proof of Nakayama's lemma

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Let $X = \{x_1, x_2, \ldots, x_n\}$ be a minimal set of generators for M, in the sense that M is not generated by any proper subset of X.

Elements of $\mathfrak{a}M$ can be written as linear combinations $\sum a_i x_i$, where $a_i \in \mathfrak{a}$.

Suppose that $|X| > 0$. Since $M = \mathfrak{a}M$, we can express x_1 as a such a linear combination:

$$x_1 = \sum a_i x_i.$$

Moving the term involving a_1 to the left, we have

$$(1 - a_1)x_1 = \sum_{i>1} a_i x_i.$$

But $a_1 \in J(R)$, so $1 - a_1$ is invertible, say with inverse b. Therefore,

$$x_1 = \sum_{i>1} b a_i x_i.$$

But this means that x_1 is redundant as a generator of M, and so M is generated by the subset $\{x_2, x_3, \ldots, x_n\}$. This contradicts the minimality of X.

We conclude that $|X| = 0$ and therefore $M = 0$.