determinant of anti-diagonal matrix

Canonical name	DeterminantOfAntidiagonalMatrix
Date of creation	2013-03-22 15:50:25
Last modified on	2013-03-22 15:50:25
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Last modified by	cvalente (11260)
Numerical id	6
Author	cvalente (11260)
Entry type	Result
Classification	msc 15-00

Let $A = \operatorname{adiag}(a_1, \dots, a_n)$ be an anti-diagonal matrix. Using the sum over all permutations formula for the determinant of a matrix and since all but possibly the anti-diagonal elements are null we get directly at the result

$$\det A = \operatorname{sgn}(n, n-1, \dots, 1) \prod_{i=1}^{n} a_i$$

so all that remains is to calculate the sign of the permutation. This can be done directly.

To bring the last element to the beginning $n-1$ permutations are needed so

$$\operatorname{sgn}(n, n-1, \dots, 1) = (-1)^{n-1} \operatorname{sgn}(1, n, n-1, \cdots, 2)$$

Now bring the last element to the second position. To do this $n-2$ permutations are needed. Repeat this procedure $n-1$ times to get the permutation $(1, \dots, n)$ which has positive sign.

Summing every permutation, it takes

$$\sum_{k=1}^{n-1} k = \frac{n(n-1)}{2}$$

permutations to get to the desired permutation.

So we get the final result that

$$\det \operatorname{adiag}(a_1, \dots, a_n) = (-1)^{\frac{n(n-1)}{2}} \prod_{i=1}^{n} a_i$$

Notice that the sign is positive if either n or $n-1$ is a multiple of 4 and negative otherwise.