UFD

Canonical name	UFD
Date of creation	2013-03-22 11:56:22
Last modified on	2013-03-22 11:56:22
Owner	drini (3)
Last modified by	drini (3)
Numerical id	19
Author	drini (3)
Entry type	Definition
Classification	msc 13G05
Synonym	unique factorization domain
Related topic	IntegralDomain
Related topic	Irreducible
Related topic	EuclideanRing
Related topic	EuclideanValuation
Related topic	ProofThatAnEuclideanDomainIsAPID
Related topic	WhyEuclideanDomains
Related topic	Y2X32
Related topic	PID
Related topic	PIDsAreUFDs
Related topic	FundamentalTheoremOfArithmetic
Defines	factorial ring
Defines	prime factor
Defines	UFD
Defines	unique factorization

An integral domain D satisfying

- Every nonzero element of D that is not a unit can be factored into a product of a finite number of irreducibles,
- If $p_1 p_2 \cdots p_r$ and $q_1 q_2 \cdots q_s$ are two factorizations of the same element a into irreducibles, then $r = s$ and we can reorder the q_j's in a way that q_j is an associate element of p_j for all j

is called a *unique factorization domain* (UFD), also a *factorial ring*.

The factors $p_1, p_2, \ldots, p_r$ are called the *prime factors* of a.

Some of the classic results about UFDs:

- On a UFD, the concept of prime element and irreducible element coincide.
- If F is a field, then $F[x]$ is a UFD.
- If D is a UFD, then $D[x]$ (the ring of polynomials on the variable x over D) is also a UFD.

  Since $R[x, y] \cong R[x][y]$, these results can be extended to rings of polynomials with a finite number of variables.
- If D is a principal ideal domain, then it is also a UFD.

  The converse is, however, not true. Let F a field and consider the UFD $F[x, y]$. Let I the ideal consisting of all the elements of $F[x, y]$ whose constant term is 0. Then it can be proved that I is not a principal ideal. Therefore not every UFD is a PID.