n'th derivative of a determinant

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Let $A = (a_{i,j})$ be a $d \times d$ matrix whose entries are real functions of t. Then,

$$\frac{d^n}{dt^n} \det(A) = \sum_{n_1+\cdots+n_d=n} \binom{n}{n_1, n_2, ..., n_d} \sum_{\pi \in S_d} \mathrm{sgn}(\pi) \prod_{i=1}^{d} \frac{d^{n_i}}{dt^{n_i}} a_{i,\pi(i)}$$

$$= \sum_{n_1+\cdots+n_d=n} \binom{n}{n_1, n_2, ..., n_d} \det \begin{pmatrix} \frac{d^{n_1}}{dt^{n_1}} a_{1,1} & \frac{d^{n_1}}{dt^{n_1}} a_{1,2} & \cdots & \frac{d^{n_1}}{dt^{n_1}} a_{1,d} \\ \vdots & \vdots & & \vdots \\ \frac{d^{n_d}}{dt^{n_d}} a_{d,1} & \frac{d^{n_d}}{dt^{n_d}} a_{d,2} & \cdots & \frac{d^{n_d}}{dt^{n_d}} a_{d,d} \end{pmatrix}$$

where $\binom{n}{n_1, n_2, ..., n_r}$ is the multinomial coefficient, S_d is the symmetric group of permutations and $\mathrm{sgn}(\pi)$ is the sign of a permutation π.