tight

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A bound is *tight* if it can be realized. A bound that is not tight is sometimes said to be *slack*.

For example, let $\mathcal{S}$ be the collection of all finite subsets of $\mathbb{R}^2$ in general position. Define a function $g\colon \mathbb{N} \to \mathbb{N}$ as follows. First, let the weight of an element S of $\mathcal{S}$ be the size of its largest convex subset, that is,

$$w(S) = \max\{|T|\colon T \subset S \text{ and } T \text{ is convex}\}.$$

The function g is defined by

$$g(n) = \min\{|S|\colon w(S) \geq n\},$$

that is, $g(n)$ is the smallest number such that any collection of $g(n)$ points in general position contains a convex n-gon. (By the `http://planetmath.org/HappyEndingProblemE` Szekeres theorem, $g(n)$ is always finite, so g is a well-defined function.) The bounds for g due to Erdős and Szekeres are

$$2^{n-2} + 1 \leq g(n) \leq \binom{2n-4}{n-2} + 1.$$

The lower bound is tight because for each n, there is a set of 2^{n-2} points in general position which contains no convex n-gon. On the other hand, the upper bound is believed to be slack. In fact, according to the Erdős-Szekeres conjecture, the formula for $g(n)$ is exactly the lower bound: $g(n) = 2^{n-2} + 1$.