ideal generators in Prüfer ring

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Let R be a Prüfer ring with total ring of fractions T. Let $\mathfrak{a}$ and $\mathfrak{b}$ be fractional ideals of R, http://planetmath.org/IdealGeneratedByASetgenerated by m and n elements of T, respectively.

- Then the sum ideal $\mathfrak{a}+\mathfrak{b}$ may, of course, be generated by $m+n$ elements.

- If $\mathfrak{a}$ or $\mathfrak{b}$ is http://planetmath.org/FractionalIdealOfCommutativeRingregular, then the http://planetmath.org/ProductOfIdealsproduct ideal $\mathfrak{ab}$ may be generated by $m+n-1$ elements, since in Prüfer rings the

  $$(a_1,\,...,\,a_m)(b_1,\,...,\,b_n) = (a_1b_1,\,a_1b_2+a_2b_1,\,a_1b_3+a_2b_2+a_3b_1,\,...,\,a_mb_n)$$

  holds.

- If both $\mathfrak{a}$ and $\mathfrak{b}$ are regular ideals, then the intersection $\mathfrak{a}\cap\mathfrak{b}$ and the quotient ideal $\mathfrak{a}\colon\mathfrak{b} = \{r \in R|\quad r\mathfrak{b} \subseteq \mathfrak{a}\}$ both may be generated by $m+n$ elements.

- If $\mathfrak{a}$ is regular, then it is also http://planetmath.org/InvertibleIdealinvertible. Its ideal has the http://planetmath.org/QuotientOfIdealsexpression

  $$\mathfrak{a}^{-1} = [R:\mathfrak{a}] = \{t \in T|\quad t\mathfrak{a} \subseteq R\}$$

  and may be generated by m elements of T (see the generators of inverse ideal).

Cf. also the two-generator property.

References

J. Pahikkala: "Some formulae for multiplying and inverting ideals". – *Annales universitatis turkuensis* 183. Turun yliopisto (University of Turku) 1982.