well quasi ordering

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Let Q be a set and $\precsim$ a quasi-order on Q. An infinite sequence in Q is referred to as *bad* if for all $i < j \in \mathbf{N}$, $a_i \not\precsim a_j$ holds; otherwise it is called *good*. Note that an antichain is obviously a bad sequence.

A quasi-ordering $\precsim$ on Q is a *well-quasi-ordering* (*wqo*) if for every infinite sequence is good. Every well-ordering is a well-quasi-ordering.

The following proposition gives equivalent definitions for well-quasi-ordering:

Proposition 1. *Given a set Q and a binary relation $\precsim$ over Q, the following conditions are equivalent:*

- *$(Q, \precsim)$ is a well-quasi-ordering;*
- *$(Q, \precsim)$ has no infinite (ω-) strictly decreasing chains and no infinite antichains.*
- *Every linear extension of $Q/_{\approx}$ is a well-order, where $\approx$ is the equivalence relation and $Q/_{\approx}$ is the set of equivalence classes induced by $\approx$.*
- *Any infinite (ω-) Q-sequence contains an increasing chain.*

The equivalence of WQO to the second and the fourth conditions is proved by the infinite version of Ramsey's theorem.