p-adic analytic

Canonical name	PadicAnalytic
Date of creation	2013-03-22 15:13:53
Last modified on	2013-03-22 15:13:53
Owner	alozano (2414)
Last modified by	alozano (2414)
Numerical id	4
Author	alozano (2414)
Entry type	Definition
Classification	msc 11S99
Classification	msc 12J12
Classification	msc 11S80
Synonym	p-adic analytic
Related topic	Analytic
Related topic	PAdicExponentialAndPAdicLogarithm
Defines	p-adic analysis
Defines	p-adic analysis

Definition. *Let* $\mathbb{C}_p$ *be the field of* `http://planetmath.org/ComplexPAdicNumbers`*complex p-adic numbers. Let* U *be a domain in* $\mathbb{C}_p$. *A function* $f : U \longrightarrow \mathbb{C}_p$ *is* p-adic analytic *if* f *has a Taylor series (with coefficients in* $\mathbb{C}_p$*) about each point* $z \in U$ *that converges to the function* f *in an open neighborhood of* z.

For example, the `http://planetmath.org/PAdicExponentialAndPAdicLogarithm`p-adic exponential function is analytic on its domain of definition:

$$U = \{z \in \mathbb{C}_p : |z|_p < \frac{1}{p^{1/(p-1)}}\}.$$

The study of p-adic analytic functions is usually called *p-adic analysis* and it is very similar to complex analysis in many respects, although there are important differences coming from the distinct topologies of $\mathbb{C}$ and $\mathbb{C}_p$.