Kodaira-Itaka dimension

Canonical name	KodairaItakaDimension
Date of creation	2013-03-22 16:12:43
Last modified on	2013-03-22 16:12:43
Owner	yark (2760)
Last modified by	yark (2760)
Numerical id	17
Author	yark (2760)
Entry type	Definition
Classification	msc 14E05
Defines	Kodaira dimension
Defines	bigness
Defines	general type

Given a projective algebraic variety X and a line bundle $L \to X$, the *Kodaira-Itaka dimension* of L is defined to be the supremum of the dimensions of the image of X by the map $\varphi_{|mL|}$ associated to the linear system $|mL|$, when m is a positive integer, namely

$$\kappa(L) = \sup_{m \in \mathbb{N}} \{\dim \varphi_{|mL|}(X)\}.$$

It is a standard fact that if we consider the graded ring

$$R(X, L) = \bigoplus_{m \in \mathbb{N}} H^0(X, mL),$$

then $\mathrm{tr.deg} R(X, L) = \kappa(L) + 1$.

When the line bundle we have is the canonical bundle K_X of X, then its Kodaira-Itaka dimension is called *Kodaira dimension* of X.

In paticular, if for some m we have $\dim \varphi_{|mL|}(X) = \dim X$ then $\kappa(L) = \dim X$ and L is called *big*.

If $\kappa(X) = \kappa(K_X) = \dim X$, then X is said to be of *general type*.