Hölder inequality

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Related topic	ConvolutionsOfComplexFunctionsOnLocallyCompactGroups
Related topic	LpNormIsDualToLq

The *Hölder inequality* concerns *vector p-norms*: given $1 \leq p,\, q \leq \infty$,

$$\text{If } \frac{1}{p} + \frac{1}{q} = 1 \text{ then } |x^T y| \leq ||\, x \,||_p ||\, y \,||_q$$

An important instance of a Hölder inequality is the *Cauchy-Schwarz inequality*.

There is a version of this result for the `http://planetmath.org/LpSpace`L^p spaces. If a function f is in $L^p(X)$, then the L^p-norm of f is denoted $||\, f \,||_p$. Given a measure space $(X, \mathfrak{B}, \mu)$, if f is in $L^p(X)$ and g is in $L^q(X)$ (with $1/p + 1/q = 1$), then the Hölder inequality becomes

$$\begin{aligned} \|fg\|_1 = \int_X |fg| \mathrm{d}\mu &\leq \left(\int_X |f|^p \mathrm{d}\mu \right)^{\frac{1}{p}} \left(\int_X |g|^q \mathrm{d}\mu \right)^{\frac{1}{q}} \\ &= \|f\|_p \, \|g\|_q \end{aligned}$$