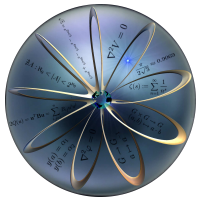

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examples of trace and norm

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Let ω be a complex root of unity different than 1. Then ω and ω^2 are the conjugate roots of the minimal polynomial x^2+x+1. Since $\mathbb{Q}(\omega)$ is the splitting field of x^2+x+1, it is Galois over $\mathbb{Q}$. Moreover the Galois group $Gal(\mathbb{Q}(\omega)/\mathbb{Q}))$ is formed by the identity and the automorphism $g(\omega)=\omega^2$ The elements of $\mathbb{Q}(\omega)$ have the form $a+b\omega$, $a,b\in\mathbb{Q}$. Then we obtain

$$N^{\mathbb{Q}}_{\mathbb{Q}(\omega)}(a+b\omega)=(a+b\omega)(a+b\omega^2)=a^2-ab+b^2, Tr^{\mathbb{Q}}_{\mathbb{Q}(\omega)}(a+b\omega)=(a+b\omega)+(a+b\omega^2)=2a-b$$