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primal element

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An element r in a commutative ring R is called *primal* if whenever $r \mid ab$, with $a, b \in R$, then there exist elements $s, t \in R$ such that

1. $r = st$,
2. $s \mid a$ and $t \mid b$.

Lemma. In a commutative ring, an element that is both irreducible and primal is a prime element.

Proof. Suppose a is irreducible and primal, and $a \mid bc$. Since a is primal, there is $x, y \in R$ such that $a = xy$, with $x \mid b$ and $y \mid c$. Since a is irreducible, either x or y is a unit. If x is a unit, with z as its inverse, then $za = zxy = y$, so that $a \mid y$. But $y \mid c$, we have that $a \mid c$. \square