special elements in a lattice

Canonical name	SpecialElementsInALattice
Date of creation	2013-03-22 16:42:29
Last modified on	2013-03-22 16:42:29
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	6
Author	CWoo (3771)
Entry type	Definition
Classification	msc 06B99
Defines	distributive element
Defines	standard element
Defines	neutral element
Defines	dually distributive
Defines	dually standard

Let L be a lattice and $a \in L$ is said to be

- *distributive* if $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$,
- *standard* if $b \wedge (a \vee c) = (b \wedge a) \vee (b \wedge c)$, or
- *neutral* if $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$

for all $b, c \in L$. There are also dual notions of the three types mentioned above, simply by exchanging $\vee$ and $\wedge$ in the definitions. So a *dually distributive* element $a \in L$ is one where $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ for all $b, c \in L$, and a *dually standard element* is similarly defined. However, a *dually neutral* element is the same as a neutral element.

Remarks For any $a \in L$, suppose P is the property in L such that $a \in P$ iff $a \vee b = a \vee c$ and $a \wedge b = a \wedge c$ imply $b = c$ for all $b, c \in L$.

- A standard element is distributive. Conversely, a distributive satisfying P is standard.
- A neutral element is distributive (and consequently dually distributive). Conversely, a distributive and dually distributive element that satisfies P is neutral.

References

[1] G. Birkhoff *Lattice Theory*, 3rd Edition, AMS Volume XXV, (1967).

[2] G. Grätzer, *General Lattice Theory*, 2nd Edition, Birkhäuser (1998).