# symmetric multilinear function

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Let $R$ be a commutative ring with identity and $M, N$ be unital $R$-modules.

Suppose that $\phi : M \times \cdots \times M \to N$ is a multilinear map, where there are $n$ copies of $M$.

Let $H$ be a subgroup of $S_n$, the symmetric group on $\{1, \ldots, n\}$, and $\chi : H \to R$ satisfy

1. $\chi(e) = 1$
2. $\chi(g_1 g_2) = \chi(g_1)\chi(g_2)$ for all $g_1, g_2 \in H$

We say that $\phi$ is *symmetric with respect to* $H$ *and* $\chi$ if

$$\phi(m_{\sigma(1)}, \ldots, m_{\sigma(n)}) = \chi(\sigma)\phi(m_1, \ldots, m_n)$$

holds for all $\sigma \in H$ and all $m_i \in M$.

Now suppose that $H = S_n$.

If $\chi = 1$ then we say that $\phi$ is a *symmetric multilinear function.* If $\chi = \epsilon$, the sign of the permutation $\sigma$, we say that $\phi$ is a *skew-symmetric multilinear function*.

For example, the permanent is a symmetric multilinear function of its rows (columns).

The determinant is a skew-symmetric multilinear function of its rows (columns).