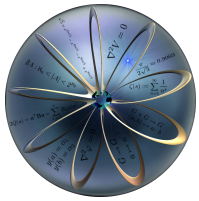

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example of Smith normal form

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As an example, we will find the Smith normal form of the following matrix over the integers.

$$\begin{pmatrix} 2 & 4 & 4 \\ -6 & 6 & 12 \\ 10 & -4 & -16 \end{pmatrix}$$

The following matrices are the intermediate steps as the algorithm is applied to the above matrix.

$$\begin{pmatrix} 2 & 0 & 0 \\ -6 & 18 & 24 \\ 10 & -24 & -36 \end{pmatrix} \to \begin{pmatrix} 2 & 0 & 0 \\ 0 & 18 & 24 \\ 0 & -24 & -36 \end{pmatrix}$$

$$\to \begin{pmatrix} 2 & 0 & 0 \\ 0 & 18 & 24 \\ 0 & -6 & -12 \end{pmatrix} \to \begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 12 \\ 0 & 18 & 24 \end{pmatrix}$$

$$\to \begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 12 \\ 0 & 0 & -12 \end{pmatrix} \to \begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 12 \end{pmatrix}$$

So the Smith normal form is

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 12 \end{pmatrix}$$

and the elementary divisors are 2, 6 and 12.