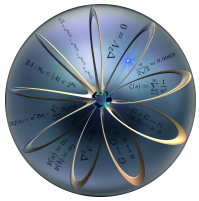

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abelian extension

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Let K be a Galois extension of F. The extension is said to be an *abelian extension* if the Galois group $\mathrm{Gal}(K/F)$ is abelian.

Examples: $\mathbb{Q}(\sqrt{2})/\mathbb{Q}$ has Galois group $\mathbb{Z}/2\mathbb{Z}$ so $\mathbb{Q}(\sqrt{2})/\mathbb{Q}$ is an abelian extension.

Let ζ_n be a `http://planetmath.org/RootOfUnity`primitive nth root of unity. Then $\mathbb{Q}(\zeta_n)/\mathbb{Q}$ has Galois group $(\mathbb{Z}/n\mathbb{Z})^*$ (the group of units of $\mathbb{Z}/n\mathbb{Z}$) so $\mathbb{Q}(\zeta_n)/\mathbb{Q}$ is abelian.