sum of powers of binomial coefficients

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Some results exist on sums of powers of binomial coefficients. Define A_s as follows:

$$A_s(n) = \sum_{i=0}^{n} \binom{n}{i}^s$$

for s a positive integer and n a nonnegative integer.

For $s = 1$, the binomial theorem implies that the sum $A_1(n)$ is simply 2^n.

For $s = 2$, the following result on the sum of the squares of the binomial coefficients $\binom{n}{i}$ holds:

$$A_2(n) = \sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}$$

that is, $A_2(n)$ is the nth central binomial coefficient.

Proof: This result follows immediately from the Vandermonde identity:

$$\binom{p+q}{k} = \sum_{i=0}^{k} \binom{p}{i}\binom{q}{k-i}$$

upon choosing $p = q = k = n$ and observing that $\binom{n}{n-i} = \binom{n}{i}$.

Expressions for $A_s(n)$ for larger values of s exist in terms of hypergeometric functions.