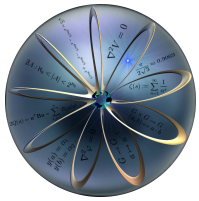

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pre-order

Canonical name	Preorder
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Synonym	semi-order
Synonym	semi-ordering
Synonym	semiorder
Synonym	semiordering
Related topic	WellQuasiOrdering
Related topic	PartialOrder
Defines	pre-ordered
Defines	preordered
Defines	semi-ordered
Defines	semiordered
Defines	quasi-ordered
Defines	quasiordered

Definition

A *pre-order* on a set S is a relation $\lesssim$ on S satisfying the following two axioms:

reflexivity: $s \lesssim s$ for all $s \in S$, and

transitivity: If $s \lesssim t$ and $t \lesssim u$, then $s \lesssim u$; for all $s, t, u \in S$.

Partial order induced by a pre-order

Given such a relation, define a new relation $s \sim t$ on S by

$$s \sim t \text{ if and only if } s \lesssim t \text{ and } t \lesssim s.$$

Then $\sim$ is an equivalence relation on S, and $\lesssim$ induces a partial order $\leq$ on the set $S/\sim$ of equivalence classes of $\sim$ defined by

$$[s] \leq [t] \text{ if and only if } s \lesssim t,$$

where $[s]$ and $[t]$ denote the equivalence classes of s and t. In particular, $\leq$ does satisfy antisymmetry, whereas $\lesssim$ may not.

Pre-orders as categories

A pre-order $\lesssim$ on a set S can be considered as a small category, in the which the objects are the elements of S and there is a unique morphism from x to y if $x \lesssim y$ (and none otherwise).