quiver representations and representation morphisms

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Let $Q = (Q_0, Q_1, s, t)$ be a quiver, i.e. Q_0 is a set of vertices, Q_1 is a set of arrows and $s, t : Q_1 \to Q_0$ are functions such that s maps each arrow to its source and t maps each arrow to its target.

A **representation** $\mathbb{V}$ of Q over a field k is a family of vector spaces $\{V_i\}_{i \in Q_0}$ over k together with a family of k-linear maps $\{f_a : V_{s(a)} \to V_{t(a)}\}_{a \in Q_1}$.

A **morphism** $F : \mathbb{V} \to \mathbb{W}$ between representations $\mathbb{V} = (V_i, g_a)$ and $\mathbb{W} = (W_i, h_a)$ is a family of k-linear maps $\{F_i : V_i \to W_i\}_{i \in Q_0}$ such that for each arrow $a \in Q_1$ the following relation holds:

$$F_{t(a)} \circ g_a = h_a \circ F_{s(a)}.$$

Obviously we can compose morphisms of representations and in this the case class of all representations and representation morphisms together with the standard composition is a category. This category is abelian.

It can be shown that for each finite quiver Q (i.e. with Q_0 finite) and field k there exists an algebra A over k such that the category of representations of Q is equivalent to the category of modules over A.

A representation $\mathbb{V}$ of Q is called **trivial** iff $V_i = 0$ for each vertex $i \in Q_0$.

A representation $\mathbb{V}$ of Q is called **locally finite-dimensional** iff $\dim_k V_i < \infty$ for each vertex $i \in Q_0$ and **finite-dimensional** iff $\mathbb{V}$ is locally finite-dimensional and $V_i = 0$ for almost all vertices $i \in Q_0$.