planetmath.org

Math for the people, by the people.

presentations of algebraic objects

Canonical name	PresentationsOfAlgebraicObjects
Date of creation	2013-03-22 16:51:27
Last modified on	2013-03-22 16:51:27
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	5
Author	CWoo (3771)
Entry type	Definition
Classification	msc 08B20
Related topic	Presentationgroup
Defines	presentation

Given an algebraic category with enough free objects one can use the general description of the free object to provide a precise description of all other objects in the category. The process is called a *presentation*.

Suppose A is an object generated by a subset X. Then if there exists a free object on X, F, then there exists a unique morphism $f : F \to A$ which matches the embedding of X in F to the embedding of X in A.

As we are in an algebraic category we have a fundamental homomorphism theorem (we take this as our definition of an algebraic category in this context). This means there is a notion of kernel K of f and quotient F/K such that F/K is isomorphic to A.

Now F is generted by X so every element of F is expressed as an informal word over X. [By *informal word* we mean whatever process encodes general elements as generated by X. For example, in groups and semigroups these are actual formal words, but in algebras these can be linear combinations of words or polynomials with indeterminants in X, etc.] Hence a set of generators for the kernel K will be expressed as words over X.

Definition 1. *A presentation of an object A is a pair of sets $\langle X|R \rangle$ where X generates A and R is a set of informal words over X such that the free object F on X and the normal subobject K of F generated by R has the property $F/K \cong A$.*

Once again, normal refers to whatever property is required for subobject to allow quotients, so normal subgroup or ideals, etc.

Existence of presentations is dependent on the category being considered. The common categories: groups, rings, and modules all have presentations.

It is generally not possible to insist that a presentation is unique. First we have the variable choice of generators. Secondly, we may choose various relations. Indeed, it is possible that the relations will generate different subobjects K such that $F/K \cong A$. In practice, presentations are a highly compactified description of an object which can hide many essential features of the object. Indeed, in the extreem case are the theorems of Boone which show that in the category of groups it is impossible to tell if an arbitrary presentation is a presentation of the trivial group. For a detailed account of these theorems refer to

Joseph Rotman, *An Introduction to the Theory of Groups*, Springer, New York, Fourth edition, 1995.