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# closure of a vector subspace in a normed space is a vector subspace

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| Canonical name | ClosureOfAVectorSubspaceInANormedSpaceIsAVectorSubspace |
| Date of creation | 2013-03-22 15:00:16 |
| Last modified on | 2013-03-22 15:00:16 |
| Owner | gumau (3545) |
| Last modified by | gumau (3545) |
| Numerical id | 7 |
| Author | gumau (3545) |
| Entry type | Result |
| Classification | msc 15A03 |
| Classification | msc 46B99 |
| Classification | msc 54A05 |
| Related topic | ClosureOfAVectorSubspaceIsAVectorSubspace2 |
| Related topic | ClosureOfSetsClosedUnderAFinitaryOperation |

Let $(V, \|\cdot\|)$ be a normed space, and $S \subset V$ a vector subspace. Then $\overline{S}$ is a vector subspace in $V$.

**Proof**

First of all, $0 \in \overline{S}$ because $0 \in S$. Now, let $x, y \in \overline{S}$, and $\lambda \in K$ (where $K$ is the ground field of the vector space $V$). Then there are two sequences in $S$, say $(x_n)_{n\in\mathbb{N}}$ and $(y_n)_{n\in\mathbb{N}}$ which converge to $x$ and $y$ respectively.

Then, the sequence $(x_n + \lambda \cdot y_n)_{n\in\mathbb{N}}$ is a sequence in $S$ (because $S$ is a vector subspace), and it's trivial (use properties of the norm) that this sequence converges to $x + \lambda \cdot y$, and so this sum is a vector which lies in $\overline{S}$.

We have proved that $\overline{S}$ is a vector subspace. QED.