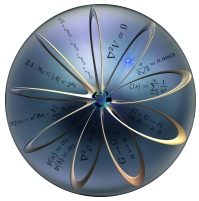

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relative complement

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Defines	relatively complemented lattice
Defines	relatively complemented

A complement of an element in a lattice is only defined when the lattice in question is http://planetmath.org/BoundedLatticebounded. In general, a lattice is not bounded and there are no complements to speak of. Nevertheless, if the sublattice of a lattice is bounded, we can speak of complements of an element *relative* to that sublattice.

Let L be a lattice, a an element of L, and $I = [b, c]$ an http://planetmath.org/LatticeInterva in L. An element $d \in L$ is said to be a complement of a *relative* to I if

$$a \vee d = c \text{ and } a \wedge d = b.$$

It is easy to see that $a \leq c$ and $b \leq a$, so $a \in I$. Similarly, $d \in I$.

An element $a \in L$ is said to be *relatively complemented* if for every interval I in L with $a \in I$, it has a complement relative to I. The lattice L itself is called a *relatively complemented lattice* if every element of L is relatively complemented. Equivalently, L is relatively complemented iff each of its interval is a complemented lattice.

Remarks.

- A relatively complemented lattice is complemented if it is bounded. Conversely, a complemented lattice is relatively complemented if it is http://planetmath.org/ModularLatticemodular.
- The notion of a relative complement of an element in a lattice has nothing to do with that found in set theory: let U be a set and A, B subsets of U, the relative complement of A in B is the set theoretic difference $B - A$. While the relative difference is necessarily a subset of B, A does not have to be a subset of B.