real analytic subvariety

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Synonym	real analytic variety
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Related topic	SmoothSubmanifoldContainedInASubvarietyOfSameDimensionIsRealAnalytic
Defines	real algebraic variety
Defines	real algebraic subvariety
Defines	local real analytic subvariety
Defines	regular point
Defines	singular point

Let $U \subset \mathbb{R}^N$ be an open set.

Definition. A closed set $X \subset U$ is called a *real analytic subvariety* of U such that for each point $p \in X$, there exists a neigbourhood V and a set $\mathcal{F}$ of real analytic functions defined in V, such that

$$X \cap V = \{p \in V \mid f(p) = 0 \text{ for all } f \in \mathcal{F}\}.$$

If $U = \mathbb{R}^N$ and all the $f \in \mathcal{F}$ are real polynomials, then X is said to be a *real algebraic subvariety*.

If X is not required to be closed, then it is said to be a *local real analytic subvariety*. Sometimes X is called a real analytic set or real analytic variety. Similarly as for complex analytic sets we can also define the regular and singular points.

Definition. A point $p \in X$ is called a *regular point* if there is a neighbourhood V of p such that $X \cap V$ is a submanifold. Any other point is called a *singular point*.

The set of regular points of X is denoted by X^- or sometimes X^*. The set of singular points is no longer a subvariety as in the complex case, though it can be sown to be semianalytic. In general, real subvarieties is far worse behaved than their complex counterparts.

References

[1] Jacek Bochnak, Michel Coste, Marie-Francoise Roy. . Springer, 1998.