



discrete valuation ring

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Defines	order

A *discrete valuation ring*  $R$  is a principal ideal domain with exactly one **nonzero** maximal ideal  $M$ . Any generator  $t$  of  $M$  is called a *uniformizer* or *uniformizing element* of  $R$ ; in other words, a uniformizer of  $R$  is an element  $t \in R$  such that  $t \in M$  but  $t \notin M^2$ .

Given a discrete valuation ring  $R$  and a uniformizer  $t \in R$ , every element  $z \in R$  can be written uniquely in the form  $u \cdot t^n$  for some unit  $u \in R$  and some nonnegative integer  $n \in \mathbb{Z}$ . The integer  $n$  is called the *order* of  $z$ , and its value is independent of the choice of uniformizing element  $t \in R$ .