Hermitian form over a division ring

Canonical name	HermitianFormOverADivisionRing
Date of creation	2013-03-22 15:41:04
Last modified on	2013-03-22 15:41:04
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	12
Author	CWoo (3771)
Entry type	Definition
Classification	msc 15A63
Defines	Hermitian form
Defines	skew Hermitian form

Let D be a division ring admitting an `http://planetmath.org/Involution2`involution $*$. Let V be a vector space over D. A *Hermitian form* over D is a function from $V \times V$ to D, denoted by $(\cdot, \cdot)$ with the following properties, for any $v, w \in V$ and $d \in D$:

1. $(\cdot, \cdot)$ is additive in each of its arguments,

2. $(du, v) = d(u, v)$,

3. $(u, dv) = (u, v)d^*$,

4. $(u, v) = (v, u)^*$.

Note that if the Hermitian form $(\cdot, \cdot)$ is non-trivial and if $*$ is the identity on D, then D is a field and $(\cdot, \cdot)$ is just a symmetric bilinear form.

If we replace the last condition by $(u, v) = -(v, u)^*$, then $(\cdot, \cdot)$ over D is called a *skew Hermitian form.*

Remark. Every skew Hermitian form over a division ring induces a Hermitian form and vice versa.