discriminator function

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Synonym	switching function
Defines	ternary discriminator
Defines	quaternary discriminator

Let A be a non-empty set. The *ternary discriminator* on A is the ternary operation t on A such that

$$t(a,b,c) := \begin{cases} a & \text{if } a \neq b, \\ c & \text{otherwise.} \end{cases}$$

In other words, t is a function that determines whether or not a pair of elements in A are the same, hence the name discriminator.

It is easy to see that, by setting two of the three variables the same, t becomes a constant function: $t(a,b,a) = a$, $t(a,a,b) = b$, and $t(a,b,b) = a$.

More generally, the *quaternary discriminator* or the *switching function* on A is the quaternary operation q on A such that

$$q(a,b,c,d) := \begin{cases} d & \text{if } a \neq b, \\ c & \text{otherwise.} \end{cases}$$

However, this generalization is really an equivalent concept in the sense that one can derive one type of discriminator from another: given q above, set $t(a,b,c) = q(a,b,c,a)$. Conversely, given t above, set $q(a,b,c,d) = t(t(a,b,c), t(a,b,d), d)$.

Remark. The following ternary functions $t_1, t_2 : A^3 \to A$ could also serve as discriminator functions:

$$t_1(a,b,c) := \begin{cases} b & \text{if } a \neq b, \\ c & \text{otherwise.} \end{cases} \qquad t_2(a,b,c) := \begin{cases} c & \text{if } a \neq b, \\ a & \text{otherwise.} \end{cases}$$

But they are really no different from *the* ternary discriminator t:

$$t_1(a,b,c) = t(b,a,c) \quad \text{and} \quad t(a,b,c) = t_1(b,a,c),$$

$$t_2(a,b,c) = t(c, t(a,b,c), a) \quad \text{and} \quad t(a,b,c) = t_2(a, t_2(a,b,c), c).$$

References

[1] G. Grätzer: *Universal Algebra*, 2nd Edition, Springer, New York (1978).

[2] S. Burris, H.P. Sankappanavar: *A Course in Universal Algebra*, Springer, New York (1981).