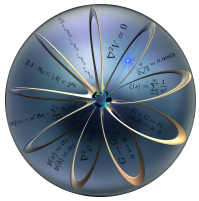

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factor theorem

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If $f(x)$ is a polynomial over a ring with identity, then $x - a$ is a factor if and only if a is a root (that is, $f(a) = 0$).

This theorem is of great help for finding factorizations of higher degree polynomials. As example, let us think that we need to factor the polynomial $p(x) = x^3 + 3x^2 - 33x - 35$. With some help of the rational root theorem we can find that $x = -1$ is a root (that is, $p(-1) = 0$), so we know $(x + 1)$ must be a factor of the polynomial. We can write then

$$p(x) = (x + 1)q(x)$$

where the polynomial $q(x)$ can be found using long or synthetic division of $p(x)$ between $x - 1$. In our case $q(x) = x^2 + 2x - 35$ which can be easily factored as $(x - 5)(x + 7)$. We conclude that

$$p(x) = (x + 1)(x - 5)(x + 7).$$