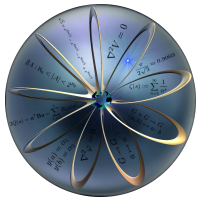

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fundamental theorem of Galois theory

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Let L/F be a Galois extension of finite degree, with Galois group $G := \mathrm{Gal}(L/F)$. There is a bijective, inclusion-reversing correspondence between subgroups of G and extensions of F contained in L, given by

- $K \mapsto \mathrm{Gal}(L/K)$, for any field K with $F \subseteq K \subseteq L$.
- $H \mapsto L^H$ (the fixed field of H in L), for any subgroup $H \leq G$.

The extension L^H/F is normal if and only if H is a normal subgroup of G, and in this case the homomorphism $G \longrightarrow \mathrm{Gal}(L^H/F)$ given by $\sigma \mapsto \sigma|_{L^H}$ induces (via the first isomorphism theorem) a natural identification $\mathrm{Gal}(L^H/F) = G/H$ between the Galois group of L^H/F and the quotient group G/H.

For the case of Galois extensions of infinite degree, see the entry on infinite Galois theory.