reduction algorithm for symmetric polynomials

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We give here an algorithm for reducing a symmetric polynomial into a polynomial in the elementary symmetric polynomials.

We define the *height* of a monomial $x_1^{e_1} \cdots x_n^{e_n}$ in $R[x_1, \ldots, x_n]$ to be $e_1 + 2e_2 + \cdots + ne_n$. The height of a polynomial is defined to be the maximum height of any of its monomial terms, or 0 if it is the zero polynomial.

Let f be a symmetric polynomial. We reduce f into elementary symmetric polynomials by induction on the height of f. Let $cx_1^{e_1} \cdots x_n^{e_n}$ be the monomial term of maximal height in f. Consider the polynomial

$$g := f - cs_1^{e_n - e_{n-1}} s_2^{e_{n-1} - e_{n-2}} \cdots s_{n-1}^{e_2 - e_1} s_n^{e_1}$$

where s_k is the k–th elementary symmetric polynomial in the n variables $x_1, \ldots, x_n$. Then g is a symmetric polynomial of lower height than f, so by the induction hypothesis, g is a polynomial in $s_1, \ldots, s_n$, and it follows immediately that f is also a polynomial in $s_1, \ldots, s_n$.