ground fields and rings

Canonical name	GroundFieldsAndRings
Date of creation	2013-03-22 15:54:22
Last modified on	2013-03-22 15:54:22
Owner	Algeboy (12884)
Last modified by	Algeboy (12884)
Numerical id	15
Author	Algeboy (12884)
Entry type	Definition
Classification	msc 08A30
Related topic	ExtensionField
Related topic	FieldAdjunction
Related topic	RingAdjunction
Defines	ground field
Defines	base field
Defines	ground ring
Defines	base ring

The following is a list of common uses of the *ground* or *base* field or ring in algebra. These are endowed with based on their context so the following list may be or may not apply uniformly.

One commonality is generally found for the use of ground ring or field: the result is a unitial subring of the original. Outside of this requirement, the constraints are specific to context.

- Given a ring R with a 1, let $\mathbb{Z}1$ be the subgroup of R generated by 1 under addition. This is consequently a subring of R of the same characteristic as R. Thus is it isomorphic to $\mathbb{Z}/c\mathbb{Z}$ where c is the characteristic of R. This is the smallest unital subring of R and so rightfully may be called the ground or base ring of R.

  When the characteristic of R is prime, $\mathbb{Z}1 \cong \mathbb{Z}/p\mathbb{Z}$ and so it may be called the ground field of R.

- Given a vector space or algebra A over a field k, then k is the ground/base field of A.

- Given a set of matrices $M_n(R)$, the ground ring is commonly the ring R, and if required as a subring of $M_n(R)$ then it is taken as the set of all scalar matrices.

- Given a field extension K/k over a field k, then k is the ground field of K in this context. For a general field where no specific subfield has been specified, the ground/base field then typically defaults to the prime subfield of K. (Recall the prime subfield is the unique smallest subfield of K.)

- Given a field K and a set of field automorphisms $f : K \to K$, the ground/base field in this context is the `http://planetmath.org/Fixed`fixed field of the automorphisms. That is, the largest subfield of K which is pointwise fixed by each f. Since a field automorphism must fix the prime subfield, this definition always produces a field containing the prime subfield.