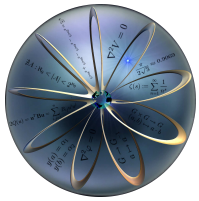

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cyclic subspace

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Defines	cyclic vector

Let V be a vector space over a field k, and $x \in V$. Let $T : V \to V$ be a linear transformation. The *T-cyclic subspace generated by x* is the smallest T-invariant subspace which contains x, and is denoted by $Z(x, T)$.

Since $x, T(x), \ldots, T^n(x), \ldots \in Z(x, T)$, we have that

$$W := \text{span}\{x, T(x), \ldots, T^n(x), \ldots\} \subseteq Z(x, T).$$

On the other hand, since W is T-invariant, $Z(x, T) \subseteq W$. Hence $Z(x, T)$ is the subspace generated by $x, T(x), \ldots, T^n(x), \ldots$ In other words, $Z(x, T) = \{p(T)(x) \mid p \in k[X]\}$.

Remark. If $Z(x, T) = V$ we say that x is a *cyclic vector* of T.