skew-symmetric matrix

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Definition:

Let A be an square matrix of order n with real entries (a_{ij}). The matrix A is skew-symmetric if $a_{ij} = -a_{ji}$ for all $1 \leq i \leq n, 1 \leq j \leq n$.

$$A = \begin{pmatrix} a_{11} = 0 & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} = 0 \end{pmatrix}$$

The main diagonal entries are zero because $a_{i,i} = -a_{i,i}$ implies $a_{i,i} = 0$.

One can see skew-symmetric matrices as a special case of complex skew-Hermitian matrices. Thus, all properties of skew-Hermitian matrices also hold for skew-symmetric matrices.

Properties:

1. The matrix A is skew-symmetric if and only if $A^t = -A$, where A^t is the matrix transpose

2. For the trace operator, we have that $\operatorname{tr}(A) = \operatorname{tr}(A^t)$. Combining this with property (1), it follows that $\operatorname{tr}(A) = 0$ for a skew-symmetric matrix A.

3. Skew-symmetric matrices form a vector space: If A and B are skew-symmetric and $\alpha, \beta \in \mathbb{R}$, then $\alpha A + \beta B$ is also skew-symmetric.

4. Suppose A is a skew-symmetric matrix and B is a matrix of same order as A. Then $B^t A B$ is skew-symmetric.

5. All eigenvalues of skew-symmetric matrices are purely imaginary or zero. This result is proven on the page for skew-Hermitian matrices.

6. According to Jacobi's Theorem, the determinant of a skew-symmetric matrix of odd order is zero.

Examples:

- $\begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}$

- $\begin{pmatrix} 0 & b & c \\ -b & 0 & e \\ -c & -e & 0 \end{pmatrix}$