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rational root theorem

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Consider the polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where all the coefficients  $a_i$  are integers.

If  $p(x)$  has a rational zero  $u/v$  where  $\gcd(u, v) = 1$ , then  $u \mid a_0$  and  $v \mid a_n$ . Thus, for finding all rational zeros of  $p(x)$ , it suffices to perform a finite number of tests.

The theorem is related to the result about monic polynomials whose coefficients belong to a unique factorization domain. Such theorem then states that any root in the fraction field is also in the base domain.