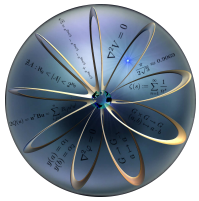

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## every orthonormal set is linearly independent

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**Theorem** : An orthonormal set of vectors in an inner product space is linearly independent.

*Proof.* We denote by $\langle \cdot, \cdot \rangle$ the inner product of $L$. Let $S$ be an orthonormal set of vectors. Let us first consider the case when $S$ is finite, i.e., $S = \{e_1, \ldots, e_n\}$ for some $n$. Suppose

$$\lambda_1 e_1 + \cdots + \lambda_n e_n = 0$$

for some scalars $\lambda_i$ (belonging to the field on the underlying vector space of $L$). For a fixed $k$ in $1, \ldots, n$, we then have

$$0 = \langle e_k, 0 \rangle = \langle e_k, \lambda_1 e_1 + \cdots + \lambda_n e_n \rangle = \lambda_1 \langle e_k, e_1 \rangle + \cdots + \lambda_n \langle e_k, e_n \rangle = \lambda_k,$$

so $\lambda_k = 0$, and $S$ is linearly independent. Next, suppose $S$ is infinite (countable or uncountable). To prove that $S$ is linearly independent, we need to show that all finite subsets of $S$ are linearly independent. Since any subset of an orthonormal set is also orthonormal, the infinite case follows from the finite case. □