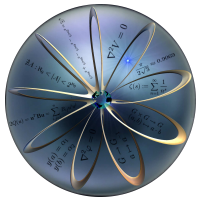

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rank of a linear mapping

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The *rank* of a linear mapping $L\colon U \to V$ is defined to be the $\dim L(U)$, the dimension of the mapping's image. Speaking less formally, the rank gives the number of independent linear constraints on $u \in U$ imposed by the equation

$$L(u) = 0.$$

Properties

1. If V is finite-dimensional, then $\operatorname{rank} L = \dim V$ if and only if L is surjective.

2. If U is finite-dimensional, then $\operatorname{rank} L = \dim U$ if and only if L is injective.

3. Composition of linear mappings does not increase rank. If $M\colon V \to W$ is another linear mapping, then

   $$\operatorname{rank} ML \leq \operatorname{rank} L$$

   and

   $$\operatorname{rank} ML \leq \operatorname{rank} M.$$

   Equality holds in the first case if and only if M is an isomorphism, and in the second case if and only if L is an isomorphism.