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ordered topological vector space

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Synonym	ordered topological linear space

Let  $k$  be either  $\mathbb{R}$  or  $\mathbb{C}$  considered as a field. An *ordered topological vector space*  $L$ , (*ordered t.v.s* for short) is

- a topological vector space over  $k$ , and
- an ordered vector space over  $k$ , such that
- the positive cone  $L^+$  of  $L$  is a closed subset of  $L$ .

The last statement can be interpreted as follows: if a sequence of non-negative elements  $x_i$  of  $L$  converges to an element  $x$ , then  $x$  is non-negative.

**Remark.** Let  $L, M$  be two ordered t.v.s., and  $f : L \rightarrow M$  a linear transformation that is monotone. Then if  $0 \leq x \in L$ ,  $0 \leq f(x) \in M$  also. Therefore  $f(L^+) \subseteq M^+$ . Conversely, a linear map that is invariant under positive cones is monotone.