graph homeomorphism

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Defines	simple subdivision

Let $G = (V, E)$ be a simple undirected graph. A *simple subdivision* is the replacement of an edge $(x, y) \in E$ with a pair of edges $(x, z), (z, y)$, z being a new vertex, *i.e.*, $z \notin V$. The reverse operation of a simple subdivision is an edge-contraction through a vertex of degree 2.

Two graphs G_1, G_2 are *homeomorphic* if G_1 can be transformed into G_2 via a finite sequence of simple subdivisions and edge-contractions through vertices of degree 2. It is easy to see that graph homeomorphism is an equivalence relation.

Equivalently, G_1 and G_2 are homeomorphic if there exists a third graph G_3 such that both G_1 and G_2 can be obtained from G_3 via a finite sequence of edge-contractions through vertices of degree 2.

If a graph G has a subgraph H which is homeomorphic to a graph G' having no vertices of degree 2, then G' is a minor of G. The vice versa is not true: as a counterexample, the Petersen graph has K_5 as a minor, but no subgraph homeomorphic to K_5. This happens because a graph homeomorphism cannot change the number of vertices of degree $d \neq 2$: since all the vertices of K_5 have degree 4 and all the vertices of the Petersen graph have degree 3, no subgraph of the Petersen graph can be homeomorphic to K_5.