graded ring

Canonical name	GradedRing
Date of creation	2013-03-22 11:45:03
Last modified on	2013-03-22 11:45:03
Owner	aplant (12431)
Last modified by	aplant (12431)
Numerical id	19
Author	aplant (12431)
Entry type	Definition
Classification	msc 13A02
Classification	msc 16W30
Classification	msc 14L15
Classification	msc 14L05
Classification	msc 12F10
Classification	msc 11S31
Classification	msc 11S15
Classification	msc 11R33
Synonym	S-graded ring
Synonym	G-graded ring
Related topic	HomogeneousIdeal
Related topic	SupportGradedRing
Defines	groupoid graded ring
Defines	semigroup graded ring
Defines	group graded ring
Defines	homogeneous element
Defines	strongly graded

Let S be a groupoid (semigroup,group) and let R be a ring (not necessarily with unity) which can be expressed as a $R = \bigoplus_{s\in S} R_s$ of additive subgroups R_s of R with $s \in S$. If $R_s R_t \subseteq R_{st}$ for all $s, t \in S$ then we say that R is *groupoid graded* (semigroup-graded, group-graded) ring.

We refer to $R = \bigoplus_{s\in S} R_s$ as an S-grading of R and the subgroups R_s as the s-components of R. If we have the stronger condition that $R_s R_t = R_{st}$ for all $s, t \in S$, then we say that the ring R is *strongly* graded by S.

Any element r_s in R_s (where $s \in S$) is said to be *homogeneous of degree* s. Each element $r \in R$ can be expressed as a unique and finite sum $r = \sum_{s\in S} r_s$ of homogeneous elements $r_s \in R_s$.

For any subset $G \subseteq S$ we have $R_G = \sum_{g\in G} R_g$. Similarly $r_G = \sum_{g\in G} r_g$. If G is a subsemigroup of S then R_G is a subring of R. If G is a left (right, two-sided) ideal of S then R_G is a left (right, two-sided) ideal of R.

Some examples of graded rings include:

Polynomial rings
Ring of symmetric functions
Generalised matrix rings
Morita contexts
Ring of Hirota derivatives
group rings
filtered algebras