intersection divisor for a quartic

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Entry type	Definition
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Defines	hyperflex
Defines	flex

Let C be a non-singular curve in the plane, defined over an algebraically closed field K, and given by a polynomial $f(x, y) = 0$ of degree 4 (i.e. C is a quartic). Let L be a (rational) line in the plane K^2. The intersection divisor of C and L is of the form:

$$(L \cdot C) = P_1 + P_2 + P_3 + P_4$$

where P_i, $i = 1, 2, 3, 4$, are points in $C(K)$. There are five possibilities:

1. The generic position: all the points P_i are distinct.

2. L is tangent to C: there exist indices $1 \leq i \neq j \leq 4$ such that $P_i = P_j$. Without loss of generality we may assume $P_1 = P_2$ and $(L \cdot C) = 2P_1 + P_3 + P_4$, and $P_3 \neq P_4$.

3. L is bitangent to C when $P_1 = P_2$ and $P_3 = P_4$ but $P_1 \neq P_3$. It may be shown that if $\mathrm{char}(K) \neq 2$ then C has exactly 28 bitangent lines.

4. L intersects C at exactly two points, thus $P_1 = P_2 = P_3 \neq P_4$. The point P_1 is called a *flex*.

5. L intersects C at exactly one point and $P_1 = P_2 = P_3 = P_4$. This point is called a *hyperflex*. A quartic C may not have any hyperflex.

References

[1] S. Flon, R. Oyono, C. Ritzenthaler, *Fast addition on non-hyperelliptic genus 3 curves*, can be found `http://eprint.iacr.org/2004/118.ps`here.