subsheaf of abelian groups

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Let $\mathcal{F}$ be a sheaf of abelian groups over a topological space X. Let $\mathcal{G}$ be a sheaf over X, such that for every open set $U \subset X$, $\mathcal{G}(U)$ is a subgroup of $\mathcal{F}(U)$. And further let the on $\mathcal{G}$ be by those on $\mathcal{F}$. Then $\mathcal{G}$ is a *subsheaf* of $\mathcal{F}$.

Suppose a sheaf of abelian groups $\mathcal{F}$ is defined as a disjoint union of stalks $\mathcal{F}_x$ over points $x \in X$, and $\mathcal{F}$ is topologized in the appropriate manner. In particular, each stalk is an abelian group and the group operations are continuous. Then a subsheaf $\mathcal{G}$ is an open subset of $\mathcal{F}$ such that $\mathcal{G}_x = \mathcal{G} \cap \mathcal{F}_x$ is a subgroup of $\mathcal{F}_x$.

When $\mathcal{G}$ is a subsheaf of $\mathcal{F}$, then $\mathcal{F}_x/\mathcal{G}_x$ is an abelian group. Considering this to be the stalk over x we have a sheaf which is denoted by $\mathcal{F}/\mathcal{G}$, with the topology being the quotient topology.

Example. *Suppose M is a complex manifold. Let $\mathcal{M}^*$ be the sheaf of germs of meromorphic functions which are not identically zero. That is, for $z \in M$, the stalk $\mathcal{M}^*_z$ is the abelian group of germs of meromorphic functions at z with the group operation being multiplication. Then $\mathcal{O}^*$, the sheaf of germs of holomorphic functions which are not identically 0 is a subsheaf of $\mathcal{M}^*$.*

The sheaf $\mathcal{M}^*/\mathcal{O}^*$ is then the sheaf of divisors. If M is of (complex) dimension 1, then $\mathcal{M}^*/\mathcal{O}^*$ is just the sheaf of functions into the integers with finite support.

References

[1] Glen E. Bredon. , Springer, 1997.

[2] Robin Hartshorne. , Springer, 1977.

[3] Lars Hörmander. , North-Holland Publishing Company, New York, New York, 1973.