representations of a bound quiver

Canonical name	RepresentationsOfABoundQuiver
Date of creation	2013-03-22 19:16:51
Last modified on	2013-03-22 19:16:51
Owner	joking (16130)
Last modified by	joking (16130)
Numerical id	6
Author	joking (16130)
Entry type	Definition
Classification	msc 14L24

Let (Q, I) be a http://planetmath.org/AdmissibleIdealsBoundQuiverAndItsAlgebrabound quiver over a field k.

Let $\mathbb{V}$ be a representation of Q over k composed by $\{f(q)\}_{q\in Q_1}$ a family of linear maps. If

$$w = (\alpha_1, \ldots, \alpha_n)$$

is a path in Q, then we have the evaluation map

$$f_w = f(\alpha_n) \circ f(\alpha_{n-1}) \circ \cdots \circ f(\alpha_2) \circ f(\alpha_1).$$

For stationary paths we define $f_{e_x} : V_x \to V_x$ by $f_{e_x} = 0$. Also, note that if ρ is a http://planetmath.org/RelationsInQuiverrelation in Q, then

$$\rho = \sum_{i=1}^{m} \lambda_i \cdot w_i$$

where all w_i's have the same source and target. Thus it makes sense to talk about evaluation in ρ, i.e.

$$f_\rho = \sum_{i=1}^{n} \lambda_i \cdot f_{w_i}.$$

In particular

$$f_\rho : V_{s(w_i)} \to V_{t(w_i)}$$

is a linear map.

Recall that the ideal I is generated by relations (see http://planetmath.org/PropertiesOfAdm entry) $\{\rho_1, \ldots, \rho_n\}$.

Definition. A representation $\mathbb{V}$ of Q over k with linear mappings $\{f(q)\}_{q\in Q_1}$ is said to be **bound by** I if

$$f_{\rho_i} = 0$$

for every $i = 1, \ldots, n$.

It can be easily checked, that this definition does not depend on the choice of (relation) generators of I.

The full subcategory of the category of all representations which is composed of all representations bound by I is denoted by $\mathrm{REP}(Q, I)$. It can be easily seen, that it is abelian.