exclusion of integer root

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Theorem. The equation

$$p(x) \ := \ a_nx^n + a_{n-1}x^{n-1} + \ldots + a_0 \ = \ 0$$

with integer coefficients a_i has no integer `http://planetmath.org/Equation`roots, if $p(0)$ and $p(1)$ are odd.

Proof. Make the antithesis, that there is an integer x_0 such that $p(x_0) = 0$. This x_0 cannot be even, because else all terms of $p(x_0)$ except a_0 were even and thus the whole sum could not have the even value 0. Consequently, x_0 and also its `http://planetmath.org/GeneralAssociativity`powers have to be odd. Since

$$2 \mid 0 = p(x_0) \quad \text{and} \quad 2 \nmid p(0) = a_0,$$

there must be among the coefficients a_n, a_{n-1}, ..., a_1 an odd amount of odd numbers. This means that

$$2 \mid a_n + a_{n-1} + \ldots + a_1 + a_0 \ = \ p(1).$$

This however contradicts the assumption on the parity of $p(1)$, whence the antithesis is wrong and the theorem .