quotient rule for arithmetic derivative

Theorem. *If the notion of arithmetic derivative is extended to rational numbers, then we have that, for every $a, b \in \mathbb{Z}$ with $b \neq 0$:*

$$\left(\frac{a}{b}\right)' = \frac{a'b - b'a}{b^2}$$

Proof. Note that

$$a' = \left(b \cdot \frac{a}{b}\right)'$$

$$= b \cdot \left(\frac{a}{b}\right)' + b' \cdot \frac{a}{b} \text{ by the Leibniz rule.}$$

Thus,

$$b \cdot \left(\frac{a}{b}\right)' = a' - b' \cdot \frac{a}{b}$$

$$= \frac{a'b - b'a}{b}.$$

It follows that

$$\left(\frac{a}{b}\right)' = \frac{a'b - b'a}{b^2}.$$

$\square$