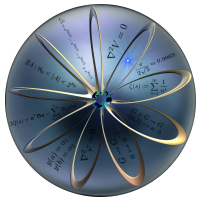

planetmath.org

Math for the people, by the people.

Picard group

Canonical name	PicardGroup
Date of creation	2013-03-22 12:52:30
Last modified on	2013-03-22 12:52:30
Owner	alozano (2414)
Last modified by	alozano (2414)
Numerical id	6
Author	alozano (2414)
Entry type	Definition
Classification	msc 14-00
Synonym	divisor class group

The *Picard group* of a variety, scheme, or more generally locally ringed space (X, O_X) is the group of locally free O_X modules of rank 1 with tensor product over O_X as the operation, usually denoted by $\mathrm{Pic}(X)$. Alternatively, the Picard group is the group of isomorphism classes of invertible sheaves on X, under tensor products.

It is not difficult to see that $\mathrm{Pic}(X)$ is isomorphic to $\mathrm{H}^1(X, O_X^*)$, the first sheaf cohomology group of the multiplicative sheaf O_X^* which consists of the units of O_X.

Finally, let $\mathrm{CaCl}(X)$ be the group of Cartier divisors on X modulo linear equivalence. If X is an integral scheme then the groups $\mathrm{CaCl}(X)$ and $\mathrm{Pic}(X)$ are isomorphic. Furthermote, if we let $\mathrm{Cl}(X)$ be the class group of Weil divisors (divisors modulo principal divisors) and X is a noetherian, integral and separated locally factorial scheme, then there is a natural isomorphism $\mathrm{Cl}(X) \cong \mathrm{Pic}(X)$. Thus, the Picard group is sometimes called the *divisor class group* of X.