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dual space separates points

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The following result is a corollary of the Hahn-Banach theorem.

Theorem - Let X be a normed vector space. Given a linearly independent set $\{x_1, \dots, x_n\} \subset X$ there exist continuous linear functionals $f_1, \dots, f_n \in X'$ such that

$$f_j(x_k) = \delta_{jk} \quad , 1 \leq j, k \leq n$$

If $x \in \text{span}\{x_1, \dots, x_n\}$, then $x = \sum_{j=1}^n f_j(x)x_j$.

The above theorem shows that if $f(x) = f(y)$ for every continuous linear functional f then $x = y$, therefore the dual space X' separates the points of X .