weakest extension of a partial ordering

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Defines	weakest extension of a partial ordering

Let A be a commutative ring with partial ordering A^+ and suppose that B is a ring that admits a partial ordering. If $f : A \to B$ is a ring monomorphism (thus we regard B as an over-ring of A), then any partial ordering of B that can contain $f(A^+)$ will also contain the set $B^+ \subset B$ defined by

$$B^+ := \{\sum_{i=1}^{n} f(a_i)b_i^2 : n \in \mathbb{N}, a_1, \ldots, a_n \in A^+\}$$

B^+ is itself a partial ordering and it is called the *weakest partial ordering of B that extends A^+ (through f)*. It is called "weakest" because this is the smallest partial ordering B^+ of B that will transform f into a poring monomorphism (i.e. a monomorphism in the category of partially ordered rings) $f : (A, A^+) \to (B, B^+)$ (for simplicity, we abuse the symbol f here).