p-ring

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Defines	strict p-ring

Definition 1. *Let R be a commutative ring with identity element equipped with a topology defined by a decreasing sequence:*

$$\ldots \subset \mathfrak{A}_3 \subset \mathfrak{A}_2 \subset \mathfrak{A}_1$$

of ideals such that $\mathfrak{A}_n \cdot \mathfrak{A}_m \subset \mathfrak{A}_{n+m}$. We say that R is a p-ring if the following conditions are satisfied:

1. *The residue ring $\overline{k} = R/\mathfrak{A}_1$ is a perfect ring of characteristic p.*

2. *The ring R is Hausdorff and complete for its topology.*

Definition 2. *A p-ring R is said to be strict (or a p-adic ring) if the topology is defined by the p-adic filtration $\mathfrak{A}_n = p^n R$, and p is not a zero-divisor of R.*

Example 1. The prototype of strict p-ring is the ring of `http://planetmath.org/PAdicIntegers`p-adic integers $\mathbb{Z}_p$ with the usual profinite topology.

References

[1] J. P. Serre, *Local Fields*, Springer-Verlag, New York.