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permutation operator

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Let V be a vector space over a field. Let $\sigma \in S_n$, the symmetric group on $\{1, \ldots, n\}$ and define a multilinear map $\phi : V \times \cdots \times V \to V^{\otimes n} = \overbrace{V \otimes \cdots \otimes V}^{n \text{ times}}$ by

$$\phi(v_1, \ldots, v_n) = v_{\sigma^{-1}(1)} \otimes \cdots \otimes v_{\sigma^{-1}(n)}.$$

Then by the http://planetmath.org/TensorProductuniversal factorization property for a http://planetmath.org/TensorProducttensor product there is a unique linear map $P(\sigma) : V^{\otimes n} \to V^{\otimes n}$ such that $P(\sigma) \otimes = \phi$. Then of course,

$$P(\sigma) v_1 \otimes \cdots \otimes v_n = v_{\sigma^{-1}(1)} \otimes \cdots \otimes v_{\sigma^{-1}(n)}.$$

$P(\sigma)$ is called the *permutation operator* associated with σ.

1 Properties

1. $P(\sigma\tau) = P(\sigma)P(\tau)$
2. $P(e) = I$, where I is the identity mapping on $V^{\otimes n}$
3. $P(\sigma)$ is nonsingular and $P(\sigma)^{-1} = P(\sigma^{-1})$