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Ramsey’s theorem

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Ramsey's theorem states that a particular arrows relation,

$$\omega \rightarrow (\omega)_k^n$$

holds for any integers n and k .

In words, if f is a function on sets of integers of size n whose range is finite then there is some infinite $X \subseteq \omega$ such that f is constant on the subsets of X of size n .

As an example, consider the case where $n = k = 2$, and $f: [\omega]^2 \rightarrow \{0, 1\}$ is defined by:

$$f(\{x, y\}) = \begin{cases} 1 & \text{if } x = y^2 \text{ or } y = x^2 \\ 0 & \text{otherwise} \end{cases}$$

Then let $X \subseteq \omega$ be the set of integers which are not perfect squares. This is clearly infinite, and obviously if $x, y \in X$ then neither $x = y^2$ nor $y = x^2$, so f is homogeneous on X .