Cauchy matrix

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Defines	Cauchy matrices

Let $x_1,\ x_2, \ldots, x_m$, and $y_1,\ y_2 \ldots, y_n$ be elements in a field F, satisfying the that

1. $x_1, \ldots, x_m$ are distinct,
2. $y_1, \ldots, y_n$ are distinct, and
3. $x_i + y_j \neq 0$ for $1 \leq i \leq m$, $1 \leq j \leq n$.

The matrix

$$\begin{bmatrix} \frac{1}{x_1+y_1} & \frac{1}{x_1+y_2} & \cdots & \frac{1}{x_1+y_n} \\ \frac{1}{x_2+y_1} & \frac{1}{x_2+y_2} & \cdots & \frac{1}{x_2+y_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{x_m+y_1} & \frac{1}{x_m+y_2} & \cdots & \frac{1}{x_m+y_n} \end{bmatrix}$$

is called a *Cauchy matrix* over F.

The determinant of a square Cauchy matrix is

$$\frac{\prod_{i<j}(x_i - x_j)(y_i - y_j)}{\prod_{ij}(x_i + y_j)}$$

Since x_i's are distinct and y_j's are distinct by definition, a square Cauchy matrix is non-singular. Any submatrix of a rectangular Cauchy matrix has full rank.