ideal inverting in Prüfer ring

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Theorem. Let $\mathfrak{a}_1, \ldots, \mathfrak{a}_n$ be invertible fractional ideals of a Prüfer ring. Then also their sum and intersection are invertible, and the inverse ideals of these are obtained by the formulae resembling de Morgan's laws:

$$(\mathfrak{a}_1 + \cdots + \mathfrak{a}_n)^{-1} = \mathfrak{a}_1^{-1} \cap \cdots \cap \mathfrak{a}_n^{-1}$$

$$(\mathfrak{a}_1 \cap \cdots \cap \mathfrak{a}_n)^{-1} = \mathfrak{a}_1^{-1} + \cdots + \mathfrak{a}_n^{-1}$$

This is due to the fact, that the sum of any ideals is the smallest ideal containing these ideals and the intersection of the ideals is the largest ideal contained in each of these ideals. Cf. sum of ideals, quotient of ideals.

References

[1] J. Pahikkala: "Some formulae for multiplying and inverting ideals". – *Annales universitatis turkuensis* **183**. Turun yliopisto (University of Turku) 1982.