



planetmath.org

Math for the people, by the people.

finitely generated torsion-free modules over
Prüfer domains

Canonical name	FinitelyGeneratedTorsionfreeModulesOverPruferDomains
Date of creation	2013-03-22 18:36:11
Last modified on	2013-03-22 18:36:11
Owner	gel (22282)
Last modified by	gel (22282)
Numerical id	4
Author	gel (22282)
Entry type	Theorem
Classification	msc 13F05
Classification	msc 13C10
Related topic	EquivalentCharacterizationsOfDedekindDomains

Theorem. *Let M be a finitely generated torsion-free module over a Prüfer domain R . Then, M is isomorphic to a <http://planetmath.org/DirectSumdirectsum>*

$$M \cong \mathfrak{a}_1 \oplus \cdots \oplus \mathfrak{a}_n$$

of finitely generated ideals $\mathfrak{a}_1, \dots, \mathfrak{a}_n$.

As invertible ideals are projective and direct sums of projective modules are themselves projective, this theorem shows that M is also a projective module. Conversely, if every finitely generated torsion-free module over an integral domain R is projective then, in particular, every finitely generated nonzero ideal of R will be projective and hence invertible. So, we get the following characterization of Prüfer domains.

Corollary. *An integral domain R is Prüfer if and only if every finitely generated torsion-free R -module is <http://planetmath.org/ProjectiveModuleprojective>.*