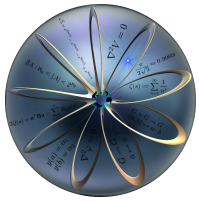

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Birkhoff Recurrence Theorem

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Let $T : X \to X$ be a continuous tranformation in a compact metric space X. Then, there exists some point $x \in X$ that is recurrent to T, that is, there exists a sequence $(n_k)_k$ such that $T^{n_k}(x) \to x$ when $k \to \infty$.

Several proofs of this theorem are available. It may be obtained from topological arguments together with Zorn's lemma. It is also a consequence of Krylov-Bogolyubov theorem, or existence of invariant probability measures theorem, which asserts that every continuous transformation in a compact metric space admits an invariant probability measure, and an application of Poincar Recurrence theorem to that invariant probability measure yields Birkhoff Recurrence theorem.

There is also a generalization of Birkhoff recurrence theorem for multiple commuting transformations, known as Birkhoff Multiple Recurrence theorem.