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basic algebra

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Let A be a finite dimensional, unital algebra over a field k. By Krull-Schmidt Theorem A can be decomposed as a (right) A-module as follows:

$$A \simeq P_1 \oplus \cdots \oplus P_k$$

where each P_i is an indecomposable module and this decomposition is unique.

Definition. The algebra A is called **(right) basic** if P_i is not isomorphic to P_j when $i \neq j$.

Of course we may easily define what does it mean for algebra to be left basic. Fortunetly these properties coincide. Let as state some known facts (originally can be found in [**?**]):

Proposition.

1. A finite algebra A over a field k is basic if and only if the algebra $A/\mathrm{rad}A$ is isomorphic to a product of fields $k \times \cdots \times k$. Thus A is right basic iff it is left basic;
2. Every simple module over a basic algebra is one-dimensional;
3. For any finite-dimensional, unital algebra A over k there exists finite-dimensional, unital, basic algebra B over k such that the category of finite-dimensional modules over A is k-linear equivalent to the category of finite-dimensional modules over B;
4. Let A be a finite-dimensional, basic and connected (i.e. cannot be written as a product of nontrivial algebras) algebra over a field k. Then there exists a bound quiver (Q, I) such that $A \simeq kQ/I$;
5. If (Q, I) is a bound quiver over a field k, then both kQ and kQ/I are basic algebras.

References

[1] I. Assem, D. Simson, A. Skowronski, *Elements of the Representation Theory of Associative Algebras, vol 1.*, Cambridge University Press 2006, 2007