orders of elements in integral domain

Canonical name	OrdersOfElementsInIntegralDomain
Date of creation	2013-03-22 15:40:28
Last modified on	2013-03-22 15:40:28
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	9
Author	pahio (2872)
Entry type	Theorem
Classification	msc 13G05
Related topic	OrderGroup
Related topic	IdealOfElementsWithFiniteOrder

Theorem. Let $(D, +, \cdot)$ be an integral domain, i.e. a commutative ring with non-zero unity 1 and no zero divisors. All non-zero elements of D have the same `http://planetmath.org/OrderGroup`order in the additive group $(D, +)$.

Proof. Let a be arbitrary non-zero element. Any `http://planetmath.org/GeneralAssociati`
na may be written as

$$na = n(1a) = \underbrace{1a + 1a + \cdots + 1a}_{n} = (\underbrace{1 + 1 + \cdots + 1}_{n})a = (n1)a.$$

Thus, because $a \neq 0$ and there are no zero divisors, an equation $na = 0$ is `http://planetmath.org/Equivalent3`equivalent with the equation $n1 = 0$. So a must have the same as the unity of D.

Note. The of the unity element is the `http://planetmath.org/Characteristic`characteristi
of the integral domain, which is 0 or a positive prime number.