quandles

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Quandles are algebraic gadgets introduced by David Joyce in [?] which can be used to define invarients of links. In the case of knots these invarients are complete up to equivalence, that is up to mirror images.

Definition 1 *A* quandle *is an algebraic structure, specifically it is a set Q with two binary operations on it, $\lhd$ and $\lhd^{-1}$ and the following axioms.*

1. $q \lhd q = q \ \forall q \in Q$
2. $(q_1 \lhd q_2) \lhd^{-1} q_2 = (q_1 \lhd^{-1} q_2) \lhd q_2 \ \forall q_1, q_2 \in Q$
3. $(q_1 \lhd q_2) \lhd q_3 = (q_1 \lhd q_3) \lhd (q_2 \lhd q_3) \ \forall q_1, q_2, q_3 \in Q$

It is useful to consider $q_1 \lhd q_2$ as 'q_2 acting on q_1'.
Examples.

1. Let Q be some group, and let n be some fixed integer. Then let $g_1 \lhd g_2 = g_2^{-n} g_1 g_2^n, \quad g_1 \lhd^{-1} g_2 = g_2^n g_1 g_2^{-n}$.
2. Let Q be some group. Then let $g_1 \lhd g_2 = g_1 \lhd^{-1} g_2 = g_2 g_1^{-1} g_2$.
3. Let Q be some module, and T some invertable linear operator on Q. Then let $m_1 \lhd m_2 = T(m_1 - m_2) + m_2, \quad m_1 \lhd^{-1} m_2 = T^{-1}(m_1 - m_2) + m_2$

Homomorphisms, isomorphisms etc. are defined in the obvious way. Notice that the third axiom gives us that the operation of a quandle element on the quandle given by $f_q \colon q' \mapsto q' \lhd q$ is a homomorphism, and the second axiom ensures that this is an isomorphism.

Definition 2 *The subgroup of the automorphism group of a quandle Q generated by the quandle operations is the* operator group *of Q.*

References

[1] D.Joyce : A Classifying Invariant Of Knots, The Knot Quandle : J.P.App.Alg 23 (1982) 37-65