ℓ-adic étale cohomology

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Let X be a scheme over a field k having algebraic closure $\overline{k}$. Let $(X \otimes_k \overline{k})_{\text{ét}}$ be the small étale site on $X \otimes_k \overline{k}$, and let $\mathbb{Z}/l^n\mathbb{Z}$ denote the sheaf on $(X \otimes_k \overline{k})_{\text{ét}}$ associated to the group scheme $\mathbb{Z}/l^n\mathbb{Z}$ for some fixed prime l. Finally, let Γ be the global sections functor on the category of étale sheaves on $(X \otimes_k \overline{k})_{\text{ét}}$.

The l-adic *étale cohomology* of X is

$$H^i_{\text{ét}}(X, \mathbb{Q}_l) = \mathbb{Q}_l \otimes_{\mathbb{Z}_l} \varprojlim_n (R^i\Gamma)(\mathbb{Z}/l^n\mathbb{Z}),.$$

where R^i denotes taking the i-th right-derived functor.

This apparently appalling definition is necessary to ensure that (for l not equal to the characteristic of k) étale cohomology is the appropriate generalization of de Rham cohomology on a complex manifold. For example, on a scheme of dimension n, the cohomology groups H^i vanish for $i > 2n$ and we have a version of Poincaré duality. Grothendieck introduced étale cohomology as a tool to prove the Weil conjectures, and indeed it is what Deligne used to prove them.

These references are approximately in order of difficulty and of generality and precision.

References

[1] J. S. Milne, *Lectures on Étale Cohomology*, 1998, available on the web at `http://www.jmilne.org/math/`http://www.jmilne.org/math/

[2] James S. Milne, *Étale cohomology*, volume 33 of *Princeton Mathematical Series*. Princeton University Press, Princeton N.J., 1980

[3] Deligne et al., *Séminaires en Gèometrie Algèbrique $4\frac{1}{2}$*, available on the web at `http://www.math.mcgill.ca/ archibal/SGA/SGA.html`http://www.math.mcgill.ca/ archibal

[4] Grothendieck et al., *Séminaires en Gèometrie Algèbrique 4*, tomes 1, 2, and 3, available on the web at `http://www.math.mcgill.ca/ archibal/SGA/SGA.html`http://www.math.mcgill.ca/ archibal