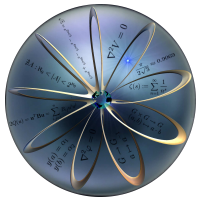

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transcendence degree

Canonical name	TranscendenceDegree
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Entry type	Definition
Classification	msc 12F20
Defines	transcendence degree of a set
Defines	transcendence degree of a field extension

The *transcendence degree* of a set S over a field K, denoted T_S, is the size of the maximal subset S' of S such that all the elements of S' are algebraically independent.

The *transcendence degree* of a field extension L over K is the transcendence degree of the minimal subset of L needed to generate L over K.

Heuristically speaking, the transcendence degree of a finite set S is obtained by taking the number of elements in the set, subtracting the number of algebraic elements in that set, and then subtracting the number of algebraic relations between distinct pairs of elements in S.

Example 1 (Computing the Transcendence Degree)**.** *The set $S = \{\sqrt{7}, \pi, \pi^2, e\}$ has transcendence $T_S \leq 2$ over $\mathbb{Q}$ since there are four elements, $\sqrt{7}$ is algebraic, and the polynomial $f(x, y) = x^2 - y$ gives an algebraic dependence between π and π^2 (i.e. (π, π^2) is a root of f), giving $T_S \leq 4 - 1 - 1 = 2$. If we assume the conjecture that e and π are algebraically independent, then no more dependencies can exist, and we can conclude that, in fact, $T_S = 2$.*