



height of a prime ideal

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Defines	rank of an ideal
Defines	codimension of an ideal

Let  $R$  be a commutative ring and  $\mathfrak{p}$  a prime ideal of  $R$ . The *height* of  $\mathfrak{p}$  is the supremum of all integers  $n$  such that there exists a chain

$$\mathfrak{p}_0 \subset \cdots \subset \mathfrak{p}_n = \mathfrak{p}$$

of distinct prime ideals. The height of  $\mathfrak{p}$  is denoted by  $h(\mathfrak{p})$ .

$h(\mathfrak{p})$  is also known as the rank of  $\mathfrak{p}$  and the codimension of  $\mathfrak{p}$ .

The Krull dimension of  $R$  is the supremum of the heights of all the prime ideals of  $R$ :

$$\sup\{h(\mathfrak{p}) \mid \mathfrak{p} \text{ prime in } R\}.$$