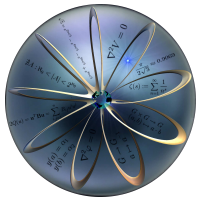

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tactical decomposition

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Let $\mathcal{I}$ be an incidence structure with point set $\mathcal{P}$ and block set $\mathcal{B}$. Let $X_\mathcal{P}$ be a partition of $\mathcal{P}$ into classes $\mathcal{P}_i$, and $X_\mathcal{B}$ a partition of $\mathcal{B}$ into classes $\mathcal{B}_j$. Let $\#(\text{P}, \mathcal{B}_j)$ denote for a moment the number of blocks in class $\mathcal{B}_j$ incident with point P, and $\#(\text{B}, \mathcal{P}_i)$ the number of points in class $\mathcal{P}_i$ incident with block B. Now the pair $(X_\mathcal{P}, X_\mathcal{B})$ is said to be

- **point-tactical** iff $\#(\text{P}, \mathcal{B}_j)$ is for any P the same for all $\mathcal{B}_j$, and is the same for all P within a class $\mathcal{P}_i$,
- **block-tactical** iff $\#(\text{B}, \mathcal{P}_i)$ is for any B the same for all $\mathcal{P}_i$, and is the same for all B within a class $\mathcal{B}_j$,
- a **tactical decomposition** if both hold.

An incidence structure admitting a tactical decomposition with a single point class $\mathcal{P}_0 = \mathcal{P}$ is called **resolvable** and $X_\mathcal{B}$ its **resolution**. Note $\#(\text{P}, \mathcal{B}_j)$ is now a constant throughout. If the constant is 1 the resolution is called a **parallelism**.

Example of point- and block-tactical: let $\mathcal{I}$ be simple (it's a hypergraph) and let $(X_\mathcal{P}, X_\mathcal{B})$ partition $\mathcal{P}$ and $\mathcal{B}$ into a single class each. This is point-tactical for a regular hypergraph, and block-tactical for a uniform hypergraph.

Example of parallelism: an affine plane (lines are the blocks, with parallel ones in the same class).

A natural example of a tactical decomposition is provided by the automorphism group G of $\mathcal{I}$. It induces a tactical decomposition with as point classes the orbits of G acting on $\mathcal{P}$ and as block classes the orbits of G acting on $\mathcal{B}$.

Trivial example of a tactical decomposition: a partition into singleton point and block classes.

The term tactical decomposition (*taktische Zerlegung* in German) was introduced by Peter Dembowski.