polynomial ring over integral domain

Canonical name	PolynomialRingOverIntegralDomain
Date of creation	2013-03-22 15:10:06
Last modified on	2013-03-22 15:10:06
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	10
Author	pahio (2872)
Entry type	Theorem
Classification	msc 13P05
Related topic	RingAdjunction
Related topic	FormalPowerSeries
Related topic	ZeroPolynomial2
Related topic	PolynomialRingOverFieldIsEuclideanDomain
Defines	coefficient ring

Theorem. If the *coefficient ring* R is an integral domain, then so is also its polynomial ring $R[X]$.

Proof. Let $f(X)$ and $g(X)$ be two non-zero polynomials in $R[X]$ and let a_f and b_g be their leading coefficients, respectively. Thus $a_f \neq 0$, $b_g \neq 0$, and because R has no zero divisors, $a_f b_g \neq 0$. But the product $a_f b_g$ is the leading coefficient of $f(X)g(X)$ and so $f(X)g(X)$ cannot be the zero polynomial. Consequently, $R[X]$ has no zero divisors, Q.E.D.

Remark. The theorem may by induction be generalized for the polynomial ring $R[X_1,\, X_2,\, \ldots,\, X_n]$.