unities of ring and subring

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Let R be a ring and S a proper subring of it. Then there exists five cases in all concerning the possible unities of R and S.

1. R and S have a common unity.

2. R has a unity but S does not.

3. R and S both have their own non-zero unities but these are distinct.

4. R has no unity but S has a non-zero unity.

5. Neither R nor S have unity.

Note: In the cases 3 and 4, the unity of the subring S must be a zero divisor of R.

Examples

1. The ring $\mathbb{Q}$ and its subring $\mathbb{Z}$ have the common unity 1.

2. The subring S of even integers of the ring $\mathbb{Z}$ has no unity.

3. Let S be the subring of all pairs $(a,\,0)$ of the ring $R = \mathbb{Z}\times\mathbb{Z}$ for which the operations "+" and "$\cdot$" are defined componentwise (i.e. $(a,\,b)+(c,\,d) = (a+c,\,b+d)$ etc.). Then S and R have the unities $(1,\,0)$ and $(1,\,1)$, respectively.

4. Let S be the subring of all pairs $(a,\,0)$ of the ring $R = \{(a,\,2b)|\ a \in \mathbb{Z} \wedge b \in \mathbb{Z}\}$ (operations componentwise). Now S has the unity $(1,\,0)$ but R has no unity.

5. Neither the ring $\{(2a,\,2b)|\ a,\,b \in \mathbb{Z}\}$ (operations componentwise) nor its subring consisting of the pairs $(2a,\,0)$ have unity.