linear manifold

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Definition Suppose V is a vector space and suppose that L is a non-empty subset of V. If there exists a $v \in V$ such that $L+v = \{v+l \mid l \in L\}$ is a vector subspace of V, then L is a **linear manifold** of V. Then we say that the dimension of L is the dimension of $L+v$ and write $\dim L = \dim(L+v)$. In the important case $\dim L = \dim V - 1$, L is called a **hyperplane**.

A linear manifold is, in other words, a linear subspace that has possibly been shifted away from the origin. For instance, in $\mathbb{R}^2$ examples of linear manifolds are points, lines (which are hyperplanes), and $\mathbb{R}^2$ itself. In $\mathbb{R}^n$ hyperplanes naturally describe tangent planes to a smooth hyper surface.

References

[1] R. Cristescu, *Topological vector spaces*, Noordhoff International Publishing, 1977.