



Euclidean vector space

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1 Definition

The term *Euclidean vector space* is synonymous with *finite-dimensional, real, positive definite, inner product space*. The canonical example is \mathbb{R}^n , equipped with the usual dot product. Indeed, every Euclidean vector space V is isomorphic to \mathbb{R}^n , up to a choice of orthonormal basis of V . As well, every Euclidean vector space V carries a natural metric space structure given by

$$d(u, v) = \sqrt{\langle u - v, u - v \rangle}, \quad u, v \in V.$$

2 Remarks.

- An analogous object with complex numbers as the base field is called a unitary space.
- Dropping the assumption of finite-dimensionality we arrive at the class of real pre-Hilbert spaces.
- If we drop the inner product and the vector space structure, but retain the metric space structure, we arrive at the notion of a Euclidean space.