conformal partitioning

Canonical name	ConformalPartitioning
Date of creation	2013-03-22 16:04:16
Last modified on	2013-03-22 16:04:16
Owner	Mathprof (13753)
Last modified by	Mathprof (13753)
Numerical id	9
Author	Mathprof (13753)
Entry type	Definition
Classification	msc 15-00
Defines	block multiplication

Let R be a ring. Let the matrices $A \in M_{m,n}(R)$ and $B \in M_{n,p}(R)$ be partitioned into submatrices $A^{i,j}$ and $B^{i,j}$ respectively as follows:

$$A = \begin{array}{c} \begin{matrix} n_1 & n_2 & \cdots & n_h \end{matrix} \\ \begin{bmatrix} A^{1,1} & A^{1,2} & \dots & A^{1,h} \\ A^{2,1} & A^{2,2} & \dots & A^{2,h} \\ \vdots & \vdots & & \vdots \\ A^{g,1} & A^{g,2} & \dots & A^{g,h} \end{bmatrix} \end{array} \begin{matrix} \}m_1 \\ \}m_2 \\ \vdots \\ \}m_g \end{matrix}$$

where $A^{i,j}$ is $m_i \times n_j, \sum_{i=1}^{g} m_i = m$, $\sum_{j=1}^{h} n_j = n$;

$$B = \begin{array}{c} \begin{matrix} p_1 & p_2 & \cdots & p_k \end{matrix} \\ \begin{bmatrix} B^{1,1} & B^{1,2} & \dots & B^{1,k} \\ B^{2,1} & B^{2,2} & \dots & B^{2,k} \\ \vdots & \vdots & & \vdots \\ B^{h,1} & B^{h,2} & \dots & B^{h,k} \end{bmatrix} \end{array} \begin{matrix} \}n_1 \\ \}n_2 \\ \vdots \\ \}n_h \end{matrix}$$

where $B^{i,j}$ is $n_i \times p_j$, $\sum_{j=1}^{k} p_j = p$. Then A and B (in this) are said to be *conformally partitioned* for multiplication.

Now suppose that A and B are conformally partitioned for multiplication. Let $C = AB$ be partitioned as follows:

$$C = \begin{array}{c} \begin{matrix} p_1 & p_2 & \cdots & p_k \end{matrix} \\ \begin{bmatrix} C^{1,1} & C^{1,2} & \dots & C^{1,k} \\ C^{2,1} & C^{2,2} & \dots & C^{2,k} \\ \vdots & \vdots & & \vdots \\ C^{g,1} & C^{g,2} & \dots & C^{g,k} \end{bmatrix} \end{array} \begin{matrix} \}m_1 \\ \}m_2 \\ \vdots \\ \}m_g \end{matrix}$$

where $C^{i,j}$ is $m_i \times p_j$, $i = 1, \cdots, g$, $j = 1, \cdots, k$. Then

$$C^{i,j} = \sum_{t=1}^{k} A^{i,t} B^{t,j}, \quad i = 1, \cdots, g, \quad j = 1, \cdots, k.$$

This method of computing AB is sometimes called *block multiplication*.