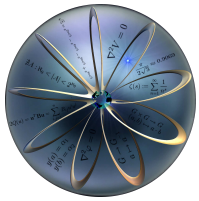

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permutation pattern

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A *permutation pattern* is simply a permutation viewed as its representation in one-line notation. Let $\pi = \pi_1\pi_2\ldots\pi_k$ be a permutation pattern on k symbols. Then for any permutation $\sigma = \sigma_1\sigma_2\ldots\sigma_n \in \mathfrak{S}_n$, we say that σ *contains* π if there is a (not necessarily contiguous) subword of σ of length k that is order-isomorphic to π. More formally, for any subset $J = \{j_1, \ldots, j_k\} \subset \{1, \ldots, n\}$ of cardinality k, write

$$\sigma_J = \sigma_{j_1}\sigma_{j_2}\ldots\sigma_{j_k}.$$

There is a `http://planetmath.org/GroupHomomorphism`isomorphism $\mathfrak{S}_J \to \mathfrak{S}_k$. We say that σ *contains* π if there is some $J \subset \{1, \ldots, n\}$ of cardinality k such that $\sigma_J \mapsto \pi$ under the isomorphism.

If a permutation σ does not contain π, then we say that σ *avoids* the pattern π.

For example, let $\pi = 132$. Then the permutation $\sigma = 1234$ avoids π, since σ is strictly increasing but π has a descent. On the other hand, $\tau = 1423$ contains π twice, once as the subword 142 and once as the subword 143.

Knuth showed that a permutation is stack-sortable if and only if it avoids the permutation pattern 231.