finite ring has no proper overrings

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The regular elements of a finite commutative ring R are the units of the ring (see the `http://planetmath.org/NonZeroDivisorsOfFiniteRing`parent of this entry). Generally, the largest overring of R, the total ring of fractions T, is obtained by forming $S^{-1}R$, the extension by localization, using as the multiplicative set S the set of all regular elements, which in this case is the unit group of R. The ring R may be considered as a subring of T, which consists formally of the fractions $\frac{a}{s} = as^{-1}$ with $a \in R$ and $s \in S$. Since every s has its own group inverse s^{-1} in S and so in R, it's evident that T no other elements than the elements of R. Consequently, $T = R$, and therefore also any overring of R coincides with R.

Accordingly, one can not extend a finite commutative ring by using a localization. Possible extensions must be made via some kind of `http://planetmath.org/RingAdjun` A more known special case is a `http://planetmath.org/AFiniteIntegralDomainIsAField`finite integral domain — it is always a field and thus closed under the divisions.