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axiomatization of dependence

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As noted by van der Waerden, it is possible to define the notion of dependence axiomatically in such a way that one can deal with linear dependence, algebraic dependence, and other sorts of dependence via a general theory. In this general theoretical framework, one can prove results about bases, dimension, and the like.

Let S be a set. The basic object of this theory is a relation D between S and the power set of S. This relation satisfies the following three axioms:

Axiom 1 *If Y is a subset of S and $x \in Y$, then $D(x, Y)$.*

Axiom 2 *If, for some set $X \subseteq S$ and some $y, z \in S$, it happens that $D(y, X \cup \{z\})$ but not $D(y, X)$, then $D(z, X \cup \{y\})$.*

Axiom 3 *If, for some sets $Y, Z \subseteq S$ and some $x \in S$, it happens that $D(x, Y)$ and, for every $y \in Y$, it is the case that $D(y, Z)$, then $D(x, Z)$.*