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half-factorial ring

Canonical name	HalffactorialRing
Date of creation	2013-03-22 18:31:14
Last modified on	2013-03-22 18:31:14
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	7
Author	pahio (2872)
Entry type	Definition
Classification	msc 13G05
Synonym	half-factorial domain
Defines	HFD

An integral domain D is called a *half-factorial ring* (HFD) if it satisfies the following conditions:

- Every nonzero element of D that is not a unit can be factored into a product of a finite number of irreducibles.
- If $p_1 p_2 \cdots p_m$ and $q_1 q_2 \cdots q_n$ are two factorizations of the same element a into irreducibles, then $m = n$.

If, in , the irreducibles p_i and q_j are always pairwise associates, then D is a factorial ring (UFD).

For example, many <http://planetmath.org/OrderInAnAlgebraorders> in the maximal order of an algebraic number field are half-factorial rings, e.g. $\mathbb{Z}[3\sqrt{2}]$ is a HFD but not a UFD (see <http://www.math.ndsu.nodak.edu/faculty/coykenda/paper/paper>).