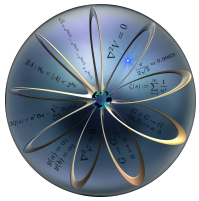

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Hilbert's irreducibility theorem

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Defines	Hilbert property
Defines	Hilbertian field

In this entry, K is a field of characteristic zero and V is an irreducible algebraic variety over K.

Definition 1. *A variety V satisfies the Hilbert property over K if $V(K)$ is not a thin algebraic set.*

Definition 2. *A field K is said to be Hilbertian if there exists an irreducible variety V/K of $\dim V \geq 1$ which has the Hilbert property.*

Theorem (Hilbert's irreducibility theorem)**.** *A number field K is Hilbertian. In particular, for every n, the affine space $\mathbb{A}^n(K)$ has the Hilbert property over K.*

However, the field of real numbers $\mathbb{R}$ and the field of p-adic rationals $\mathbb{Q}_p$ are not Hilbertian.

References

[1] J.-P. Serre, *Topics in Galois Theory*, Research Notes in Mathematics, Jones and Barlett Publishers, London.