finite field cannot be algebraically closed

Canonical name	FiniteFieldCannotBeAlgebraicallyClosed
Date of creation	2013-03-22 16:29:09
Last modified on	2013-03-22 16:29:09
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	6
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Entry type	Theorem
Classification	msc 12F05
Related topic	AlgebraicClosureOfAFiniteField

Theorem. *A finite field cannot be algebraically closed.*

Proof. The proof proceeds by the method of contradiction. Assume that a field F is both finite and algebraically closed. Consider the polynomial $p(x) = x^2 - x$ as a function from F to F. There are two elements which any field (in particular, F) must have — the additive identity 0 and the multiplicative identity 1. The polynomial p maps both of these elements to 0. Since F is finite and the function $p\colon F \to F$ is not one-to-one, the function cannot map onto F either, so there must exist an element a of F such that $x^2 - x \neq a$ for all $x \in F$. In other words, the polynomial $x^2 - x - a$ has no root in F, so F could not be algebraically closed. $\square$