sectionally complemented lattice

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Proposition 1. *Let L be a lattice with the least element 0. Then the following are equivalent:*

1. *Every pair of elements have a* `http://planetmath.org/DifferenceOfLatticeElementsdiff`
2. *for any $a \in L$, the lattice interval $[0, a]$ is a complemented lattice.*

Proof. Suppose first that every pair of elements have a difference. Let $b \in [0, a]$ and let c be a difference between a and b. So $0 = b \wedge c$ and $c \vee b = b \vee a = a$, since $b \leq a$. This shows that c is a complement of b in $[0, a]$.

Next suppose that $[0, a]$ is complemented for every $a \in L$. Let $x, y \in L$ be any two elements in L. Let $a = x \vee y$. Since $[0, a]$ is complemented, y has a complement, say $z \in [0, a]$. This means that $y \wedge z = 0$ and $y \vee z = a = x \vee y$. Therefore, z is a difference of x and y. $\square$

Definition. A lattice L with the least element 0 satisfying either of the two equivalent conditions above is called a *sectionally complemented lattice*.

Every relatively complemented lattice is sectionally complemented. Every sectionally complemented distributive lattice is relatively complemented.

Dually, one defines a *dually sectionally complemented lattice* to be a lattice L with the top element 1 such that for every $a \in L$, the interval $[a, 1]$ is complemented, or, equivalently, the lattice dual L^{∂} is sectionally complemented.

References

[1] G. Grätzer, *General Lattice Theory*, 2nd Edition, Birkhäuser (1998)