homogeneous elements of a graded ring

Canonical name	HomogeneousElementsOfAGradedRing
Date of creation	2013-03-22 14:14:52
Last modified on	2013-03-22 14:14:52
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	6
Author	mathcam (2727)
Entry type	Definition
Classification	msc 13A02
Related topic	HomogeneousIdeal
Defines	homogeneous element
Defines	homogeneous degree
Defines	irrelevant ideal
Defines	homogeneous union

Let k be a field, and let R be a connected commutative k-algebra http://planetmath.org/Grad by $\mathbb{N}^m$. Then via the grading, we can decompose R into a direct sum of vector spaces: $R = \coprod_{\omega \in \mathbb{N}^m} R_\omega$, where $R_0 = k$.

For an arbitrary ring element $x \in R$, we define the *homogeneous degree* of x to be the value ω such that $x \in R_\omega$, and we denote this by $\deg(x) = \omega$. (See also homogeneous ideal)

A set of some importance (ironically), is the *irrelevant ideal* of R, denoted by R^+, and given by

$$R_+ = \coprod_{\omega \neq 0} R_\omega.$$

Finally, we often need to consider the elements of such a ring R without using the grading, and we do this by looking at the *homogeneous union* of R:

$$\mathcal{H}(R) = \bigcup_{\omega} R_\omega.$$

In particular, in defining a homogeneous system of parameters, we are looking at elements of $\mathcal{H}(R_+)$.

References

[1] Richard P. Stanley, *Combinatorics and Commutative Algebra*, Second edition, Birkhauser Press. Boston, MA. 1986.