redundancy of two-sidedness in definition of group

Canonical name	RedundancyOfTwosidednessInDefinitionOfGroup
Date of creation	2015-01-20 17:28:03
Last modified on	2015-01-20 17:28:03
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Last modified by	pahio (2872)
Numerical id	3
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Entry type	Definition

In the definition of group, one usually supposes that there is a two-sided identity element and that any element has a two-sided inverse (cf. `http://planetmath.org/Group`group).

The group may also be defined without the two-sidednesses:

A *group* is a pair of a non-empty set G and its associative binary operation $(x, y) \mapsto xy$ such that

1) the operation has a right identity element e;

2) any element x of G has a right inverse x^{-1}.

We have to show that the right identity e is also a left identity and that any right inverse is also a left inverse.

Let the above assumptions on G be true. If a^{-1} is the right inverse of an arbitrary element a of G, the calculation

$$a^{-1}a = a^{-1}ae = a^{-1}aa^{-1}(a^{-1})^{-1} = a^{-1}e(a^{-1})^{-1} = a^{-1}(a^{-1})^{-1} = e$$

shows that it is also the left inverse of a. Using this result, we then can write

$$ea = (aa^{-1})a = a(a^{-1}a) = ae = a,$$

whence e is a left identity element, too.