Lindemann-Weierstrass theorem

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If $\alpha_1, \ldots, \alpha_n$ are linearly independent algebraic numbers over $\mathbb{Q}$, then $e^{\alpha_1}, \ldots, e^{\alpha_n}$ are algebraically independent over $\mathbb{Q}$.

An equivalent version of the theorem that if $\alpha_1, \ldots, \alpha_n$ are distinct algebraic numbers over $\mathbb{Q}$, then $e^{\alpha_1}, \ldots, e^{\alpha_n}$ are linearly independent over $\mathbb{Q}$.

Some immediate consequences of this theorem:

- If α is a non-zero algebraic number over $\mathbb{Q}$, then e^{α} is transcendental over $\mathbb{Q}$.
- e is transcendental over $\mathbb{Q}$.
- π is transcendental over $\mathbb{Q}$. As a result, it is impossible to "square the circle"!

It is easy to see that π is transcendental over $\mathbb{Q}(e)$ iff e is transcendental over $\mathbb{Q}(\pi)$ iff π and e are algebraically independent. However, whether π and e are algebraically independent is still an open question today.

Schanuel's conjecture is a generalization of the Lindemann-Weierstrass theorem. If Schanuel's conjecture were proven to be true, then the algebraic independence of e and π over $\mathbb{Q}$ can be shown.