properties of diagonally dominant matrix

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1)(Levy-Desplanques theorem) A strictly diagonally dominant matrix is non-singular.

Proof. Let A be a strictly diagonally dominant matrix and let's assume A is singular, that is, $\lambda = 0 \in \sigma(A)$. Then, by Gershgorin's circle theorem, an index i exists such that:

$$\sum_{j \neq i} |a_{ij}| \geq |\lambda - a_{ii}| = |a_{ii}|,$$

which is in contrast with strictly diagonally dominance definition. □

2)() $|\det(A)| \geq \prod_{i=1}^{n} \left(|a_{ii}| - \sum_{j=1, j \neq i} |a_{ij}| \right)$ (See `http://planetmath.org/ProofOfDeterminar` for a proof.)

3) A Hermitian diagonally dominant matrix with real nonnegative diagonal entries is positive semidefinite.

Proof. Let A be a Hermitian diagonally dominant matrix with real nonnegative diagonal entries; then its eigenvalues are real and, by Gershgorin's circle theorem, for each eigenvalue an index i exists such that:

$$\lambda \in [a_{ii} - \sum_{j \neq i} |a_{ij}|, a_{ii} + \sum_{i \neq j} |a_{ij}|],$$

which implies, by definition of diagonally dominance,$\lambda \geq 0$. □