Bezout domain

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Related topic	GcdDomain
Related topic	DivisibilityByProduct
Defines	Bezout identity

A *Bezout domain* D is an integral domain such that every finitely generated ideal of D is http://planetmath.org/PIDprincipal.

Remarks.

- A PID is obviously a Bezout domain.
- Furthermore, a Bezout domain is a gcd domain. To see this, suppose D is a Bezout domain with $a, b \in D$. By definition, there is a $d \in D$ such that $(d) = (a, b)$, the ideal generated by a and b. So $a \in (d)$ and $b \in (d)$ and therefore, $d \mid a$ and $d \mid b$. Next, suppose $c \in D$ and that $c \mid a$ and $c \mid b$. Then both $a, b \in (c)$ and so $d \in (c)$. This means that $c \mid d$ and we are done.
- From the discussion above, we see in a Bezout domain D, a greatest common divisor exists for every pair of elements. Furthermore, if $\gcd(a, b)$ denotes one such greatest common divisor between $a, b \in D$, then for some $r, s \in D$:

  $$\gcd(a, b) = ra + sb.$$

  The above equation is known as the *Bezout identity*, or Bezout's Lemma.