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diagonalization of quadratic form

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A quadratic form may be diagonalized by the following procedure:

1. Find a variable x such that x^2 appears in the quadratic form. If no such variable can be found, perform a linear change of variable so as to create such a variable.
2. By completing the square, define a new variable x' such that there are no cross-terms involving x' .
3. Repeat the procedure with the remaining variables.

Example Suppose we have been asked to diagonalize the quadratic form

$$Q = x^2 + xy - 3xz - y^2/4 + yz - 9z^2/4$$

in three variables. We could proceed as follows:

- Since x^2 appears, we do not need to perform a change of variables.
- We have the cross terms xy and $-3xz$. If we define $x' = x + y/2 - 3z/2$, then

$$x'^2 = x^2 + xy - 3xz + y^2/4 + 9z^2/4 - 3yz/2$$

Hence, we may re-express Q as

$$Q = x'^2 - yz/2$$

- We must now repeat the procedure with the remaining variables, y and z . Since neither y^2 nor z^2 appears, we must make a change of variable. Let us define $z' = z + 2y$.

$$Q = x'^2 - y^2 - yz'/2$$

- We have a cross term $-yz'/2$. To eliminate this term, make a change of variable $y' = y + z'/4$. Then we have

$$y'^2 = y^2 + yz'/2 + z'^2/16$$

and hence

$$Q = x'^2 - y'^2 + z'^2/16$$

The quadratic form is now diagonal, so we are done. We see that the form has rank 3 and signature 2.