Hilbert matrix

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1 Hilbert Matrix

A *Hilbert matrix* H of order n is a square matrix defined by

$$H_{ij} = \frac{1}{i+j-1}$$

An example of a Hilbert matrix when $n = 5$ is

$$\begin{bmatrix} \frac{1}{1} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \end{bmatrix}$$

Hilbert matrices are ill-conditioned.

2 Inverse

The inverse of a Hilbert matrix $H^{-1} \in M_N(\mathbb{R})$ is given by

$$H_{ij}^{-1} = (-1)^{i+j}(i+j-1)\binom{N+i-1}{N-j}\binom{N+j-1}{N-i}\binom{i+j-2}{i-1}^2$$

An example of an inverted Hilbert matrix when $n = 5$ case is:

$$\begin{bmatrix} 25 & -300 & 1050 & -1400 & 630 \\ -300 & 4800 & -18900 & 26880 & -12600 \\ 1050 & -18900 & 79380 & -117600 & 56700 \\ -1400 & 26880 & -117600 & 179200 & -88200 \\ 630 & -12600 & 56700 & -88200 & 44100 \end{bmatrix}$$

For more fun with Hilbert matrices, see [**?**].

References

[1] Choi, Man-Duen. Tricks or Treats with the Hilbert Matrix. American Mathematical Monthly 90, 301-312, 1983.