



Math for the people, by the people.

## Cartier divisor

Canonical name	CartierDivisor
Date of creation	2013-03-22 13:52:29
Last modified on	2013-03-22 13:52:29
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	6
Author	mathcam (2727)
Entry type	Definition
Classification	msc 14A99

On a scheme  $X$ , a Cartier divisor is a global section of the sheaf  $\mathcal{K}^*/\mathcal{O}^*$ , where  $\mathcal{K}^*$  is the multiplicative sheaf of meromorphic functions, and  $\mathcal{O}^*$  the multiplicative sheaf of invertible regular functions (the units of the structure sheaf).

More explicitly, a Cartier divisor is a choice of open cover  $U_i$  of  $X$ , and meromorphic functions  $f_i \in \mathcal{K}^*(U_i)$ , such that  $f_i/f_j \in \mathcal{O}^*(U_i \cap U_j)$ , along with two Cartier divisors being the same if the open cover of one is a refinement of the other, with the same functions attached to open sets, or if  $f_i$  is replaced by  $gf_i$  with  $g \in \mathcal{O}_*$ .

Intuitively, the only thing carried by Cartier divisor is where it vanishes, and the order it does there. Thus, a Cartier divisor should give us a Weil divisor, and vice versa. On “nice” (for example, nonsingular over an algebraically closed field) schemes, it does.