topic entry on the algebraic foundations of mathematics

Canonical name	TopicEntryOnTheAlgebraicFoundationsOfMathematics
Date of creation	2013-03-22 18:14:02
Last modified on	2013-03-22 18:14:02
Owner	bci1 (20947)
Last modified by	bci1 (20947)
Numerical id	35
Author	bci1 (20947)
Entry type	Topic
Classification	msc 08A99
Classification	msc 08A70
Classification	msc 18E05
Classification	msc 18-00
Classification	msc 03-00
Classification	msc 08A05
Synonym	Algebraic Foundations of Mathematics
Related topic	Algebras
Related topic	Graph
Related topic	Hypergraph
Related topic	TopicEntryOnAlgebra
Related topic	IndexOfCategoryTheory
Related topic	NonAbelianStructures
Related topic	JordanBanachAndJordanLieAlgebras
Related topic	AbelianCategory
Related topic	AxiomsForAnAbelianCategory
Related topic	GeneralizedVanKampenTheoremsHigherDimensional
Related topic	AxiomaticTheoryOfSupercategories
Related topic	Categ
Defines	universal algebra
Defines	algebraic structure
Defines	logic algebra
Defines	co-algebra
Defines	gebra
Defines	K-algebra
Defines	quantum algebra
Defines	lattice algebra

This is a contributed topic on the algebraic foundations of mathematics. This topic of algebraic foundations in mathematics will cover a wide range of concepts and areas of mathematics, ranging from universal algebras, algebraic topology to algebraic geometry, number theory and logic algebras.

a. *Universal (or general) algebra* : is defined as *the (meta) mathematical study of general theories of algebraic structures* rather than the study of specific cases, or models of algebraic structures.

b. Various, specifically selected algebraic structures, such as :

1. Boolean algebra
2. Logic lattice algebras or many-valued (MV) logic algebras
3. Quantum logic algebras
4. Quantum operator algebras (such as : involution, *-algebras, or $*$-algebras, von Neumann algebras, JB- and JL- algebras, Poisson and C^* - or C*- algebras,
5. Algebra over a set
6. Sigma-algebra and T-algebras of monads
7. K-algebras
8. Group algebras
9. Graphs generated by free groups
10. Groupoid algebras and Groupoid C^*-convolution algebras
11. Hypergraphs generated by free groupoids
12. Double algebras
13. Index of algebras
14. Categorical algebra
15. F-algebra/coalgebra in category theory
16. Category of categories as a foundation for mathematics: `http://planetmath.org/FunctorCa` Categories and `http://planetmath.org/2Category`2-category

17. http://planetmath.org/IndexOfCategoryTheoryIndex of category theory

18. super-categories and topological 'supercategories'

19. Higher dimensional algebras (HDA) –such as: algebroids, double algebroids, categorical algebroids, double groupoid convolution algebroids, groupoid C^* -convolution algebroids, etc., and Supercategorical algebras (SA) as concrete interpretations of the theory of elementary abstract supercategories (ETAS)

20. Index of supercategories

21. http://planetmath.org/IndexOfCategoriesIndex of categories

22. Index of HDA

Remark The last items of HDA and SA are more precisely understood in the context of, or as generalizations/ extensions of, universal algebras.

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