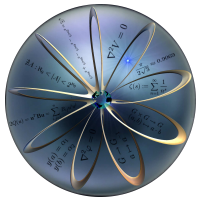

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trace

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Let K/F be a Galois extension, and let $x \in K$. The *trace* $\mathrm{Tr}_F^K(x)$ of x is defined to be the sum of all the elements of the orbit of x under the group action of the Galois group $\mathrm{Gal}(K/F)$ on K; taken with multiplicities if K/F is a finite extension.

In the case where K/F is a finite extension,

$$\mathrm{Tr}_F^K(x) := \sum_{\sigma \in \mathrm{Gal}(K/F)} \sigma(x)$$

The trace of x is always an element of F, since any element of $\mathrm{Gal}(K/F)$ permutes the orbit of x and thus fixes $\mathrm{Tr}_F^K(x)$.

The name "trace" derives from the fact that, when K/F is finite, the trace of x is simply the trace of the linear transformation $T : K \longrightarrow K$ of vector spaces over F defined by $T(v) := xv$.