complex conjugate

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1 Definition

1.1 Scalar Complex Conjugate

Let z be a complex number with real part a and imaginary part b,

$$z = a + bi$$

Then the *complex conjugate* of z is

$$\bar{z} = a - bi$$

Complex conjugation represents a reflection about the real axis on the Argand diagram representing a complex number.

Sometimes a star $(*)$ is used instead of an overline, e.g. in physics you might see

$$\int_{-\infty}^{\infty} \Psi^* \Psi dx = 1$$

where Ψ^* is the complex conjugate of a wave .

1.2 Matrix Complex Conjugate

Let $A = (a_{ij})$ be a $n \times m$ matrix with complex entries. Then the *complex conjugate* of A is the matrix $\overline{A} = (\overline{a_{ij}})$. In particular, if $v = (v^1, \ldots, v^n)$ is a complex row/column vector, then $\overline{v} = (\overline{v^1}, \ldots, \overline{v^n})$.

Hence, the matrix complex conjugate is what we would expect: the same matrix with all of its scalar components conjugated.

2 Properties of the Complex Conjugate

2.1 Scalar Properties

If u, v are complex numbers, then

1. $\overline{uv} = (\overline{u})(\overline{v})$
2. $\overline{u+v} = \overline{u} + \overline{v}$
3. $\left(\overline{u}\right)^{-1} = \overline{u^{-1}}$

4. $\overline{(\overline{u})} = u$

5. If $v \neq 0$, then $\overline{(\frac{u}{v})} = \overline{u}/\overline{v}$

6. Let $u = a + bi$. Then $\overline{u}u = u\overline{u} = a^2 + b^2 \geq 0$ (the complex modulus).

7. If z is written in polar form as $z = re^{i\phi}$, then $\overline{z} = re^{-i\phi}$.

2.2 Matrix and Vector Properties

Let A be a matrix with complex entries, and let v be a complex row/column vector.

Then

1. $\overline{A^T} = \left(\overline{A}\right)^T$

2. $\overline{Av} = \overline{A}\overline{v}$, and $\overline{vA} = \overline{v}\overline{A}$. (Here we assume that A and v are compatible size.)

Now assume further that A is a complex square matrix, then

1. $\operatorname{trace}\overline{A} = \overline{(\text{trace } A)}$

2. $\det\overline{A} = \overline{(\det A)}$

3. $\left(\overline{A}\right)^{-1} = \overline{A^{-1}}$