atom

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Defines	atomic poset
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Defines	atomistic

Let P be a poset, partially ordered by $\leq$. An element $a \in P$ is called an *atom* if it covers some minimal element of P. As a result, an atom is never minimal. A poset P is called *atomic* if for every element $p \in P$ that is not minimal has an atom a such that $a \leq p$.

Examples.

1. Let A be a set and $P = 2^A$ its power set. P is a poset ordered by $\subseteq$ with a unique minimal element $\varnothing$. Thus, all singleton subsets $\{a\}$ of A are atoms in P.

2. $\mathbb{Z}^+$ is partially ordered if we define $a \leq b$ to mean that $a \mid b$. Then 1 is a minimal element and any prime number p is an atom.

Remark. Given a lattice L with underlying poset P, an element $a \in L$ is called an *atom* (of L) if it is an atom in P. A lattice is a called an *atomic lattice* if its underlying poset is atomic. An *atomistic lattice* is an atomic lattice such that each element that is not minimal is a join of atoms. If a is an atom in a semimodular lattice L, and if a is not under x, then $a \vee x$ is an atom in any interval lattice I where $x = \bigwedge I$.

Examples.

1. $P = 2^A$, with the usual intersection and union as the lattice operations meet and join, is atomistic: every subset B of A is the union of all the singleton subsets of B.

2. $\mathbb{Z}^+$, partially ordered as above, with lattice binary operations defined by $a \wedge b = \gcd(a, b)$, and $a \vee b = \operatorname{lcm}(a, b)$, is a lattice that is atomic, as we have seen earlier. But it is not atomistic: 4 is not a join of 2's; 36 is not a join of 2 and 3 are just two counterexamples.