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grouping method for factoring polynomials

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Factoring a given polynomial may in certain special cases be using the following *grouping method*:

1. .
2. Factorize the separately.
3. The whole polynomial may then possibly be written in form of a product.

Examples

$$\begin{aligned} \text{a) } x^3 - x^2 - x + 1 &= \{x^3 - x^2\} + \{-x + 1\} = x^2(x - 1) - 1(x - 1) = \\ &= (x - 1)(x^2 - 1) \\ &= (x - 1)^2(x + 1) \end{aligned}$$

$$\begin{aligned} \text{b) } x^4 + 3x^3 - 3x - 1 &= \{x^4 - 1\} + \{3x^3 - 3x\} = (x^2 + 1)(x^2 - 1) + 3x(x^2 - 1) \\ &= (x^2 - 1)(x^2 + 1 + 3x) = (x - 1)(x + 1)(x^2 + 3x + 1) \end{aligned}$$

$$\begin{aligned} \text{c) } x^4 + 4 &= \{x^4 + 4x^2 + 4\} - 4x^2 = (x^2 + 2)^2 - (2x)^2 = (x^2 + 2 + 2x)(x^2 + \\ &= (x^2 + 2x + 2)(x^2 - 2x + 2) \end{aligned}$$

$$\begin{aligned} \text{d) } x^4 + x^2 + 1 &= \{x^4 + 2x^2 + 1\} - x^2 = (x^2 + 1)^2 - x^2 = (x^2 + 1 + x)(x^2 + 1 - x) \\ &= (x^2 + x + 1)(x^2 - x + 1) \end{aligned}$$

The trinomials x^2+3x+1 , $x^2\pm 2x+2$ and $x^2\pm x+1$ are irreducible polynomials.