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normal irreducible varieties are nonsingular
in codimension 1

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Theorem 1. *Let X be a normal irreducible variety. The singular set $S \subset X$ has codimension 2 or more.*

Proof. Assume not. We may assume X is affine, since codimension is local. Now let \mathfrak{u} be the ideal of functions vanishing on S . This is an ideal of height 1, so the local ring of Y , $\mathcal{O}_S = A(X)_{\mathfrak{u}}$, where $A(X)$ is the affine ring of X , is a 1-dimensional local ring, and integrally closed, since X is normal. Any integrally closed 1-dimensional local domain is a DVR, and thus regular. But S is the singular set, so its local ring is not regular, a contradiction. \square