fibre product

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Let S be a scheme, and let $i : X \longrightarrow S$ and $j : Y \longrightarrow S$ be schemes over S. A *fibre product* of X and Y over S is a scheme $X \times_S Y$ together with morphisms

$$p : X \times_S Y \longrightarrow X$$
$$q : X \times_S Y \longrightarrow Y$$

such that given any scheme Z with morphisms

$$x : Z \longrightarrow X$$
$$y : Z \longrightarrow Y$$

where $i \circ x = j \circ y$, there exists a unique morphism

$$(x, y) : Z \longrightarrow X \times_S Y$$

making the diagram

$$\begin{tikzcd}
Z \arrow[drr, bend left, "x"] \arrow[dr, dotted, "{(x,y)}"] \arrow[ddr, bend right, "y"'] & & \\
& X \times_S Y \arrow[r, "p"'] \arrow[d, "q"] & X \arrow[d, "i"] \\
& Y \arrow[r, "j"'] & S
\end{tikzcd}$$

commute. In other words, a fiber product is an object $X \times_S Y$, **together with** morphisms p, q making the diagram commute, with the universal property that any other collection (Z, x, y) forming such a commutative diagram maps into $(X \times_S Y, p, q)$.

Fibre products of schemes always exist and are unique up to canonical isomorphism.

Other notes Fibre products are also called pullbacks and can be defined in any category using the same definition (but need not exist in general). For example, they always exist in the category of modules over a fixed ring, as well as in the category of groups.