proof of Ramsey's theorem

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$$\omega \to (\omega)^n_k$$

is proven by induction on n.

If $n = 1$ then this just states that any partition of an infinite set into a finite number of subsets must include an infinite set; that is, the union of a finite number of finite sets is finite. This is simple enough to prove: since there are a finite number of sets, there is a largest set of size x. Let the number of sets be y. Then the size of the union is no more than xy.

If

$$\omega \to (\omega)^n_k$$

then we can show that

$$\omega \to (\omega)^{n+1}_k$$

Let f be some coloring of $[S]^{n+1}$ by k where S is an infinite subset of ω. Observe that, given an $x < \omega$, we can define $f^x \colon [S \setminus \{x\}]^n \to k$ by $f^x(X) = f(\{x\} \cup X)$. Since S is infinite, by the induction hypothesis this will have an infinite homogeneous set.

Then we define a sequence of integers $\langle n_i \rangle_{i \in \omega}$ and a sequence of infinite subsets of ω, $\langle S_i \rangle_{i \in \omega}$ by induction. Let $n_0 = 0$ and let $S_0 = \omega$. Given n_i and S_i for $i \leq j$ we can define S_j as an infinite homogeneous set for $f^{n_i} \colon [S_{j-1}]^n \to k$ and n_j as the least element of S_j.

Obviously $N = \bigcup \{n_i\}$ is infinite, and it is also homogeneous, since each n_i is contained in S_j for each $j \leq i$.