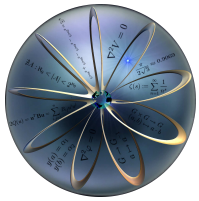

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Zorn's lemma

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Related topic	KuratowskisLemma
Related topic	EveryRingHasAMaximalIdeal
Related topic	InductivelyOr

If X is a partially ordered set such that every chain in X has an upper bound, then X has a maximal element.

Note that the empty chain in X has an upper bound in X if and only if X is non-empty. Because this case is rather different from the case of non-empty chains, Zorn's Lemma is often stated in the following form: If X is a non-empty partially ordered set such that every non-empty chain in X has an upper bound, then X has a maximal element. (In other words: Any non-empty inductively ordered set has a maximal element.)

In ZF, Zorn's Lemma is equivalent to the `http://planetmath.org/AxiomOfChoice`Axiom of Choice.