Faltings' theorem

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Let K be a number field and let C/K be a non-singular curve defined over K and genus g. When the genus is 0, the curve is isomorphic to $\mathbb{P}^1$ (over an algebraic closure $\overline{K}$) and therefore $C(K)$ is either empty or equal to $\mathbb{P}^1(K)$ (in particular $C(K)$ is infinite). If the genus of C is 1 and $C(K)$ contains at least one point over K then C/K is an elliptic curve and the Mordell-Weil theorem shows that $C(K)$ is a finitely generated abelian group (in particular, $C(K)$ may be finite or infinite). However, if $g \geq 2$, Mordell conjectured in 1922 that $C(K)$ cannot be infinite. This was first proven by Faltings in 1983.

Theorem (Faltings' Theorem (Mordell's conjecture))**.** *Let K be a number field and let C/K be a non-singular curve defined over K of genus $g \geq 2$. Then $C(K)$ is finite.*

The reader may also be interested in Siegel's theorem.