example of using Eisenstein criterion

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For showing the `http://planetmath.org/IrreduciblePolynomial2`irreducibility of the polynomial

$$P(x) \;:=\; x^5+5x+11$$

one would need a prime number dividing its other coefficients except the first one, but there is no such prime. However, a suitable $x := y+a$ may change the situation. Since the binomial coefficients of $(y-1)^5$ except the first and the last one are divisible by 5 and $11 \equiv 1 \pmod{5}$, we try

$$x \;:=\; y-1.$$

Then

$$P(y-1) \;=\; y^5-5y^4+10y^3-10y^2+10y+5.$$

Thus the prime 5 divides other coefficients except the first one and the square of 5 does not divide the constant term of this polynomial in y, whence the Eisenstein criterion says that $P(y-1)$ is irreducible (in the field $\mathbb{Q}$ of its coefficients). Apparently, also $P(x)$ must be irreducible.

It would be easy also to see that $P(x)$ does not have `http://planetmath.org/RationalRootThe`
zeroes.