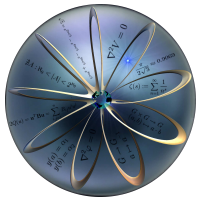

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reducible matrix

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An $n \times n$ matrix A is said to be a *reducible matrix* if and only if for some permutation matrix P, the matrix $P^T AP$ is block upper triangular. If a square matrix is not reducible, it is said to be an *irreducible matrix*.

The following conditions on an $n \times n$ matrix A are equivalent.

1. A is an irreducible matrix.
2. The digraph associated to A is strongly connected.
3. For each i and j, there exists some k such that $(A^k)_{ij} > 0$.
4. For any partition $J \sqcup K$ of the index set $\{1, 2, \ldots, n\}$, there exist $j \in J$ and $k \in K$ such that $a_{jk} \neq 0$.

For certain applications, irreducible matrices are more useful than reducible matrices. In particular, the Perron-Frobenius theorem gives more information about the spectra of irreducible matrices than of reducible matrices.