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minor (of a matrix)

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Related topic LaplaceExpansion Related topic CauchyBinetFormula

Defines principal minor

Defines cofactor

Given an $n \times m$ matrix A with entries a_{ij} , a minor of A is the determinant of a smaller matrix formed from its entries by selecting only some of the rows and columns. Let $K = \{k_1, k_2, \ldots, k_p\}$ and $L = \{l_1, l_2, \ldots, l_p\}$ be subsets of $\{1, 2, \ldots, n\}$ and $\{1, 2, \ldots, m\}$, respectively. The indices are chosen such that $k_1 < k_2 < \cdots < k_p$ and $l_1 < l_2 < \cdots < l_p$. The p-th order minor defined by K and L is the following determinant

$$A\begin{pmatrix} k_1 & k_2 & \cdots & k_p \\ l_1 & l_2 & \cdots & l_p \end{pmatrix} = \begin{vmatrix} a_{k_1l_1} & a_{k_1l_2} & \cdots & a_{k_1l_p} \\ a_{k_2l_1} & a_{k_2l_2} & \cdots & a_{k_2l_p} \\ \vdots & \vdots & \ddots & \vdots \\ a_{k_pl_1} & a_{k_pl_2} & \cdots & a_{k_pk_p} \end{vmatrix}.$$

If p exceeds either m or n, then the minor is automatically zero. When p = m = n, the minor is simply the determinant of the matrix. If K = L, then the minor is called *principal*. The word *minor* may also refer to just the matrix formed from the selected rows and columns, not necessarily its determinant. The precise meaning is usually clear from context.

There does not seem to be a standard notation for matrix minors. Another possible notation is $[A]_{K,L}$.

Some authors reserve the term *minor* for the case when only one row and one column are removed. This use is in conjunction with the concept of a http://planetmath.org/LaplaceExpansioncofactor.