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directed graph

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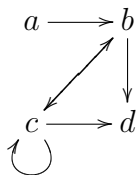
A *directed graph* or *digraph* is a pair $G = (V, E)$ where V is a set of *vertices* and E is a subset of $V \times V$ called *edges* or *arcs*.

If E is symmetric (i.e., $(u, v) \in E$ if and only if $(v, u) \in E$), then the digraph is isomorphic to an ordinary (that is, undirected) graph.

Digraphs are generally drawn in a similar manner to graphs with arrows on the edges to indicate a sense of direction. For example, the digraph

$$(\{a, b, c, d\}, \{(a, b), (b, d), (b, c), (c, b), (c, c), (c, d)\})$$

may be drawn as



Since the graph is directed, one has the concept of the number of edges originating or terminating at a given vertex v . The out-degree, $d_{\text{out}}(v)$ of a vertex v is the number of edges having v as their originating vertex; similarly, the in-degree, $d_{\text{in}}(v)$ is the number of edges having v as their terminating vertex.

If the graph has a finite number of vertices, say v_1, \dots, v_n , then obviously

$$\sum_{i=1}^n d_{\text{in}}(v_i) = \sum_{i=1}^n d_{\text{out}}(v_i)$$

A *directed path* in a digraph G is a sequence of edges e_1, \dots, e_k such that the end vertex of e_i is the start vertex of e_{i+1} for $i = 1, 2, \dots, k-1$. Such a path is called a *directed circuit* if, in addition, the end vertex of e_k is the start vertex of e_1 .

A digraph is *connected* (or, sometimes, *strongly connected*) if for every pair of vertices u and v there is a directed path from u to v . In addition, a digraph $G = (V, E)$ is said to have a *root* $r \in V$ if every vertex $v \in V$ is reachable from r , i.e. if there is a directed path from r to v .

A digraph is called a *directed tree* if it has a root and if the underlying (undirected) graph is a tree. That is, it must morphologically look like a tree, and the structure imposed by the directional arrows must flow “away” from the root.

If H is a subgraph of a digraph G , then H is said to be a *directed spanning tree* of G if H is a directed tree and H contains all vertices of G . This is a direct analog of the concept of spanning trees for undirected graphs. Note that if r is the root of H , then r is clearly a root of G . Also, if r is any root of G , it is possible to construct a directed spanning tree of G with root r : construct H edge by edge starting from r . At each vertex, add any edge from G whose terminus is a vertex not yet in H . Since r is a root of G , this process is guaranteed to include each vertex in G ; since we are choosing at each step only vertices not yet visited, we are guaranteed to end up with a tree.