kernel of a linear mapping

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Let $T\colon V \to W$ be a linear mapping between vector spaces.

The set of all vectors in V that T maps to 0 is called the *kernel* (or *nullspace*) of T, and is denoted $\ker T$. So

$$\ker T = \{\, x \in V \mid T(x) = 0 \,\}.$$

The kernel is a vector subspace of V, and its `http://planetmath.org/Dimension2`dimension is called the nullity of T.

The function T is injective if and only if $\ker T = \{0\}$ (see the `http://planetmath.org/Operato` proof). In particular, if the dimensions of V and W are equal and finite, then T is invertible if and only if $\ker T = \{0\}$.

If U is a vector subspace of V, then we have

$$\ker T|_U = U \cap \ker T,$$

where $T|_U$ is the `http://planetmath.org/RestrictionOfAFunction`restriction of T to U.

When the linear mappings are given by means of matrices, the kernel of the matrix A is

$$\ker A = \{\, x \in V \mid Ax = 0 \,\}.$$