bounded complete

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Let P be a poset. Recall that a subset S of P is called *bounded from above* if there is an element $a \in P$ such that, for every $s \in S$, $s \leq a$.

A poset P is said to be *bounded complete* if every subset which is bounded from above has a supremum.

Remark. Since it is not required that the subset be non-empty, we see that P has a bottom. This is because the empty set is vacuously bounded from above, and therefore has a supremum. However, this supremum is less than or equal to every member of P, and hence it is the least element of P.

Clearly, any complete lattice is bounded complete. An example of a non-complete bounded complete poset is any closed subset of $\mathbb{R}$ of the form $[a, \infty)$, where $a \in \mathbb{R}$. In addition, arbitrary products of bounded complete posets is also bounded complete.

It can be shown that a poset is a bounded complete dcpo iff it is a complete semilattice.

Remark. A weaker concept is that of *Dedekind completeness*: A poset P is *Dedekind complete* if every *non-empty* subset bounded from above has a supremum. An obvious example is $\mathbb{R}$, which is Dedekind complete but not bounded complete (as it has no bottom). Dedekind completeness is more commonly known as the least upper bound property.