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lexicographic order

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Defines	dictionary order

Let A be a set equipped with a total order $<$, and let $A^n = A \times \cdots \times A$ be the n-fold Cartesian product of A. Then the *lexicographic order* $<$ on A^n is defined as follows:

If $a = (a_1, \ldots, a_n) \in A^n$ and $b = (b_1, \ldots, b_n) \in A^n$, then $a < b$ if $a_1 < b_1$ or

$$\begin{array}{rcl} a_1 & = & b_1, \\ & \vdots & \\ a_k & = & b_k, \\ a_{k+1} & < & b_{k+1} \end{array}$$

for some $k = 1, \ldots, n-1$.

Examples

- The lexicographic order yields a total order on the field of complex numbers.
- The lexicographic order of words of finite length consisting of letters $'\ '$ (space) $< a < b < \cdots < y < z$ is the *dictionary order*. To compare words of different length, one simply pads the shorter with $'\ '$s from the right. For example, prove $<$ proved $<$ proven.

Properties

- The lexicographic order is a total order.
- If the original set is well-ordered, the lexicographic ordering on the product is also a well-ordering.