generalized inverse

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Let A be an $m \times n$ matrix with entries in $\mathbb{C}$. A *generalized inverse*, denoted by A^-, is an $n \times m$ matrix with entries in $\mathbb{C}$, such that

$$AA^-A = A.$$

Examples

1. Let
$$A = \begin{pmatrix} 2 & 3 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$
Then any matrix of the form
$$A^- = \begin{pmatrix} 2 & -3 & a \\ -1 & 2 & b \\ c & d & e \end{pmatrix},$$
where a, b, c, d and $e \in \mathbb{C}$, is a generalized inverse.

2. Using the same example from above, if $a = b = c = d = e = 0$, then we have an example of the *Moore-Penrose generalized inverse*, which is a unique matrix.

3. Again, using the example from above, if $a = b = c = d = 0$ and e is any complex number, we have an example of a *Drazin inverse.*

Remark Generalized inverse of a matrix has found many applications in statistics. For example, in general linear model, one solves the set of normal equations
$$\mathbf{X}^{\mathrm{T}}\mathbf{X}\boldsymbol{\beta} = \mathbf{X}^{\mathrm{T}}\mathbf{Y},$$
to get the MLE $\hat{\boldsymbol{\beta}}$ of the parameter vector $\boldsymbol{\beta}$. If the design matrix $\mathbf{X}$ is not of full rank (this occurs often when the model is either an ANOVA or ANCOVA type) and hence $\mathbf{X}^{\mathrm{T}}\mathbf{X}$ is singular. Then the MLE can be given by

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^-\mathbf{X}^{\mathrm{T}}\mathbf{Y}.$$