supremum

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Related topic	InfimumAndSupremumForRealNumbers
Related topic	ExistenceOfSquareRootsOfNonNegativeRealNumbers
Related topic	LinearContinuum
Related topic	NondecreasingSequenceWithUpperBound
Related topic	EssentialSupremum

The *supremum* of a set X having a partial order is the least upper bound of X (if it exists) and is denoted $\sup X$.

Let A be a set with a partial order $\leqslant$, and let $X \subseteq A$. Then $s = \sup X$ if and only if:

1. For all $x \in X$, we have $x \leqslant s$ (i.e. s is an upper bound).

2. If s' meets condition **1**, then $s \leqslant s'$ (s is the *least* upper bound).

There is another useful definition which works if $A = \mathbb{R}$ with $\leqslant$ the usual order on $\mathbb{R}$, supposing that s is an upper bound:

$$s = \sup X \text{ if and only if } \forall \varepsilon > 0, \exists x \in X : s - \varepsilon < x.$$

Note that it is not necessarily the case that $\sup X \in X$. Suppose $X =]0, 1[$, then $\sup X = 1$, but $1 \notin X$.

Note also that a set may not have an upper bound at all.