modular inequality

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In any http://planetmath.org/latticelattice the self-dual *modular inequality* is true: if $x \leq z$ then $x \vee (y \wedge z) \leq (x \vee y) \wedge z$.

Proof. $x \leq x \vee y$ and we are given that $x \leq z$, so $x \leq (x \vee y) \wedge z$. Also, $y \wedge z \leq y \leq x \vee y$ and $y \wedge z \leq z$ imply that $y \wedge z \leq (x \vee y) \wedge z$. Therefore, $x \vee (y \wedge z) \leq (x \vee y) \wedge z$. $\square$