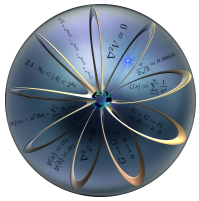

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abelian variety

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Definition 1. *An* abelian variety *over a field k is a proper group scheme over* Spec k *that is a variety.*

This extremely terse definition needs some further explanation.

Proposition 1. *The group law on an abelian variety is commutative.*

This implies that for every ring R, the R-points of an abelian variety form an abelian group.

Proposition 2. *An abelian variety is projective.*

If C is a curve, then the Jacobian of C is an abelian variety. This example motivated the development of the theory of abelian varieties, and many properties of curves are best understood by looking at the Jacobian.

If E is an elliptic curve, then E is an abelian variety (and in fact E is naturally isomorphic to its Jacobian).

See Mumford's excellent book *Abelian Varieties*. The bibliography for algebraic geometry has details and other books.