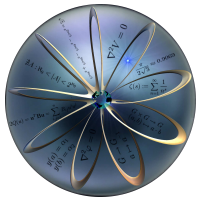

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limit of nth root of n

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The `http://planetmath.org/NthRoot`nth root of n tends to 1 as n tends to infinity, i.e. the real number sequence

$$\sqrt[1]{1},\ \sqrt[2]{2},\ \sqrt[3]{3},\ \ldots,\ \sqrt[n]{n},\ \ldots$$

converges to the limit

$$\lim_{n\to\infty}\sqrt[n]{n} = 1. \tag{1}$$

Proof. If we denote $\sqrt[n]{n} := 1+\delta_n$, we may write by the binomial theorem that

$$n = (1+\delta_n)^n = 1 + \binom{n}{1}\delta_n + \binom{n}{2}\delta_n^2 + \ldots + \binom{n}{n}\delta_n^n.$$

This implies, since all hand side are positive (when $n > 1$), that

$$n > \binom{n}{2}\delta_n^2 = \frac{n(n-1)}{2!}\delta_n^2, \qquad \delta_n^2 < \frac{2}{n-1}, \qquad 0 < \delta_n < \sqrt{\frac{2}{n-1}},$$

whence $\lim\limits_{n\to\infty}\delta_n = 0$. Accordingly,

$$\lim_{n\to\infty}\sqrt[n]{n} = \lim_{n\to\infty}(1+\delta_n) = 1,$$

Q.E.D.

Note. (1) follows also from the corollary 3 in the entry growth of exponential function.