# matrix resolvent properties

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The matrix resolvent norm for a complex-valued $s$ is related to the proximity of such value to the spectrum of $A$; more precisely, the following simple yet meaningful property holds:

$$\|R_A(s)\| \geq \frac{1}{\operatorname{dist}(s, \sigma_A)},$$

where $\|.\|$ is any self consistent matrix norm, $\sigma_A$ is the spectrum of $A$ and the distance between a complex point and the discrete set of the eigenvalues $\lambda_i$ is defined as $\operatorname{dist}(s, \sigma_A) = \min\limits_{1 \leq i \leq n} |s - \lambda_i|$.

From this fact it comes immediately, for any $1 \leq i \leq n$,

$$\lim_{s \to \lambda_i} \|R_A(s)\| = +\infty.$$

*Proof.* Let ($\lambda_i$,$\mathbf{v}$) be an eigenvalue-eigenvector pair of $A$; then

$$(sI - A)v = sv - Av = (s - \lambda_i)v$$

which shows $(s - \lambda_i)$ to be an eigenvalue of $(sI - A)$; $(s - \lambda_i)^{-1}$ is then an eigenvalue of $(sI - A)^{-1}$ and , since for any self consistent norm $|\lambda| \leq \|A\|$, we have:

$$\max_{1 \leq i \leq n} \frac{1}{|s - \lambda_i|} \leq \|(sI - A)^{-1}\|$$

whence

$$\|(sI - A)^{-1}\| \geq \frac{1}{\min\limits_{1 \leq i \leq n} |s - \lambda_i|} = \frac{1}{\operatorname{dist}(s, \sigma_A)}.$$

$\square$