torsion (space curve)

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Let $I \subset R$ be an interval and let $\gamma : I \to \mathbb{R}^3$ be a parameterized space curve, assumed to be http://planetmath.org/SpaceCurveregular and free of points of inflection. We interpret $\gamma(t)$ as the trajectory of a particle moving through 3-dimensional space. Let $T(t), N(t), B(t)$ denote the corresponding moving trihedron. The speed of this particle is given by $\|\gamma'(t)\|$.

In order for a moving particle to escape the osculating plane, it is necessary for the particle to "roll" along the axis of its tangent vector, thereby lifting the normal acceleration vector out of the osculating plane. The "rate of roll", that is to say the rate at which the osculating plane rotates about the tangent vector, is given by $B(t) \cdot N'(t)$; it is a number that depends on the speed of the particle. The rate of roll relative to the particle's speed is the quantity

$$\tau(t) = \frac{B(t) \cdot N'(t)}{\|\gamma'(t)\|} = \frac{(\gamma'(t) \times \gamma''(t)) \cdot \gamma'''(t)}{\|\gamma'(t) \times \gamma''(t)\|^2},$$

called the torsion of the curve, a quantity that is invariant with respect to reparameterization. The torsion $\tau(t)$ is, therefore, a measure of an intrinsic property of the oriented space curve, another real number that can be covariantly assigned to the point $\gamma(t)$.