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unit vector

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A *unit vector* is a unit-length element of Euclidean space. Equivalently, one may say that the norm of a unit vector is equal to 1, and write $\|\mathbf{u}\| = 1$, where \mathbf{u} is the vector in question.

Let \mathbf{v} be a non-zero vector. To *normalize* \mathbf{v} is to find the unique unit vector with the same direction as \mathbf{v} . This is done by multiplying \mathbf{v} by the reciprocal of its length; the corresponding unit vector is given by $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$.

Note: The concept of a unit vector and normalization makes sense in any vector space equipped with a real or complex norm. Thus, in quantum mechanics one represents states as unit vectors belonging to a (possibly) infinite-dimensional Hilbert space. To obtain an expression for such states one normalizes the results of a calculation.

Example: Consider \mathbb{R}^3 and the vector $\mathbf{v} = (1, 2, 3)$. The norm (length) is $\sqrt{14}$. Normalizing, we obtain the unit vector \mathbf{u} pointing in the same direction, namely $\mathbf{u} = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$.