lemma on projection of countable sets

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Suppose $\mathbb{F}$ is an infinite field and S is an infinite subset of $\mathbb{F}^n$. Then there exists a line L such that the projection of S on L is infinite.

Proof: This proof will proceed by an induction on n. The case $n = 1$ is trivial since a one-dimensional linear space is a line.

Consider two cases:

Case I: There exists a proper subspace of $\mathbb{F}^n$ which contains an infinite number of points of S.

In this case, we can restrict attention to this subspace. By the induction hypothesis, there exists a line in the subspace such that the projection of points in the subspace to this line is already infinite.

Case II: Every proper subspace of $\mathbb{F}^n$ contains at most a finite number of points of S.

In this case, any line will do. By definition, one constructs a projection by dropping hyperplanes perpendicular to the line passing through the points of the set. Since each of these hyperplanes will contain a finite number of elements of S, an infinite number of hyperplanes will be needed to contain all the points of S, hence the projection will be infinite.