block determinants

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If A and D are square matrices

- If A^{-1} exists, then

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(A) \det(D - CA^{-1}B)$$

- If D^{-1} exists, then

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(D) \det(A - BD^{-1}C)$$

The matrices $D - CA^{-1}B$ and $A - BD^{-1}C$ are called the Schur complements of A and D, respectively.
Mention that

- If A, D are square matrices, then

$$\det \begin{pmatrix} A & B \\ O & D \end{pmatrix} = \det(A) \det(D)$$

  , where O is a zero matrix.

- Also we have that

$$\det \begin{pmatrix} A & O \\ O & B \end{pmatrix} = \det(A) \det(B).$$

- Another useful result for block determinants is the following. As $J = \begin{pmatrix} O & I \\ -I & O \end{pmatrix}$ is a symplectic matrix, we have that $\det J = 1$. Using now the fact that $\det MN = \det(M) \det(N)$ for any M, N square matrices, we have that

$$\det \begin{pmatrix} O & A \\ B & O \end{pmatrix} = \det \begin{pmatrix} O & A \\ B & O \end{pmatrix} \det J = -\det(A) \det(B)$$

This holds for any square matrices A, B and for the last point A, B have also the same order. They do not need to be invertible.