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ring adjunction

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Let R be a commutative ring and E an extension ring of it. If $\alpha \in E$ and commutes with all elements of R, then the smallest subring of E containing R and α is denoted by $R[\alpha]$. We say that $R[\alpha]$ is obtained from R by adjoining α to R via *ring adjunction*.

By the about "evaluation homomorphism",

$$R[\alpha] = \{f(\alpha) \mid f(X) \in R[X]\},$$

where $R[X]$ is the polynomial ring in one indeterminate over R. Therefore, $R[\alpha]$ consists of all expressions which can be formed of α and elements of the ring R by using additions, subtractions and multiplications.

Examples: The polynomial rings $R[X]$, the ring $\mathbb{Z}[i]$ of the Gaussian integers, the ring $\mathbb{Z}[\frac{-1+i\sqrt{3}}{2}]$ of Eisenstein integers.