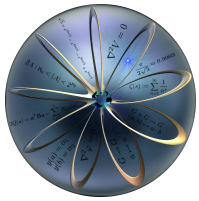

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equitable matrix

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Equitable matrices have been used in economics and group theory [**?**].

Definition 1. *An $n \times n$ matrix $M = (m_{ij})$ is an* equitable matrix *if all m_{ij} are positive, and $m_{ij} = m_{ik}m_{kj}$ for all $i, j, k = 1, \ldots, n$.*

Setting $i = j = k$ yields $m_{ii} = m_{ii}m_{ii}$ so diagonal elements of equitable matrices equal 1. Next, setting $i = j$ yields $m_{ii} = m_{ik}m_{ki}$, so $m_{ik} = 1/m_{ki}$.

Examples

1. An example of an equitable matrix of order n is

$$\begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix}.$$

   This example shows that equitable matrices exist for all n.

2. The most general equitable matrix of orders 2 and 3 are

$$\begin{pmatrix} 1 & a \\ 1/a & 1 \end{pmatrix},$$

   and

$$\begin{pmatrix} 1 & a & ab \\ 1/a & 1 & b \\ 1/ab & 1/b & 1 \end{pmatrix},$$

   where $a, b, c > 0$.

Properties

1. A $n \times n$ matrix $M = (m_{ij})$ is equitable if and only if it can be expressed in the form

$$m_{ij} = \exp(\lambda_i - \lambda_j)$$

   for real numbers $\lambda_1, \lambda_2, \ldots, \lambda_n$ with $\lambda_1 = 0$. (`http://planetmath.org/ParameterizationOf`

2. An equitable matrix is completely determined by its first row. If m_{1i}, $i = 1, \ldots, n$ are known, then

$$m_{ij} = \frac{m_{1j}}{m_{1i}}.$$

3. If M is an $n \times n$ equitable matrix, then

$$\exp(M) = I + \frac{e^n - 1}{n} M,$$

where exp is the matrix exponential.

4. Equitable matrices form a group under the Hadamard product [**?**].

5. If M is an $n \times n$ equitable matrix and $s\colon \{1, \ldots, r\} \to \{1, \ldots, n\}$ is a mapping, then

$$K_{ab} = M_{s(a)\, s(b)}, \quad a, b = 1, \ldots, r$$

is an equitable $r \times r$ matrix. In particular, striking the l:th row and column in an equitable matrix yields a new equitable matrix.

See [**?**] for further properties and references.

References

[1] H. Eves, *Elementary Matrix Theory*, Dover publications, 1980.