example of construction of a Schauder basis

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Consider an uniformly continuous function $f : [0,1] \to \mathbb{R}$. A Schauder basis $\{f_n(x)\}_0^\infty \in C[0,1]$ is constructed. For this purpose we set $f_0(x) = 1$, $f_1(x) = x$. Let us consider the sequence of semi-open intervals in $[0,1]$

$$I_n = [2^{-k}(2n-2), 2^{-k}(2n-1)), \qquad J_n = [2^{-k}(2n-1), 2^{-k}2n),$$

where $2^{k-1} < n \leq 2^k$, $k \geq 1$. Define now

$$f_n(x) = \begin{cases} 2^k[x - (2^{-k}(2n-2) - 1)] & \text{if } x \in I_n, \\ 1 - 2^k[x - (2^{-k}(2n-1) - 1)] & \text{if } x \in J_n, \\ 0 & \text{otherwise.} \end{cases}$$

Geometrically these functions form a sequence of triangular functions of height *one* and width $2^{-(k-1)}$, sweeping $[0,1]$. So that if $f \in C([0,1])$, it is expressible in Fourier series $f(x) \sim \sum_{n=0}^{\infty} c_n f_n(x)$ and computing the coefficients c_n by equating the values of $f(x)$ and the series at the points $x = 2^{-k}m$, $m = 0, 1, \ldots, 2^k$. The resulting series converges uniformly to $f(x)$ by the imposed premise.