every PID is a UFD - alternative proof

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Proposition. If R is a principal ideal domain, then R is a unique factorization domain.

Proof. Recall, that due to Kaplansky Theorem (see `http://planetmath.org/EquivalentDefin` article for details) it is enough to show that every nonzero prime ideal in R contains a prime element.

On the other hand, recall that an element $p \in R$ is prime if and only if an ideal (p) generated by p is nonzero and prime.

Thus, if P is a nonzero prime ideal in R, then (since R is a PID) there exists $p \in R$ such that $P = (p)$. This completes the proof. $\square$