decomposable curve

Canonical name	DecomposableCurve
Date of creation	2013-03-22 19:19:38
Last modified on	2013-03-22 19:19:38
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	7
Author	pahio (2872)
Entry type	Definition
Classification	msc 08A40
Classification	msc 26A09
Related topic	Hyperbola2
Defines	decomposable
Defines	decomposable surface

An algebraic curve

$$f(x,\, y) \;=\; 0$$

is *decomposable*, if the polynomial $f(x,\, y)$ is in $\mathbb{R}[x,\, y]$; that is, if there are polynomials $g(x,\, y)$ and $h(x,\, y)$ with positive degree in $\mathbb{R}[x,\, y]$ such that

$$f(x,\, y) \;=\; g(x,\, y)\, h(x,\, y).$$

Example. The quadratic curve

$$\frac{x^2}{a^2}-\frac{y^2}{b^2} \;=\; 0 \tag{1}$$

is decomposable, since the equation may be written

$$\left(\frac{x}{a}+\frac{y}{b}\right)\left(\frac{x}{a}-\frac{y}{b}\right) \;=\; 0$$

or equivalently

$$\frac{x}{a}+\frac{y}{b} \;=\; 0 \quad \vee \quad \frac{x}{a}-\frac{y}{b} \;=\; 0.$$

Thus the curve (1) consists of two intersecting lines.

Analogically, one can say that an algebraic surface

$$g(x,\, y,\, z) \;=\; 0$$

is *decomposable*, e.g. $(x+y+z)^2-1 = 0$ which consists of two parallel planes.