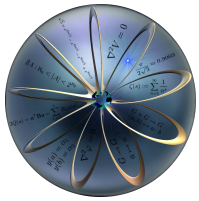

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variant of Cardano's derivation

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By a linear change of variable, a cubic polynomial over $\mathbb{C}$ can be given the form $x^3 + 3bx + c$. To find the zeros of this cubic in the form of surds in b and c, make the substitution $x = y^{1/3} + z^{1/3}$, thus replacing one unknown with two, and then write down identities which are suggested by the resulting equation in two unknowns. Specifically, we get

$$y + 3(y^{1/3} + z^{1/3})y^{1/3}z^{1/3} + z + 3b(y^{1/3} + z^{1/3}) + c = 0. \tag{1}$$

This will be true if

$$y + z + c = 0 \tag{2}$$

$$3y^{1/3}z^{1/3} + 3b = 0, \tag{3}$$

which in turn requires

$$yz = -b^3. \tag{4}$$

The pair of equations (2) and (4) is a quadratic system in y and z, readily solved. But notice that (3) puts a restriction on a certain choice of cube roots.