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bilinear map

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Entry type	Definition
Classification	msc 13C99
Synonym	bilinear function
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Related topic	Multilinear
Related topic	BilinearForm
Related topic	ScalarMap
Defines	bilinear

Let R be a ring, and let M, N and P be modules over R. A function $f\colon M \times N \to P$ is said to be a *bilinear map* if for each $b \in M$ the function $h\colon N \to P$ defined by $h(y) = f(b, y)$ for all $y \in N$ is http://planetmath.org/LinearTransformati (that is, an R-module homomorphism), and for each $c \in N$ the function $g\colon M \to P$ defined by $g(x) = f(x, c)$ for all $x \in M$ is linear. Sometimes we may say that the function is R-*bilinear*, .

A common case is a bilinear map $V \times V \to V$, where V is a vector space over a field K; the vector space with this operation then forms an algebra over K.

If R is a commutative ring, then every R-bilinear map $M \times N \to P$ corresponds in a natural way to a linear map $M \otimes N \to P$, where $M \otimes N$ is the tensor product of M and N (over R).