conjugated roots of equation

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The rules

$$\overline{w_1 + w_2} = \overline{w_1} + \overline{w_2} \quad \text{and} \quad \overline{w_1 w_2} = \overline{w_1}\,\overline{w_2},$$

concerning the complex conjugates of the sum and product of two complex numbers, may be by induction generalised for arbitrary number of complex numbers w_k. Since the complex conjugate of a real number is the same real number, we may write

$$\overline{a_k z^k} = a_k \overline{z}^k$$

for real numbers a_k $(k = 0, 1, 2, \ldots)$. Thus, for a polynomial $P(x) := a_0x^n + a_1x^{n-1} + \ldots + a_n$ we obtain

$$\overline{P(z)} = \overline{a_0z^n + a_1z^{n-1} + \ldots + a_n} = a_0\overline{z}^n + a_1\overline{z}^{n-1} + \ldots + a_n = P(\overline{z}).$$

I.e., the values of a polynomial with real coefficients computed at a complex number and its complex conjugate are complex conjugates of each other.

If especially the value of a polynomial with real coefficients vanishes at some complex number z, it vanishes also at $\overline{z}$. So the roots of an algebraic equation

$$P(x) = 0$$

with real coefficients are pairwise complex conjugate numbers.

Example. The roots of the binomial equation

$$x^3 - 1 = 0$$

are $x = 1,\ x = \frac{-1 \pm i\sqrt{3}}{2},$ the third roots of unity.