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difference of lattice elements

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Let $\mathfrak{A}$ is a lattice with least element 0.

Let $a, b \in \mathfrak{A}$. A of a and b is an element $c \in \mathfrak{A}$ that $b \cap c = 0$ and $a \cup b = b \cup c$. When there is only one difference of a and b, it is denoted $a \setminus b$.

One immediate property is: 0 is the unique difference of any element a and itself ($a \setminus a = 0$). For if c is such a difference, then $a \cap c = 0$ and $a = a \cup c$. So $c \leq a$ by the second equation, and hence that $c = a \cap c = 0$ by the first equation.

For arbitrary lattices of two given elements do not necessarily exist. For some lattices there may be more than one difference of two given elements.

For a distributive lattice with bottom element 0, the difference of two elements, if it exists, must be unique. To see this, let c and d be two differences of a and b. Then

- $b \cap c = b \cap d = 0$, and
- $a \cup b = b \cup c = b \cup d$.

So $c = c \cap (b \cup c) = c \cap (b \cup d) = (c \cap b) \cup (c \cap d) = 0 \cup (c \cap d) = c \cap d$. Similarly, $d = d \cap c$. As a result, $c = c \cap d = d \cap c = d$.