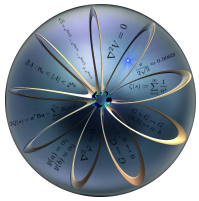

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existence of extensions of field isomorphisms to splitting fields

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The following theorem implies the essential uniqueness of splitting fields and algebraic closures.

Theorem. *Let $\sigma : F \to F'$ be an isomorphism of fields, $S = \{f_\alpha : \alpha \in A\}$ a set of non-constant polynomials in $F[X]$, and $S' = \{\sigma(f_\alpha) : \alpha \in A\}$ the corresponding set of polynomials in $F'[X]$. If K is a splitting field of S over F and K' a splitting field of S' over F', then σ may be extended to an isomorphism of K and K'.*

Corollary. *If F is a field and S a set of non-constant polynomials in $F[X]$, then any two splitting fields of S over F are F-isomorphic. In particular, any two algebraic closures of F are F-isomorphic.*