eigenvalues of an involution

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Proof. For the first claim suppose λ is an eigenvalue corresponding to an eigenvector x of A. That is, $Ax = \lambda x$. Then $A^2 x = \lambda Ax$, so $x = \lambda^2 x$. As an eigenvector, x is non-zero, and $\lambda = \pm 1$. Now property (1) follows since the determinant is the product of the eigenvalues. For property (2), suppose that $A - \lambda I = -\lambda A(A - 1/\lambda I)$, where A and λ are as above. Taking the determinant of both sides, and using part (1), and the properties of the determinant, yields

$$\det(A - \lambda I) = \pm \lambda^n \det(A - \frac{1}{\lambda} I).$$

Property (2) follows. $\square$