Kurosh-Ore theorem

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Theorem 1 (Kurosh-Ore). *Let L be a modular lattice and suppose that $a \in L$ has two irredundant decompositions of joins of join-irreducible elements:*

$$a = x_1 \vee \cdots \vee x_m = y_1 \vee \cdots \vee y_n.$$

Then

1. *$m = n$, and*
2. *every x_i can be replaced by some y_j, so that*

$$a = x_1 \vee \cdots \vee x_{i-1} \vee y_j \vee x_{i+1} \vee \cdots \vee x_m.$$

There is also a dual statement of the above theorem in terms of meets.

Remark. Additionally, if L is a distributive lattice, then the second property above (known the *replacement property*) can be strengthened: each x_i is equal to some y_j. In other words, except for the re-ordering of elements in the decomposition, the above join is unique.