matrix representation of a bilinear form

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Given a bilinear form, $B : U \times V \to K$, we show how we can represent it with a matrix, with respect to a particular pair of bases for U and V

Suppose U and V are finite-dimensional and we have chosen bases, $\mathcal{B}_1 = \{e_1, \dots\}$ and $\mathcal{B}_2 = \{f_1, \dots\}$. Now we define the matrix C with entries $C_{ij} = B(e_i, f_j)$. This will be the matrix associated to B with respect to this basis as follows; If we write $x, y \in V$ as column vectors in terms of the chosen bases, then check $B(x, y) = x^T C y$. Further if we choose the corresponding dual bases for U^* and V^* then C and C^T are the corresponding matrices for B_R and B_L, respectively (in the sense of linear maps). Thus we see that a symmetric bilinear form is represented by a symmetric matrix, and similarly for skew-symmetric forms.

Let $\mathcal{B}_1'$ and $\mathcal{B}_2'$ be new bases, and P and Q the corresponding change of basis matrices. Then the new matrix is $C' = P^T C Q$.