rank-nullity theorem

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Let V and W be vector spaces over the same field. If $\phi\colon V \to W$ is a linear mapping, then

$$\dim V = \dim(\ker \phi) + \dim(\operatorname{im} \phi).$$

In other words, the dimension of V is equal to the http://planetmath.org/CardinalArithmeticsu of the http://planetmath.org/RankLinearMappingrank and nullity of ϕ.

Note that if U is a subspace of V, then this (applied to the canonical mapping $V \to V/U$) says that

$$\dim V = \dim U + \dim(V/U),$$

that is,

$$\dim V = \dim U + \operatorname{codim} U,$$

where codim denotes codimension.

An alternative way of stating the rank-nullity theorem is by saying that if

$$0 \to U \to V \to W \to 0$$

is a short exact sequence of vector spaces, then

$$\dim(V) = \dim(U) + \dim(W).$$

In fact, if

$$0 \to V_1 \to \cdots \to V_n \to 0$$

is an exact sequence of vector spaces, then

$$\sum_{i=1}^{\lfloor n/2 \rfloor} V_{2i} = \sum_{i=1}^{\lceil n/2 \rceil} V_{2i-1},$$

that is, the sum of the dimensions of even-numbered terms is the same as the sum of the dimensions of the odd-numbered terms.