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equality

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Defines	identic equation

In any set S, the *equality*, denoted by "=", is a binary relation which is reflexive, symmetric, transitive and antisymmetric, i.e. it is an antisymmetric equivalence relation on S, or which is the same thing, the equality is a symmetric partial order on S.

In fact, for any set S, the smallest equivalence relation on S is the equality (by smallest we that it is contained in every equivalence relation on S). This offers a definition of "equality". From this, it is clear that there is only one equality relation on S. Its equivalence classes are all singletons $\{x\}$ where $x \in S$.

The concept of equality is essential in almost all branches of mathematics. A few examples will suffice:

$$\begin{aligned} 1+1 &= 2 \\ e^{i\pi} &= -1 \\ \mathbb{R}[i] &= \mathbb{C} \end{aligned}$$

(The second example is Euler's identity.)

Remark 1. Although the four characterising , reflexivity, http://planetmath.org/Symmetric transitivity and http://planetmath.org/Antisymmetricantisymmetry, determine the equality on S uniquely, they cannot be thought to form the definition of the equality, since the concept of antisymmetry already the equality.

Remark 2. An equality (equation) in a set S may be true regardless to the values of the variables involved in the equality; then one speaks of an *identity* or *identic equation* in this set. E.g. $(x+y)^2 = x^2+y^2$ is an identity in a field with http://planetmath.org/Characteristiccharacteristic 2.