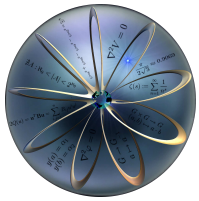

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thin algebraic set

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Definition 1. *Let V be an irreducible algebraic variety (we assume it to be integral and quasi-projective) over a field K with characteristic zero. We regard V as a topological space with the usual Zariski topology.*

1. *A subset $A \subset V(K)$ is said to be of type C_1 if there is a closed subset $W \subset V$, with $W \neq V$, such that $A \subset W(K)$. In other words, A is not dense in V (with respect to the Zariski topology).*

2. *A subset $A \subset V(K)$ is said to be of type C_2 if there is an irreducible variety V' of the same dimension as V, and a (generically) surjective algebraic morphism $\phi\colon V' \to V$ of degree ≥ 2, with $A \subset \phi(V'(K))$*

Example. Let K be a field and let $V(K) = \mathbb{A}(K) = \mathbb{A}^1(K) = K$ be the 1-dimensional affine space. Then, the only Zariski-closed subsets of V are finite subsets of points. Thus, the only subsets of type C_1 are subsets formed by a finite number of points.

Let $V'(K) = \mathbb{A}(K)$ be affine space and define:

$$\phi\colon V' \to V$$

by $\phi(k) = k^2$. Then $\deg(\phi) = 2$. Thus, the subset:

$$A = \{k^2 : k \in \mathbb{A}(K)\}$$

, i.e. A is the subset of perfect squares in K, is a subset of type C_2.

Definition 2. *A subset A of an irreducible variety V/K is said to be a thin algebraic set (or thin set, or "mince" set) if it is a union of a finite number of subsets of type C_1 and type C_2.*

References

[1] J.-P. Serre, *Topics in Galois Theory*, Research Notes in Mathematics, Jones and Barlett Publishers, London.