disjunction property of Wallman

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A partially ordered set $\mathfrak{A}$ with a least element 0 has the *disjunction property of Wallman* if for every pair (a, b) of elements of the poset, either $b \leq a$ or there exists an element $c \leq b$ such that $c \neq 0$ and c has no nontrivial common predecessor with a. That is, in the latter case, the only x with $x \leq a$ and $x \leq c$ is $x = 0$.

For the case if the poset $\mathfrak{A}$ is a $\cap$-semilattice *disjunction property of Wallman* is equivalent to every of the following three formulas:

1. $\forall a, b \in \mathfrak{A} : (\{c \in \mathfrak{A} | c \cap a \neq 0\} = \{c \in \mathfrak{A} | c \cap b \neq 0\} \Rightarrow a = b)$;
2. $\forall a, b \in \mathfrak{A} : (\{c \in \mathfrak{A} | c \cap a \neq 0\} \subseteq \{c \in \mathfrak{A} | c \cap b \neq 0\} \Rightarrow a \subseteq b)$;
3. $\forall a, b \in \mathfrak{A} : (a \subset b \Rightarrow \{c \in \mathfrak{A} | c \cap a \neq 0\} \subset \{c \in \mathfrak{A} | c \cap b \neq 0\})$.

The proof of this equivalence can be found in `http://www.mathematics21.org/binaries/filt` online article.