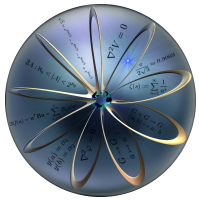

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examples of ramification of archimedean places

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Example 1. Let $K = \mathbb{Q}(\sqrt{-d})$ be a quadratic imaginary number field. Then K has only two embeddings which, in fact, are complex-conjugate embeddings:

$$\psi\colon K \to \mathbb{C}, \sqrt{-d} \to \sqrt{-d}$$

$$\overline{\psi}\colon K \to \mathbb{C}, \sqrt{-d} \to -\sqrt{-d}$$

The archimedean place $w = (\psi, \overline{\psi})$ is lying above the unique archimedean place of $\mathbb{Q}$:

$$\phi\colon \mathbb{Q} \to \mathbb{R}$$

and therefore, the place $v = \phi$ ramifies in K.

Example 2. Let K be a CM-field i.e. K is a `http://planetmath.org/TotallyRealAndImaginary` imaginary quadratic extension of a totally real field K^+. Then we claim that the extension K/K^+ is totally ramified at the archimedean (or infinite) places. Indeed, let v be an archimedean place of K^+. By assumption, K^+ is a totally real field, thus all its places are real, and so, v is real. Let w be any archimedean place of K lying above v (i.e. extending v to K). Since K is totally imaginary, the place w is a pair of complex embeddings, and therefore v ramifies in K/K^+. Thus, all archimedean places of K^+ ramify in K and $e(w|v) = 2$ for all $w|v$.