# binary Golay code

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The *binary Golay Code* $\mathcal{G}_{23}$ is a perfect linear binary [23,12,7]-code with a plethora of different constructions.

## Sample Constructions

- **Lexicographic Construction:** Let $v_0$ be the all-zero word in $\mathbb{F}_2^{23}$, and inductively define $v_j$ to be the smallest word (smallest with respect to the lexicographic ordering on $\mathbb{F}_2^{23}$ that differs from $v_i$ in at least 7 places for all $i < j$.

- **Construction:** $\mathcal{G}_{23}$ is the quadratic residue code of length 23.

The *extended binary Golay Code* $\mathcal{G}_{24}$ is obtained by appending a zero-sum check digit to the end of every word in $\mathcal{G}_{23}$.

Both the binary Golay code and the extended binary Golay code have some remarkable .

## Properties

- $\mathcal{G}_{24}$ has 4096 codewords: 1 of weight 0, 759 of weight 8, 2576 of weight 12, 759 of weight 18, and 1 of weight 24.

- The automorphism group of $\mathcal{G}_{24}$ is the Mathieu group $M_{24}$, one of the sporadic groups.

- The Golay Code is used to define the Leech Lattice, one of the most efficient sphere-packings known to date.

- The optimal strategy to the mathematical game called Mogul is to always revert the current position to one corresponding to a word of the Golay code.

- The words of weight 8 in $\mathcal{G}_{24}$ form a $S(5, 8, 24)$ Steiner system. In fact, this property uniquely determines the code.