field

Canonical name	Field
Date of creation	2013-03-22 11:48:43
Last modified on	2013-03-22 11:48:43
Owner	djao (24)
Last modified by	djao (24)
Numerical id	9
Author	djao (24)
Entry type	Definition
Classification	msc 12E99
Classification	msc 03A05

A *field* is a set F together with two binary operations on F, called addition and multiplication, and denoted $+$ and $\cdot$, satisfying the following properties, for all $a, b, c \in F$:

1. $a + (b + c) = (a + b) + c$ (associativity of addition)
2. $a + b = b + a$ (commutativity of addition)
3. $a + 0 = a$ for some element $0 \in F$ (existence of zero element)
4. $a + (-a) = 0$ for some element $-a \in F$ (existence of additive inverses)
5. $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ (associativity of multiplication)
6. $a \cdot b = b \cdot a$ (commutativity of multiplication)
7. $a \cdot 1 = a$ for some element $1 \in F$, with $1 \neq 0$ (existence of unity element)
8. If $a \neq 0$, then $a \cdot a^{-1} = 1$ for some element $a^{-1} \in F$ (existence of multiplicative inverses)
9. $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ (distributive property)

Equivalently, a field is a commutative ring F with identity such that:

- $1 \neq 0$
- If $a \in F$, and $a \neq 0$, then there exists $b \in F$ with $a \cdot b = 1$.