ordered ring

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An *ordered ring* is a commutative ring R with a total ordering $\leq$ such that, for every $a, b, c \in R$:

1. If $a \leq b$, then $a + c \leq b + c$
2. If $a \leq b$ and $0 \leq c$, then $c \cdot a \leq c \cdot b$

An *ordered field* is an ordered ring $(R, \leq)$ where R is also a field.

Examples of ordered rings include:

- The integers $\mathbb{Z}$, under the standard ordering $\leq$.
- The real numbers $\mathbb{R}$ under the standard ordering.
- The polynomial ring $\mathbb{R}[x]$ in one variable over $\mathbb{R}$, under the relation $f \leq g$ if and only if $g - f$ has nonnegative leading coefficient.

Examples of rings which do not admit any ordering relation making them into an ordered ring include:

- The complex numbers $\mathbb{C}$.
- The finite field $\mathbb{Z}/p\mathbb{Z}$, where p is any prime.