subquiver and image of a quiver

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Let $Q = (Q_0, Q_1, s, t)$ be a quiver.

Definition. A quiver $Q' = (Q'_0, Q'_1, s', t')$ is said to be a **subquiver** of Q, if

$$Q'_0 \subseteq Q_0, \quad Q'_1 \subseteq Q_1$$

are such that if $\alpha \in Q'_1$, then $s(\alpha), t(\alpha) \in Q'_0$. Furthermore

$$s'(\alpha) = s(\alpha), \quad t'(\alpha) = t(\alpha).$$

In this case we write $Q' \subseteq Q$.

A subquiver $Q' \subseteq Q$ is called **full** if for any $x, y \in Q'_0$ and any $\alpha \in Q_1$ such that $s(\alpha) = x$ and $t(\alpha) = y$ we have that $\alpha \in Q'_1$. In other words a subquiver is full if it ,,inherits" all arrows between points.

If Q' is a subquiver of Q, then the mapping

$$i = (i_0, i_1)$$

where both i_0, i_1 are inclusions is a morphism of quivers. In this case i is called **the inclusion morphism**.

If $F : Q \to Q'$ is any morphism of quivers $Q = (Q_0, Q_1, s, t)$ and $Q' = (Q'_0, Q'_1, s', t')$, then the quadruple

$$\mathrm{Im}(F) = (\mathrm{Im}(F_0), \mathrm{Im}(F_1), s'', t'')$$

where s'', t'' are the restrictions of s', t' to $\mathrm{Im}(F_1)$ is called **the image of** F. It can be easily shown, that $\mathrm{Im}(F)$ is a subquiver of Q'.