conductor of an elliptic curve

Canonical name	ConductorOfAnEllipticCurve
Date of creation	2013-03-22 13:49:51
Last modified on	2013-03-22 13:49:51
Owner	alozano (2414)
Last modified by	alozano (2414)
Numerical id	9
Author	alozano (2414)
Entry type	Definition
Classification	msc 14H52
Synonym	conductor
Related topic	EllipticCurve
Related topic	LSeriesOfAnEllipticCurve
Related topic	ArithmeticOfEllipticCurves
Defines	conductor of an elliptic curve

Let E be an elliptic curve over $\mathbb{Q}$. For each prime $p \in \mathbb{Z}$ define the quantity f_p as follows:

$$f_p = \begin{cases} 0, \text{ if } E \text{ has good reduction at } p, \\ 1, \text{ if } E \text{ has multiplicative reduction at } p, \\ 2, \text{ if } E \text{ has additive reduction at } p, \text{ and } p \neq 2, 3, \\ 2 + \delta_p, \text{ if } E \text{ has additive reduction at } p = 2 \textit{ or } 3. \end{cases}$$

where δ_p depends on wild ramification in the action of the inertia group at p of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ on the Tate module $T_p(E)$.

Definition. *The conductor* $N_{E/\mathbb{Q}}$ *of* $E/\mathbb{Q}$ *is defined to be:*

$$N_{E/\mathbb{Q}} = \prod_p p^{f_p}$$

where the product is over all primes and the exponent f_p *is defined as above.*

Example. Let $E/\mathbb{Q}\colon y^2 + y = x^3 - x^2 + 2x - 2$. The primes of bad reduction for E are $p = 5$ and 7. The reduction at $p = 5$ is additive, while the reduction at $p = 7$ is multiplicative. Hence $N_{E/\mathbb{Q}} = 25 \cdot 7 = 175$.

References

[1] James Milne, *Elliptic Curves*, `http://www.jmilne.org/math/CourseNotes/math679.html`onli course notes.

[2] Joseph H. Silverman, *The Arithmetic of Elliptic Curves*. Springer-Verlag, New York, 1986.

[3] Joseph H. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves*. Springer-Verlag, New York, 1994.