de Moivre identity

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From the Euler relation

$$e^{i\theta} = \cos\theta + i\sin\theta$$

it follows that

$$e^{i\theta \cdot n} = (e^{i\theta})^n$$
$$\cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^n$$

where $n \in \mathbb{Z}$. This is called *de Moivre's formula*, and besides being generally useful, it's a convenient way to remember double- (and higher-multiple-) angle formulas. For example,

$$\cos 2\theta + i\sin 2\theta = (\cos\theta + i\sin\theta)^2 = \cos^2\theta + 2i\sin\theta\cos\theta - \sin^2\theta.$$

Since the imaginary parts and real parts on each side must be equal, we must have

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

and

$$\sin 2\theta = 2\sin\theta\cos\theta.$$