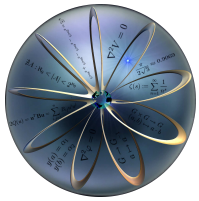

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inverse of matrix with small-rank adjustment

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Suppose that an $n \times n$ matrix B is obtained by adding a small-rank adjustment XRY^T to matrix A,

$$B = A + XRY^T,$$

where X and Y are $n \times r$ matrices, and R is an $r \times r$ matrix. Assume that the inverse of A is known and r is much smaller than n. The following formula for B^{-1} is often useful,

$$B^{-1} = A^{-1} - A^{-1}X(R^{-1} + Y^T A^{-1} X)^{-1} Y^T A^{-1}$$

provided that all inverses in the formula exist.

In particular, when $r = 1$ and $A = I$, we have

$$(I + xy^T)^{-1} = I - \frac{xy^T}{1 + y^T x}$$

for any $n \times 1$ column vectors x and y such that $1 + y^T x \neq 0$.