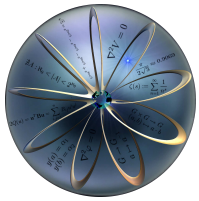

planetmath.org

Math for the people, by the people.

formal power series converges if and only if it converges along every line

Canonical name	FormalPowerSeriesConvergesIfAndOnlyIfItConvergesAlongEveryLine
Date of creation	2013-03-22 17:42:11
Last modified on	2013-03-22 17:42:11
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	5
Author	jirka (4157)
Entry type	Theorem
Classification	msc 13H05
Classification	msc 13B35
Classification	msc 13J05
Classification	msc 13F25

Suppose $T(x)$ denotes the formal power series $\sum_\alpha a_\alpha x^\alpha$, using the multi-index notation, where $x = (x_1, \ldots, x_N)$ and $a_\alpha \in \mathbb{C}$. Fixing $v \in \mathbb{R}^N$ and we can also talk of the formal power series in $t \in \mathbb{R}$

$$\begin{aligned} T(tv) &= \sum_\alpha a_\alpha (tv)^\alpha \\ &= \sum_\alpha a_\alpha v^\alpha t^{|\alpha|} \\ &= \sum_{k=0}^{\infty} \left(\sum_{|\alpha|=k} a_\alpha v^\alpha \right) t^k. \end{aligned}$$

Theorem. *Suppose $T(x)$ is a formal power series in $x \in \mathbb{R}^N$. Suppose $T(tv)$ is a convergent power series in $t \in \mathbb{R}$ for all $v \in \mathbb{R}^N$. Then T is convergent.*

The other direction, if $T(x)$ converges then $t \mapsto T(tv)$ converges, is obvious.

References

[1] M. Salah Baouendi, Peter Ebenfelt, Linda Preiss Rothschild. , Princeton University Press, Princeton, New Jersey, 1999.