the compositum of a Galois extension and another extension is Galois

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Theorem 1. *Let E/K be a Galois extension of fields, let F/K be an arbitrary extension and assume that E and F are both subfields of some other larger field T. The compositum of E and F is here denoted by EF. Then:*

1. *EF is a Galois extension of F and E is Galois over $E \cap F$;*

2. *Let $H = \mathrm{Gal}(EF/F)$. The restriction map:*

$$\begin{aligned} H = \mathrm{Gal}(EF/F) &\longrightarrow \mathrm{Gal}(E/E \cap F) \\ \sigma &\longrightarrow \sigma|_E \end{aligned}$$

   is an isomorphism, where $\sigma|_E$ denotes the restriction of σ to E.

Remark 1. *Notice, however, that if E/F and F/K are both Galois extensions, the extension E/K need not be Galois. See example of normal extension for a counterexample.*