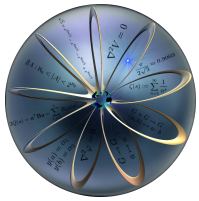

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integrality is transitive

Canonical name	IntegralityIsTransitive
Date of creation	2013-03-22 17:01:25
Last modified on	2013-03-22 17:01:25
Owner	rm50 (10146)
Last modified by	rm50 (10146)
Numerical id	6
Author	rm50 (10146)
Entry type	Theorem
Classification	msc 13B21

Let $C \subset B \subset A$ be rings. If B is integral over C and A is integral over B, then A is integral over C.

Proof. Choose $u \in A$. Then $u^n + b_1 u^{n-1} + \cdots + b_n = 0, b_i \in B$. Thus $C[b_1, \ldots, b_n, u]$ is integral and thus module-finite over $C[b_1, \ldots, b_n]$. Each b_i is integral over C, so $C[b_1, \ldots, b_n]$ is integral hence module-finite over C. Thus $C[b_1, \ldots, b_n, u]$ is module-finite, hence integral, over C, so u is integral over C.