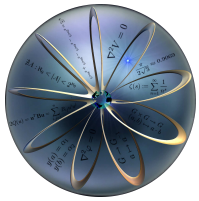

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cyclic decomposition theorem

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Defines	admissible subspace

Let k be a field, V a finite dimensional vector space over k and T a linear operator over V. Call a subspace $W \subseteq V$ *T-admissible* if W is T-invariant and for any polynomial $f(X) \in k[X]$ with $f(T)(v) \in W$ for $v \in V$, there is a $w \in W$ such that $f(T)(v) = f(T)(w)$.

Let W_0 be a proper T-admissible subspace of V. There are non zero vectors $x_1, ..., x_r$ in V with respective annihilator polynomials $p_1, ..., p_r$ such that

1. $V = W_0 \oplus Z(x_1, T) \oplus \cdots \oplus Z(x_r, T)$ (See the cyclic subspace definition)

2. p_k divides p_{k-1} for every $k = 2, ..., r$

Moreover, the integer r and the `http://planetmath.org/MinimalPolynomialEndomorphismminim` polynomials $p_1, ..., p_r$ are uniquely determined by (1),(2) and the fact that none of x_k is zero.

This is "one of the deepest results in linear algebra" (Hoffman & Kunze)