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Gelfand–Tornheim theorem

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Defines	normed field

Theorem. Any normed field is isomorphic either to the field \mathbb{R} of real numbers or to the field \mathbb{C} of complex numbers.

The *normed field* means a field K having a subfield R isomorphic to \mathbb{R} and satisfying the following: There is a mapping $\|\cdot\|$ from K to the set of non-negative reals such that

- $\|a\| = 0$ iff $a = 0$
- $\|ab\| \leq \|a\| \cdot \|b\|$
- $\|a + b\| \leq \|a\| + \|b\|$
- $\|ab\| = |a| \cdot \|b\|$ when $a \in R$ and $b \in K$

Using the Gelfand–Tornheim theorem, it can be shown that the only fields with archimedean valuation are isomorphic to subfields of \mathbb{C} and that the valuation is the usual absolute value (modulus) or some positive power of the absolute value.

References

- [1] Emil Artin: . Lecture notes. Mathematisches Institut, Göttingen (1959).