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p-extension

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Definition 1. Let p be a prime number. A Galois extension of fields E/F , with $G = \text{Gal}(E/F)$, is said to be a p -extension if G is a p -group.

Example 1. Let d be a square-free integer. Then the field extension $\mathbb{Q}(\sqrt{d})/\mathbb{Q}$ is a 2-extension.

Example 2. Let $p > 2$ be a prime and, for any n , let ζ_{p^n} be a primitive p^n th root of unity. The cyclotomic extension:

$$\mathbb{Q}(\zeta_{p^n})/\mathbb{Q}(\zeta_p)$$

is a p -extension. Indeed:

$$G_n = \text{Gal}(\mathbb{Q}(\zeta_{p^n})/\mathbb{Q}) \cong (\mathbb{Z}/p^n\mathbb{Z})^\times$$

Thus, $|G_n| = \varphi(p^n) = p^{(n-1)}(p-1)$ and $|G_1| = \varphi(p) = p-1$, where φ is the Euler phi function. Therefore the extension above is of degree $p^{(n-1)}$.