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companion matrix

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Given a monic polynomial $p(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ the *companion matrix* of $p(x)$, denoted $\mathcal{C}_{p(x)}$, is the $n \times n$ matrix with 1's down the first subdiagonal and minus the coefficients of $p(x)$ down the last column, or alternatively, as the transpose of this matrix. Adopting the first convention this is simply

$$\mathcal{C}_{p(x)} = \begin{pmatrix} 0 & 0 & \dots & \dots & \dots & -a_0 \\ 1 & 0 & \dots & \dots & \dots & -a_1 \\ 0 & 1 & \dots & \dots & \dots & -a_2 \\ 0 & 0 & \ddots & & & \vdots \\ \vdots & \vdots & & \ddots & & \vdots \\ 0 & 0 & \dots & \dots & 1 & -a_{n-1} \end{pmatrix}.$$

Regardless of which convention is used the `http://planetmath.org/MinimalPolynomialEndom` polynomial of $\mathcal{C}_{p(x)}$ equals $p(x)$, and the characteristic polynomial of $\mathcal{C}_{p(x)}$ is just $(-1)^n p(x)$. The $(-1)^n$ is needed because we have defined the characteristic polynomial to be $\det(\mathcal{C}_{p(x)} - xI)$. If we had instead defined the characteristic polynomial to be $\det(xI - \mathcal{C}_{p(x)})$ then this would not be needed.