vector subspace

Canonical name	VectorSubspace
Date of creation	2013-03-22 11:55:24
Last modified on	2013-03-22 11:55:24
Owner	yark (2760)
Last modified by	yark (2760)
Numerical id	20
Author	yark (2760)
Entry type	Definition
Classification	msc 15-00
Synonym	subspace
Synonym	linear subspace
Related topic	VectorSpace
Related topic	LinearManifold
Defines	dimension theorem for subspaces
Defines	proper vector subspace

Definition Let V be a vector space over a field F, and let W be a subset of V. If W is itself a vector space, then W is said to be a *vector subspace* of V. If in addtition $V \neq W$, then W is a *proper vector subspace* of V.

If W is a nonempty subset of V, then a necessary and sufficient condition for W to be a subspace is that $a + \gamma b \in W$ for all $a, b \in W$ and all $\gamma \in F$.

0.0.1 Examples

1. Every vector space is a vector subspace of itself.

2. In every vector space, $\{0\}$ is a vector subspace.

3. If S and T are vector subspaces of a vector space V, then the vector sum
$$S + T = \{s + t \in V \mid s \in S, t \in T\}$$
and the intersection
$$S \cap T = \{u \in V \mid u \in S, u \in T\}$$
are vector subspaces of V.

4. Suppose S and T are vector spaces, and suppose L is a linear mapping $L\colon S \to T$. Then $\operatorname{Im} L$ is a vector subspace of T, and $\operatorname{Ker} L$ is a vector subspace of S.

5. If V is an inner product space, then the orthogonal complement of any subset of V is a vector subspace of V.

0.0.2 Results for vector subspaces

Theorem 1 [**?**] Let V be a finite dimensional vector space. If W is a vector subspace of V and $\dim W = \dim V$, then $W = V$.

Theorem 2 [**?**] (Dimension theorem for subspaces) Let V be a vector space with subspaces S and T. Then

$$\dim(S + T) + \dim(S \cap T) = \dim S + \dim T.$$

References

[1] S. Lang, *Linear Algebra*, Addison-Wesley, 1966.

[2] W.E. Deskins, *Abstract Algebra*, Dover publications, 1995.