvious is the fact that $T(1, k) = k$ because there are k ways to choose just one item from a collection of k items; likewise, $T(k - 1, k) = k$ because there are k ways to choose all but one item from a collection of k items since leaving out one item in turn can only be done k times in such a collection.

From the foregoing, row 1 of Pascal's triangle is 1, 1, row 2 is 1, 2, 1 and row 3 is 1, 3, 3, 1. From Pascal's rule it follows that even-numbered rows (with an odd number of columns, and their highest, central value at $T(\frac{k}{2}, k)$) will be symmetrical along the central value if the previous row was also symmetrical, while odd-numbered rows (with an even number of columns, and the highest, central value at both $T(\frac{k-1}{2}, k)$ and $T(\frac{k+1}{2}, k)$ will be symmetrical about the central values if the previous row was symmetrical. Since the first three rows are symmetrical, all the following rows are also symmetrical.