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## Dieudonné theorem on linear preservers of the singular matrices

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Let  $\mathbb{F}$  be an arbitrary field. Consider  $\mathcal{M}_n(\mathbb{F})$ , the vector space of all  $n \times n$  matrices over  $\mathbb{F}$ . Moreover, let  $\mathcal{GL}_n(\mathbb{F})$  be the full linear group of nonsingular  $n \times n$  matrices over  $\mathbb{F}$ .

**Theorem 1.** *For a linear automorphism  $\varphi : \mathcal{M}_n(\mathbb{F}) \longrightarrow \mathcal{M}_n(\mathbb{F})$  the following conditions are equivalent:*

- (i)  $\forall A \in \mathcal{M}_n(\mathbb{F}) : \det(A) = 0 \Rightarrow \det(\varphi(A)) = 0$ ,
- (ii) *either  $\exists P, Q \in \mathcal{GL}_n(\mathbb{F}) \forall A \in \mathcal{M}_n(\mathbb{F}) : \varphi(A) = PAQ$ , or  $\exists P, Q \in \mathcal{GL}_n(\mathbb{F}) \forall A \in \mathcal{M}_n(\mathbb{F}) : \varphi(A) = PQA$*

The original proof [?] of the nontrivial implication (i)  $\Rightarrow$  (ii) is based on the fundamental theorem of projective geometry.

## References

- [D] J. Dieudonné, Sur une généralisation du groupe orthogonal à quatre variables, *Arch. Math.* **1**: 282–287 (1949).