an application of Z-matrix in a mobile radio system

Canonical name	AnApplicationOfZmatrixInAMobileRadioSystem
Date of creation	2013-03-22 16:14:16
Last modified on	2013-03-22 16:14:16
Owner	kshum (5987)
Last modified by	kshum (5987)
Numerical id	6
Author	kshum (5987)
Entry type	Application
Classification	msc 15A99

The following is an application of Z-matrix in wireless communication called power balancing problem.

Consider n pairs of mobile users and receiving antennae. For $i = 1, \ldots, n$, mobile user i transmits radio signal to antenna i. Mobile user i transmits at power P_i. The radio channel attenuate the signal and user i's signal is received at antenna i with power $G_{ii}P_i$, where G_{ii} denote the channel gain. The radio signals also interfere each other. At antenna i, the interference due to user j has power $G_{ij}P_j$. The receiver noise power at antenna i is denoted by n_i. The signal to interference plus noise at receiver i is

$$\Gamma_i = \frac{G_{ii}P_i}{\sum_{j\neq i} G_{ij}P_j + n_i}$$

To guarantee the quality of received signal, it is required that the signal to interference plus noise ratio Γ_i is equal to a predefined constant γ_i for all i. Given γ_i, $i = 1, \ldots, n$, we want to find $P_1, \ldots, P_n$ such that the above equation holds for $i = 1, \ldots, n$. Let A be the $n \times n$ matrix with zero diagonal and (i, j)-entry $(G_{ij}\gamma_i)/G_{ii}$ for $i \neq j$. We want to solve

$$(I - A)\mathbf{p} = \mathbf{n}$$

where $\mathbf{p} = (P_1, \ldots, P_n)^T$ is the power vector and $\mathbf{n} = (n_i\gamma_i/G_{ii})_{i=1}^n$. The matrix $I - A$ is a Z-matrix, since all G_{ij} and γ_i are positive constants. The required power vector is $(I - A)^{-1}\mathbf{n}$ if $I - A$ is invertible. We also required that the components of $\mathbf{p}$ to be positive as power cannot be negative. The resulting power vector $(I - A)^{-1}\mathbf{n}$ has positive components if $(I - A)^{-1}$ is a non-negative matrix. In such case, $I - A$ is an M-matrix.