generating function for the reciprocal central binomial coefficients

Canonical name	GeneratingFunctionForTheReciprocalCentralBinomialCoefficients
Date of creation	2013-03-22 18:58:09
Last modified on	2013-03-22 18:58:09
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Last modified by	juanman (12619)
Numerical id	12
Author	juanman (12619)
Entry type	Result
Classification	msc 05A19
Classification	msc 11B65
Classification	msc 05A10
Classification	msc 05A15
Synonym	convergent series

It is well known that the sequence called central binomial coefficients is defined by $\binom{2n}{n}$ and whose initial terms are $1, 2, 6, 20, 70, 252, ...$ has a generating function $\frac{1}{\sqrt{1-4x}}$. But it is less known the fact that the function

$$\frac{4\left(\sqrt{4-x}+\sqrt{x}\ \arcsin(\frac{\sqrt{x}}{2})\right)}{\sqrt{(4-x)^3}}$$

has ordinary power series

$$1+\frac{x}{2}+\frac{x^2}{6}+\frac{x^3}{20}+\frac{x^4}{70}+\frac{x^5}{252}+...$$

This means that such a function is a generating function for the reciprocals $\binom{2n}{n}^{-1}$.

From that expression we can see that the numerical series $\sum_{n=0}^{\infty}\binom{2n}{n}^{-1}$ sums $\frac{4\left(\sqrt{3}+\frac{\pi}{6}\right)}{3\sqrt{3}}$ which has the approximate value 1,7363998587187151.

Reference:

1) Renzo Sprugnoli, *Sum of reciprocals of the Central Binomial Coefficients*, Integers: electronic journal of combinatorial number theory, 6 (2006) A27, 1-18