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Mordell-Weil theorem

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Let  $K$  be a number field and let  $E$  be an elliptic curve over  $K$ . By  $E(K)$  we denote the set of points in  $E$  with coordinates in  $K$ .

**Theorem 1** (Mordell-Weil).  $E(K)$  is a finitely generated abelian group.

*Proof.* The proof of this theorem is fairly involved. The main two ingredients are the so called <http://planetmath.org/WeakMordellWeilTheorem> weak Mordell-Weil theorem, the concept of height function for abelian groups and the “<http://planetmath.org/HeightFunctiondescent>” theorem.

See [?], Chapter VIII, page 189. □

## References

- [1] James Milne, *Elliptic Curves*, online course notes.  
<http://www.jmilne.org/math/CourseNotes/math679.html><http://www.jmilne.org/math/CourseNotes/math679.html>
- [2] Joseph H. Silverman, *The Arithmetic of Elliptic Curves*. Springer-Verlag, New York, 1986.
- [3] Joseph H. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves*. Springer-Verlag, New York, 1994.
- [4] Goro Shimura, *Introduction to the Arithmetic Theory of Automorphic Functions*. Princeton University Press, Princeton, New Jersey, 1971.