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## quotient rule for arithmetic derivative

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**Theorem.** *If the notion of arithmetic derivative is extended to rational numbers, then we have that, for every  $a, b \in \mathbb{Z}$  with  $b \neq 0$ :*

$$\left(\frac{a}{b}\right)' = \frac{a'b - b'a}{b^2}$$

*Proof.* Note that

$$\begin{aligned} a' &= \left(b \cdot \frac{a}{b}\right)' \\ &= b \cdot \left(\frac{a}{b}\right)' + b' \cdot \frac{a}{b} \text{ by the Leibniz rule.} \end{aligned}$$

Thus,

$$\begin{aligned} b \cdot \left(\frac{a}{b}\right)' &= a' - b' \cdot \frac{a}{b} \\ &= \frac{a'b - b'a}{b}. \end{aligned}$$

It follows that

$$\left(\frac{a}{b}\right)' = \frac{a'b - b'a}{b^2}.$$

□