# characteristic

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Let $(F, +, \cdot)$ be a field. The *characteristic* $\mathrm{Char}(F)$ of $F$ is commonly given by one of three equivalent definitions:

- if there is some positive integer $n$ for which the result of adding any element to itself $n$ times yields 0, then the characteristic of the field is the least such $n$. Otherwise, $\mathrm{Char}(F)$ is defined to be 0.
- if $f : \mathbb{Z} \to F$ is defined by $f(n) = n \cdot 1$ then $\mathrm{Char}(F)$ is the least strictly positive generator of $\ker(f)$ if $\ker(f) \neq \{0\}$; otherwise it is 0.
- if $K$ is the prime subfield of $F$, then $\mathrm{Char}(F)$ is the size of $K$ if this is finite, and 0 otherwise.

Note that the first definition also applies to arbitrary rings, and not just to fields.

The characteristic of a field (or more generally an integral domain) is always prime. For if the characteristic of $F$ were composite, say $mn$ for $m, n > 1$, then in particular $mn$ would equal zero. Then either $m$ would be zero or $n$ would be zero, so the characteristic of $F$ would actually be smaller than $mn$, contradicting the minimality condition.