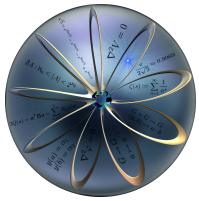

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examples of torsion subgroups of elliptic curves

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Mazur's theorem shows that given an elliptic curve defined over the rationals, the only possible torsion subgroups are the following:

$$\mathbb{Z}/N\mathbb{Z} \quad \text{with } 1 < N < 11 \text{ or } N = 12$$

$$\mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2N\mathbb{Z} \text{ with } 0 < N < 5$$

Here we show examples of curves with the torsion subgroups mentioned above:

CURVE	**TORSION SUBGROUP**	**GENERATORS**
$y^2 = x^3 - 2$	trivial	$\mathcal{O}$
$y^2 = x^3 + 8$	$\mathbb{Z}/2\mathbb{Z}$	$[[-2, 0]]$
$y^2 = x^3 + 4$	$\mathbb{Z}/3\mathbb{Z}$	$[[0, 2]]$
$y^2 = x^3 + 4x$	$\mathbb{Z}/4\mathbb{Z}$	$[[2, 4]]$
$y^2 - y = x^3 - x^2$	$\mathbb{Z}/5\mathbb{Z}$	$[[0, 1]]$
$y^2 = x^3 + 1$	$\mathbb{Z}/6\mathbb{Z}$	$[[2, 3]]$
$y^2 = x^3 - 43x + 166$	$\mathbb{Z}/7\mathbb{Z}$	$[[3, 8]]$
$y^2 + 7xy = x^3 + 16x$	$\mathbb{Z}/8\mathbb{Z}$	$[[-2, 10]]$
$y^2 + xy + y = x^3 - x^2 - 14x + 29$	$\mathbb{Z}/9\mathbb{Z}$	$[[3, 1]]$
$y^2 + xy = x^3 - 45x + 81$	$\mathbb{Z}/10\mathbb{Z}$	$[[0, 9]]$
$y^2 + 43xy - 210y = x^3 - 210x^2$	$\mathbb{Z}/12\mathbb{Z}$	$[[0, 210]]$
$y^2 = x^3 - 4x$	$\mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$	$[[2, 0], [0, 0]]$
$y^2 = x^3 + 2x^2 - 3x$	$\mathbb{Z}/4\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$	$[[3, 6], [0, 0]]$
$y^2 + 5xy - 6y = x^3 - 3x^2$	$\mathbb{Z}/6\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$	$[[-3, 18], [2, -2]]$
$y^2 + 17xy - 120y = x^3 - 60x^2$	$\mathbb{Z}/8\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$	$[[30, -90], [-40, 400]]$