Smarandache n-structure

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In any of knowledge, a *Smarandache n-structure*, for $n \geqslant 2$, on a set S means a weak structure w_0 on S such that there exists a chain of proper subsets $P_{n-1} \subset P_{n-2} \subset \cdots \subset P_2 \subset P_1 \subset S$ whose corresponding structures satisfy the inverse inclusion chain $w_{n-1} \succ w_{n-2} \succ \cdots \succ w_2 \succ w_1 \succ w_0$, where $\succ$ signifies strictly stronger (i.e., structure satisfying more axioms).

By *proper subset* one understands a subset different from the empty set, from the idempotent if any, and from the whole set.

Now one defines the *weak structure*:

Let A be a set, B a proper subset of it, ϕ an operation on A, and $a_1, a_2, \ldots, a_k, a_{k+1}, \ldots, a_{k+m}$ be $k+m$ independent axioms, where $k, m \geqslant 1$. If the operation ϕ on the set A satisfies the axioms $a_1, a_2, \ldots, a_k$ and does not satisfy the axioms $a_{k+1}, \ldots, a_{k+m}$, while on the subset B the operation ϕ satisfies the axioms $a_1, a_2, \ldots, a_k, a_{k+1}, \ldots, a_{k+m}$, one says that structure $w_A = (A, \phi)$ is *weaker* than structure $w_B = (B, \phi)$ and one writes $w_A \prec w_B$, or one says that w_B is *stronger* than structure w_A and one writes $w_B \succ w_A$. But if ϕ satisfies the same axioms on A as on B one says that structures w_A and w_B are equal and one writes $w_A = w_B$.
When ϕ satisfies the same axioms or less axioms on A than on B one says that structures w_A is *weaker than or equal* to structure w_B and one writes $w_A \preceq w_B$, or w_B is *stronger than or equal* to w_B and one writes $w_B \succeq w_A$.
For example a semigroup is a structure weaker than a group structure.

This definition can be extended to structures with many operations $(A, \phi_1, \phi_2, \ldots, \phi_r)$ for $r \geqslant 2$. Thus, let A be a set and B a proper subset of it.
a) If $(A, \phi_i) \preceq (B, \phi_i)$ for all $1 \leq i \leq r$, then $(A, \phi_1, \phi_2, \ldots, \phi_r) \preceq (B, \phi_1, \phi_2, \ldots, \phi_r)$.
b) If $\exists i_0 \in \{1, 2, \ldots, r\}$ such that $(A, \phi_{i_0}) \prec (B, \phi_{i_0})$ and $(A, \phi_i) \preceq (B, \phi_i)$ for all $i \neq i_0$, then $(A, \phi_1, \phi_2, \ldots, \phi_r) \prec (B, \phi_1, \phi_2, \ldots, \phi_r)$.
In this case, for two operations, a ring is a structure weaker than a field structure.

This definition comprises large classes of structures, some more important than others.

As a particular case, in abstract algebra, a *Smarandache 2-algebraic structure* (two levels only of structures in algebra) on a set S, is a weak algebraic structure w_0 on S such that there exists a proper subset P of S, which is embedded with a stronger algebraic structure w_1.
For example: a *Smarandache semigroup* is a semigroup (different from a group) which has a proper subset that is a group.
Other examples: a *Smarandache groupoid of first order* is a groupoid (different from a semigroup) which has a proper subset that is a semigroup,

while a *Smarandache groupoid of second order* is a groupoid (different from a semigroup) which has a proper subset that is a group. And so on.

References:

1. `http://www.gallup.unm.edu/ smarandache/eBooks-otherformats.htm`Digital Library of Science:

2. W. B. Vasantha Kandasamy, *Smarandache Algebraic Structures*, book : (Vol. I: Groupoids; Vol. II: Semigroups; Vol. III: Semirings, Semifields, and Semivector Spaces; Vol. IV: Loops; Vol. V: Rings; Vol. VI: Near-rings; Vol. VII: Non-associative Rings; Vol. VIII: Bialgebraic Structures; Vol. IX: Fuzzy Algebra; Vol. X: Linear Algebra), Am. Res. Press & Bookman, Martinsville, 2002-2003.