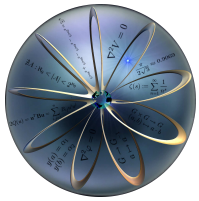

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general linear group scheme

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Definition 1 *Fix a positive integer n. We define the* general linear group scheme GL_n *as the affine scheme defined by*

$$\mathbb{Z}[Y, X_{11}, \ldots, X_{1n}, \ldots, X_{n1}, \ldots, X_{nn}]/\left\langle Y \det \begin{pmatrix} X_{11} & \cdots & X_{1n} \\ \vdots & \ddots & \vdots \\ X_{n1} & \cdots & X_{nn} \end{pmatrix} - 1 \right\rangle$$

Observe that if R is any commutative ring, as `http://planetmath.org/ExampleOfFunctorOfPo` with schemes, an R-point of GL_n is given by specifying, for each i and j, an element r_{ij} that is the image of X_{ij}, and by specifying one other element r such that

$$r \det \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & r_{nn} \end{pmatrix} = 1.$$

In other words, an R-point of GL_n is an invertible matrix with entries in R.

As usual with schemes, we denote the R-points of GL_n by $\mathrm{GL}_n(R)$; we see that this notion does not lead to confusion, since it is exactly what is meant by the usual usage of this notation (see entry General Linear Group).