Ockham algebra

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A lattice L is called an *Ockham algebra* if

1. L is distributive
2. L is bounded, with 0 as the bottom and 1 as the top
3. there is a unary operator $\neg$ on L with the following properties:
   (a) $\neg$ satisfies the de Morgan's laws; this means that:
      - $\neg(a \vee b) = \neg a \wedge \neg b$ and
      - $\neg(a \wedge b) = \neg a \vee \neg b$

   (b) $\neg 0 = 1$ and $\neg 1 = 0$

Such a unary operator is an example of a dual endomorphism. When applied, $\neg$ interchanges the operations of $\vee$ and $\wedge$, and 0 and 1.

An Ockham algebra is a generalization of a Boolean algebra, in the sense that $\neg$ replaces $'$, the complement operator, on a Boolean algebra.

Remarks.

- An intermediate concept is that of a De Morgan algebra, which is an Ockham algebra with the additional requirement that $\neg(\neg a) = a$.
- In the category of Ockham algebras, the morphism between any two objects is a $\{0,1\}$-`http://planetmath.org/LatticeHomomorphism`lattice homomorphism f that preserves $\neg$: $f(\neg a) = \neg f(a)$. In fact, $f(0) = f(\neg 1) = \neg f(1) = \neg 1 = 0$, so that it is safe to drop the assumption that f preserves 0.

References

[1] T.S. Blyth, J.C. Varlet, *Ockham Algebras*, Oxford University Press, (1994).

[2] T.S. Blyth, *Lattices and Ordered Algebraic Structures*, Springer, New York (2005).