theorems on continuation

Canonical name	TheoremsOnContinuation
Date of creation	2013-03-22 17:59:51
Last modified on	2013-03-22 17:59:51
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	10
Author	pahio (2872)
Entry type	Theorem
Classification	msc 12J20
Classification	msc 13A18
Classification	msc 13F30
Classification	msc 11R99
Synonym	theorems on continuations of exponents

Theorem 1. When ν_0 is an exponent valuation of the field k and K/k is a finite field extension, ν_0 has a continuation to the extension field K.

Theorem 2. If the `http://planetmath.org/ExtensionField`degree of the field extension K/k is n and ν_0 is an arbitrary `http://planetmath.org/ExponentValuation2ex` of k, then ν_0 has at most n continuations to the extension field K.

Theorem 3. Let ν_0 be an exponent valuation of the field k and $\mathfrak{o}$ the ring of the exponent ν_0. Let K/k be a finite extension and $\mathfrak{O}$ the integral closure of $\mathfrak{o}$ in K. If $\nu_1, \ldots, \nu_m$ are all different continuations of ν_0 to the field K and $\mathfrak{O}_1, \ldots, \mathfrak{O}_m$ `http://planetmath.org/RingOfExponent`their rings, then

$$\mathfrak{O} = \bigcap_{i=1}^{m} \mathfrak{O}_i.$$

The proofs of those theorems are found in [1], which is available also in Russian (original), English and French.

Corollary. The ring $\mathfrak{O}$ (of theorem 3) is a UFD. The exponents of K, which are determined by the pairwise coprime prime elements of $\mathfrak{O}$, coincide with the continuations $\nu_1, \ldots, \nu_m$ of ν_0. If $\pi_1, \ldots, \pi_m$ are the pairwise coprime prime elements of $\mathfrak{O}$ such that $\nu_i(\pi_1) = 1$ for all i's and if the prime element p of the ring $\mathfrak{o}$ has the

$$p = \varepsilon \pi_1^{e_1} \cdots \pi_m^{e_m}$$

with ε a unit of $\mathfrak{O}$, then e_i is the ramification index of the exponent ν_i with respect to ν_0 $(i = 1, \ldots, m)$.

References

[1] S. Borewicz & I. Safarevic: *Zahlentheorie.* Birkhäuser Verlag. Basel und Stuttgart (1966).