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perfect field

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A *perfect field* is a field K such that every algebraic extension field L/K is separable over K.

All fields of characteristic 0 are perfect, so in particular the fields $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{Q}$ are perfect. If K is a field of characteristic p (with p a prime number), then K is perfect if and only if the Frobenius endomorphism F on K, defined by

$$F(x) = x^p \quad (x \in K),$$

is an automorphism of K. Since the Frobenius map is always injective, it is sufficient to check whether F is surjective. In particular, all finite fields are perfect (any injective endomorphism is also surjective). Moreover, any field whose characteristic is nonzero that is `http://planetmath.org/AlgebraicExtension`algebraic over its prime subfield is perfect. Thus, the only fields that are not perfect are those whose characteristic is nonzero and are transcendental over their prime subfield.

Similarly, a ring R of characteristic p is perfect if the endomorphism $x \mapsto x^p$ of R is an *automorphism* (i.e., is surjective).