Vandermonde identity

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Theorem 1 ([**?**] 24.1.1 formula A. II.)**.** *For any p and q and any k with $0 \leq k \leq p+q$,*

$$\binom{p+q}{k} = \sum_{i=0}^{k} \binom{p}{i}\binom{q}{k-i}. \tag{*}$$

Proof. Let P and Q be disjoint sets with $|P| = p$ and $|Q| = q$. Then the left-hand side of Equation (*) is equal to the number of subsets of $P \cup Q$ of size k. To build a subset of $P \cup Q$ of size k, we first decide how many elements, say i with $0 \leq i \leq k$, we will select from P. We can then select those elements in $\binom{p}{i}$ ways. Once we have done so, we must select the remaining $k-i$ elements from Q, which we can do in $\binom{q}{k-i}$ ways. Thus there are $\binom{p}{i}\binom{q}{k-i}$ ways to select a subset of $P \cup Q$ of size k subject to the restriction that exactly i elements come from P. Summing over all possible i completes the proof. $\square$

References

[1] Abramowitz, M., and I. A. Stegun, eds. *Handbook of Mathematical Functions*. National Bureau of Standards, Dover, New York, 1974.