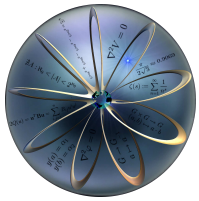

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extension field

Canonical name	ExtensionField
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Entry type	Definition
Classification	msc 12F99
Synonym	extension
Related topic	FiniteExtension
Related topic	PerfectField
Related topic	GroundFieldsAndRings
Related topic	EquivalentConditionsForNormalityOfAFieldExtension
Defines	degree
Defines	field extension
Defines	base field

We say that a field K is an *extension* of F if F is a subfield of K.

We usually denote K being an extension of F by $F \subset K,\ F \leq K,\ K/F$ or

$$\begin{array}{c} K \\ | \\ F \end{array}$$

One may speak of the *field extension* K/F and call F the *base field*.

If K is an extension of F, we can regard K as a vector space over F. The dimension of this space (which could possibly be infinite) is denoted $[K : F]$, and called the *degree* of the extension.[1]

One of the classic theorems on extensions states that if $F \subset K \subset L$, then

$$[L : F] = [L : K][K : F]$$

(in other words, degrees are multiplicative in towers).

[1]The term "degree" reflects the fact that, in the more general setting of Dedekind domains and scheme-theoretic algebraic curves, the degree of an extension of function fields equals the algebraic degree of the polynomial defining the projection map of the underlying curves.