Kuratowski's theorem

A finite graph is planar if and only if it contains no subgraph that is isomorphic to or is a subdivision of K_5 or $K_{3,3}$, where K_5 is the complete graph of order 5 and $K_{3,3}$ is the complete bipartite graph with 3 vertices in each of the halfs. Wagner's theorem is an equivalent later result.

References

[1] Kazimierz Kuratowski. Sur le problème des courbes gauches en topologie. *Fund. Math.*, 15:271–283, 1930.