



**planetmath.org**

Math for the people, by the people.

## Mordell curve

Canonical name	MordellCurve
Date of creation	2013-03-22 13:49:57
Last modified on	2013-03-22 13:49:57
Owner	alozano (2414)
Last modified by	alozano (2414)
Numerical id	5
Author	alozano (2414)
Entry type	Definition
Classification	msc 14H52
Related topic	EllipticCurve
Related topic	BirchAndSwinnertonDyerConjecture
Related topic	ArithmeticOfEllipticCurves
Defines	Mordell curve

A *Mordell curve* is an elliptic curve  $E/K$ , for some field  $K$ , which admits a model by a Weierstrass equation of the form:

$$y^2 = x^3 + k, \quad k \in K$$

**Examples:**

1. Let  $E_1/\mathbb{Q}$ :  $y^2 = x^3 + 2$ , this is a Mordell curve with Mordell-Weil group  $E_1(\mathbb{Q}) \simeq \mathbb{Z}$  and generated by  $(-1, 1)$ .
2. Let  $E_2/\mathbb{Q}$ :  $y^2 = x^3 + 109858299531561$ , then  $E_2(\mathbb{Q}) \simeq \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}^5$ . See <http://math.bu.edu/people/alozano/Torsion.html> generators here.
3. In general, a Mordell curve of the form  $y^2 = x^3 + n^2$  has torsion group isomorphic to  $\mathbb{Z}/3\mathbb{Z}$  generated by  $(0, n)$ .
4. Let  $E_3/\mathbb{Q}$ :  $y^2 = x^3 + 496837487681$  then this is a Mordell curve with  $E_3(\mathbb{Q}) \simeq \mathbb{Z}^8$ . See <http://math.bu.edu/people/alozano/Mordell.html> generators here.
5. <http://www.maths.nott.ac.uk/personal/pmxtow/mordellc.htm> Here you can find a list of the minimal-known positive and negative  $k$  for Mordell curves of given rank, and the Mordell curves with maximum rank known (see BS-D conjecture).