Hermitian matrix

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Defines	Hermitian operator

For a complex matrix A, let $A^* = \overline{A}^T$, where A^T is the transpose, and $\bar{A}$ is the complex conjugate of A.

Definition A complex square matrix A is *Hermitian*, if

$$A = A^*.$$

Properties

1. The eigenvalues of a Hermitian matrix are real.
2. The diagonal elements of a Hermitian matrix are real.
3. The complex conjugate of a Hermitian matrix is a Hermitian matrix.
4. If A is a Hermitian matrix, and B is a complex matrix of same order as A, then BAB^* is a Hermitian matrix.
5. A matrix is symmetric if and only if it is real and Hermitian.
6. Hermitian matrices are a vector subspace of the vector space of complex matrices. The real symmetric matrices are a subspace of the Hermitian matrices.
7. Hermitian matrices are also called *self-adjoint* since if A is Hermitian, then in the usual inner product of $\mathbb{C}^n$, we have
$$\langle u, Av \rangle = \langle Au, v \rangle$$
for all $u, v \in \mathbb{C}^n$.

Example

1. For any $n \times m$ matrix A, the $n \times n$ matrix AA^* is Hermitian.
2. For any square matrix A, the Hermitian part of A, $\frac{1}{2}(A + A^*)$ is Hermitian. See `http://planetmath.org/DirectSumOfHermitianAndSkewHermitianMatrices`this page.
3. $$\begin{bmatrix} 1 & 1+i & 1+2i & 1+3i \\ 1-i & 2 & 2+2i & 2+3i \\ 1-2i & 2-2i & 3 & 3+3i \\ 1-3i & 2-3i & 3-3i & 4 \end{bmatrix}$$

The first two examples are also examples of normal matrices.

Notes

1. Hermitian matrices are named after Charles Hermite (1822-1901) [**?**], who proved in 1855 that the eigenvalues of these matrices are always real [**?**].

2. Hermitian, or self-adjoint operators on a Hilbert space play a fundamental role in quantum theories as their eigenvalues are observable, or measurable; such Hermitian operators can be represented by Hermitian matrices.

References

[1] H. Eves, *Elementary Matrix Theory*, Dover publications, 1980.

[2] The MacTutor History of Mathematics archive, `http://www-gap.dcs.st-and.ac.uk/ history/Mathematicians/Hermite.html`Charles Hermite