# long division

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In this entry we treat two cases of long division.

# 1 Integers

**Theorem 1** (Integer Long Division)**.** *For every pair of integers $a, b \neq 0$ there exist unique integers $q$ and $r$ such that:*

1. $a = b \cdot q + r$,
2. $0 \leq r < |b|$.

**Example 1.** Let $a = 10$ and $b = -3$. Then $q = -3$ and $r = 1$ correspond to the long division:

$$10 = (-3) \cdot (-3) + 1.$$

**Definition 1.** *The number $r$ as in the theorem is called the remainder of the division of $a$ by $b$. The numbers $a$, $b$ and $q$ are called the dividend, divisor and quotient respectively.*

# 2 Polynomials

**Theorem 2** (Polynomial Long Division)**.** *Let $R$ be a commutative ring with non-zero unity and let $a(x)$ and $b(x)$ be two polynomials in $R[x]$, where the leading coefficient of $b(x)$ is a unit of $R$. Then there exist unique polynomials $q(x)$ and $r(x)$ in $R[x]$ such that:*

1. $a(x) = b(x) \cdot q(x) + r(x)$,
2. $0 \leq \deg(r(x)) < \deg b(x)$ *or* $r(x) = 0$.

**Example 2.** Let $R = \mathbb{Z}$ and let $a(x) = x^3 + 3$, $b(x) = x^2 + 1$. Then $q(x) = x$ and $r(x) = -x + 3$, so that:

$$x^3 + 3 = x(x^2 + 1) - x + 3.$$

**Example 3.** The theorem is not true in general if the leading coefficient of $b(x)$ is not a unit. For example, if $a(x) = x^3 + 3$ and $b(x) = 3x^2 + 1$ then there are no $q(x)$ and $r(x)$ **with coefficients in** $\mathbb{Z}$ with the required properties.