solid angle

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Defines	full solid angle
Defines	steradian
Defines	trihedral angle

A conical surface may contain a certain portion Ω of the space $\mathbb{R}^3$. This portion is called *solid angle* or *space angle*. If the conical surface contains a portion A of a spherical surface with radius R and with `http://planetmath.org/Sphere`centre P in the of the solid angle, then the magnitude of the solid angle is given by

$$\Omega = \frac{A}{R^2}$$

which is on the radius R. The spherical surface can be replaced by any surface a, through which all the half-lines originating from P and being contained in the solid angle go. Then the solid angle may be computed from the

$$\Omega = -\int_a d\vec{a} \cdot \nabla \frac{1}{r}, \tag{1}$$

where r is the length of the position vector $\vec{r}$ for the points on the surface a. The *full solid angle*, consisting of all points of $\mathbb{R}^3$, has the magnitude 4π.

The SI of solid angle, analogous to the angle radian, is the *steradian* (= 1 sr). The steradian takes a proportion $\frac{1}{4\pi}$, or approximately 7.957747 %, of the surface area of a sphere.

If the solid angle is bounded by three planes having exactly one common point, it may be called a *trihedral angle*; cf. the example 2!

Example 1. The solid angle determined by a right circular cone with the angle α between its axis and is equal to $2\pi(1-\cos\alpha)$, i.e. $4\pi\sin^2\dfrac{\alpha}{2}$.

Example 2. Let $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$ be the position vectors of three points in $\mathbb{R}^3$ and r_1, r_2, r_3 their lengths. Then the solid angle Ω of the tetrahedron by the vectors $\vec{r}_i$ is obtained from the equation

$$\tan\frac{\Omega}{2} = \frac{\vec{r}_1\times\vec{r}_2\cdot\vec{r}_3}{(\vec{r}_1\cdot\vec{r}_2)r_3 + (\vec{r}_2\cdot\vec{r}_3)r_1 + (\vec{r}_3\cdot\vec{r}_1)r_2 + r_1r_2r_3}, \tag{2}$$

where the numerator of the is the triple scalar product of the vectors. This equation is expressed simplier using the unit vectors $\vec{u}_i$ corresponding $\vec{r}_i$:

$$\tan\frac{\Omega}{2} = \frac{\vec{u}_1\times\vec{u}_2\cdot\vec{u}_3}{1+\vec{u}_2\cdot\vec{u}_3+\vec{u}_3\cdot\vec{u}_1+\vec{u}_1\cdot\vec{u}_2}$$

The result (2) is due to van Oosterom and Strackee 1983.

Example 3. Using (2), one can easily get the http://planetmath.org/ConeInMathbbR3apical solid angle of a http://planetmath.org/ConeInMathbbR3right pyramid with square base:

$$\Omega \;=\; 4\arctan\frac{a^2}{2h\sqrt{2a^2+4h^2}} \;=\; 4\arcsin\frac{a^2}{a^2+4h^2}$$

Here a is the side of the base square and h is the http://planetmath.org/ConeInMathbbR3height of the pyramid. Cf. the solid angle of rectangular pyramid.

References

[1] A. van Oosterom, J. Strackee: A solid angle of a plane triangle. – *IEEE Trans. Biomed. Eng.* **30**:2 (1983); 125–126.

[2] M. S. Gossman, A. J. Pahikkala, M. B. Rising, P. H. McGinley: Providing Solid Angle Formalism for Skyshine Calculations. – *Journal of Applied Clinical Medical Physics.* **11**:4 (2010); 278–282.