identity matrix

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The $n \times n$ *identity matrix* I (or I_n) over a ring R (with an identity 1) is the square matrix with coefficients in R given by

$$I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 1 \end{bmatrix},$$

where the numeral "1" and "0" respectively represent the multiplicative and additive identities in R.

0.0.1 Properties

The identity matrix I_n serves as the multiplicative identity in the ring of $n \times n$ matrices over R with standard matrix multiplication. For any $n \times n$ matrix M, we have $I_n M = M I_n = M$, and the identity matrix is uniquely defined by this property. In addition, for any $n \times m$ matrix A and $m \times n$ B, we have $IA = A$ and $BI = B$.

The $n \times n$ identity matrix I satisfy the following properties

- For the determinant, we have $\det I = 1$, and for the trace, we have $\operatorname{tr} I = n$.
- The identity matrix has only one eigenvalue $\lambda = 1$ of multiplicity n. The corresponding eigenvectors can be chosen to be $v_1 = (1, 0, \ldots, 0), \ldots, v_n = (0, \ldots, 0, 1)$.
- The matrix exponential of I gives $e^I = eI$.
- The identity matrix is a diagonal matrix.