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second order tensor: symmetric and skew-symmetric parts

Canonical name	SecondOrderTensorSymmetricAndSkewsymmetricParts
Date of creation	2013-03-22 15:51:32
Last modified on	2013-03-22 15:51:32
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Last modified by	rspuzio (6075)
Numerical id	18
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Entry type	Theorem
Classification	msc 15A69

We shall prove the following theorem on existence and uniqueness. (Here, we assime that the ground field has characteristic different from 2. This hypothesis is satisfied for the cases of greatest interest, namely real and complex ground fields.)

Theorem 1. *Every covariant and contravariant tensor of second rank may be expressed univocally as the sum of a symmetric and skew-symmetric tensor.*

Proof. Let us consider a contravariant tensor.

1. Existence. Put

$$U^{ij} = \frac{1}{2}(T^{ij} + T^{ji}), \qquad W^{ij} = \frac{1}{2}(T^{ij} - T^{ji}).$$

Then $U^{ij} = U^{ji}$ is symmetric, $W^{ij} = -W^{ji}$ is skew-symmetric, and

$$T^{ij} = U^{ij} + W^{ij}.$$

2. Uniqueness. Let us suppose that T^{ij} admits the decompositions

$$T^{ij} = U^{ij} + W^{ij} = U'^{ij} + W'^{ij}.$$

By taking the transposes

$$T^{ji} = U^{ji} + W^{ji} = U'^{ji} + W'^{ji},$$

we separate the symmetric and skew-symmetric parts in both equations and making use of their symmetry properties, we have

$$\begin{aligned} U^{ij} - U'^{ij} &= W'^{ij} - W^{ij} \\ = U^{ji} - U'^{ji} &= W'^{ji} - W^{ji} \\ = W^{ij} - W'^{ij} &= U'^{ij} - U^{ij} \\ = -(U^{ij} - U'^{ij}) &= 0, \end{aligned}$$

which shows uniqueness of each part. *mutatis mutandis* for a covariant tensor T_{ij}. □