norm

Canonical name	Norm
Date of creation	2013-03-22 12:18:02
Last modified on	2013-03-22 12:18:02
Owner	djao (24)
Last modified by	djao (24)
Numerical id	5
Author	djao (24)
Entry type	Definition
Classification	msc 12F05

Let K/F be a Galois extension, and let $x \in K$. The *norm* $\mathrm{N}_F^K(x)$ of x is defined to be the product of all the elements of the orbit of x under the group action of the Galois group $\mathrm{Gal}(K/F)$ on K; taken with multiplicities if K/F is a finite extension.

In the case where K/F is a finite extension, the norm of x can be defined to be the determinant of the linear transformation $[x] : K \to K$ given by $[x](k) := xk$, where K is regarded as a vector space over F. This definition does not require that K/F be Galois, or even that K be a field—for instance, it remains valid when K is a division ring (although F does have to be a field, in order for determinant to be defined). Of course, for finite Galois extensions K/F, this definition agrees with the previous one, and moreover the formula

$$\mathrm{N}_F^K(x) := \prod_{\sigma \in \mathrm{Gal}(K/F)} \sigma(x)$$

holds.

The norm of x is always an element of F, since any element of $\mathrm{Gal}(K/F)$ permutes the orbit of x and thus fixes $\mathrm{N}_F^K(x)$.