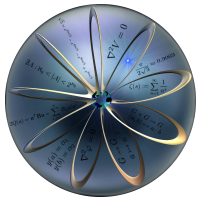

planetmath.org

Math for the people, by the people.

flat morphism

Canonical name	FlatMorphism
Date of creation	2013-03-22 14:11:10
Last modified on	2013-03-22 14:11:10
Owner	archibal (4430)
Last modified by	archibal (4430)
Numerical id	4
Author	archibal (4430)
Entry type	Definition
Classification	msc 14A15
Synonym	flat
Related topic	Scheme
Related topic	EtaleMorphism
Defines	flat sheaf

Let $f\colon X \to Y$ be a morphism of schemes. Then a sheaf $\mathcal{F}$ of $\mathcal{O}_X$-modules is *flat over* Y *at a point* $x \in X$ if $\mathcal{F}_x$ is a `http://planetmath.org/FlatModule`flat $\mathcal{O}_{Y,f(x)}$-module by way of the map $f^\sharp\colon \mathcal{O}_Y \to \mathcal{O}_X$ associated to f.

The morphism f itself is said to be *flat* if $\mathcal{O}_X$ is flat over Y at every point of X.

This is the natural condition for X to form a "continuous family" over Y. That is, for each $y \in Y$, the fiber X_y of f over y is a scheme. We can consider X as a family of schemes parameterized by Y. If the morphism f is flat, then this family should be thought of as a "continuous family". In particular, this means that certain cohomological invariants remain constant on the fibers of X.

References

[1] Robin Hartshorne, *Algebraic Geometry*, Springer–Verlag, 1977 (GTM **52**).