contraction

Canonical name	Contraction
Date of creation	2013-03-22 13:37:28
Last modified on	2013-03-22 13:37:28
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	4
Author	mathcam (2727)
Entry type	Definition
Classification	msc 15A75
Classification	msc 58A10

Definition Let ω be a smooth k-form on a smooth manifold M, and let ξ be a smooth vector field on M. The *contraction* of ω with ξ is the smooth $(k-1)$-form that maps $x \in M$ to $\omega_x(\xi_x, \cdot)$. In other words, ω is point-wise evaluated with ξ in the first slot. We shall denote this $(k-1)$-form by $\iota_\xi \omega$. If ω is a 0-form, we set $\iota_\xi \omega = 0$ for all ξ.

Properties Let ω and ξ be as above. Then the following properties hold:

1. For any real number k
$$\iota_{k\xi}\omega = k\iota_\xi \omega.$$

2. For vector fields ξ and η
$$\begin{aligned} \iota_{\xi+\eta}\omega &= \iota_\xi \omega + \iota_\eta \omega, \\ \iota_\xi \iota_\eta \omega &= -\iota_\eta \iota_\xi \omega, \\ \iota_\xi \iota_\xi \omega &= 0. \end{aligned}$$

3. Contraction is an anti-derivation [**?**]. If ω^1 is a p-form, and ω^2 is a q-form, then
$$\iota_\xi\big(\omega^1 \wedge \omega^2\big) = (\iota_\xi \omega^1) \wedge \omega^2 + (-1)^p\, \omega^1 \wedge (\iota_\xi \omega^2).$$

References

[1] T. Frankel, *Geometry of physics*, Cambridge University press, 1997.