torsion element

Canonical name	TorsionElement
Date of creation	2013-03-22 13:54:41
Last modified on	2013-03-22 13:54:41
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	7
Author	mathcam (2727)
Entry type	Definition
Classification	msc 13C12
Defines	torsion submodule
Defines	torsion module

Let R be a commutative ring, and M an R-module. We call an element $m \in M$ a *torsion element* if there exists a non-zero-divisor $\alpha \in R$ such that $\alpha \cdot m = 0$. The set is denoted by $tor(M)$.

$tor(M)$ is not empty since $0 \in tor(M)$. Let $m, n \in tor(M)$, so there exist $\alpha, \beta \neq 0 \in R$ such that $0 = \alpha \cdot m = \beta \cdot n$. Since $\alpha\beta \cdot (m - n) = \beta \cdot \alpha \cdot m - \alpha \cdot \beta \cdot n = 0, \alpha\beta \neq 0$, this implies that $m - n \in tor(M)$. So $tor(M)$ is a subgroup of M. Clearly $\tau \cdot m \in tor(M)$ for any non-zero $\tau \in R$. This shows that $tor(M)$ is a submodule of M, the **torsion submodule** of M. In particular, a module that equals its own torsion submodule is said to be a *torsion module*.