

The n th *Takeuchi number* T_n is the value of the function $T(n, 0, n + 1)$ which measures how many times the Takeuchi function $t(x, y, z)$ has to call itself to give the answer starting with $x = n, y = 0, z = n + 1$. For example, the second Takeuchi number is 4, since $t(2, 0, 3)$ requires four recursions to obtain the answer 2. The first few Takeuchi numbers are 0, 1, 4, 14, 53, 223, 1034, 5221, 28437, listed in A000651 of Sloane's OEIS. Prellberg gives a formula for the asymptotic growth of the Takeuchi numbers:

$$T_n \sim c B_n \exp\left(\frac{1}{2W(n)^2}\right)$$

, where c is the Takeuchi-Prellberg constant (approximately 2.2394331), B_n is the n th Bernoulli number and $W(x)$ is Lambert's W function.

References

- [1] Steven R. Finch *Mathematical Constants* New York: Cambridge University Press (2003): 321
- [2] T. Prellberg, "On the asymptotics of Takeuchi numbers", *Symbolic computation, number theory, special functions, physics and combinatorics*, Dordrecht: Kluwer Acad. Publ. (2001): 231 - 242.