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multiplication ring

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Let R be a commutative ring with non-zero unity. If $\mathfrak{a}$ and $\mathfrak{b}$ are two `http://planetmath.org/FractionalIdealOfCommutativeRing`*fractional ideals* of R with $\mathfrak{a} \subseteq \mathfrak{b}$ and if $\mathfrak{b}$ is `http://planetmath.org/FractionalIdealOfCommutativeRing`inve
then there is a $\mathfrak{c}$ of R such that $\mathfrak{a} = \mathfrak{b}\mathfrak{c}$ (one can choose $\mathfrak{c} = \mathfrak{b}^{-1}\mathfrak{a}$).

Definition. Let R be a commutative ring with non-zero unity and let $\mathfrak{a}$ and $\mathfrak{b}$ be ideals of R. The ring R is a *multiplication ring* if $\mathfrak{a} \subseteq \mathfrak{b}$ always implies that there exists a $\mathfrak{c}$ of R such that $\mathfrak{a} = \mathfrak{b}\mathfrak{c}$.

Theorem. Every Dedekind domain is a multiplication ring. If a multiplication ring has no zero divisors, it is a Dedekind domain.

References

[1] M. Larsen & P. McCarthy: *Multiplicative theory of ideals.* Academic Press. New York (1971).