Hadamard product

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Defines	Oppenheim inequality

Definition Suppose $A = (a_{ij})$ and $B = (b_{ij})$ are two $n \times m$-matrices with entries in some field. Then their *Hadamard product* is the entry-wise product of A and B, that is, the $n \times m$-matrix $A \circ B$ whose (i, j)th entry is $a_{ij}b_{ij}$.

Properties

Suppose A, B, C are matrices of the same size and λ is a scalar. Then

$$\begin{aligned} A \circ B &= B \circ A, \\ A \circ (B + C) &= A \circ B + A \circ C, \\ A \circ (\lambda B) &= \lambda(A \circ B), \end{aligned}$$

- If A, B are diagonal matrices, then $A \circ B = AB$.
- (*Oppenheim inequality*) [**?**]: If A, B are positive definite matrices, and (a_{ii}) are the diagonal entries of A, then

  $$\det A \circ B \geq \det B \prod a_{ii}$$

  with equality if and only if A is a diagonal matrix.

Remark

There is also a Hadamard product for two power series: Then the Hadamard product of $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ is $\sum_{i=1}^{\infty} a_i b_i$.

References

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[3] B. Mond, J. E. Pecaric, *Inequalities for the Hadamard product of matrices*, SIAM Journal on Matrix Analysis and Applications, Vol. 19, Nr. 1, pp. 66-70. `http://epubs.siam.org/sam-bin/dbq/article/30295`(link)