biops

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Entry type	Definition
Classification	msc 08A99
Defines	semigroup
Defines	monoid
Defines	group
Defines	rig
Defines	ring
Defines	quasigroup
Defines	loop

Let S be a set and $n \in \mathbf{N}$. Set $\mathbf{N}_n := \{i \in \mathbf{N} | i < n\}$. If there exists a map $\cdot : \mathbf{N}_n \to (S^2 \to S) : i \mapsto \cdot_i$ where $\cdot_i : S^2 \to S : (a, b) \mapsto a \cdot_i b$ is a binary operation, then I shall say that $(S, \cdot)$ is an n-*biops*. In other words, an n-biops is an algebraic system with n binary operations defined on it, and the operations are labelled $0, 1, \ldots, n-1$.

Let $(S, \cdot)$ be an n-biops. If $\cdot$ has the property p, then I shall say that $(S, \cdot)$ is a p n-biops.

For example if $(S, \cdot)$ is an n-biops and $\cdot$ is 0-commutative, 0-associative, 0-alternative or $(0, 1)$-distributive, then I shall say that $(S, \cdot)$ is a 0-commutative n-biops, 0-associative n-biops, 0-alternative n-biops or $(0, 1)$-distributive n-biops respectively.

If an n-biops B is i-p for each $i \in \mathbf{N}_n$ then I shall say that B is a p n-biops.

A 0-associative 1-biops is called a semigroup. A semigroup with identity element is called a monoid. A monoid with inverses is called a group.

A $(0, 1)$-distributive 2-biops $(S, +, \cdot)$, such that both $(S, +)$ and $(S, \cdot)$ are monoids, is called a rig.

A $(0, 1)$-distributive 2-biops $(S, +, \cdot)$, such that $(S, +)$ is a group and $(S, \cdot)$ is a monoid, is called a ring.

A rig with 0-inverses is a ring.

A 0-associative 2-biops $(S, \cdot, /)$ with 0-identity such that for every $\{a, b\} \subset S$ we have

$$b = (b/a) \cdot a = (b \cdot a)/a$$

is called a group.

A 3-biops $(S, \cdot, /, \backslash)$ such that for every $\{a, b\} \subset S$ we have

$$a \backslash (a \cdot b) = a \cdot (a \backslash b) = b = (b/a) \cdot a = (b \cdot a)/a$$

is called a quasigroup.

A quasigroup such that for every $\{a, b\} \subset S$ we have $a/a = b \backslash b$ is called a loop.

A 0-associative loop is a group.