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radical extension

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Synonym	radical tower

A *radical tower* is a field extension L/F which has a filtration

$$F = L_0 \subset L_1 \subset \cdots \subset L_n = L$$

where for each i, $0 \leq i < n$, there exists an element $\alpha_i \in L_{i+1}$ and a natural number n_i such that $L_{i+1} = L_i(\alpha_i)$ and $\alpha_i^{n_i} \in L_i$.

A *radical extension* is a field extension K/F for which there exists a radical tower L/F with $L \supset K$. The notion of radical extension coincides with the informal concept of solving for the roots of a polynomial by radicals, in the sense that a polynomial over K is solvable by radicals if and only if its splitting field is a radical extension of F.