module

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Let R be a ring with identity. A *left module* M over R is a set with two binary operations, $+ : M \times M \longrightarrow M$ and $\cdot : R \times M \longrightarrow M$, such that

1. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ for all $\mathbf{u}, \mathbf{v}, \mathbf{w} \in M$
2. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ for all $\mathbf{u}, \mathbf{v} \in M$
3. There exists an element $\mathbf{0} \in M$ such that $\mathbf{u} + \mathbf{0} = \mathbf{u}$ for all $\mathbf{u} \in M$
4. For any $\mathbf{u} \in M$, there exists an element $\mathbf{v} \in M$ such that $\mathbf{u} + \mathbf{v} = \mathbf{0}$
5. $a \cdot (b \cdot \mathbf{u}) = (a \cdot b) \cdot \mathbf{u}$ for all $a, b \in R$ and $\mathbf{u} \in M$
6. $a \cdot (\mathbf{u} + \mathbf{v}) = (a \cdot \mathbf{u}) + (a \cdot \mathbf{v})$ for all $a \in R$ and $\mathbf{u}, \mathbf{v} \in M$
7. $(a + b) \cdot \mathbf{u} = (a \cdot \mathbf{u}) + (b \cdot \mathbf{u})$ for all $a, b \in R$ and $\mathbf{u} \in M$

A left module M over R is called *unitary* or *unital* if $1_R \cdot \mathbf{u} = \mathbf{u}$ for all $\mathbf{u} \in M$.

A (unitary or unital) *right module* is defined analogously, except that the function $\cdot$ goes from $M \times R$ to M and the scalar multiplication operations act on the right. If R is commutative, there is an equivalence of categories between the category of left R–modules and the category of right R–modules.