partial order

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Related topic	Relation
Related topic	TotalOrder
Related topic	Poset
Related topic	BinarySearch
Related topic	SortingProblem
Related topic	ChainCondition
Related topic	PartialOrderWithChainConditionDoesNotCollapseCardinals
Related topic	QuasiOrder
Related topic	CategoryAssociatedToAPartialOrder
Related topic	OrderingRelation
Related topic	HasseDiagram
Related topic	NetsAndClosuresOfSubspaces

A *partial order* (often simply referred to as an *order* or *ordering*) is a relation $\leq \subset A \times A$ that satisfies the following three properties:

1. Reflexivity: $a \leq a$ for all $a \in A$
2. Antisymmetry: If $a \leq b$ and $b \leq a$ for any $a, b \in A$, then $a = b$
3. Transitivity: If $a \leq b$ and $b \leq c$ for any $a, b, c \in A$, then $a \leq c$

A *total order* is a partial order that satisfies a fourth property known as *comparability*:

- Comparability: For any $a, b \in A$, either $a \leq b$ or $b \leq a$.

A set and a partial order on that set define a poset.

Remark. In some literature, especially those dealing with the foundations of mathematics, a partial order $\leq$ is defined as a transitive irreflexive binary relation (on a set). As a result, if $a \leq b$, then $b \not\leq a$, and therefore $\leq$ is antisymmetric.