proof of Dedekind domains with finitely many primes are PIDs

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Proof. Let $\mathfrak{p}_1, \ldots, \mathfrak{p}_k$ be all the primes of a Dedekind domain R. If I is any ideal of R, then by the Weak Approximation Theorem we can choose $x \in R$ such that $\nu_{\mathfrak{p}_i}((x)) = \nu_{\mathfrak{p}_i}(I)$ for all i (where $\nu_{\mathfrak{p}}$ is the $\mathfrak{p}$-adic valuation). But since R is Dedekind, ideals have unique factorization; since (x) and I have identical factorizations, we must have $(x) = I$ and I is principal. $\square$