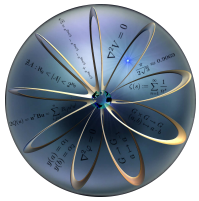

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zero rule of product

Canonical name	ZeroRuleOfProduct
Date of creation	2013-03-22 15:06:46
Last modified on	2013-03-22 15:06:46
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	11
Author	pahio (2872)
Entry type	Result
Classification	msc 13G05
Synonym	product to zero rule
Related topic	CancellationRing
Related topic	EulersDerivationOfTheQuarticFormula
Related topic	GroupingMethodForFactorizingPolynomials
Related topic	HyperbolasOrthogonalToEllipses

For real and complex numbers, and more generally for elements of an integral domain, a product equals to zero if and only if at least one of the equals to zero. For two elements a and b, we have

$$ab = 0 \iff a = 0 \vee b = 0.$$

For example, this rule can be used in solving polynomial equations:

$$x^3-x^2-2x+2 = 0$$

$$(x^3-x^2)+(-2x+2) = 0$$

$$x^2(x-1)-2(x-1) = 0$$

$$(x-1)(x^2-2) = 0$$

$$x-1 = 0 \vee x^2-2 = 0$$

$$x = 1 \vee x = \pm\sqrt{2}$$

The used sign "$\vee$" is the logical or.