equivalent valuations

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Defines	equivalence of valuations

Let K be a field. The *equivalence of valuations* $|\cdot|_1$ and $|\cdot|_2$ of K may be defined so that

1. $|\cdot|_1$ is not the trivial valuation;
2. if $|a|_1 < 1$ then $|a|_2 < 1 \qquad \forall a \in K$.

It it easy to see that these conditions imply for both valuations (use $\frac{1}{a}$). Also, we have always

$$|a|_1 \leqq 1 \Leftrightarrow |a|_2 \leqq 1;$$

so both valuations have a common valuation ring in the case they are non-archimedean. (The of the more general Krull valuations is defined to that they have common valuation rings.) Further, both valuations determine a common metric on K.

Theorem. Two valuations (of `http://planetmath.org/KrullValuation`rank one) $|\cdot|_1$ and $|\cdot|_2$ of K are iff one of them is a positive power of the other,

$$|a|_1 = |a|_2^c \qquad \forall a \in K,$$

where c is a positive .