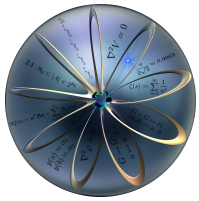

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## discrete valuation

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|---|---|
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A *discrete valuation* on a field $K$ is a valuation $|\cdot| : K \to \mathbb{R}$ whose image is a discrete subset of $\mathbb{R}$.

For any field $K$ with a discrete valuation $|\cdot|$, the set

$$R := \{x \in K : |x| \leq 1\}$$

is a subring of $K$ with sole maximal ideal

$$M := \{x \in K : |x| < 1\},$$

and hence $R$ is a discrete valuation ring. Conversely, given any discrete valuation ring $R$, the field of fractions $K$ of $R$ admits a discrete valuation sending each element $x \in R$ to $c^n$, where $0 < c < 1$ is some arbitrary fixed constant and $n$ is the order of $x$, and extending multiplicatively to $K$.

**Note:** Discrete valuations are often written additively instead of multiplicatively; under this alternate viewpoint, the element $x$ maps to $\log_c |x|$ (in the above notation) instead of just $|x|$. This transformation reverses the order of the absolute values (since $c < 1$), and sends the element $0 \in K$ to $\infty$. It has the advantage that every valuation can be normalized by a suitable scalar multiple to take values in the integers.