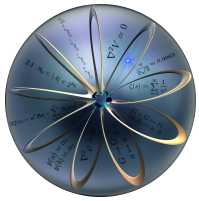

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some formulas involving rising factorial

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Recall that, for $a \in \mathbb{C}$ and n a nonnegative integer, the rising factorial $(a)_n$ is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k).$$

The following results hold regarding the rising factorial:

- For all $a \in \mathbb{C}$, we have $(a)_0 = 1$.
- For all nonnegative integers n, we have $(1)_n = n!$.
- The binomial coefficients are given by

$$\binom{a}{n} = \frac{(-1)^n (-a)_n}{n!}.$$

- The rising factorial relates to the gamma function. One relation is given by the formula

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}.$$

  This formula can be used to extend the definition of rising factorial so that n can be any complex number provided that $a+n$ is not a nonpositive integer.

- Another relation between the rising factorial and the gamma function is given by

$$\Gamma(a) = \lim_{n \to \infty} \frac{n!\, n^{a-1}}{(a)_n}.$$