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order in an algebra

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Entry type	Definition
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Related topic	ComplexMultiplication
Defines	order
Defines	maximal order
Defines	conductor of an order

Let A be an algebra (not necessarily commutative), finitely generated over \mathbb{Q} . An *order* R of A is a subring of A which is finitely generated as a \mathbb{Z} -module and which satisfies $R \otimes \mathbb{Q} = A$.

Examples:

1. The ring of integers in a number field is an order, known as the *maximal order*.
2. Let K be a quadratic imaginary field and \mathcal{O}_K its ring of integers. For each integer $n \geq 1$ the ring $\mathcal{O} = \mathbb{Z} + n\mathcal{O}_K$ is an order of K (in fact it can be proved that every order of K is of this form). The number n is called the *conductor* of the order \mathcal{O} .

Reference: Joseph H. Silverman, *The arithmetic of elliptic curves*, Springer-Verlag, New York, 1986.