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cyclic rings of behavior one

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Theorem. *A cyclic ring has a multiplicative identity if and only if it has behavior one.*

Proof. For a proof that a cyclic ring with a multiplicative identity has behavior one, see <http://planetmath.org/MultiplicativeIdentityOfACyclicRingMustBeAGenerator> theorem.

Let R be a cyclic ring with behavior one. Let r be a <http://planetmath.org/Generator> of the additive group of R such that $r^2 = r$. Let $s \in R$. Then there exists $a \in R$ with $s = ar$. Since $rs = r(ar) = ar^2 = ar = s$ and multiplication in cyclic rings is commutative, then r is a multiplicative identity. \square