Segre map

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Synonym	Segre embedding

The *Segre map* is an embedding $s : \mathbb{P}^n \times \mathbb{P}^m \to \mathbb{P}^{nm+n+m}$ of the product of two projective spaces into a larger projective space. It is important since it makes the product of two projective varieties into a projective variety again. Invariantly, it can described as follows. Let V, W be (finite dimensional) vector spaces; then

$$\begin{array}{cccccc} s: & \mathbb{P}V & \times & \mathbb{P}W & \longrightarrow & \mathbb{P}(V \otimes W) \\ & [x] & , & [y] & \longmapsto & [x \otimes y] \end{array}$$

In homogeneous coordinates, the pair of points $[x_0 : x_1 : \cdots : x_n]$, $[y_0 : y_1 : \cdots : y_m]$ maps to

$$[x_0 y_0 : x_1 y_0 : \cdots : x_n y_0 : x_0 y_1 : x_1 y_1 : \cdots : x_n y_m].$$

If we imagine the target space as the projectivized version of the space of $(n+1) \times (m+1)$ matrices, then the image is exactly the set of matrices which have rank 1; thus it is the common zero locus of the equations

$$\begin{vmatrix} a_{ij} & a_{il} \\ a_{kj} & a_{kl} \end{vmatrix} = a_{ij} a_{kl} - a_{il} a_{kj} = 0$$

for all $0 \leq i < k \leq n$, $0 \leq j < l \leq m$. Varieties of this form (defined by vanishing of minors in some space of matrices) are usually called *determinantal varieties.*