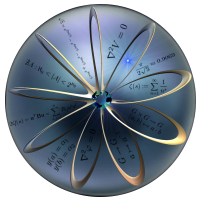

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derivation of quadratic formula

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Suppose A, B, C are real numbers, with $A \neq 0$, and suppose

$$Ax^2 + Bx + C = 0.$$

Since A is nonzero, we can divide by A and obtain the equation

$$x^2 + bx + c = 0,$$

where $b = \frac{B}{A}$ and $c = \frac{C}{A}$. This equation can be written as

$$x^2 + bx + \frac{b^2}{4} - \frac{b^2}{4} + c = 0,$$

so completing the square, i.e., applying the identity $(p+q)^2 = p^2 + 2pq + q^2$, yields

$$\left(x + \frac{b}{2}\right)^2 = \frac{b^2}{4} - c.$$

Then, taking the square root of both sides, and solving for x, we obtain the solution formula

$$\begin{aligned} x &= -\frac{b}{2} \pm \sqrt{\frac{b^2}{4} - c} \\ &= \frac{B}{2A} \pm \sqrt{\frac{B^2}{4A^2} - \frac{C}{A}} \\ &= \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \end{aligned}$$

and the derivation is completed.

A slightly less intuitive but more aesthetically pleasing approach to this derivation can be achieved by multiplying both sides of the equation

$$ax^2 + bx + c = 0$$

by $4a$, resulting in the equation

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac,$$

in which the left-hand side can be expressed as $(2ax + b)^2$. From here, the proof is identical.