modular lattice

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A lattice L is said to be *modular* if $x \vee (y \wedge z) = (x \vee y) \wedge z$ for all $x, y, z \in L$ such that $x \leq z$. In fact it is sufficient to show that $x \vee (y \wedge z) \geq (x \vee y) \wedge z$ for all $x, y, z \in L$ such that $x \leq z$, as the reverse inequality holds in all lattices (see modular inequality).

There are a number of other equivalent conditions for a lattice L to be modular:

- $(x \wedge y) \vee (x \wedge z) = x \wedge (y \vee (x \wedge z))$ for all $x, y, z \in L$.
- $(x \vee y) \wedge (x \vee z) = x \vee (y \wedge (x \vee z))$ for all $x, y, z \in L$.
- For all $x, y, z \in L$, if $x < z$ then either $x \wedge y < z \wedge y$ or $x \vee y < z \vee y$.

The following are examples of modular lattices.

- All `http://planetmath.org/DistributiveLattice`distributive lattices.
- The lattice of normal subgroups of any group.
- The lattice of submodules of any `http://planetmath.org/Module`module. (See modular law.)

A finite lattice L is modular if and only if it is graded and its rank function ρ satisfies $\rho(x) + \rho(y) = \rho(x \wedge y) + \rho(x \vee y)$ for all $x, y \in L$.