subfield criterion

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Let K be a skew field and S its subset. For S to be a subfield of K, it's necessary and sufficient that the following three conditions are fulfilled:

1. S a non-zero element of K.
2. $a - b \in S$ always when $a, b \in S$.
3. $ab^{-1} \in S$ always when $a, b \in S$ and $b \neq 0$.

Proof. Because the conditions are fulfilled in every skew field, they are necessary. For proving the sufficience, suppose now that the subset S these conditions. The condition 1 guarantees that S is not empty and the condition 2 that $(S, +)$ is an subgroup of $(K, +)$; thus all the required properties of addition for a skew field hold in S. If b is a non-zero element of S, then, according to the condition 3, we have $0 \neq 1 = bb^{-1} \in S$. Moreover, $a \cdot 1 = 1 \cdot a = a \in S$ for all $a \in S \subseteq K$. The laws of multiplication (associativity and left and distributivity over addition) hold in S since they hold in whole K. So S fulfils all the postulates for a skew field.