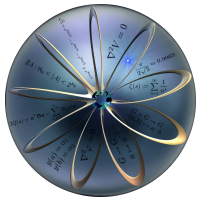

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there are no non-square doubly stochastic matrices

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Suppose $A = (a_{ij})$ is a $n \times m$ matrix with nonnegative entries such that

$$\sum_{j=1}^{m} a_{ij} = 1, \quad i = 1, \ldots, n, \tag{1}$$

$$\sum_{i=1}^{n} a_{ij} = 1, \quad j = 1, \ldots, m. \tag{2}$$

Then $n = m$.

This is seen by summing equation (**??**) over $i = 1, \ldots, n$ and equation (**??**) over $j = 1, \ldots, m$. Then

$$\sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} = n,$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} = m,$$

and since the right hand sides coincide, it follows that $n = m$.