semialgebraic set

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Defines	semialgebraic
Defines	dimension of a semialgebraic set

Definition. Consider the $A \subset \mathbb{R}^n$, defined by real polynomials $p_{j\ell}$, $j = 1, \ldots, k$, $\ell = 1, \ldots, m$, and the relations $\epsilon_{j\ell}$ where $\epsilon_{j\ell}$ is $>$, $=$, or $<$.

$$A = \bigcup_{\ell=1}^{m} \{x \in \mathbb{R}^n \mid p_{j\ell}(x)\ \epsilon_{j\ell}\ 0,\ j = 1, \ldots, k\}. \tag{1}$$

Sets of this form are said to be *semialgebraic*.

Similarly as algebraic subvarieties, finite union and intersection of semialgebraic sets is still a semialgebraic set. Furthermore, unlike subvarieties, the complement of a semialgebraic set is again semialgebraic. Finally, and most importantly, the Tarski-Seidenberg theorem says that they are also closed under projection.

On a dense open subset of A, A is (locally) a submanifold, and hence we can easily define the *dimension* of A to be the largest dimension at points at which A is a submanifold. It is not hard to see that a semialgebraic set lies inside an algebraic subvariety of the same dimension.

References

[1] Edward Bierstone and Pierre D. Milman, *Semianalytic and subanalytic sets*, Inst. Hautes Études Sci. Publ. Math. (1988), no. 67, 5–42. `http://www.ams.org/mathscinet-getitem?mr=89k:32011`MR 89k:32011