bound for the rank of an elliptic curve

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Theorem. *Let $E/\mathbb{Q}$ be an elliptic curve given by the equation:*

$$E\colon y^2 = x(x-t)(x-s), \text{ with } t, s \in \mathbb{Z}$$

and suppose that E has $s = m+a$ primes of bad reduction, with m and a being the number of primes with multiplicative and additive reduction respectively. Then the rank of E, denoted by R_E, satisfies:

$$R_E \leq m + 2a - 1$$

Example. As an application of the theorem above, we can prove that $E_1\colon y^2 = x^3 - x$ has only finitely many rational solutions. Indeed, the discriminant of E_1, $\Delta = 64$, is only divisible by $p = 2$, which is a prime of (bad) multiplicative reduction. Therefore $R_{E_1} = 0$. Moreover, the Nagell-Lutz theorem implies that the only torsion points on E_1 are those of order 2. Hence, the only rational points on E_1 are:

$$\{\mathcal{O}, (0,0), (1,0), (-1,0)\}.$$

References

[1] James Milne, *Elliptic Curves*, online course notes.
`http://www.jmilne.org/math/CourseNotes/math679.html`http://www.jmilne.org/math/Co