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partitioned matrix

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A *partitioned matrix*, or a *block matrix*, is a matrix M that has been constructed from other smaller matrices. These smaller matrices are called *blocks* or *sub-matrices* of M.

For instance, if we partition the below 5×5 matrix as follows

$$L = \left(\begin{array}{cc|ccc} 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 1 & 2 & 3 \\ \hline 2 & 3 & 9 & 9 & 9 \\ 2 & 3 & 9 & 9 & 9 \\ 2 & 3 & 9 & 9 & 9 \end{array}\right),$$

then we can define the matrices

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, C = \begin{pmatrix} 2 & 3 \\ 2 & 3 \\ 2 & 3 \end{pmatrix}, D = \begin{pmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{pmatrix}$$

and write L as

$$L = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \text{ or } L = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right).$$

If $A_1, \ldots, A_n$ are square matrices (of possibly different sizes), then we define the *direct sum* of the matrices $A_1, \ldots, A_n$ as the partitioned matrix

$$\operatorname{diag}(A_1, \ldots, A_n) = \left(\begin{array}{c|c|c} A_1 & & \\ \hline & \ddots & \\ \hline & & A_n \end{array}\right),$$

where the off-diagonal blocks are zero.

If A and B are matrices of the same size partitioned into blocks of the same size, the partition of the sum is the sum of the partitions.

If A and B are $m \times n$ and $n \times k$ matrices, respectively, then if the blocks of A and B are of the correct size to be multiplied, then the blocks of the product are the products of the blocks.