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multiplicity

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Entry type	Definition
Classification	msc 12D10
Synonym	order of the zero
Related topic	OrderOfVanishing
Related topic	DerivativeOfPolynomial
Defines	zero of order
Defines	multiple zero
Defines	simple zero
Defines	simple

If a polynomial $f(x)$ in $\mathbb{C}[x]$ is divisible by $(x-a)^m$ but not by $(x-a)^{m+1}$ (a is some complex number, $m \in \mathbb{Z}_+$), we say that $x = a$ is a *zero of the polynomial with multiplicity* m or alternatively a *zero of order* m.

Generalization of the multiplicity to http://planetmath.org/RealFunctionreal and complex functions (by rspuzio): If the function f is continuous on some open set D and $f(a) = 0$ for some $a \in D$, then the zero of f at a is said to be of multiplicity m if $\frac{f(z)}{(z-a)^m}$ is continuous in D but $\frac{f(z)}{(z-a)^{m+1}}$ is not.

If $m \geqq 2$, we speak of a *multiple zero*; if $m = 1$, we speak of a *simple zero*. If $m = 0$, then actually the number a is not a zero of $f(x)$, i.e. $f(a) \neq 0$.

Some properties (from which 2, 3 and 4 concern only polynomials):

1. The zero a of a polynomial $f(x)$ with multiplicity m is a zero of the $f'(x)$ with multiplicity $m-1$.

2. The zeros of the polynomial $\gcd(f(x), f'(x))$ are same as the multiple zeros of $f(x)$.

3. The quotient $\dfrac{f(x)}{\gcd(f(x), f'(x))}$ has the same zeros as $f(x)$ but they all are .

4. The zeros of any irreducible polynomial are .