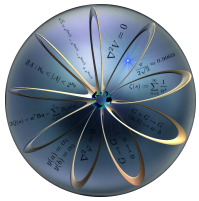

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partially ordered ring

Canonical name	PartiallyOrderedRing
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Entry type	Definition
Classification	msc 13J25
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Synonym	po-ring
Synonym	l-ring
Synonym	lattice-ordered ring
Defines	lattice ordered ring
Defines	positive cone

A ring R that is a poset at the same time is called a *partially ordered ring*, or a *po-ring*, if, for $a, b, c \in R$,

- $a \leq b$ implies $a + c \leq b + c$, and
- $0 \leq a$ and $0 \leq b$ implies $0 \leq ab$.

Note that R does not have to be associative.

If the underlying poset of a po-ring R is in fact a lattice, then R is called a *lattice-ordered ring*, or an *l-ring* for short.

Remark. The underlying abelian group of a po-ring (with addition being the binary operation) is a po-group. The same is true for l-rings.

Below are some examples of po-rings:

- Clearly, any (totally) ordered ring is a po-ring.
- The ring of continuous functions over a topological space is an l-ring.
- Any matrix ring over an ordered field is an l-ring if we define $(a_{ij}) \leq (b_{ij})$ whenever $a_{ij} \leq b_{ij}$ for all i, j.

Remark. Let R be a po-ring. The set $R^+ := \{r \in R \mid 0 \leq r\}$ is called the *positive cone* of R.

References

[1] G. Birkhoff *Lattice Theory*, 3rd Edition, AMS Volume XXV, (1967).