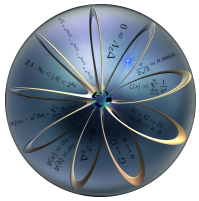

planetmath.org

Math for the people, by the people.

cubic formula

Canonical name	CubicFormula
Date of creation	2013-03-22 12:10:25
Last modified on	2013-03-22 12:10:25
Owner	djao (24)
Last modified by	djao (24)
Numerical id	10
Author	djao (24)
Entry type	Theorem
Classification	msc 12D10
Synonym	cubic equation
Related topic	QuarticFormula
Related topic	GaloisTheoreticDerivationOfTheQuarticFormula
Related topic	FerrariCardanoDerivationOfTheQuarticFormula
Related topic	FundamentalTheoremOfGaloisTheory

The three roots r_1, r_2, r_3 of a cubic polynomial equation $x^3+ax^2+bx+c = 0$ are given by

$$
\begin{aligned}
r_1 &= -\frac{a}{3} + \left(\frac{-2a^3 + 9ab - 27c + \sqrt{(2a^3 - 9ab + 27c)^2 + 4(-a^2 + 3b)^3}}{54} \right)^{1/3} \\
&\quad + \left(\frac{-2a^3 + 9ab - 27c - \sqrt{(2a^3 - 9ab + 27c)^2 + 4(-a^2 + 3b)^3}}{54} \right)^{1/3} \\
r_2 &= -\frac{a}{3} - \frac{1 + i\sqrt{3}}{2} \left(\frac{-2a^3 + 9ab - 27c + \sqrt{(2a^3 - 9ab + 27c)^2 + 4(-a^2 + 3b)^3}}{54} \right)^{1/3} \\
&\quad + \frac{-1 + i\sqrt{3}}{2} \left(\frac{-2a^3 + 9ab - 27c - \sqrt{(2a^3 - 9ab + 27c)^2 + 4(-a^2 + 3b)^3}}{54} \right)^{1/3} \\
r_3 &= -\frac{a}{3} + \frac{-1 + i\sqrt{3}}{2} \left(\frac{-2a^3 + 9ab - 27c + \sqrt{(2a^3 - 9ab + 27c)^2 + 4(-a^2 + 3b)^3}}{54} \right)^{1/3} \\
&\quad - \frac{1 + i\sqrt{3}}{2} \left(\frac{-2a^3 + 9ab - 27c - \sqrt{(2a^3 - 9ab + 27c)^2 + 4(-a^2 + 3b)^3}}{54} \right)^{1/3}
\end{aligned}
$$