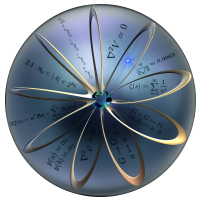

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Ostrowski's valuation theorem

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The field of rational numbers has no other http://planetmath.org/EquivalentValuationsnon equivalent valuations than

- the trivial valuation,
- the absolute value, i.e. the complex modulus $|\cdot|_\infty$ and
- the p-adic valuations $|\cdot|_p$ when p goes through all positive primes.

Note. Any valuation $|\cdot|$ of the field $\mathbb{Q}$ defines a metric $d(x,\, y) = |x - y|$ in the field, but $\mathbb{Q}$ is http://planetmath.org/Completecomplete only with respect to (the "trivial metric" defined by) the trivial valuation. The field has the proper completions with respect to its other valuations: the field of reals $\mathbb{R}$ and the fields $\mathbb{Q}_p$ of http://planetmath.org/PAdicIntegers$p$-adic numbers; cf. also http://planetmath.org/PAdicCanonicalForm$p$-adic canonical form.