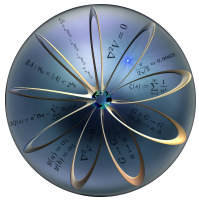

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ring of S-integers

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Definition. *Let K be a number field and let S be a finite set of absolute values of K, containing all archimedean valuations. The ring of S-integers of K, usually denoted by R_S, is the ring:*

$$R_S = \{k \in K : \nu(k) \geq 0 \text{ for all valuations } \nu \notin S\}.$$

Notice that, for any set S as above, the ring of integers of K, $\mathcal{O}_K$, is always contained in R_S.

Example. Let $K = \mathbb{Q}$ and let $S = \{\nu_p, |\cdot|\}$ where p is a prime and ν_p is the usual p-adic valuation, and $|\cdot|$ is the usual absolute value. Then

$$R_S = \mathbb{Z}\left[\frac{1}{p}\right]$$

, i.e. R_S is the result of adjoining (as a new ring element) $1/p$ to $\mathbb{Z}$ (i.e. we allow to invert p).