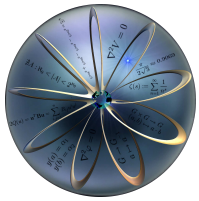

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non-degenerate quadratic form

Canonical name	NondegenerateQuadraticForm
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Entry type	Definition
Classification	msc 15A63
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Synonym	non degenerate quadratic form
Synonym	non singular quadratic form
Defines	non-degenerate quadratic form
Defines	non-singular quadratic form
Defines	regular quadratic form

Let k be a field of characteristic not 2. Then a quadratic form Q over a vector space V (over a field k) is said to be , if its associated bilinear form:

$$B(x, y) = \frac{1}{2}(Q(x + y) - Q(x) - Q(y))$$

is non-degenerate.

If we fix a basis $\boldsymbol{b}$ for V, then $Q(x)$ can be written as

$$Q(x) = x^T Ax$$

for some symmetric matrix A over k. Then it's not hard to see that Q is non-degenerate iff A is non-singular. Because of this, a non-degenerate quadratic form is also known as a *non-singular* quadratic form. A third name for a non-degenerate quadratic form is that of a *regular quadratic form*.