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stufe of a field

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Defines	stufe

The *stufe* of a field F is the least number n such that -1 can be expressed as a sum of n squares:

$$-1 = a_1^2 + \cdots + a_n^2,$$

where each $a_i \in F$. If no such an n exists, then we say that the *stufe* of F is ∞ .

Remarks.

- The word “stufe”, meaning “level” in German, is attributed to mathematician Albrecht Pfister.
- A theorem of Pfister asserts that in a field F , if -1 can be expressed as a finite sum of squares, then the stufe of F is a power of 2.

References

- [1] A. Pfister, *Zur Darstellung definiter Funktionen als Summe von Quadraten*, Inventiones Mathematicae (1967).
- [2] A. R. Rajwade, *Squares*, Cambridge University Press (1993).