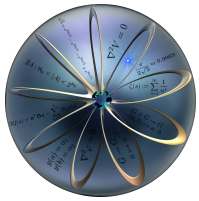

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determining integer contraharmonic means

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For determining effectively values c of integer contraharmonic means of two positive integers u and v ($1 < u < v$), it's convenient to start from the (7) in the `http://planetmath.org/IntegerContraharmonicMeans`parent entry:

$$v \;=\; \frac{2u^2}{w} - u \tag{1}$$

where w is any positive factor of $2u^2$ less than u. Substituting the above expression of v to the defining expression

$$c \;=\; \frac{u^2+v^2}{u+v}$$

of c, this gets the form

$$c \;=\; \frac{2u^2}{w} - 2u + w. \tag{2}$$

Hence one can use the formulae (1) and (2), giving in them for each desired u the values w of the positive factors of $2u^2$, beginning from $w := 1$ and stopping before $w = u$.

The for the integer harmonic mean, corresponding (2), is simply

$$h \;=\; 2u - w. \tag{3}$$

Example. In the following table one sees for $u = 36$ all possible values of the parametre w and the corresponding values of c and h; the pertinent values of v are given, too.

w	1	2	3	4	6	8	9	12	16	18	24	27	32
v	2556	1260	828	612	396	288	252	180	126	108	72	60	45
c	2521	1226	795	580	366	260	225	156	106	90	60	51	41
h	71	70	69	68	66	64	63	60	56	54	48	45	40

As one sees, the contraharmonic and the harmonic mean may differ considerably, but also the difference 1 is possible.

References

[1] J. Pahikkala: "On contraharmonic mean and Pythagorean triples". – *Elemente der Mathematik* **65**:2 (2010).