linearly independent

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Let V be a vector space over a field F. We say that $v_1, \ldots, v_k \in V$ are *linearly dependent* if there exist scalars $\lambda_1, \ldots, \lambda_k \in F$, not all zero, such that

$$\lambda_1 v_1 + \cdots + \lambda_k v_k = 0.$$

If no such scalars exist, then we say that the vectors are *linearly independent*. More generally, we say that a (possibly infinite) subset $S \subset V$ is linearly independent if all finite subsets of S are linearly independent.

In the case of two vectors, linear dependence means that one of the vectors is a scalar multiple of the other. As an alternate characterization of dependence, we also have the following.

Proposition 1. *Let $S \subset V$ be a subset of a vector space. Then, S is linearly dependent if and only if there exists a $v \in S$ such that v can be expressed as a linear combination of the vectors in the set $S \setminus \{v\}$ (`http://planetmath.org/SetDifference`all the vectors in S other than v).*

Remark. Linear independence can be defined more generally for modules over rings: if M is a (left) module over a ring R. A subset S of M is linearly independent if whenever $r_1 m_1 + \cdots + r_n m_n = 0$ for $r_i \in R$ and $m_i \in M$, then $r_1 = \cdots = r_n = 0$.