global dimension

Canonical name	GlobalDimension
Date of creation	2013-03-22 14:51:51
Last modified on	2013-03-22 14:51:51
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	5
Author	CWoo (3771)
Entry type	Definition
Classification	msc 13D05
Classification	msc 16E10
Classification	msc 18G20
Synonym	homological dimension

For any ring R, the *left global dimension* of R is defined to be the supremum of projective dimensions of left modules of R:

$$l. \operatorname{Gd}(R) := \sup\{\operatorname{pd}_R(M) \mid M \text{ is a left R-module } \}.$$

Similarly, the *right global dimension* of R is:

$$r. \operatorname{Gd}(R) := \sup\{\operatorname{pd}_R(M) \mid M \text{ is a right R-module } \}.$$

If R is commutative, then $l. \operatorname{Gd}(R) = r. \operatorname{Gd}(R)$ and we may drop l and r and simply use $\operatorname{Gd}(R)$ to mean the *global dimension* of R.

Remarks.

1. For a ring R, $l. \operatorname{Gd}(R) = 0$ iff $r. \operatorname{Gd}(R) = 0$ (see the first example below). However, in general, $l. \operatorname{Gd}(R)$ is not necessarily the same as $r. \operatorname{Gd}(R)$.

2. The left (right) global dimension of a ring can also be defined in terms of injective dimensions. For example, for right global dimension of R, we have: $r. \operatorname{Gd}(R) = \sup\{\operatorname{id}_R(M) \mid M \text{ is a right R-module } \}$. This definition turns out to be equivalent to the one using projective dimensions.

Examples.

1. $l. \operatorname{Gd}(R) = 0$ iff R is a semisimple ring iff $r. \operatorname{Gd}(R) = 0$.

2. $r. \operatorname{Gd}(R) = 1$ iff R is a right hereditary ring that is not semisimple.