Tarski-Seidenberg theorem

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Theorem (Tarski-Seidenberg)**.** *The set of semialgebraic sets is closed under projection.*

That is, if $A \subset \mathbb{R}^n \times \mathbb{R}^m$ is a semialgebraic set, and if π is the projection onto the first n coordinates, then $\pi(A)$ is also semialgebraic.

Łojasiewicz generalized this theorem further. For this we need a bit of notation.

Let $U \subset \mathbb{R}^n$. Suppose $\mathcal{A}(U)$ is any ring of real valued functions on U. Define $\mathcal{S}(\mathcal{A}(U))$ to be the smallest set of subsets of U, which contain the sets $\{x \in U \mid f(x) > 0\}$ for all $f \in \mathcal{A}(U)$, and is closed under finite union, finite intersection and complement. Let $\mathcal{A}(U)[t]$ denote the ring of polynomials in $t \in \mathbb{R}^m$ with coefficients in $\mathcal{A}(U)$.

Theorem (Tarski-Seidenberg-Łojasiewicz)**.** *Suppose that $V \subset U \times \mathbb{R}^m \subset \mathbb{R}^{n+m}$, is such that $V \in \mathcal{S}(\mathcal{A}(U)[t])$. Then the projection of V onto the first n variables is in $\mathcal{S}(\mathcal{A}(U))$.*

References

[1] Edward Bierstone and Pierre D. Milman, *Semianalytic and subanalytic sets*, Inst. Hautes Études Sci. Publ. Math. (1988), no. 67, 5–42. `http://www.ams.org/mathscinet-getitem?mr=89k:32011`MR 89k:32011