purely inseparable

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Let F be a field of characteristic $p > 0$ and let α be an element which is algebraic over F. Then α is said to be *purely inseparable* over F if $\alpha^{p^n} \in F$ for some $n \geq 0$.

An algebraic field extension K/F is *purely inseparable* if each element of K is purely inseparable over F.

Purely inseparable extensions have the following property: if K/F is purely inseparable, and A is an algebraic closure of F which contains K, then any homomorphism $K \to A$ which fixes F necessarily fixes K.

Let K/F be an arbitrary algebraic extension. Then there is an intermediate field E such that K/E is purely inseparable, and E/F is separable.

Example. Let s be an indeterminate, and let $K = \mathbb{F}_3(s)$ where $\mathbb{F}_3$ is the finite field with 3 elements. Let $F = \mathbb{F}_3(s^6)$. Then K/F is neither separable, nor purely inseparable. Let $E = \mathbb{F}_3(s^3)$. Then E/F is separable, and K/E is purely inseparable.