extension by localization

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Synonym	ring extension by localization
Related topic	TotalRingOfFractions
Related topic	ClassicalRingOfQuotients
Related topic	FiniteRingHasNoProperOverrings
Defines	ring of fractions
Defines	ring of quotients

Let R be a commutative ring and let S be a non-empty multiplicative subset of R. Then the `http://planetmath.org/Localization`localisation of R at S gives the commutative ring $S^{-1}R$ but, generally, it has no subring isomorphic to R. Formally, $S^{-1}R$ consists of all elements $\frac{a}{s}$ $(a \in R,\, s \in S)$. Therefore, $S^{-1}R$ is called also a *ring of quotients* of R. If $0 \in S$, then $S^{-1}R = \{0\};$ we assume now that $0 \notin S$.

- The mapping $a \mapsto \frac{as}{s}$, where s is any element of S, is well-defined and a homomorphism from R to $S^{-1}R$. All elements of S are mapped to units of $S^{-1}R$.

- If, especially, S contains no zero divisors of the ring R, then the above mapping is an isomorphism from R to a certain subring of $S^{-1}R$, and we may think that $S^{-1}R \supseteq R$. In this case, the ring of fractions of R is an extension ring of R; this concerns of course the case that R is an integral domain. But if R is a finite ring, then $S^{-1}R = R,$ and no proper extension is obtained.