ideal

Canonical name	Ideal
Date of creation	2013-03-22 11:49:27
Last modified on	2013-03-22 11:49:27
Owner	djao (24)
Last modified by	djao (24)
Numerical id	18
Author	djao (24)
Entry type	Definition
Classification	msc 14K99
Classification	msc 16D25
Classification	msc 11N80
Classification	msc 13A15
Classification	msc 54C05
Classification	msc 54C08
Classification	msc 54J05
Classification	msc 54D99
Classification	msc 54E05
Classification	msc 54E15
Classification	msc 54E17
Related topic	Subring
Related topic	PrimeIdeal
Defines	left ideal
Defines	right ideal
Defines	2-sided ideal
Defines	two-sided ideal

Let R be a ring. A *left ideal* (resp., *right ideal*) I of R is a nonempty subset $I \subset R$ such that:

- $a - b \in I$ for all $a, b \in I$
- $r \cdot a \in I$ (resp. $a \cdot r \in I$) for all $a \in I$ and $r \in R$

A *two-sided ideal* is a left ideal I which is also a right ideal. If R is a commutative ring, then these three notions of ideal are equivalent. Usually, the word "ideal" by itself means two-sided ideal.

The name "ideal" comes from the study of number theory. When the failure of unique factorization in number fields was first noticed, one of the solutions was to work with so-called "ideal numbers" in which unique factorization did hold. These "ideal numbers" were in fact ideals, and in Dedekind domains, unique factorization of ideals does indeed hold. The term "ideal number" is no longer used; the term "ideal" has replaced and generalized it.