fundamental theorem of algebra result

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Author	rspuzio (6075)
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This leads to the following theorem:

Given a polynomial $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ of degree $n \geq 1$ where $a_i \in \mathbb{C}$, there are exactly n roots in $\mathbb{C}$ to the equation $p(x) = 0$ if we count multiple roots.

Proof The non-constant polynomial $a_1 x - a_0$ has one root, $x = a_0/a_1$. Next, assume that a polynomial of degree $n - 1$ has $n - 1$ roots.

The polynomial of degree n has then by the fundamental theorem of algebra a root z_n. With polynomial division we find the unique polynomial $q(x)$ such that $p(x) = (x - z_n)q(x)$. The original equation has then $1 + (n - 1) = n$ roots. By induction, every non-constant polynomial of degree n has exactly n roots.

For example, $x^4 = 0$ has four roots, $x_1 = x_2 = x_3 = x_4 = 0$.