scalar map

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Defines	scalar map

Given a ring R, a left R-module U, a right R-module V and a two-sided R-module W then a map $b : U \times V \to W$ is an R-*scalar map* if

1. b is biadditive, that is $b(u+u', v) = b(u, v) + b(u', v)$ and $b(u, v+v') = b(u, v) + b(u, v')$ for all $u, u' \in U$ and $v, v' \in V$;

2. $b(ru, v) = rb(u, v)$ and $b(u, vr) = b(u, v)r$ for all $u \in U$, $v \in V$ and $r \in R$.

Such maps can also be called *outer linear*.

Unlike bilinear maps, scalar maps do not force a commutative multiplication on R even when the map is non-degenerate and the modules are faithful. For example, if A is an associative ring then the multiplication of A, $b : A \times A \to A$ is a A-outer linear:

$$b(xy, z) = (xy)z = x(yz) = xb(y, z)$$

and likewise $b(x, yz) = b(x, y)z$. Using a non-commutative ring A confirms the claim.

It is immediate however that $\langle b(U, V)\rangle$ is in fact an R-bimodule. This is because:

$$s(b(u, v)r) = sb(u, vr) = b(su, vr) = sb(u, vr) = (sb(u, v))r$$

for all $u \in U$, $v \in V$ and $s, r \in R$. Therefore it is not uncommon to require that indeed all of W be an R-bimodule.