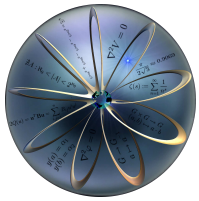

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finite dimensional proper subspaces of a normed space are nowhere dense

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- Let V be a normed space. If $S \subseteq V$ is a finite dimensional proper subspace, then S is nowhere dense.

Proof :

It is known that for any topological vector space (in particular, normed spaces) http://planetmath.org/ProperSubspacesOfATopologicalVectorSpaceHaveEmptyInte proper subspace has empty interior.

From the http://planetmath.org/EveryFiniteDimensionalSubspaceOfANormedSpaceIsClo we also know that finite dimensional subspaces of V are closed.

Then, $\text{int}(\overline{S}) = \text{int}(S) = \varnothing$, which shows that S is nowhere dense. $\square$