ring without irreducibles

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An integral domain may not any irreducible elements. One such example is the ring of all algebraic integers. Any nonzero non-unit ϑ of this ring satisfies an equation

$$x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n = 0$$

with integer coefficients a_j, since it is an algebraic integer; moreover, we can assume that $a_n = \mathrm{N}(\vartheta) \neq \pm 1$ (see norm and trace of algebraic number: 2). The element ϑ has the

$$\vartheta = \sqrt{\vartheta}\cdot\sqrt{\vartheta}.$$

Here, $\sqrt{\vartheta}$ belongs to the ring because it satisfies the equation

$$x^{2n}+a_1x^{2n-2}+\cdots+a_{n-1}x^2+a_n = 0,$$

and it is no unit. Thus the element ϑ is not irreducible.