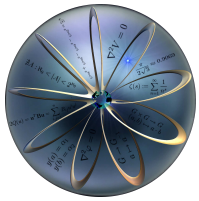

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a finite ring is cyclic if and only its order and characteristic are equal

Canonical name	AFiniteRingIsCyclicIfAndOnlyItsOrderAndCharacteristicAreEqual
Date of creation	2013-03-22 13:30:30
Last modified on	2013-03-22 13:30:30
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	12
Author	mathcam (2727)
Entry type	Theorem
Classification	msc 13A99

. A finite ring is cyclic if and only if its http://planetmath.org/OrderRingorder and characteristic are equal.

Proof. If R is a cyclic ring and r is a http://planetmath.org/Generatorgenerator of the additive group of R, then $|r| = |R|$. Since, for every $s \in R$, $|s|$ divides $|R|$, then it follows that char $R = |R|$. Conversely, if R is a finite ring such that char $R = |R|$, then the exponent of the additive group of R is also equal to $|R|$. Thus, there exists $t \in R$ such that $|t| = |R|$. Since $\langle t \rangle$ is a subgroup of the additive group of R and $|\langle t \rangle| = |t| = |R|$, it follows that R is a cyclic ring. $\square$