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nonsingular variety

Canonical name	NonsingularVariety
Date of creation	2013-03-22 12:03:47
Last modified on	2013-03-22 12:03:47
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	10
Author	CWoo (3771)
Entry type	Definition
Classification	msc 14-00
Synonym	non-singular variety
Defines	nonsingular
Defines	non-singular
Defines	singular point
Defines	nonsingular point
Defines	non-singular point

A variety over an algebraically closed field k is *nonsingular* at a point x if the local ring \mathcal{O}_x is a regular local ring. Equivalently, if around the point one has an open affine neighborhood wherein the variety is cut out by certain polynomials F_1, \dots, F_n of m variables x_1, \dots, x_m , then it is nonsingular at x if the Jacobian has maximal rank at that point. Otherwise, x is a *singular point*.

A variety is *nonsingular* if it is nonsingular at each point.

Over the real or complex numbers, nonsingularity corresponds to “smoothness”: at nonsingular points, varieties are locally real or complex manifolds (this is simply the implicit function theorem). Singular points generally have “corners” or self intersections. Typical examples are the curves $x^2 = y^3$, which has a cusp at $(0,0)$ and is nonsingular everywhere else, and $x^2(x+1) = y^2$, which has a self-intersection at $(0,0)$ and is nonsingular everywhere else.