multiplicative encoding

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The *multiplicative encoding* of a finite sequence a of positive integers k long is the product of the first k primes with the members of the sequence used as exponents, thus

$$\prod_{i=1}^{k} {p_i}^{a_i},$$

with p_i being the ith prime number. For example, the fourth row of Pascal's triangle is 1, 3, 3, 1. The multiplicative encoding is $2^1 3^3 5^3 7^1 = 47250$.

Encryption is not the purpose of multiplicative encoding, as the original sequence is easily retrieved with trial division. However, there are applications in combinatorics. Neil Sloane, for example, encodes the most famous number triangles as multiplicative encodings of the rows in order. While the resulting sequence for Pascal's triangle does not consist of squarefree numbers (save the first two), it does contain only singly even numbers.

Another use is in logic, such as Kurt Gödel encoding a logical proposition as a single integer. As an example, Nagel and Newman convert $(\exists x)(x = sy)$ to the integer sequence 8, 4, 13, 9, 8, 13, 5, 7, 17, 9, and by multiplicative encoding to the single integer 172225505803959398742621651659678877886965404082311908389214945

References

[1] Ernest Nagel & James Newman, *Gödel's Proof.* New York: New York University Press (2001): 75 - 76

[2] Neil Sloane, *The Encyclopedia of Integer Sequences.* New York: Academic Press (1995): M1722