elliptic surface

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Definition 1. *Let k be a field and let C/k be a smooth projective curve defined over the field k and has genus g. The function field of C/k will be denoted by $K = k(C)$. An elliptic surface $\mathcal{E}$ over the curve C is, by definition, a two-dimensional projective variety together with:*

1. *A morphism $\pi : \mathcal{E} \to C$ such that for all but finitely many points $t \in C(\bar{k})$, the fiber $\mathcal{E}_t = \pi^{-1}(t)$ is a non-singular curve of genus 1,*

2. *A section to π (the zero section) $\sigma_0 : C \to \mathcal{E}$.*

With this definition, $\mathcal{E}/K$ may be regarded as an elliptic curve over the field K.

Example 1. The surface $y^2 = x^3 + t$ is an elliptic surface over the curve $\mathbb{P}^1(\mathbb{Q})$. It may be regarded as an elliptic curve over the function field $\mathbb{Q}(t)$.

References

[1] R. Miranda, *The basic theory of elliptic surfaces*, Dottorato di Ricerca in Matematica, Dipartimento di Mathematica dell' Universit di Pisa, ETS Editrice Pisa, 1989.

[2] J. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves*, Graduate Texts in Mathematics 151, Springer-Verlag, New York.