Tschirnhaus transformations

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A polynomial transformation which transforms a polynomial to another with certain zero-coefficients is called a *Tschirnhaus Transformation*. It is thus an invertible transformation of the form $x \mapsto g(x)/h(x)$ where g, h are polynomials over the base field K (or some subfield of the splitting field of the polynomial being transformed). If $\gcd(h(x), f(x)) = 1$ then the Tschirnhaus transformation becomes a polynomial transformation mod f.

Specifically, it concerns a substitution that reduces finding the roots of the polynomial

$$\mathrm{p} = T^n + a_1 T^{n-1} + ... + a_n = \prod_{i=1}^{n} (T - r_i) \in k[T]$$

to finding the roots of another q - with less parameters - and solving an auxiliary polynomial equation s, with $\deg(s) < \deg(p \cap q)$.

Historically, the transformation was applied to reduce the general quintic equation, to simpler resolvents. Examples due to Hermite and Klein are respectively: The principal resolvent

$$K(X) := X^5 + a_0 X^2 + a_1 X + a_3$$

and the Bring-Jerrard form

$$K(X) := X^5 + a_1 X + a_2$$

Tschirnhaus transformations are also used when computing Galois groups to remove repeated roots in resolvent polynomials. Almost any transformation will work but it is extremely hard to find an efficient algorithm that can be proved to work.