alternative proof that a finite integral domain is a field

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Proof. Let R be a finite integral domain and $a \in R$ with $a \neq 0$. Since R is finite, there exist positive integers j and k with $j < k$ such that $a^j = a^k$. Thus, $a^k - a^j = 0$. Since $j < k$ and j and k are positive integers, $k - j$ is a positive integer. Therefore, $a^j(a^{k-j} - 1) = 0$. Since $a \neq 0$ and R is an integral domain, $a^j \neq 0$. Thus, $a^{k-j} - 1 = 0$. Hence, $a^{k-j} = 1$. Since $k - j$ is a positive integer, $k - j - 1$ is a nonnegative integer. Thus, $a^{k-j-1} \in R$. Note that $a \cdot a^{k-j-1} = a^{k-j} = 1$. Hence, a has a multiplicative inverse in R. It follows that R is a field. $\square$