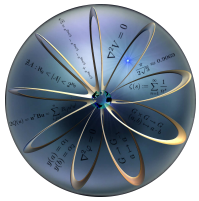

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Krull valuation

Canonical name	KrullValuation
Date of creation	2013-03-22 14:54:39
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Entry type	Definition
Classification	msc 13F30
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Related topic	OrderedGroup
Related topic	TrivialValuation
Related topic	IsolatedSubgroup
Related topic	ValueGroupOfCompletion
Related topic	PlaceOfField
Related topic	OrderValuation
Related topic	AlternativeDefinitionOfValuation2
Related topic	UniquenessOfDivisionAlgorithmInEuclideanDomain
Defines	value group
Defines	rank of Krull valuation
Defines	rank of valuation

Definition. The mapping $|.|: K \to G$, where K is a field and G an ordered group equipped with zero, is a *Krull valuation* of K, if it has the properties

1. $|x| = 0 \Leftrightarrow x = 0$;
2. $|xy| = |x| \cdot |y|$;
3. $|x + y| \leqq \max\{|x|, |y|\}$.

Thus the Krull valuation is more general than the usual `http://planetmath.org/Valuation`val which is also characterized as and which has real values. The image $|K \setminus \{0\}|$ is called the *value group* of the Krull valuation; it is abelian. In general, the *rank of Krull valuation* the `http://planetmath.org/IsolatedSubgroup`rank of the value group.

We may say that a Krull valuation is `http://planetmath.org/Valuation`non-archimedean.

Some values

- $|1| = 1$ because the Krull valuation is a group homomorphism from the multiplicative group of K to the ordered group.
- $|-1| = 1$ because $1 = |(-1)^2| = |-1|^2$ and 1 is the only element of the ordered group being its own inverse ($S \cap S^{-1} = \varnothing$).
- $|-x| = |(-1)x| = |-1| \cdot |x| = |x|$

References

[1] Emil Artin: *Theory of Algebraic Numbers*. Lecture notes. Mathematisches Institut, Göttingen (1959).

[2] P. Jaffard: *Les systèmes d'idéaux*. Dunod, Paris (1960).