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algebraic and geometric multiplicity do not coincide

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| Canonical name | AlgebraicAndGeometricMultiplicityDoNotCoincide |
| Date of creation | 2013-03-22 15:15:18 |
| Last modified on | 2013-03-22 15:15:18 |
| Owner | matte (1858) |
| Last modified by | matte (1858) |
| Numerical id | 5 |
| Author | matte (1858) |
| Entry type | Example |
| Classification | msc 15A18 |

Zero is an eigenvalue of

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

with algebraic multiplicity 2 and geometric multiplicity 1.

Indeed, as

$$\det(A - \lambda I) = \lambda^2$$

it follows that 0 is an eigenvalue of A with algebraic multiplicity 2. To find the geometric multiplicity of A we need to calculate $\ker A$. Thus, suppose

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

This implies $b = 0$, so

$$\ker A = \operatorname{span} \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

and the geometric multiplicity of 0 is 1.