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sheafification

Canonical name	Sheafification1
Date of creation	2013-03-22 14:13:08
Last modified on	2013-03-22 14:13:08
Owner	archibal (4430)
Last modified by	archibal (4430)
Numerical id	4
Author	archibal (4430)
Entry type	Theorem
Classification	msc 14F20
Classification	msc 18F10
Classification	msc 18F20
Related topic	Sheafification
Related topic	Site
Related topic	Sheaf2
Related topic	Sheaf
Defines	sheafification

Let T be a site. Let P_T denote the category of presheaves on T (with values in the category of abelian groups), and S_T the category of sheaves on T . There is a natural inclusion functor $\iota: S_T \rightarrow P_T$.

Theorem 1 *The functor ι has a left adjoint $\sharp: P_T \rightarrow S_T$, that is, for any sheaf F and presheaf G , we have*

$$\mathrm{Hom}_{S_T}(G^\sharp, F) \cong \mathrm{Hom}_{P_T}(G, \iota F).$$

This functor \sharp is called sheafification, and G^\sharp is called the sheafification of G .

One can readily check that this description in terms of adjoints characterizes \sharp completely, and that this definition reduces to the usual definition of <http://planetmath.org/Sheafificationsheafification> when T is the Zariski site. It also allows derivation of various exactness properties of \sharp and ι .

References

- [1] Grothendieck et al., *Séminaires en Géométrie Algébrique* 4, tomes 1, 2, and 3, available on the web at <http://www.math.mcgill.ca/archibal/SGA/SGA.html><http://www.math.mcgill.ca/archibal/SGA/SGA.html>