irreducible polynomial

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Let $f(x) = a_0+a_1x+\cdots+a_nx^n$ be a polynomial with complex coefficients a_ν and with the http://planetmath.org/Polynomialdegree $n > 0$. If $f(x)$ can not be written as product of two polynomials with positive degrees and with coefficients in the field $\mathbb{Q}(a_0,\, a_1,\, \ldots,\, a_n)$, then the polynomial $f(x)$ is said to be . Otherwise, $f(x)$ is *reducible*.

Examples. All linear polynomials are . The polynomials x^2-3, x^2+1 and x^2-i are (although they split in linear factors in the fields $\mathbb{Q}(\sqrt{3})$, $\mathbb{Q}(i)$ and $\mathbb{Q}(\frac{1+i}{\sqrt{2}})$, respectively). The polynomials x^4+4 and x^6+1 are not .

The above definition of polynomial is special case of the more general setting where $f(x)$ is a non-constant polynomial in the polynomial ring $K[x]$ of a field K; if $f(x)$ is not expressible as product of two polynomials with positive degrees in the ring $K[x]$, then $f(x)$ is (in $K[x]$).

Example. If K is the Galois field with two elements (0 and 1), then the trinomial x^2+x+1 of $K[x]$ is (because an equation $x^2+x+1 = (x+a)(x+b)$ would imply the two conflicting conditions $a+b = 1$ and $ab = 1$).