circulant matrix

Canonical name	CirculantMatrix
Date of creation	2013-03-22 13:53:38
Last modified on	2013-03-22 13:53:38
Owner	bwebste (988)
Last modified by	bwebste (988)
Numerical id	9
Author	bwebste (988)
Entry type	Definition
Classification	msc 15-01
Classification	msc 15A99

A square matrix $M : A \times A \to C$ is said to be g-*circulant* for an integer g if each row other than the first is obtained from the preceding row by shifting the elements cyclically g columns to the right (g¿0) or -g columns to the left (g ¡ 0).

That is, if $A = [a_{ij}]$ then $a_{i,j} = a_{i+1,j+g}$ where the subscripts are computed modulo d. A 1-circulant is commonly called a circulant and a -1-circulant is called a back circulant.

More explicitly, a matrix of the form

$$\begin{bmatrix} M_1 & M_2 & M_3 & \dots & M_d \\ M_d & M_1 & M_2 & \dots & M_{d-1} \\ M_{d-1} & M_d & M_1 & \dots & M_{d-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ M_2 & M_3 & M_4 & \dots & M_1 \end{bmatrix}$$

is called circulant.

Because the `http://planetmath.org/JordanCanonicalFormTheorem`Jordan decomposition of a circulant matrix is rather simple, circulant matrices have some interest in connection with the approximation of eigenvalues of more general matrices. In particular, they have become part of the standard apparatus in the computerized analysis of signals and images.