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monomial

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Defines	degree of a monomial

A *monomial* is a product of non-negative powers of variables. It may also include an optional coefficient (which is sometimes ignored when discussing particular properties of monomials). A polynomial can be thought of as a sum over a set of monomials.

For example, the following are monomials.

$$1 \quad x \quad x^2y \\ xyz \quad 3x^4y^2z^3 \quad -z$$

If there are  $n$  variables from which a monomial may be formed, then a monomial may be represented without its coefficient as a vector of  $n$  naturals. Each position in this vector would correspond to a particular variable, and the value of the element at each position would correspond to the power of that variable in the monomial. For instance, the monomial  $x^2yz^3$  formed from the set of variables  $\{w, x, y, z\}$  would be represented as  $(0 \ 2 \ 1 \ 3)^T$ . A constant would be a zero vector.

Given this representation, we may define a few more concepts. First, the *degree of a monomial* is the sum of the elements of its vector representation. Thus, the degree of  $x^2yz^3$  is  $0 + 2 + 1 + 3 = 6$ , and the degree of a constant is 0. If a polynomial is represented as a sum over a set of monomials, then the degree of a polynomial can be defined as the degree of the monomial of largest degree belonging to that polynomial.