Hasse principle

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Defines	Hasse condition
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Let V be an algebraic variety defined over a field K. By $V(K)$ we denote the set of points on V defined over K. Let $\bar{K}$ be an algebraic closure of K. For a valuation ν of K, we write K_ν for the completion of K at ν. In this case, we can also consider V defined over K_ν and talk about $V(K_\nu)$.

Definition 1.

1. *If $V(K)$ is not empty we say that V is* soluble *in K.*

2. *If $V(K_\nu)$ is not empty then we say that V is* locally soluble *at ν.*

3. *If V is locally soluble for all ν then we say that V satisfies the* Hasse condition*, or we say that V/K is* everywhere locally soluble.

The *Hasse Principle* is the idea (or desire) that an everywhere locally soluble variety V must have a rational point, i.e. a point defined over K. Unfortunately this is not true, there are examples of varieties that satisfy the Hasse condition but have no rational points.

Example: A quadric (of any dimension) satisfies the Hasse condition. This was proved by Minkowski for quadrics over $\mathbb{Q}$ and by Hasse for quadrics over a number field.

References

[1] Swinnerton-Dyer, *Diophantine Equations: Progress and Problems*, `http://swc.math.arizona.edu/notes/files/DLSSw-Dyer1.pdf`online notes.