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normal complex analytic variety

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Defines	normal analytic space
Defines	normal analytic subvariety

Let V be a local complex analytic variety (or a complex analytic space). A point $p \in V$ is normal if and only if every weakly holomorphic function through V extends to be holomorphic in V near p .

In particular, if $V \subset \mathbb{C}^n$ is a complex analytic subvariety, it is normal at p if and only if every weakly holomorphic function through V extends to be holomorphic in a neighbourhood of p in \mathbb{C}^n .

To see that this definition is equivalent to the usual one, that is, that V is normal at p if and only if \mathcal{O}_p (the ring of germs of holomorphic functions at p) is integrally closed, we need the following theorem. Let \mathcal{M}_p be the total quotient ring of \mathcal{O}_p , that is, the ring of germs of meromorphic functions.

Theorem. *Let V be a local complex analytic variety. Then $\mathcal{O}_p^w(V)$ is the integral closure of $\mathcal{O}_p(V)$ in \mathcal{M}_p .*

References

- [1] Hassler Whitney. . Addison-Wesley, Philippines, 1972.