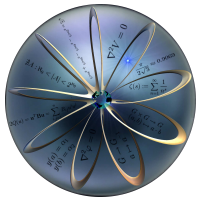

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regular local ring

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A local ring R of dimension n is *regular* if and only if its maximal ideal $\mathfrak{m}$ is generated by n elements.

Equivalently, R is regular if $\dim_{R/\mathfrak{m}} \mathfrak{m}/\mathfrak{m}^2 = \dim R$, where the first dimension is that of a vector space, and the latter is the Krull dimension, since by Nakayama's lemma, elements generate $\mathfrak{m}$ if and only if their images under the projection generate $\mathfrak{m}/\mathfrak{m}^2$.

By Krull's principal ideal theorem, $\mathfrak{m}$ cannot be generated by fewer than n elements, so the maximal ideals of regular local rings have a minimal number of generators.