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complete lattice

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Defines	countably-complete lattice
Defines	κ-complete
Defines	κ-complete lattice

Complete lattices

A *complete lattice* is a poset P such that every subset of P has both a supremum and an infimum in P.

For a complete lattice L, the supremum of L is denoted by 1, and the infimum of L is denoted by 0. Thus L is a bounded lattice, with 1 as its greatest element and 0 as its least element. Moreover, 1 is the infimum of the empty set, and 0 is the supremum of the empty set.

Generalizations

A *countably complete lattice* is a poset P such that every countable subset of P has both a supremum and an infimum in P.

Let κ be an infinite cardinal. A κ-complete lattice is a lattice L such that for every subset $A \subseteq L$ with $|A| \leq \kappa$, both $\bigvee A$ and $\bigwedge A$ exist. (Note that an $\aleph_0$-complete lattice is the same as a countably complete lattice.)

Every complete lattice is a for every infinite cardinal κ, and in particular is a countably complete lattice. Every countably complete lattice is a bounded lattice.