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## proof of Bezout's Theorem

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Let D be an integral domain with an Euclidean valuation. Let  $a,b \in D$  not both 0. Let  $(a,b) = \{ax+by|x,y \in D\}$ . (a,b) is an ideal in  $D \neq \{0\}$ . We choose  $d \in (a,b)$  such that  $\mu(d)$  is the smallest positive value. Then (a,b) is generated by d and has the property d|a and d|b. Two elements x and y in D are associate if and only if  $\mu(x) = \mu(y)$ . So d is unique up to a unit in D. Hence d is the greatest common divisor of a and b.