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proof of finite extensions of Dedekind domains are Dedekind

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Let  $R$  be a Dedekind domain with field of fractions  $K$ . If  $L/K$  is a finite extension of fields and  $A$  is the integral closure of  $R$  in  $L$ , then we show that  $A$  is also a Dedekind domain.

We proceed by splitting the proof up into the separable and purely inseparable cases. Letting  $F$  consist of all elements of  $L$  which are separable over  $K$ , then  $F/K$  is a separable extension and  $L/F$  is a purely inseparable extension.

First, the integral closure  $B$  of  $R$  in  $F$  is a Dedekind domain (see proof of finite separable extensions of Dedekind domains are Dedekind). Then, as  $A$  is integrally closed and contains  $B$ , it is equal to the integral closure of  $B$  in  $L$  and, therefore, is a Dedekind domain (see proof of finite inseparable extensions of Dedekind domains are Dedekind).