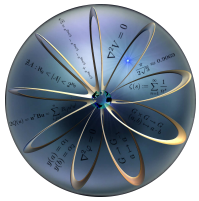

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harmonic function

Canonical name	HarmonicFunction1
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A real or complex-valued function $f : V \to \mathbb{R}$ or $f : V \to \mathbb{C}$ defined on the vertices V of a graph $G = (V, E)$ is called *harmonic* at $v \in V$ if its value at v is its average value at the neighbours of v:

$$f(v) = \frac{1}{\deg(v)} \sum_{\{u,v\} \in E} f(u).$$

It is called harmonic *except on* A, for some $A \subseteq V$, if it is harmonic at each $v \in V \setminus A$, and harmonic if it is harmonic at each $v \in V$.

Any harmonic $f : \mathbb{Z}^n \to \mathbb{R}$, where $\mathbb{Z}^n$ is the n-dimensional grid, is if below (or above). However, this is not necessarily true on other graphs.