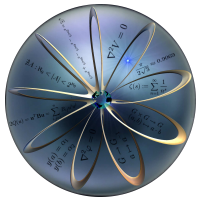

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Frobenius homomorphism

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Defines	Frobenius automorphism

Let F be a field of characteristic $p > 0$. Then for any $a, b \in F$,

$$\begin{aligned} (a+b)^p &= a^p + b^p, \\ (ab)^p &= a^p b^p. \end{aligned}$$

Thus the map

$$\begin{aligned} \phi : F &\to F \\ a &\mapsto a^p \end{aligned}$$

is a field homomorphism, called the *Frobenius homomorphism*, or simply the *Frobenius map* on F. If it is surjective then it is an automorphism, and is called the *Frobenius automorphism*.

Note: This morphism is sometimes also called the "small Frobenius" to distinguish it from the map $a \mapsto a^q$, with $q = p^n$. This map is then also referred to as the "big Frobenius" or the "power Frobenius map".