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## complete ring of quotients of reduced commutative rings

Canonical name	CompleteRingOfQuotientsOfReducedCommutativeRings
Date of creation	2013-03-22 18:27:33
Last modified on	2013-03-22 18:27:33
Owner	jocaps (12118)
Last modified by	jocaps (12118)
Numerical id	6
Author	jocaps (12118)
Entry type	Theorem
Classification	msc 13B30
Related topic	CompleteRingOfQuotients
Related topic	essentialmonomorphism
Defines	rational extension

There is a characterization of complete ring of quotients of reduced commutative rings. Let  $A$  be a <http://planetmath.org/ReducedRing>reduced commutative ring, then if  $B$  is an overring of  $A$  and if for any element  $b \in B \setminus \{0\}$  there is an  $a \in A$  such that  $ab \in A \setminus \{0\}$ , then  $B$  is said to be a *rational extension* of  $A$ . See how similar this is with the definition of essential extension in the category of rings, obviously all rational extensions of reduced commutative rings are also essential extensions. Furthermore there is a maximum (upto  $A$ -isomorphism) rational extension of  $A$  and this is in fact the complete ring of quotients of  $A$ .