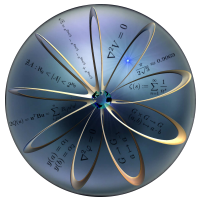

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projective variety

Canonical name	ProjectiveVariety
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Defines	quasi-projective variety

Given a homogeneous polynomial F of degree d in $n+1$ variables $X_0, \ldots, X_n$ and a point $[x_0 : \cdots : x_n]$, we cannot evaluate F at that point, because it has multiple such representations, but since $F(\lambda x_0, \ldots, \lambda x_n) = \lambda^d F(x_0, \ldots, x_n)$ we can say whether any such representation (and hence all) vanish at that point.

A *projective variety* over an algebraically closed field k is a subset of some projective space $\mathbb{P}^n_k$ over k which can be described as the common vanishing locus of finitely many homogeneous polynomials with coefficients in k, and which is not the union of two such smaller loci. Also, a *quasi-projective variety* is an open subset of a projective variety.