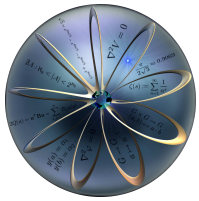

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properties of quadratic equation

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The quadratic equation

$$ax^2+bx+c=0$$

or

$$x^2+px+q=0$$

with rational, real, `http://planetmath.org/AlgebraicNumber`algebraic or complex coefficients ($a \neq 0$) has the following properties:

- It has in $\mathbb{C}$ two roots (which may be equal), since the complex numbers form an algebraically closed field containing the coefficients.
- The sum of the roots is equal to $-\frac{b}{a}$, i.e. $-p$.
- The product of the roots is equal to $\frac{c}{a}$, i.e. q.

Corollary. If the leading coefficient and the constant are equal, then the roots are inverse numbers of each other.

Without solving the equation, the value of any symmetric polynomial of the roots can be calculated.

Example. If one has to $x_1^3+x_2^3$, when x_1 and x_2 are the roots of the equation $x^2-4x+9=0$, we have $x_1+x_2=4$ and $x_1x_2=9$. Because

$$(x_1+x_2)^3 = x_1^3+3x_1^2x_2+3x_1x_2^2+x_2^3 = (x_1^3+x_2^3)+3x_1x_2(x_1+x_2),$$

we obtain

$$x_1^3+x_2^3 = (x_1+x_2)^3-3x_1x_2(x_1+x_2) = 4^3-3\cdot 9\cdot 4 = -44.$$

Note. If one wants to write easily a quadratic equation with rational roots, one could take such one that the sum of the coefficients is zero (then one root is always 1). For instance, the roots of the equation $5x^2+11x-16=0$ are 1 and $-\frac{16}{5}$.