



compound matrix

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Entry type	Definition
Classification	msc 15-00
Defines	rth adjugate
Defines	Sylvester -Franke theorem

Suppose that A is an $m \times n$ matrix with entries from a field F and $1 \leq r \leq \min(m, n)$. The r^{th} *compound matrix* or r^{th} of A is the $\binom{m}{r} \times \binom{n}{r}$ matrix whose entries are $\det A[\alpha, \beta]$, $\alpha \in Q_{r,m}$ and $\beta \in Q_{r,n}$, arranged in lexicographic order and we use submatrix notation. The notation for this matrix is $C_r(A)$.

Properties

1. $C_r(AB) = C_r(A)C_r(B)$ when r is less than or equal to the number of rows or columns of A and B
2. If A is nonsingular, the $C_r(A)^{-1} = C_r(A^{-1})$.
3. If A has complex entries, then $C_r(A^*) = (C_r(A))^*$.
4. $C_r(A^T) = (C_r(A))^T$
5. $C_r(\overline{A}) = \overline{C_r(A)}$
6. For any $k \in F$ $C_r(kA) = k^r C_r(A)$
7. $C_r(I_n) = I_{\binom{n}{r}}$
8. $\det(C_r(A)) = \det(A)^{\binom{n-1}{r-1}}$ (Sylvester — Franke theorem)