example of Cramer's rule

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Say we want to solve the system

$$\begin{aligned}
3x + 2y + z - 2w &= 4 \\
2x - y + 2z - 5w &= 15 \\
4x + 2y \phantom{{}+2z} - 5w &= 1 \\
3x \phantom{{}+2y} - 2z - 4w &= 1.
\end{aligned}$$

The associated matrix is

$$\begin{pmatrix} 3 & 2 & 1 & -2 \\ 2 & -1 & 2 & -5 \\ 4 & 2 & 0 & -1 \\ 3 & 0 & -2 & -4 \end{pmatrix}$$

whose determinant is $\Delta = -65$. Since the determinant is non-zero, we can use Cramer's rule. To obtain the value of the k-th variable, we replace the k-th column of the matrix above by the column vector

$$\begin{pmatrix} 4 \\ 15 \\ 1 \\ 1 \end{pmatrix},$$

the determinant of the obtained matrix is divided by Δ and the resulting value is the wanted solution.

So

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 4 & 2 & 1 & -2 \\ 15 & -1 & 2 & -5 \\ 1 & 2 & 0 & -1 \\ 1 & 0 & -2 & -4 \end{vmatrix}}{-65} = \frac{-65}{-65} = 1$$

$$y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 4 & 1 & -2 \\ 2 & 15 & 2 & -5 \\ 4 & 1 & 0 & -1 \\ 3 & 1 & -2 & -4 \end{vmatrix}}{-65} = \frac{130}{-65} = 2$$

$$z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 3 & 2 & 4 & -2 \\ 2 & -1 & 15 & -5 \\ 4 & 2 & 1 & 1 \\ 3 & 0 & 1 & -4 \end{vmatrix}}{-65} = \frac{-195}{-65} = 3$$

$$w = \frac{\Delta_4}{\Delta} = \frac{\begin{vmatrix} 3 & 2 & 1 & 4 \\ 2 & -1 & 2 & 15 \\ 4 & 2 & 0 & 1 \\ 3 & 0 & -2 & 1 \end{vmatrix}}{-65} = \frac{65}{-65} = -1$$