# zero of polynomial

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Let $R$ be a subring of a commutative ring $S$. If $f$ is a polynomial in $R[X]$, it defines an evaluation homomorphism from $S$ to $S$. Any element $\alpha$ of $S$ satisfying

$$f(\alpha) = 0$$

is a *zero of the polynomial* $f$.

If $R$ also is equipped with a non-zero unity, then the polynomial $f$ is in $S[X]$ divisible by the binomial $X-\alpha$ (cf. the factor theorem). In this case, if $f$ is divisible by $(X-\alpha)^n$ but not by $(X-\alpha)^{n+1}$, then $\alpha$ is a zero of the *order* $n$ of the polynomial $f$. If this order is 1, then $\alpha$ is a *simple zero* of $f$.

For example, the real number $\sqrt{2}$ $(\in \mathbb{R})$ is a zero of the polynomial $X^2-2$ of the polynomial ring $\mathbb{Q}[X]$.