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grounded relation

Canonical name	GroundedRelation
Date of creation	2013-03-22 17:48:38
Last modified on	2013-03-22 17:48:38
Owner	Jon Awbrey (15246)
Last modified by	Jon Awbrey (15246)
Numerical id	11
Author	Jon Awbrey (15246)
Entry type	Definition
Classification	msc 08A70
Classification	msc 08A02
Classification	msc 03G15
Classification	msc 03E20
Classification	msc 03C05
Classification	msc 03B10
Related topic	Relation

A **grounded relation** over a sequence of sets is a mathematical object consisting of two components. The first component is a subset of the cartesian product taken over the given sequence of sets, which sets are called the *domains* of the relation. The second component is just the cartesian product itself.

For example, if L is a grounded relation over a finite sequence of sets, $X_1, \ldots, X_k$, then L has the form $L = (F(L), G(L))$, where $F(L) \subseteq G(L) = X_1 \times \ldots \times X_k$.

1 Remarks

- In various language that is used, $F(L)$ may be called the *figure* or the *graph* of L, while $G(L)$ may be called the *ground* of L.

- The default assumption in almost all applications is that the domains of the grounded relation are nonempty sets, hence departures from this assumption need to be noted explicitly.

- In many applications all relations are considered relative to explicitly specified grounds. In these settings it is conventional to refer to grounded relations somewhat more simply as "relations".

- One often hears or reads the usage "$L \subseteq X_1 \times \ldots \times X_k$" when the speaker or writer really means "$F(L) \subseteq X_1 \times \ldots \times X_k$". Be charitable in your interpretations.

- The cardinality of $G(L)$ is referred to as the *adicity* or the *arity* of the relation. For example, in the finite case, L may be described as *k-adic* or *k-ary*.

- The set $\mathrm{dom}_j(L) := X_j$ is referred to as the jth *domain* of the relation.

- In the special case where $k = 2$, the set X_1 is called "the domain" and the set X_2 is called "the codomain" of the relation.