behavior exists uniquely (infinite case)

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The following is a proof that behavior exists uniquely for any infinite http://planetmath.org/CyclicRing3cyclic ring R.

Proof. Let r be a http://planetmath.org/Generatorgenerator of the additive group of R. Then there exists $z \in \mathbb{Z}$ with $r^2 = zr$. If $z \geq 0$, then z is a behavior of R. Assume $z < 0$. Note that $-z > 0$ and $-r$ is also a generator of the additive group of R. Since $(-r)^2 = (-1)^2 r^2 = (-1)^2(zr) = (-z)(-r)$, it follows that $-z$ is a behavior of R. Thus, existence of behavior has been proven.

Let a and b be behaviors of R. Then there exist generators s and t of the additive group of R such that $s^2 = as$ and $t^2 = bt$. If $s = t$, then $as = s^2 = t^2 = bt = bs$, causing $a = b$. If $s \neq t$, then it must be the case that $t = -s$. (This follows from the fact that 1 and -1 are the only generators of $\mathbb{Z}$.) Thus, $as = s^2 = (-1)^2 s^2 = (-s)^2 = t^2 = bt = b(-s) = -bs$, causing $a = -b$. Since a and b are nonnegative, it follows that $a = b = 0$. Thus, uniqueness of behavior has been proven. $\square$