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algebraic extension

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Related topic	Algebraic
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Related topic	EquivalentConditionsForNormalityOfAFieldExtension
Defines	examples of field extension
Defines	transcendental extension

Definition 1. *Let L/K be an extension of fields. L/K is said to be an* algebraic extension *of fields if every element of L is algebraic over K. If L/K is not algebraic then we say that it is a transcendental extension of fields.*

Examples:

1. Let $L = \mathbb{Q}(\sqrt{2})$. The extension $L/\mathbb{Q}$ is an algebraic extension. Indeed, any element $\alpha \in L$ is of the form

$$\alpha = q + t\sqrt{2} \in L$$

for some $q, t \in \mathbb{Q}$. Then $\alpha \in L$ is a root of

$$X^2 - 2qX + q^2 - 2t^2 = 0$$

2. The field extension $\mathbb{R}/\mathbb{Q}$ is not an algebraic extension. For example, $\pi \in \mathbb{R}$ is a transcendental number over $\mathbb{Q}$ (see pi). So $\mathbb{R}/\mathbb{Q}$ is a transcendental extension of fields.

3. Let K be a field and denote by $\overline{K}$ the algebraic closure of K. Then the extension $\overline{K}/K$ is algebraic.

4. In general, a finite extension of fields is an algebraic extension. However, the converse is not true. The extension $\overline{\mathbb{Q}}/\mathbb{Q}$ is *far* from finite.

5. The extension $\mathbb{Q}(\pi)/\mathbb{Q}$ is transcendental because π is a transcendental number, i.e. π is not the root of any polynomial $p(x) \in \mathbb{Q}[x]$.