category of quivers is concrete

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Let $\mathcal{Q}$ denote the category of all quivers and quiver morphisms with standard composition. If $Q=(Q_0,Q_1,s,t)$ is a quiver, then we can associate with Q the set

$$S(Q)=Q_0\sqcup Q_1$$

where ,,$\sqcup$" denotes the disjoint union of sets.

Furthermore, if $F:Q\to Q'$ is a morphism of quivers, then F induces function

$$S(F):S(Q)\to S(Q')$$

by putting $S(F)(a)=F_0(a)$ if $a\in Q_0$ and $S(F)(\alpha)=F_1(\alpha)$ if $\alpha\in Q_1$.

Proposition. The category $\mathcal{Q}$ together with $S:\mathcal{Q}\to\mathcal{SET}$ is a concrete category over the category of all sets $\mathcal{SET}$.

Proof. The fact that S is a functor we leave as a simple exercise. Now assume, that $F,G:Q\to Q'$ are morphisms of quivers such that $S(F)=S(G)$. It follows, that for any vertex $a\in Q_0$ and any arrow $\alpha\in Q_1$ we have

$$F_0(a)=S(F)(a)=S(G)(a)=G_0(a);$$

$$F_1(\alpha)=S(F)(\alpha)=S(G)(\alpha)=G_1(\alpha)$$

which clearly proves that $F=G$. This completes the proof. $\square$

Remark. Note, that if $F:Q\to Q'$ is a morphism of quivers, then F is injective in $(\mathcal{Q},S)$ (see `http://planetmath.org/InjectiveAndSurjectiveMorphismsInConcreteCa` entry for details) if and only if both F_0, F_1 are injective. The same holds if we replace word ,,injective" with ,,surjective".