primary ideal

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An ideal Q in a commutative ring R is a *primary ideal* if for all elements $x, y \in R$, we have that if $xy \in Q$, then either $x \in Q$ or $y^n \in Q$ for some $n \in \mathbb{N}$.

This is clearly a generalization of the notion of a prime ideal, and (very) loosely mirrors the relationship in $\mathbb{Z}$ between prime numbers and prime powers.

It is clear that every prime ideal is primary.

Example. Let $Q = (25)$ in $R = \mathbb{Z}$. Suppose that $xy \in Q$ but $x \notin Q$. Then $25|xy$, but 25 does not divide x. Thus 5 must divide y, and thus some power of y (namely, y^2), must be in Q.

The radical of a primary ideal is always a prime ideal. If P is the radical of the primary ideal Q, we say that Q is *P-primary*.