



quartic formula

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The four roots r_1, r_2, r_3, r_4 of a quartic polynomial equation $x^4 + ax^3 + bx^2 + cx + d = 0$ are given by

$$\begin{aligned}
r_1 &= \frac{-a}{4} - \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)}} \\
r_2 &= \frac{-a}{4} - \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)}} \\
r_3 &= \frac{-a}{4} + \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)}} \\
r_4 &= \frac{-a}{4} + \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}\right)}}
\end{aligned}$$

The formulas for the roots are much too unwieldy to be used for solving quartic equations by radicals, even with the help of a computer. A practical algorithm for solving quartic equations by radicals is given in the concluding paragraph of the Galois-theoretic derivation of the quartic formula.