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separated scheme

Canonical name	SeparatedScheme
Date of creation	2013-03-22 12:50:25
Last modified on	2013-03-22 12:50:25
Owner	djao (24)
Last modified by	djao (24)
Numerical id	6
Author	djao (24)
Entry type	Definition
Classification	msc 14A15
Defines	separated

A scheme X is defined to be a *separated scheme* if the morphism

$$d : X \to X \times_{\mathrm{Spec}\,\mathbb{Z}} X$$

into the fibre product $X \times_{\mathrm{Spec}\,\mathbb{Z}} X$ which is induced by the identity maps $i : X \longrightarrow X$ in each coordinate is a closed immersion.

Note the similarity to the definition of a Hausdorff topological space. In the situation of topological spaces, a space X is Hausdorff if and only if the diagonal morphism $X \longrightarrow X \times X$ is a closed embedding of topological spaces. The definition of a separated scheme is very similar, except that the topological product is replaced with the scheme fibre product.

More generally, if X is a scheme over a base scheme Y, the scheme X is defined to be *separated* over Y if the diagonal embedding

$$d : X \to X \times_Y X$$

is a closed immersion.