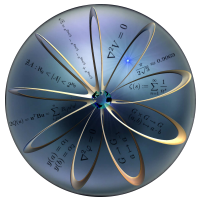

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countable algebraic sets

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An algebraic set over an uncountably infinite base field $\mathbb{F}$ (like the real or complex numbers) cannot be countably infinite.

Proof: Let S be a countably infinite subset of $\mathbb{F}^n$. By a cardinality argument (see the attachment), there must exist a line such that the projection of this set to the line is infinite. Since the projection of an algebraic set to a linear subspace is an algebraic set, the projection of S to this line would be an algebraic subset of the line. However, an algebraic subset of a line is the locus of zeros of some polynomial, hence must be finite. Therefore, S could not be algebraic since that would lead to a contradiction.