smooth submanifold contained in a subvariety of same dimension is real analytic

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This theorem seems to usually be attributed to Malgrange in literature as it appeared in his book[**?**].

Theorem (Malgrange). *Suppose $M \subset \mathbb{R}^N$ is a connected smooth (C^∞) submanifold and $V \subset \mathbb{R}^N$ is a real analytic subvariety of the same dimension as M, such that $M \subset V$. Then M is a real analytic submanifold.*

The condition that M is smooth cannot be relaxed to C^k for $k < \infty$. For example, note that in $\mathbb{R}^2$, the subvariety $y^3 - x^8 = 0$, which is the graph of the C^1 function $y = |x|^{\frac{8}{3}}$, is not a real analytic submanifold.

References

[1] Bernard Malgrange. . Oxford University Press, 1966.