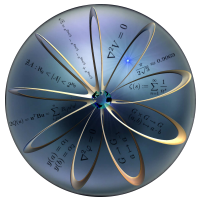

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positive definite

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Introduction

The definiteness of a matrix is an important property that has use in many areas of mathematics and physics. Below are some examples:

1. In optimizing problems, the definiteness of the Hessian matrix determines the quality of an extremal value. The full details can be found on `http://planetmath.org/RelationsBetweenHessianMatrixAndLocalExtrema`this page.

Definition [**?**] Suppose A is an $n \times n$ square Hermitian matrix. If, for any non-zero vector x, we have that

$$x^* A x > 0,$$

then A a *positive definite* matrix. (Here $x^* = \overline{x}^t$, where $\overline{x}$ is the complex conjugate of x, and x^t is the transpose of x.)

One can show that a Hermitian matrix is positive definite if and only if all its eigenvalues are positive [**?**]. Thus the determinant of a positive definite matrix is positive, and a positive definite matrix is always invertible. The Cholesky decomposition provides an economical method for solving linear equations involving a positive definite matrix. Further conditions and properties for positive definite matrices are given in [**?**].

References

[1] M. C. Pease, *Methods of Matrix Algebra*, Academic Press, 1965

[2] C.R. Johnson, *Positive definite matrices*, American Mathematical Monthly, Vol. 77, Issue 3 (March 1970) 259-264.