Jordan canonical form theorem

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A **Jordan block** or **Jordan matrix** is a matrix of the form

$$\begin{pmatrix} \lambda & 1 & 0 & \cdots & 0 \\ 0 & \lambda & 1 & \cdots & 0 \\ 0 & 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & 0 & \cdots & \lambda \end{pmatrix}$$

with a constant value λ along the diagonal and 1's on the superdiagonal. Some texts the 1's on the subdiagonal instead.

Theorem. *Let V be a finite-dimensional vector space over a field F and $t : V \to V$ be a linear transformation. Then, if the characteristic polynomial factors completely over F, there will exist a basis of V with respect to which the matrix of t is of the form*

$$\begin{pmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \cdots & 0 \\ & & \cdots & \\ 0 & 0 & \cdots & J_k \end{pmatrix}$$

where each J_i is a Jordan block in which $\lambda = \lambda_i$.

The matrix in Theorem 1 is called a *Jordan canonical form* for the transformation t.