group

Canonical name	Group
Date of creation	2013-03-22 11:42:53
Last modified on	2013-03-22 11:42:53
Owner	drini (3)
Last modified by	drini (3)
Numerical id	34
Author	drini (3)
Entry type	Definition
Classification	msc 14F99
Classification	msc 08A99
Classification	msc 20A05
Classification	msc 20-00
Classification	msc 83C99
Classification	msc 32C05
Related topic	Subgroup
Related topic	CyclicGroup
Related topic	Simple
Related topic	SymmetricGroup
Related topic	FreeGroup
Related topic	Ring
Related topic	Field
Related topic	GroupHomomorphism
Related topic	LagrangesTheorem
Related topic	IdentityElement
Related topic	ProperSubgroup
Related topic	Groupoid
Related topic	FundamentalGroup
Related topic	TopologicalGroup
Related topic	LieGroup
Related topic	ProofThatGInGImpliesThatLangleGRangleLeG
Related topic	GeneralizedCyclicGroup
Defines	identity
Defines	inverse
Defines	neutralizing element
Defines	non-trivial element
Defines	nontrivial element
Defines	group operation

Group.

A group is a pair $(G, *)$, where G is a non-empty set and "$*$" is a binary operation on G, such that the following conditions hold:

- For any a, b in G, $a * b$ belongs to G. (The operation "$*$" is closed).
- For any $a, b, c \in G$, $(a * b) * c = a * (b * c)$. (Associativity of the operation).
- There is an element $e \in G$ such that $g * e = e * g = g$ for any $g \in G$. (Existence of identity element).
- For any $g \in G$ there exists an element h such that $g * h = h * g = e$. (Existence of inverses).

If G is a group under *, then * is referred to as the *group operation* of G.

Usually, the symbol "$*$" is omitted and we write ab for $a * b$. Sometimes, the symbol "+" is used to represent the operation, especially when the group is *abelian*.

It can be proved that there is only one identity element, and that for every element there is only one inverse. Because of this we usually denote the inverse of a as a^{-1} or $-a$ when we are using additive notation. The identity element is also called *neutral element* due to its behavior with respect to the operation, and thus a^{-1} is sometimes (although uncommonly) called the *neutralizing element* of a. An element of a group besides the identity element is sometimes called a *non-trivial element*.

Groups often arise as the symmetry groups of other mathematical objects; the study of such situations uses group actions. In fact, much of the study of groups themselves is conducted using group actions.