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Laver table

Canonical name	LaverTable
Date of creation	2013-03-22 16:26:13
Last modified on	2013-03-22 16:26:13
Owner	PrimeFan (13766)
Last modified by	PrimeFan (13766)
Numerical id	6
Author	PrimeFan (13766)
Entry type	Definition
Classification	msc 05C38

A *Laver table* L_n for a given integer $n > 0$ has 2^n rows i and columns j with each entry being determined thus: $L_n(i, j) = i \star j$, with $i \star 1 = (i \bmod 2^n) + 1$ for the first column. Subsequent rows are calculated with $i \star (j \star k) := (i \star j) \star (i \star k)$.

For example, L_2 is

$$\begin{bmatrix} 2 & 4 & 2 & 4 \\ 3 & 4 & 3 & 4 \\ 4 & 4 & 4 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

There is no known closed formula to calculate the entries of a Laver table directly, and it is in fact suspected that such a formula does not exist.

The entries repeat with a certain periodicity m . This periodicity is always a power of 2; the first few periodicities are 1, 1, 2, 4, 4, 8, 8, 8, 8, 16, 16, ... (see A098820 in Sloane's OEIS). The sequence is increasing, and it was proved in 1995 by Richard Laver that under the assumption that there exists a rank-into-rank, it actually tends towards infinity. Nevertheless, it grows extremely slowly; Randall Dougherty showed that the first n for which the table entries' period can possibly be 32 is $A(9, A(8, A(8, 255)))$, where A denotes the Ackermann function.

References

- [1] P. Dehornoy, "Das Unendliche als Quelle der Erkenntnis", *Spektrum der Wissenschaft Spezial* 1/2001: 86 - 90
- [2] R. Laver, "On the Algebra of Elementary Embeddings of a Rank into Itself", *Advances in Mathematics* **110** (1995): 334

This entry based entirely on a Wikipedia entry from a PlanetMath member.