linear continuum

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Defines	linear continuum

Let X be a totally-ordered set under an order $<$ having at least two distinct points. Then X is said to be a *linear continuum* if the following two conditions are satisfied:

1. The order relation $<$ is a dense total order (i.e., for every $x, y \in X$ with $x < y$ there exists $z \in X$ such that $x < z < y$).

2. Every non-empty subset of X that is bounded above has a least upper bound (i.e., X has the least upper bound property).

Some examples of ordered sets that are linear continua include $\mathbb{R}$, the set $[0,1] \times [0,1]$ in the dictionary order, and the so-called long line $\Omega \times [0,1)$ in the dictionary topology. (The third example is a special case of a general result on well-ordered sets and linear continua.)

Proposition. *If X is a well-ordered set, then the set $X \times [0,1)$ is a linear continua in the dictionary order topology.*

Linear continua are of special interest when they are made into topological spaces under the order topology, and the following two establish some useful properties of such spaces:

Proposition. *If X is a linear continuum in the order topology, then X is* `http://planetmath.org/node/941`*connected and so are intervals in X.*

As a corollary of the preceding , we obtain the result that $\mathbb{R}$ is in its usual topology, as are the intervals $[a, b]$ and (a, b), where $a < b \in \mathbb{R}$.

Proposition. *If X is a linear continuum in the order topology, then every closed interval in X is compact.*

Proof. This is essentially a slightly generalized version of the Heine-Borel Theorem for $\mathbb{R}$, and the proof is almost identical. $\square$