maximal ideal

Canonical name	MaximalIdeal
Date of creation	2013-03-22 11:50:57
Last modified on	2013-03-22 11:50:57
Owner	djao (24)
Last modified by	djao (24)
Numerical id	8
Author	djao (24)
Entry type	Definition
Classification	msc 13A15
Classification	msc 16D25
Classification	msc 81R50
Classification	msc 46M20
Classification	msc 18B40
Classification	msc 22A22
Classification	msc 46L05
Related topic	ProperIdeal
Related topic	Module
Related topic	Comaximal
Related topic	PrimeIdeal
Related topic	EveryRingHasAMaximalIdeal

Let R be a ring with identity. A proper left (right, two-sided) ideal $\mathfrak{m} \subsetneq R$ is said to be *maximal* if $\mathfrak{m}$ is not a proper subset of any other proper left (right, two-sided) ideal of R.

One can prove:

- A left ideal $\mathfrak{m}$ is maximal if and only if $R/\mathfrak{m}$ is a simple left R-module.
- A right ideal $\mathfrak{m}$ is maximal if and only if $R/\mathfrak{m}$ is a simple right R-module.
- A two-sided ideal $\mathfrak{m}$ is maximal if and only if $R/\mathfrak{m}$ is a simple ring.

All maximal ideals are prime ideals. If R is commutative, an ideal $\mathfrak{m} \subset R$ is maximal if and only if the quotient ring $R/\mathfrak{m}$ is a field.