# reciprocal polynomial

| | |
|---|---|
| Canonical name | ReciprocalPolynomial |
| Date of creation | 2013-03-22 13:36:33 |
| Last modified on | 2013-03-22 13:36:33 |
| Owner | rspuzio (6075) |
| Last modified by | rspuzio (6075) |
| Numerical id | 12 |
| Author | rspuzio (6075) |
| Entry type | Definition |
| Classification | msc 12D10 |
| Related topic | CharacteristicPolynomialOfASymplecticMatrixIsAReciprocalPolynomial |

**Definition** [?] Let $p : \mathbb{C} \to \mathbb{C}$ be a polynomial of degree $n$ with complex (or real) coefficients. Then $p$ is a *reciprocal polynomial* if

$$p(z) = \pm z^n p(1/z)$$

for all $z \in \mathbb{C}$.

Examples of reciprocal polynomials are Gaussian polynomials, as well as the characteristic polynomials of orthogonal matrices (including the identity matrix as a special case), symplectic matrices, `http://planetmath.org/LinearInvolution`involut matrices, and the Pascal matrices [?].

It is clear that if $z$ is a zero for a reciprocal polynomial, then $1/z$ is also a zero. This property motivates the name. This means that the spectra of matrices of above type is symmetric with respect to the unit circle in $\mathbb{C}$; if $\lambda \in \mathbb{C}$ is an eigenvalue, so is $1/\lambda$.

The sum, difference, and product of two reciprocal polynomials is again a reciprocal polynomial. Hence, reciprocal polynomials form an algebra over the complex numbers.

# References


[1] H. Eves, *Elementary Matrix Theory*, Dover publications, 1980.

[2] N.J. Higham, *Accuracy and Stability of Numerical Algorithms*, 2nd ed., SIAM, 2002.