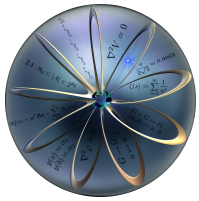

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proof of Weyl's inequality

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Let λ_i be the i-th eigenvalue of $A + E$. Then, by the Courant-Fisher min-max theorem and being $x^H E x \geq 0$ by hypothesis, we have:

$$\lambda_i(A+E) = \max_{S, dim S = i} \min_{\|x\| \neq 0} \frac{x^H (A+E) x}{x^H x} =$$

$$= \max_{S, \dim S = i} \min_{\|x\| \neq 0} \left(\frac{x^H A x}{x^H x} + \frac{x^H E x}{x^H x} \right) \geq \max_{S, \dim S = i} \min_{\|x\| \neq 0} \frac{x^H A x}{x^H x} = \lambda_i(A).$$