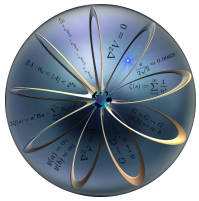

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uniqueness of additive identity in a ring

Canonical name	UniquenessOfAdditiveIdentityInARing
Date of creation	2013-03-22 14:14:06
Last modified on	2013-03-22 14:14:06
Owner	alozano (2414)
Last modified by	alozano (2414)
Numerical id	6
Author	alozano (2414)
Entry type	Theorem
Classification	msc 13-00
Classification	msc 16-00
Classification	msc 20-00

Lemma 1. *Let R be a ring. There exists a unique element 0 in R such that for all a in R:*

$$0 + a = a + 0 = a.$$

Proof. By the definition of ring, there exists at least one identity in R, call it 0_1. Suppose $0_2 \in R$ is an element which also the of additive identity. Thus,

$$0_2 + 0_1 = 0_2$$

On the other hand, 0_1 is an additive identity, therefore:

$$0_2 + 0_1 = 0_1 + 0_2 = 0_1$$

Hence $0_2 = 0_1$, i.e. there is a unique additive identity. □