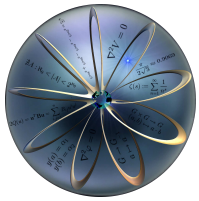

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linear complex structure

Canonical name	LinearComplexStructure
Date of creation	2013-03-22 16:16:48
Last modified on	2013-03-22 16:16:48
Owner	Mazzu (14365)
Last modified by	Mazzu (14365)
Numerical id	12
Author	Mazzu (14365)
Entry type	Definition
Classification	msc 15-00
Related topic	ComplexificationOfVectorSpace
Defines	linear complex structure

A on a real vector space V, with $\dim(V) = m$, is a linear automorphism $J \in \mathrm{Aut}(V)$ such that $J^2 = J \circ J = -\mathrm{id}_V$. With a complex structure J we can consider V as a complex vector space with the product $\mathbb{C} \times V \to V$ given by

$$(x + iy)\mathbf{v} = x\mathbf{v} + yJ(\mathbf{v}), \quad \forall x, y \in \mathbb{R},\ \mathbf{v} \in V.$$

This implies that the dimension m of V must be even.

A common example is $V = \mathbb{R}^{2n}$ with the standard basis $\mathbf{e}_1, ..., \mathbf{e}_n, \mathbf{f}_1, ..., \mathbf{f}_n$, for which we can obtain a complex structure $J_0 \in \mathrm{Aut}(\mathbb{R}^{2n})$ represented by the matrix

$$\begin{pmatrix} \mathbf{0} & \mathbf{I}_n \\ -\mathbf{I}_n & \mathbf{0} \end{pmatrix}.$$

Here $\mathbf{I}_n \in \mathrm{M}_n(\mathbb{R})$ is the identity $n \times n$ matrix and $\mathbf{0} \in \mathrm{M}_n(\mathbb{R})$ is the zero $n \times n$ matrix.