bounded lattice

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Defines	top
Defines	bottom
Defines	bounded poset

A lattice L is said to be if there is an element $0 \in L$ such that $0 \leq x$ for all $x \in L$. Dually, L is if there exists an element $1 \in L$ such that $x \leq 1$ for all $x \in L$. A *bounded lattice* is one that is both from above and below.

For example, any finite lattice L is bounded, as $\bigvee L$ and $\bigwedge L$, being join and meet of finitely many elements, exist. $\bigvee L = 1$ and $\bigwedge L = 0$.

Remarks. Let L be a bounded lattice with 0 and 1 as described above.

- $0 \wedge x = 0$ and $0 \vee x = x$ for all $x \in L$.
- $1 \wedge x = x$ and $1 \vee x = 1$ for all $x \in L$.
- As a result, 0 and 1, if they exist, are necessarily unique. For if there is another such a pair $0'$ and $1'$, then $0 = 0 \wedge 0' = 0' \wedge 0 = 0'$. Similarly $1 = 1'$.
- 0 is called the *bottom* of L and 1 is called the *top* of L.
- L is a lattice interval and can be written as $[0, 1]$.

Remark. More generally, a poset P is said to be *bounded* if it has both a greatest element 1 and a least element 0.