Zariski topology

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Let $\mathbb{A}_k^n$ denote the affine space k^n over a field k. The *Zariski topology* on $\mathbb{A}_k^n$ is defined to be the topology whose closed sets are the sets

$$V(I) := \{x \in \mathbb{A}_k^n \mid f(x) = 0 \text{ for all } f \in I\} \subset \mathbb{A}_k^n,$$

where $I \subset k[X_1, \ldots, X_n]$ is any ideal in the polynomial ring $k[X_1, \ldots, X_n]$. For any affine variety $V \subset \mathbb{A}_k^n$, the *Zariski topology* on V is defined to be the subspace topology induced on V as a subset of $\mathbb{A}_k^n$.

Let $\mathbb{P}_k^n$ denote n–dimensional projective space over k. The *Zariski topology* on $\mathbb{P}_k^n$ is defined to be the topology whose closed sets are the sets

$$V(I) := \{x \in \mathbb{P}_k^n \mid f(x) = 0 \text{ for all } f \in I\} \subset \mathbb{P}_k^n,$$

where $I \subset k[X_0, \ldots, X_n]$ is any homogeneous ideal in the graded k–algebra $k[X_0, \ldots, X_n]$. For any projective variety $V \subset \mathbb{P}_k^n$, the *Zariski topology* on V is defined to be the subspace topology induced on V as a subset of $\mathbb{P}_k^n$.

The Zariski topology is the predominant topology used in the study of algebraic geometry. Every regular morphism of varieties is continuous in the Zariski topology (but not every continuous map in the Zariski topology is a regular morphism). In fact, the Zariski topology is the weakest topology on varieties making points in $\mathbb{A}_k^1$ closed and regular morphisms continuous.