irreducible

Canonical name	Irreducible1
Date of creation	2013-03-22 12:03:30
Last modified on	2013-03-22 12:03:30
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	14
Author	mathcam (2727)
Entry type	Definition
Classification	msc 14A15
Classification	msc 14A10
Classification	msc 54B05
Related topic	IrreducibleComponent
Related topic	HyperconnectedSpace
Defines	reducible

A subset F of a topological space X is *reducible* if it can be written as a union $F = F_1 \cup F_2$ of two closed proper subsets F_1, F_2 of F (closed in the subspace topology). That is, F is reducible if it can be written as a union $F = (G_1 \cap F) \cup (G_2 \cap F)$ where G_1,G_2 are closed subsets of X, neither of which contains F.

A subset of a topological space is *irreducible* (or *hyperconnected*) if it is not reducible.

As an example, consider $\{(x, y) \in \mathbb{R}^2 : xy = 0\}$ with the subspace topology from $\mathbb{R}^2$. This space is a union of two lines $\{(x, y) \in \mathbb{R}^2 : x = 0\}$ and $\{(x, y) \in \mathbb{R}^2 : y = 0\}$, which are proper closed subsets. So this space is reducible, and thus not irreducible.