affine space

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Definition. *Let K be a field and let n be a positive integer. In algebraic geometry we define affine space (or affine n-space) to be the set*

$$\{(k_1, \dots, k_n) : k_i \in K\}.$$

Affine space is usually denoted by K^n or $\mathbb{A}^n$ (or $\mathbb{A}^n(K)$ if we want to emphasize the field of definition).

In Algebraic Geometry, we consider affine space as a topological space, with the usual Zariski topology (see also algebraic set, affine variety). The polynomials in the ring $K[x_1, \dots, x_n]$ are regarded as functions (algebraic functions) on $\mathbb{A}^n(K)$. "Gluing" several copies of affine space one obtains a projective space.

Lemma. *If K is algebraically closed, affine space $\mathbb{A}^n(K)$ is an irreducible algebraic variety.*

References

[1] R. Hartshorne, *Algebraic Geometry*, Springer-Verlag, New York.