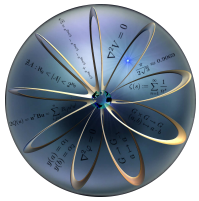

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arithmetical ring

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Theorem. If R is a commutative ring, then the following three conditions are equivalent:

- For all ideals $\mathfrak{a}$, $\mathfrak{b}$ and $\mathfrak{c}$ of R, one has $\mathfrak{a} \cap (\mathfrak{b} + \mathfrak{c}) = (\mathfrak{a} \cap \mathfrak{b}) + (\mathfrak{a} \cap \mathfrak{c})$.
- For all ideals $\mathfrak{a}$, $\mathfrak{b}$ and $\mathfrak{c}$ of R, one has $\mathfrak{a} + (\mathfrak{b} \cap \mathfrak{c}) = (\mathfrak{a} + \mathfrak{b}) \cap (\mathfrak{a} + \mathfrak{c})$.
- For each maximal ideal $\mathfrak{p}$ of R the set of all ideals of $R_{\mathfrak{p}}$, the `http://planetmath.org/Localiz` of R at $R \backslash \mathfrak{p}$, is totally ordered by set inclusion.

The ring R satisfying the conditions of the theorem is called an *arithmetical ring*.