AB is conjugate to BA

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Proposition 1. *Given square matrices A and B where one is invertible then AB is conjugate to BA.*

Proof. If A is invertible then $A^{-1}ABA = BA$. Similarly if B is invertible then B serves to conjugate BA to AB. $\square$

The result of course applies to any ring elements a and b where one is invertible. It also holds for all group elements.

Remark 2. *This is a partial generalization to the observation that the Cayley table of an abelian group is symmetric about the main diagonal. In abelian groups this follows because $AB = BA$. But in non-abelian groups AB is only conjugate to BA. Thus the conjugacy class of a group are symmetric about the main diagonal.*

Corollary 3. *If A or B is invertible then AB and BA have the same eigenvalues.*

This leads to an alternate proof of `http://planetmath.org/ABAndBAAreAlmostIsospectral`A... and BA being almost isospectral. If A and B are both non-invertible, then we restrict to the non-zero eigenspaces E of A so that A is invertible on E. Thus $(AB)|_E$ is conjugate to $(BA)|_E$ and so indeed the two transforms have identical non-zero eigenvalues.