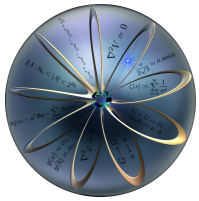

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directed set

Canonical name	DirectedSet
Date of creation	2013-03-22 12:54:00
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Owner	yark (2760)
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Numerical id	11
Author	yark (2760)
Entry type	Definition
Classification	msc 06A06
Synonym	upward-directed set
Synonym	upward directed set
Related topic	Cofinality
Related topic	AccumulationPointsAndConvergentSubnets
Defines	residual
Defines	cofinal
Defines	downward-directed set
Defines	downward directed set
Defines	filtered set

A *directed set* is a partially ordered set $(A, \leq)$ such that whenever $a, b \in A$ there is an $x \in A$ such that $a \leq x$ and $b \leq x$.

A subset $B \subseteq A$ is said to be *residual* if there is $a \in A$ such that $b \in B$ whenever $a \leq b$, and *cofinal* if for each $a \in A$ there is $b \in B$ such that $a \leq b$.

A directed set is sometimes called an *upward-directed set*. We may also define the dual notion: a *downward-directed set* (or *filtered set*) is a partially ordered set $(A, \leq)$ such that whenever $a, b \in A$ there is an $x \in A$ such that $x \leq a$ and $x \leq b$.

Note: Many authors do not require $\leq$ to be antisymmetric, so that it is only a pre-order (rather than a partial order) with the given property. Also, it is common to require A to be non-empty.