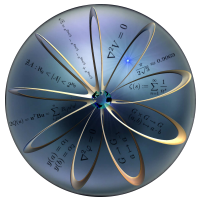

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derivative of matrix

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Suppose I is an open set of $\mathbb{R}$, and for each $t \in I$, $A(t)$ is an $n \times m$ matrix. If each element in $A(t)$ is a differentiable function of t, we say that A is a differentiable, and define the derivative of A componentwise. This derivative we shall write as $\frac{d}{dt}A$ or $\frac{dA}{dt}$.

Properties

In the below we assume that all matrices are dependent on a parameter t and the matrices are differentiable with respect to t.

1. For any $n \times m$ matrix A,

$$\left(\frac{dA}{dt}\right)^T = \frac{d}{dt}\left(A^T\right),$$

   where ^{T} is the matrix transpose.

2. If $A(t), B(t)$ are matrices such that AB is defined, then

$$\frac{d}{dt}(AB) = \frac{dA}{dt}B + A\frac{dB}{dt}.$$

3. When $A(t)$ is invertible,

$$\frac{d}{dt}(A^{-1}) = -A^{-1}\frac{dA}{dt}A^{-1}.$$

4. For a square matrix $A(t)$,

$$\operatorname{tr}(\frac{dA}{dt}) = \frac{d}{dt}\operatorname{tr}(A),$$

   where tr is the matrix trace.

5. If $A(t), B(t)$ are $n \times m$ matrices and $A \circ B$ is the Hadamard product of A and B, then

$$\frac{d}{dt}(A \circ B) = \frac{dA}{dt} \circ B + A \circ \frac{dB}{dt}.$$