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Young tableau

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Synonym	Young tableaux
Defines	semi-standard tableau
Defines	semi-standard tableaux
Defines	standard Young tableau
Defines	standard Young tableaux

Let  $Y$  be a Young diagram. A *filling* of  $Y$  is a labelling of the boxes in  $Y$  by positive integers. For example, consider the Young diagram with shape  $\lambda = (4, 4, 2, 1) \vdash 11$ .


One filling of this Young diagram is

2	4	1	1
5	2	3	7
1	4		
1			

A filling is a *Young tableau* if it includes each label from 1 to  $n$  exactly once. One Young tableau with shape  $\lambda$  is

5	11	8	10
2	3	7	6
9	4		
1			

Each Young tableau with shape  $\lambda \vdash n$  corresponds to a <http://planetmath.org/Partitionset> partition of  $[n] = \{1, \dots, n\}$ .

A filling is a *semi-standard tableau* if the labels monotonically increase in each row and strictly increase in each column. One semi-standard tableau with shape  $\lambda$  is

1	1	2	3
2	4	4	5
5	6		
8			

Finally, a semi-standard tableau is a *standard Young tableau* if it includes each label from 1 to  $n$  exactly once. Hence a standard Young tableau is both a semi-standard tableau and a Young tableau. One standard Young tableau with shape  $\lambda$  is

1	3	6	8
2	4	7	9
5	10		
11			

There is some variation in this terminology. For example, Fulton uses the terms tableau and Young tableau interchangeably for what we call a semi-standard Young tableau.

## References

- [1] William Fulton. *Young tableaux: with applications to representation theory and geometry*. Cambridge University Press, 1997.
- [2] Bruce E. Sagan. *The symmetric group: representations, combinatorial algorithms, and symmetric functions*, 2nd ed. Springer, 2001.
- [3] Richard P. Stanley. *Enumerative combinatorics, volume 2*. Cambridge University Press, 1999.