supersingular

Canonical name	Supersingular
Date of creation	2013-03-22 12:18:30
Last modified on	2013-03-22 12:18:30
Owner	nerdy2 (62)
Last modified by	nerdy2 (62)
Numerical id	5
Author	nerdy2 (62)
Entry type	Definition
Classification	msc 14H52

An elliptic curve E over a field of characteristic p defined by the cubic equation $f(w,x,y)=0$ is called *supersingular* if the coefficient of $(wxy)^{p-1}$ in $f(w,x,y)^{p-1}$ is zero.

A supersingular elliptic curve is said to have Hasse invariant 0; an ordinary (i.e. non-supersingular) elliptic curve is said to have Hasse invariant 1.

This is equivalent to many other conditions. E is supersingular iff the invariant differential is exact. Also, E is supersingular iff $F^*: H^1(E,\mathcal{O}_E) \to H^1(E,\mathcal{O}_E)$ is nonzero where F^* is induced from the Frobenius morphism $F: E \to E$.