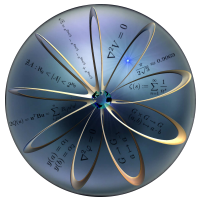

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unramified action

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Let K be a number field and let ν be a discrete valuation on K (this might be, for example, the valuation attached to a prime ideal $\mathfrak{P}$ of K).

Let K_ν be the completion of K at ν, and let $\mathcal{O}_\nu$ be the ring of integers of K_ν, i.e.

$$\mathcal{O}_\nu = \{k \in K_\nu \mid \nu(k) \geq 0\}$$

The maximal ideal of $\mathcal{O}_\nu$ will be denoted by

$$\mathcal{M} = \{k \in K_\nu \mid \nu(k) > 0\}$$

and we denote by k_ν the residue field of K_ν, which is

$$k_\nu = \mathcal{O}_\nu / \mathcal{M}$$

We will consider three different global Galois groups, namely

$$G_{\overline{K}/K} = \mathrm{Gal}(\overline{K}/K)$$

$$G_{\overline{K_\nu}/K_\nu} = \mathrm{Gal}(\overline{K_\nu}/K_\nu)$$

$$G_{\overline{k_\nu}/k_\nu} = \mathrm{Gal}(\overline{k_\nu}/k_\nu)$$

where $\overline{K}, \overline{K_\nu}, \overline{k_\nu}$ are algebraic closures of the corresponding field. We also define notation for the inertia group of $G_{\overline{K_\nu}/K_\nu}$

$$I_\nu \subseteq G_{\overline{K_\nu}/K_\nu}$$

Definition 1. *Let $\mathcal{S}$ be a set and suppose there is a group action of $Gal(\overline{K_\nu}/K_\nu)$ on $\mathcal{S}$. We say that $\mathcal{S}$ is* unramified *at ν, or the action of $G_{\overline{K_\nu}/K_\nu}$ on $\mathcal{S}$ is unramified at ν, if the action of I_ν on $\mathcal{S}$ is trivial, i.e.*

$$\sigma(s) = s \quad \forall \sigma \in I_\nu, \quad \forall s \in \mathcal{S}$$

Remark: By Galois theory we know that, K_ν^{nr}, the fixed field of I_ν, the inertia subgroup, is the maximal unramified extension of K_ν, so

$$I_\nu \cong \mathrm{Gal}(\overline{K_\nu}/K_\nu^{\mathrm{nr}})$$