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localization

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Let R be a commutative ring and let S be a nonempty multiplicative subset of R . The *localization* of R at S is the ring $S^{-1}R$ whose elements are equivalence classes of $R \times S$ under the equivalence relation $(a, s) \sim (b, t)$ if $r(at - bs) = 0$ for some $r \in S$. Addition and multiplication in $S^{-1}R$ are defined by:

- $(a, s) + (b, t) = (at + bs, st)$
- $(a, s) \cdot (b, t) = (a \cdot b, s \cdot t)$

The equivalence class of (a, s) in $S^{-1}R$ is usually denoted a/s . For $a \in R$, the localization of R at the minimal multiplicative set containing a is written as R_a . When S is the complement of a prime ideal \mathfrak{p} in R , the localization of R at S is written $R_{\mathfrak{p}}$.