structure sheaf

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Let X be an irreducible algebraic variety over a field k, together with the Zariski topology. Fix a point $x \in X$ and let $U \subset X$ be any affine open subset of X containing x. Define

$$\mathfrak{o}_x := \{f/g \in k(U) \mid f, g \in k[U],\ g(x) \neq 0\},$$

where $k[U]$ is the coordinate ring of U and $k(U)$ is the fraction field of $k[U]$. The ring $\mathfrak{o}_x$ is independent of the choice of affine open neighborhood U of x.

The *structure sheaf* on the variety X is the sheaf of rings whose sections on any open subset $U \subset X$ are given by

$$\mathcal{O}_X(U) := \bigcap_{x \in U} \mathfrak{o}_x,$$

and where the restriction map for $V \subset U$ is the inclusion map $\mathcal{O}_X(U) \hookrightarrow \mathcal{O}_X(V)$.

There is an equivalence of categories under which an affine variety X with its structure sheaf corresponds to the prime spectrum of the coordinate ring $k[X]$. In fact, the topological embedding $X \hookrightarrow \mathrm{Spec}(k[X])$ gives rise to a lattice–preserving bijection[1] between the open sets of X and of $\mathrm{Spec}(k[X])$, and the sections of the structure sheaf on X are isomorphic to the sections of the sheaf $\mathrm{Spec}(k[X])$.

[1]Those who are fans of topos theory will recognize this map as an isomorphism of topos.