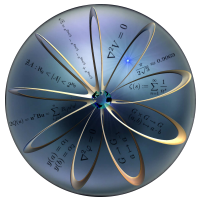

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quadratic formula

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The roots of the quadratic equation

$$ax^2 + bx + c = 0 \qquad a, b, c \in \mathbb{R}, a \neq 0$$

are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The number $\Delta = b^2 - 4ac$ is called the *discriminant* of the equation. If $\Delta > 0$, there are two different real roots, if $\Delta = 0$ there is a single real root, and if $\Delta < 0$ there are no real roots (but two different complex roots).

Let's work a few examples.

First, consider $2x^2 - 14x + 24 = 0$. Here $a = 2$, $b = -14$, and $c = 24$. Substituting in the formula gives us

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4 \cdot 2 \cdot 24}}{2 \cdot 2} = \frac{14 \pm \sqrt{4}}{4} = \frac{14 \pm 2}{4} = \frac{7 \pm 1}{2}.$$

So we have two solutions (depending on whether we take the sign + or −): $x = \frac{8}{2} = 4$ and $x = \frac{6}{2} = 3$.

Now we will solve $x^2 - x - 1 = 0$. Here $a = 1$, $b = -1$, and $c = -1$, so

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2},$$

and the solutions are $x = \frac{1+\sqrt{5}}{2}$ and $x = \frac{1-\sqrt{5}}{2}$.