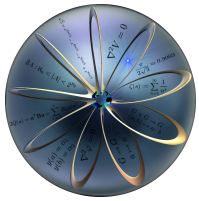

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Kummer theory

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The following theorem is usually referred to as *Kummer theory*.

Theorem 1 (Kummer Theory)**.** *Let n be a positive integer and let K be a field of characteristic not dividing n which contains the n-th roots of unity. Then:*

1. *The extension $K(\sqrt[n]{a})$ for $a \in K$ is a cyclic extension over K of degree dividing n.*

2. *Any cyclic extension of degree n over K is of the form $K(\sqrt[n]{a})$ for some $a \in K$.*

Definition 1. *Let n be a positive integer and let K be a field of characteristic not dividing n which contains the n-th roots of unity. An extension of K of the form:*

$$K(\sqrt[n]{a_1}, \sqrt[n]{a_2}, \ldots, \sqrt[n]{a_k})$$

with $a_i \in K^\times$ is called a **Kummer extension** *of K. Notice that the Galois group of the extension is of* `http://planetmath.org/Exponent`*exponent n.*