dual of Dilworth's theorem

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Theorem 1. *Let P be a poset of height h. Then P can be partitioned into h antichains and furthermore at least h antichains are required.*

Proof. Induction on h. If $h = 1$, then no elements of P are comparable, so P is an antichain. Now suppose that P has height $h \geq 2$ and that the theorem is true for $h - 1$. Let A_1 be the set of maximal elements of P. Then A_1 is an antichain in P and $P - A_1$ has height $h - 1$ since we have removed precisely one element from every chain. Hence, $P - A_1$ can be paritioned into $h - 1$ antichains $A_2, A_3, \ldots A_h$. Now we have the partition $A_1 \cup A_2 \cup \cdots \cup A_h$ of P into h antichains as desired.

The necessity of h antichains is trivial by the pigeonhole principle; since P has height h, it has a chain of length h, and each element of this chain must be placed in a different antichain of our partition. $\square$