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position vector

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In the space $\mathbb{R}^3$, the vector

$$\vec{r} \;:=\; (x,\, y,\, z) \;=\; x\vec{i} + y\vec{j} + z\vec{k}$$

directed from the origin to a point $(x,\, y,\, z)$ is the *position vector* of this point. When the point is , $\vec{r}$ a vector field and its

$$r \;:=\; \sqrt{x^2+y^2+z^2}$$

a scalar .

The

- $\nabla\cdot\vec{r} \;=\; 3$
- $\nabla\times\vec{r} \;=\; \vec{0}$
- $\nabla r \;=\; \dfrac{\vec{r}}{r} \;=\; \vec{r}^{\,0}$
- $\nabla\dfrac{1}{r} \;=\; -\dfrac{\vec{r}}{r^3} \;=\; -\dfrac{\vec{r}^{\,0}}{r^2}$
- $\nabla^2\dfrac{1}{r} \;=\; 0$

are valid, where $\vec{r}^{\,0}$ is the unit vector having the direction of $\vec{r}$.

If $\vec{c}$ is a vector, $\vec{U}\colon \mathbb{R}^3 \to \mathbb{R}^3$ a vector function and $f\colon \mathbb{R} \to \mathbb{R}$ is a twice differentiable function, then the formulae

- $\nabla(\vec{c}\cdot\vec{r}) \;\; = \vec{c}$
- $\nabla\cdot(\vec{c}\times\vec{r}) \;=\; 0$
- $(\vec{U}\cdot\nabla)\vec{r} \;=\; \vec{U}$
- $(\vec{U}\times\nabla)\cdot\vec{r} \;=\; 0$
- $(\vec{U}\times\nabla)\times\vec{r} \;=\; -2\vec{U}$
- $\nabla f(r) \;=\; f'(r)\,\vec{r}^{\,0}$
- $\nabla^2 f(r) \;=\; f''(r) + \dfrac{2}{r} f'(r)$

hold.

References

[1] K. Väisälä: *Vektorianalyysi*. Werner Söderström Osakeyhtiö, Helsinki (1961).