



planetmath.org

Math for the people, by the people.

trivial valuation

Canonical name	TrivialValuation
Date of creation	2013-03-22 14:20:23
Last modified on	2013-03-22 14:20:23
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	16
Author	pahio (2872)
Entry type	Definition
Classification	msc 12J20
Classification	msc 11R99
Related topic	IndependenceOfTheValuations
Related topic	KrullValuation

The *trivial valuation* of a field K is the Krull valuation $|\cdot|$ of K such that $|0| = 0$ and $|x| = 1$ for other elements x of K .

Properties

1. Every field has the trivial valuation.
2. The trivial valuation is non-archimedean.
3. The valuation ring of the trivial valuation is the whole field and the corresponding maximal ideal is the zero ideal.
4. The field is <http://planetmath.org/Complete> complete with respect to (the metric given by) its trivial valuation.
5. A finite field has only the trivial valuation. (Let a be the primitive element of the multiplicative group of the field, which is <http://planetmath.org/CyclicGroup>. If $|\cdot|$ is any valuation of the field, then one must have $|a| = 1$ since otherwise $|1| \neq 1$. Consequently, $|x| = |a^m| = |a|^m = 1^m = 1$ for all non-zero elements x .)
6. Every algebraic extension of finite fields has only the trivial valuation, but every field of characteristic 0 has non-trivial valuations.