flag variety

Canonical name	FlagVariety
Date of creation	2013-03-22 13:27:56
Last modified on	2013-03-22 13:27:56
Owner	bwebste (988)
Last modified by	bwebste (988)
Numerical id	6
Author	bwebste (988)
Entry type	Definition
Classification	msc 14M15
Synonym	flag manifold
Defines	complete flag variety
Defines	partial flag variety

Let k be a field, and let V be a vector space over k of dimension n and choose an increasing sequence $\mathbf{i} = (i_1, \ldots, i_m)$, with $1 \leq i_1 < \cdots < i_m \leq n$. Then the *(partial) flag variety* $\mathcal{F}\ell(V, \mathbf{i})$ associated to this data is the set of all flags $\{0\} \leq V_1 \subset \cdots \subset V_n$ with $\dim V_j = i_j$. This has a natural embedding into the product of Grassmannians $G(V, i_1) \times \cdots G(V, i_m)$, and its image here is closed, making $\mathcal{F}\ell(V, \mathbf{i})$ into a projective variety over k. If $k = \mathbb{C}$ these are often called *flag manifolds*.

The group $\mathrm{Sl}(V)$ acts transtively on $\mathcal{F}\ell(V, \mathbf{i})$, and the stabilizer of a point is a parabolic subgroup. Thus, as a homogeneous space, $\mathcal{F}\ell(V, \mathbf{i}) \cong \mathrm{Sl}(V)/P$ where P is a parabolic subgroup of $\mathrm{Sl}(V)$. In particular, the complete flag variety is isomorphic to $\mathrm{Sl}(V)/B$, where B is the Borel subgroup.