dense total order

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A total order $(S, <)$ is *dense* if whenever $x < z$ in S, there exists at least one element y of S such that $x < y < z$. That is, each nontrivial closed interval has nonempty interior.

A subset T of a total order S is *dense in* S if for every $x, z \in S$ such that $x < z$, there exists some $y \in T$ such that $x < y < z$. Because of this, a dense total order S is sometimes said to be *dense in itself*.

For example, the integers with the usual order are not dense, since there is no integer strictly between 0 and 1. On the other hand, the rationals $\mathbb{Q}$ are dense, since whenever r and s are rational numbers, it follows that $(r+s)/2$ is a rational number strictly between r and s. Also, both $\mathbb{Q}$ and the irrationals $\mathbb{R} \setminus \mathbb{Q}$ are dense in $\mathbb{R}$.

It is usually convenient to assume that a dense order has at least two elements. This allows one to avoid the trivial cases of the one-point order and the empty order.