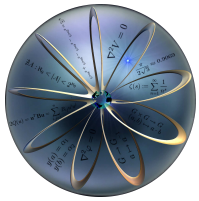

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analytic algebraic function

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Synonym	k-analytic algebraic function
Synonym	analytic algebraic
Defines	holomorphic algebraic function
Defines	real-analytic algebraic function
Defines	Nash function
Defines	analytic algebraic mapping

Let k be a field, and let $k\{x_1, \ldots, x_n\}$ be the ring of convergent power series in n variables. An element in this ring can be thought of as a function defined in a neighbourhood of the origin in k^n to k. The most common cases for k are $\mathbb{C}$ or $\mathbb{R}$, where the convergence is with respect to the standard euclidean metric. These definitions can also be generalized to other fields.

Definition. A function $f \in k\{x_1, \ldots, x_n\}$ is said to be *k-analytic algebraic* if there exists a nontrivial polynomial $p \in k[x_1, \ldots, x_n, y]$ such that $p(x, f(x)) \equiv 0$ for all x in a neighbourhood of the origin in k^n. If $k = \mathbb{C}$ then f is said to be *holomorphic algebraic* and if $k = \mathbb{R}$ then f is said to be *real-analytic algebraic* or a *Nash function*.

The same definition applies near any other point other then the origin by just translation.

Definition. A mapping $f\colon U \subset k^n \to k^m$ where U is a neighbourhood of the origin is said to be k-analytic algebraic if each component function is analytic algebraic.

References

[1] M. Salah Baouendi, Peter Ebenfelt, Linda Preiss Rothschild. , Princeton University Press, Princeton, New Jersey, 1999.