planetmath.org

Math for the people, by the people.

opposite group

Canonical name	OppositeGroup
Date of creation	2013-03-22 17:09:56
Last modified on	2013-03-22 17:09:56
Owner	Wkbj79 (1863)
Last modified by	Wkbj79 (1863)
Numerical id	10
Author	Wkbj79 (1863)
Entry type	Definition
Classification	msc 08A99
Classification	msc 20-00

Let G be a group under the operation $*$. The *opposite group* of G, denoted G^{op}, has the same underlying set as G, and its group operation is $*'$ defined by $g_1 *' g_2 = g_2 * g_1$.

If G is abelian, then it is equal to its opposite group. Also, every group G (not necessarily abelian) is isomorphic to its opposite group: The `http://planetmath.org/GroupIsomorphism`isomorphism $\varphi\colon G \to G^{\text{op}}$ is given by $\varphi(x) = x^{-1}$. More generally, any anti-automorphism $\psi\colon G \to G$ gives rise to a corresponding isomorphism $\psi'\colon G \to G^{\text{op}}$ via $\psi'(g) = \psi(g)$, since $\psi'(g * h) = \psi(g * h) = \psi(h) * \psi(g) = \psi(g) *' \psi(h) = \psi'(g) *' \psi'(h)$.

Opposite groups are useful for converting a right action to a left action and vice versa. For example, if G is a group that acts on X on the , then a left action of G^{op} on X can be defined by $g^{\text{op}}x = xg$.

constructions occur in opposite ring and opposite category.