Vieta's formula

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Suppose $P(x)$ is a polynomial of degree n with roots $r_1, r_2, \ldots, r_n$ (not necessarily distinct). For $1 \leq k \leq n$, define S_k by

$$S_k = \sum_{1 \leq \alpha_1 < \alpha_2 < \ldots \alpha_k \leq n} r_{\alpha_1} r_{\alpha_2} \ldots r_{\alpha_k}$$

For example,

$$S_1 = r_1 + r_2 + r_3 + \ldots + r_n$$

$$S_2 = r_1 r_2 + r_1 r_3 + r_1 r_4 + r_2 r_3 + \ldots + r_{n-1} r_n$$

Then writing $P(x)$ as

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots a_1 x + a_0,$$

we find that

$$S_i = (-1)^i \frac{a_{n-i}}{a_n}$$

For example, if $P(x)$ is a polynomial of degree 1, then $P(x) = a_1 x + a_0$ and clearly $r_1 = -\frac{a_0}{a_1}$.

If $P(x)$ is a polynomial of degree 2, then $P(x) = a_2 x^2 + a_1 x + a_0$ and $r_1 + r_2 = -\frac{a_1}{a_2}$ and $r_1 r_2 = \frac{a_0}{a_2}$. Notice that both of these formulas can be determined from the quadratic formula.

More intrestingly, if $P(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$, then $r_1 + r_2 + r_3 = -\frac{a_2}{a_3}$, $r_1 r_2 + r_2 r_3 + r_3 r_1 = \frac{a_1}{a_3}$, and $r_1 r_2 r_3 = -\frac{a_0}{a_3}$.