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Galois group of the compositum of two Galois extensions

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Theorem 1. *Let E and F be Galois extensions of a field K . Then:*

1. *The intersection $E \cap F$ is Galois over K .*
2. *The compositum EF is Galois over K . Moreover, the Galois group $\text{Gal}(EF/K)$ is isomorphic to the subgroup H of the direct product $G = \text{Gal}(E/K) \times \text{Gal}(F/K)$ given by:*

$$H = \{(\sigma, \psi) : \sigma|_{E \cap F} = \psi|_{E \cap F}\}$$

- i. e. H consists of pairs of elements of G whose restrictions to $E \cap F$ are equal.*

Corollary 1. *Let E and F be Galois extensions of a field K such that $E \cap F = K$. Then EF is Galois over K and the Galois group is isomorphic to the direct product:*

$$\text{Gal}(EF/K) \cong \text{Gal}(E/K) \times \text{Gal}(F/K).$$