cardinality of monomials

Canonical name	CardinalityOfMonomials
Date of creation	2013-03-22 16:34:42
Last modified on	2013-03-22 16:34:42
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	10
Author	rspuzio (6075)
Entry type	Theorem
Classification	msc 12-00

Theorem 1. *If S is a finite set of variable symbols, then the number of monomials of degree n constructed from these symbols is $\binom{n+m-1}{n}$, where m is the cardinality of S.*

Proof. The proof proceeds by inducion on the cardinality of S. If S has but one element, then there is but one monomial of degree n, namely the sole element of S raised to the n-th power. Since $\binom{n+1-1}{n} = 1$, the conclusion holds when $m = 1$.

Suppose, then, that the result holds whenver $m < M$ for some M. Let S be a set with exactly M elements and let x be an element of S. A monomial of degree n constructed from elements of S can be expressed as the product of a power of x and a monomial constructed from the elements of $S \setminus \{x\}$. By the induction hypothesis, the number of monomials of degree k constructed from elements of $S \setminus \{x\}$ is $\binom{k+M-2}{k}$. Summing over the possible powers to which x may be raised, the number of monomials of degree n constructed from the elements of S is as follows:

$$\sum_{k=0}^{n} \binom{k+M-2}{k} = \binom{k+M-1}{k}$$

□

Theorem 2. *If S is an infinite set of variable symbols, then the number of monomials of degree n constructed from these symbols equals the cardinality of S.*