proof of Pascal's rule

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Owner	drini (3)
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Author	drini (3)
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The definition of $\binom{n}{k}$ is the number of k-subsets out from an n-set. Using this combinatorial meaning the proof is straightforward.

Let x a distinct element from the n-set. As previously stated, $\binom{n}{k}$ counts the number of subsets with k elements, chosen from the set with n elements. Now, some of these subsets will contain x and some others don't.

The number of k-subsets not containing x is $\binom{n-1}{k}$, since we need to choose k elements from the $n-1$ elements different from x.

The number of k-subsets containing x is $\binom{n-1}{k-1}$, because if it is given that x is in the subset, we only need to choose the remaining $k-1$ elements from the $n-1$ elements that are different from x.

Thus

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}.$$