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genus

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Entry type Definition Classification msc 14H99 "Genus" has number of distinct but compatible definitions.

In topology, if S is an orientable surface, its genus g(S) is the number of "handles" it has. More precisely, from the classification of surfaces, we know that any orientable surface is a sphere, or the connected sum of n tori. We say the sphere has genus 0, and that the connected sum of n tori has genus n (alternatively, genus is additive with respect to connected sum, and the genus of a torus is 1). Also, $g(S) = 1 - \chi(S)/2$ where $\chi(S)$ is the Euler characteristic of S.

In algebraic geometry, the genus of a smooth projective curve X over a field k is the dimension over k of the vector space $\Omega^1(X)$ of global regular differentials on X. Recall that a smooth complex curve is also a Riemann surface, and hence topologically a surface. In this case, the two definitions of genus coincide.