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annihilator of vector subspace

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Defines	annihilator
Defines	annihilated subspace

If V is a vector space, and S is any subset of V, the *annihilator* of S, denoted by S^0, is the subspace of the dual space V^* that kills every vector in S:

$$S^0 = \{\phi \in V^* : \phi(v) = 0 \text{ for all } v \in S\}\,.$$

Similarly, if Λ is any subset of V^*, the *annihilated subspace* of Λ is

$$\Lambda^{-0} = \{v \in V : \phi(v) = 0 \text{ for all } \phi \in \Lambda\} = \bigcap_{\phi \in \Lambda} \ker \phi\,.$$

(Note: this may not be the standard notation.)

1 Properties

Assume V is finite-dimensional. Let W and Φ denote subspaces of V and V^*, respectively, and let $\widehat{\ }$ denote the natural isomorphism from V to its double dual V^{**}.

i. $S^0 = (\operatorname{span} S)^0$

ii. $\Lambda^{-0} = (\operatorname{span} \Lambda)^{-0}$

iii. $W^{00} = \widehat{W}$

iv. $(\Phi^{-0})^0 = \Phi$

v. $(W^0)^{-0} = W$

vi. $\dim W + \dim W^0 = \dim V$ (a dimension theorem)

vii. $\dim \Phi + \dim \Phi^{-0} = \dim V^* = \dim V$

viii. $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$, where $W_1 + W_2$ denotes the sum of two subspaces of V.

ix. If $T : V \to V$ is a linear operator, and $W = \ker T$, then the image of the pullback $T^* : V^* \to V^*$ is W^0.

References

[1] Friedberg, Insel, Spence. *Linear Algebra.* Prentice-Hall, 1997.