



planetmath.org

Math for the people, by the people.

homogeneous system of parameters

Canonical name	HomogeneousSystemOfParameters
Date of creation	2013-03-22 14:14:55
Last modified on	2013-03-22 14:14:55
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	5
Author	mathcam (2727)
Entry type	Definition
Classification	msc 13A02
Defines	partial homogeneous system of parameters
Defines	complete homogeneous system of parameters
Defines	homogeneous M -sequence
Defines	depth
Defines	depth of a module

Let k be a field, let R be an \mathbb{N}^m -graded k -algebra, and let M be a \mathbb{Z}^m -graded R -module.

Let $\mathcal{H}(R_+)$ be the homogeneous union of the irrelevant ideal of R .

A *partial homogeneous system of parameters* for M is a finite sequence of elements $\theta_1, \theta_2, \dots, \theta_r \in \mathcal{H}(R_+)$ such that

$$\dim \left(M / \left(\sum_{i=1}^r \theta_i M \right) \right) = \dim(M) - r,$$

where \dim gives the Krull dimension.

A *homogeneous system of parameters* is a partial homogeneous system of parameters such that $r = \dim(M)$.

A sequence $\theta_1, \dots, \theta_r \in \mathcal{H}(R_+)$ is a *M -sequence* if for all i with $0 \leq i < r$, we have that θ_{i+1} is not a zero-divisor in

$$M / \left(\sum_{j=1}^i \theta_j M \right).$$

Finally, view M as being \mathbb{Z} -graded by using any specialization of the above \mathbb{Z}^m -grading. Then we define the *depth* of M to be the length of the longest homogeneous M -sequence.

References

- [1] Richard P. Stanley, *Combinatorics and Commutative Algebra*, Second edition, Birkhauser Press. Boston, MA. 1986.