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Weil divisors on schemes

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Defines	prime divisor
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Defines	regular in codimension one

Let  $X$  be a noetherian integral separated scheme such that every local ring  $\mathcal{O}_x$  of  $X$  of dimension one is regular (such a scheme  $X$  is said to be regular in codimension one, or non-singular in codimension one).

**Definition.** *A prime divisor on  $X$  is a closed integral subscheme  $Y$  of codimension one. We define an abelian group  $\text{Div}(X)$  generated by the prime divisors on  $X$ . A Weil divisor is an element of  $\text{Div}(X)$ . Thus, a Weil divisor  $\mathcal{W}$  can be written as:*

$$\mathcal{W} = \sum n_Y Y$$

*where the sum is over all the prime divisors  $Y$  of  $X$ , the  $n_Y$  are integers and only finitely many of them are non-zero. A degree of a divisor is defined to be  $\deg(\mathcal{W}) = \sum n_Y$ . Finally, a divisor is said to be effective if  $n_Y \geq 0$  for all the prime divisors  $Y$ .*

For more information, see Hartshorne's book listed in the bibliography for algebraic geometry.