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matrix condition number

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1 Matrix Condition Number

The *condition number for matrix inversion* with respect to a matrix norm $\|\cdot\|$ of a square matrix A is defined by

$$\kappa(A) = \|A\| \|A^{-1}\| \,,$$

if A is non-singular; and $\kappa(A) = +\infty$ if A is singular.

The condition number is a measure of stability or sensitivity of a matrix (or the linear system it represents) to numerical operations. In other words, we may not be able to trust the results of computations on an ill-conditioned matrix.

Matrices with condition numbers near 1 are said to be *well-conditioned*. Matrices with condition numbers much greater than one (such as around 10^5 for a 5×5 Hilbert matrix) are said to be *ill-conditioned*.

If $\kappa(A)$ is the condition number of A, then $\kappa(A)$ measures a sort of inverse distance from A to the set of singular matrices, normalized by $\|A\|$. Precisely, if A is invertible, and $\|B - A\| < \|A^{-1}\|^{-1}$, then B must also be invertible. On the other hand, in the case of the 2-norm, there always exists a singular matrix B such that $\|B - A\|_2 = \|A^{-1}\|_2^{-1}$ (so the distance estimate is sharp).

References

[1] Golub and Van Loan. *Matrix Computations*, 3rd edition. Johns Hopkins University Press, 1996.