j-invariant

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Entry type	Definition
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Related topic	EllipticCurve
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Defines	j-invariant
Defines	discriminant of an elliptic curve
Defines	invariant differential

Let E be an elliptic curve over $\mathbb{Q}$ with Weierstrass equation:

$$y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$$

with coefficients $a_i \in \mathbb{Q}$. Let:

$$\begin{aligned}
b_2 &= a_1^2 + 4a_2, \\
b_4 &= 2a_4 + a_1a_3, \\
b_6 &= a_3^2 + 4a_6, \\
b_8 &= a_1^2a_6 + 4a_2a_6 - a_1a_3a_4 + a_3^2a_2 - a_4^2, \\
c_4 &= b_2^2 - 24b_4, \\
c_6 &= -b_2^3 + 36b_2b_4 - 216b_6
\end{aligned}$$

Definition 1.

1. *The* discriminant *of E is defined to be*

$$\Delta = -b_2^2b_8 - 8b_4^3 - 27b_6^2 + 9b_2b_4b_6$$

2. *The* j-invariant *of E is*

$$j = \frac{c_4^3}{\Delta}$$

3. *The* invariant differential *is*

$$\omega = \frac{dx}{2y + a_1x + a_3} = \frac{dy}{3x^2 + 2a_2x + a_4 - a_1y}$$

Example:

If E has a Weierstrass equation in the simplified form $y^2 = x^3 + Ax + B$ then

$$\Delta = -16(4A^3 + 27B^2), \quad j = -\frac{1728(4A)^3}{\Delta}$$

Note: The discriminant Δ coincides in this case with the usual notion of `http://planetmath.org/Discriminant`discriminant of the polynomial $x^3 + Ax + B$.