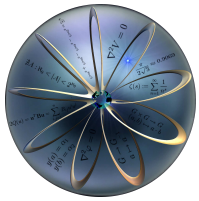

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$V(I) = \emptyset$ **implies** $I = R$

Canonical name	VIemptysetImpliesIR
Date of creation	2013-03-22 16:07:43
Last modified on	2013-03-22 16:07:43
Owner	Wkbj79 (1863)
Last modified by	Wkbj79 (1863)
Numerical id	10
Author	Wkbj79 (1863)
Entry type	Theorem
Classification	msc 14A15
Related topic	ProofThatOperatornameSpecRIsQuasiCompact

Note that most of the notation used here is defined in the entry prime spectrum.

Theorem. *If R is a commutative ring with identity and I is an ideal of R with $V(I) = \emptyset$, then $I = R$.*

Proof. Let R be a commutative ring with identity and I be an ideal of R with $I \neq R$. Then, by `http://planetmath.org/EveryRingHasAMaximalIdeal`this theorem, there exists a maximal ideal M of R containing I. Since M is , then M is a proper prime ideal of R. Thus, $M \in V(I)$. The theorem follows. $\square$