total order

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Synonym	linear order
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Related topic	PartialOrder
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Related topic	ProofOfGeneralizedIntermediateValueTheorem
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Defines	totally ordered set
Defines	linearly ordered set
Defines	comparability
Defines	totally ordered
Defines	linearly ordered
Defines	chain
Defines	totally-ordered set
Defines	linearly-ordered set
Defines	totally-ordered
Defines	linearly-ordered

A *totally ordered set* (or *linearly ordered set*) is a poset $(T, \leq)$ which has the property of *comparability*:

- for all $x, y \in T$, either $x \leq y$ or $y \leq x$.

In other words, a totally ordered set is a set T with a binary relation $\leq$ on it such that the following hold for all $x, y, z \in T$:

- $x \leq x$. (*reflexivity*)
- If $x \leq y$ and $y \leq x$, then $x = y$. (*antisymmetry*)
- If $x \leq y$ and $y \leq z$, then $x \leq z$. (*transitivity*)
- Either $x \leq y$ or $y \leq x$. (*comparability*)

The binary relation $\leq$ is then called a *total order* or a *linear order* (or *total ordering* or *linear ordering*). A totally ordered set is also sometimes called a *chain*, especially when it is considered as a subset of some other poset. If every nonempty subset of T has a least element, then the total order is called a `http://planetmath.org/WellOrderedSet`well-order.

Some people prefer to define the binary relation $<$ as a total order, rather than $\leq$. In this case, $<$ is required to be `http://planetmath.org/Transitive3`transitive and to obey the law of trichotomy. It is straightforward to check that this is equivalent to the above definition, with the usual relationship between $<$ and $\leq$ (that is, $x \leq y$ if and only if either $x < y$ or $x = y$).

A totally ordered set can also be defined as a lattice $(T, \vee, \wedge)$ in which the following property holds:

- for all $x, y \in T$, either $x \wedge y = x$ or $x \wedge y = y$.

Then totally ordered sets are `http://planetmath.org/DistributiveLattice`distributive lattices.