triple cross product

Canonical name	TripleCrossProduct
Date of creation	2013-03-22 14:15:53
Last modified on	2013-03-22 14:15:53
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	28
Author	pahio (2872)
Entry type	Definition
Classification	msc 15A72
Synonym	vector triple product
Synonym	triple vector product
Related topic	PhysicalVector
Defines	Lagrange's formula

The cross product of a vector with a cross product is called the *triple cross product.*

The of the triple cross product or *Lagrange's* is

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

("exterior dot far times near minus exterior dot near times far" — this works also when "exterior" is the last).

The the vectors $\vec{b}$ and $\vec{c}$ (when these are not parallel).

Note that the use of parentheses in the triple cross products is necessary, since the cross product operation is not `http://planetmath.org/GeneralAssociativity`associativ i.e., generally we have

$$(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$$

(for example: $(\vec{i} \times \vec{i}) \times \vec{j} = \vec{0}$ but $\vec{i} \times (\vec{i} \times \vec{j}) = -\vec{j}$ when $(\vec{i}, \vec{j}, \vec{k})$ is a right-handed orthonormal basis of $\mathbb{R}^3$). So the `http://planetmath.org/AlgebraicSystem`system $(\mathbb{R}^3, +, \times)$ is not a ring.

A direct consequence of the is the *Jacobi identity*

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0},$$

which is one of the properties making $(\mathbb{R}^3, +, \times)$ a Lie algebra.

It follows from the also that

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = (\vec{a}\vec{b}\vec{d})\vec{c} - (\vec{a}\vec{b}\vec{c})\vec{d}$$

where $(\vec{u}\vec{v}\vec{w})$ means the triple scalar product of $\vec{u}$, $\vec{v}$ and $\vec{w}$.