generating function for the reciprocal Catalan numbers

Canonical name	GeneratingFunctionForTheReciprocalCatalanNumbers
Date of creation	2013-03-22 19:05:12
Last modified on	2013-03-22 19:05:12
Owner	juanman (12619)
Last modified by	juanman (12619)
Numerical id	10
Author	juanman (12619)
Entry type	Derivation
Classification	msc 05A19
Classification	msc 05A15
Classification	msc 05A10
Related topic	CatalanNumbers

The series

$$1 + x + \frac{x^2}{2} + \frac{x^3}{5} + \frac{x^4}{14} + \frac{x^5}{42} + \frac{x^6}{132} + \frac{x^7}{429} + \cdots$$

whose coefficients are the reciprocal of the Catalan numbers $\frac{\binom{2n}{n}}{n+1}$, has as a generating function

$$\frac{2\left(\sqrt{4-x}\,(8+x) + 12\sqrt{x}\,\arctan(\frac{\sqrt{x}}{\sqrt{4-x}})\right)}{\sqrt{(4-x)^5}}$$

To deduce such a formula the easy way, one starts from the generating function of the reciprocal central binomial coefficients and having into account the relation

$$\frac{d}{dx}\left(\frac{x^{n+1}}{\binom{2n}{n}}\right) = \frac{(n+1)x^n}{\binom{2n}{n}}$$

for each term in the corresponding series and applied to the function in the region of uniform convergence. Another method is almost exaclty the same like in the derivation of the generating function for the reciprocal central binomial coefficients.