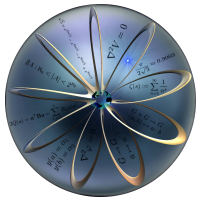

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counter example to Nakayama's lemma for non-finitely generated modules

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The hypothesis that the module M be finitely generated is really necessary. For example, the field of p-adic numbers $\mathbb{Q}_p$ is not finitely generated over its ring of integers $\mathbb{Z}_p$ and $(p)\mathbb{Q}_p = \mathbb{Q}_p$.

In one sense, the reason why $\mathbb{Q}_p$ is "bad" is that is has no proper sub module which is also maximal. Thus $\mathbb{Q}_p$ has no non-zero simple quotient. This explains why the following `http://planetmath.org/ProofOfNakayamasLemma2`Proof of Nakayama's Lemma does not work for non-finitely generated modules.