spectral values classification

Canonical name	SpectralValuesClassification
Date of creation	2013-03-22 18:52:01
Last modified on	2013-03-22 18:52:01
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Last modified by	fernsanz (8869)
Numerical id	5
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Entry type	Definition
Classification	msc 15A18
Synonym	eigenvalues
Synonym	spectrum
Related topic	Eigenvalue
Related topic	SpectrumOfAMuI
Related topic	InvertibleLinearTransformation
Defines	spectrum
Defines	point spectrum
Defines	residual spectrum
Defines	continuous spectrum
Defines	resolvent set
Defines	eigenvalues
Defines	puntual spectrum
Defines	point spectral value
Defines	residual spectral value
Defines	continuous spectral value
Defines	resolvent set value

Definition 1. Let X a topological vector space and $A : X \supset D_A \longrightarrow X$ a linear transformation with domain D_A. Depending on the properties of[1] $(\lambda - A)$ the following definitions apply:

$(\lambda - A)^{-1}$	Boundness of $(\lambda - A)^{-1}$	$R(\lambda - A)$	Set to which λ belongs
exists	bounded	dense in X	resolvent set $\rho(A)$
exists	unbounded	dense in X	continuous spectrum $C\sigma(A)$
exists	bounded or unbounded in X	not dense in X	residual spectrum $R\sigma(A)$
not exists		dense or not dense in X	puntual spectrum $P\sigma(A)$

Remark 1. It is obvious that, if F is the field of possible values for λ (usually $F = \mathbb{C}$ or $F = \mathbb{R}$) then $F = \rho(A) \cup C\sigma(A) \cup R\sigma(A) \cup P\sigma(A)$, that is, these definitions cover all the possibilities for λ. The complement of the resolvent set is called *spectrum* of the operator A, i.e., $\sigma(A) = C\sigma(A) \cup R\sigma(A) \cup P\sigma(A)$

Remark 2. In the finite dimensional case if $(\lambda - A)^{-1}$ exists it must be bounded, since all finite dimensional linear mappings are bounded. This existence also implies that the range of $(\lambda - A)$ must be the whole X. So, in the finite dimensional case the only spectral values we can encounter are point spectrum values (eigenvalues).

[1] the notation $(\lambda - A)$ is to be understood as $\lambda I - A$ with I the identity transformation and $R(\lambda - A)$ is the range of $(\lambda - A)$