Stone space

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Defines	dual algebra

A *Stone space*, also called a *Boolean space*, is a topological space that is zero-dimensional, `http://planetmath.org/T0Space`T_0 and compact. Equivalently, a Stone space is a totally disconnected compact Hausdorff space.

Given a stone space X, one may associate a Boolean algebra X^* by taking the set of all of its clopen sets. The set theoretic operations of intersection, union, and complement makes X^* a Boolean algebra. X^* is known as the *dual algebra* of X.

The significance of Stone spaces stems from *Stone duality*: a pervasive equivalence between the algebraic notions and theorems of Boolean algebras on one hand, and the topological notions and theorems of Stone spaces on the other. This equivalence comprises the content and consequences of M. H. Stone's representation theorem.

There is a bijective correspondence between the following

1. The class consisting of all Boolean spaces

2. The class consisting of all Boolean algebras

3. The class consisting of all Boolean rings

4. The class consisting of all prime spectra of von Neumann regular rings

In fact, viewing each class as a category equipped with the appropriate class of morphisms, the categories are naturally equivalent between one another.

More to come: partial proofs, constructions, categorical equivalence between the first three items due to Stone's Representation Theorem and references . . . .

References

[1] Paul R. Halmos, *Lectures on Boolean Algebras*, D. Van Nostrand Company, Inc., 1963.

[2] Peter T. Johnstone, *Stone Spaces*, Cambridge University Press, 1982.