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Takeuchi function

Canonical name	TakeuchiFunction
Date of creation	2013-03-22 17:33:07
Last modified on	2013-03-22 17:33:07
Owner	PrimeFan (13766)
Last modified by	PrimeFan (13766)
Numerical id	4
Author	PrimeFan (13766)
Entry type	Definition
Classification	msc 05A16

The *Takeuchi function* is a triply recursive 3-parameter function originally defined by Ichiro Takeuchi in 1978 as $t(x, y, z) = y$ if $x \leq y$ and $t(x, y, z) = t(t(x - 1, y, z), t(y - 1, z, x), t(z - 1, x, y))$ otherwise. Later John McCarthy simplified the definition of the function as $t(x, y, z) = y$ if $x \leq y$, $t(x, y, z) = z$ if $y \leq z$ and $t(x, y, z) = x$ in all other cases.

For example, $t(194, 13, 5) = 194$ since 194 is not less than 13, and 13 is not less than 5. The return value of the function “is on no practical significance,” but the function itself “is useful for benchmark testing of programming languages.” (Finch, 2003) The function $T(x, y, z)$ is the number of times t calls itself to obtain the return value. A properly optimized implementation of the function in a given programming language should not require more recursion than T indicates.

References

- [1] Steven R. Finch *Mathematical Constants* New York: Cambridge University Press (2003): 321