# group variety

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Let $G$ be a variety (an http://planetmath.org/AffineVarietyaffine, http://planetmath.org/ProjectiveVarietyprojective, or quasi-projective variety). We say $G$ is a *group variety* if $G$ is provided with morphisms of varieties:

$$\begin{aligned} \mu : G \times G &\to G \\ (g_1, g_2) &\mapsto g_1 g_2, \end{aligned}$$

$$\begin{aligned} \iota : G &\to G \\ g &\mapsto g^{-1}, \end{aligned}$$

and

$$\begin{aligned} \epsilon : \{*\} &\to G \\ * &\mapsto e, \end{aligned}$$

and if these morphisms make the elements of $G$ into a group.

In short, $G$ should be a group object in the category of varieties. Examples include the general linear group of dimension $n$ on $k$ and elliptic curves.

Group varieties that are actually projective are in fact abelian groups (although this is not obvious) and are called abelian varieties; their study is of interest to number theorists (among others).

Just as schemes generalize varieties, group schemes generalize group varieties. When dealing with situations in positive characteristic, or with families of group varieties, often they are more appropriate.

There is also a (not very closely related) concept in group theory of a "http://planetmath.org/VarietyOfGroupsvariety of groups".