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path algebra of a quiver

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Let $Q = (Q_0, Q_1, s, t)$ be a quiver, i.e. Q_0 is a set of vertices, Q_1 is a set of arrows, $s : Q_1 \to Q_0$ is a source function and $t : Q_1 \to Q_0$ is a target function.

Recall that a **path** of length $l \geqslant 1$ from x to y in Q is a sequence of arrows $(a_1, \dots, a_l)$ such that

$$s(a_1) = x; \quad t(a_l) = y;$$

$$t(a_i) = s(a_{i+1})$$

for any $i = 1, 2, \dots, l-1, l$.

Also we allow paths of length 0, i.e. stationary paths.

If $a = (a_1, \dots, a_l)$ and $b = (b_1, \dots, b_k)$ are two paths such that $t(a_l) = s(b_1)$ then we say that a and b are **compatibile** and in this case we can form another path from a and b, namely

$$a \circ b = (a_1, \dots, a_l, b_1, \dots, b_k).$$

Note, that the length of $a \circ b$ is a sum of lengths of a and b. Also a path $a = (a_1, \dots, a_l)$ of positive length is called a **cycle** if $t(a_l) = s(a_1)$. In this case we can compose a with itself to produce new path.

Also if a is a path from x to y and e_x, e_y are stationary paths in x and y respectively, then we define $a \circ e_y = a$ and $e_x \circ a = a$.

Let kQ be a vector space with a basis consisting of all paths (including stationary paths). For paths a and b define multiplication as follows:

If a and b are compatible, then put $ab = a \circ b$ and put $ab = 0$ otherwise. This operation extendes bilinearly to entire kQ and it can be easily checked that kQ becomes an associative algebra in this manner called the **path algebra** of Q over k.