theorem for the direct sum of finite dimensional vector spaces

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Theorem Let S and T be subspaces of a finite dimensional vector space V. Then V is the direct sum of S and T, i.e., $V = S \oplus T$, if and only if $\dim V = \dim S + \dim T$ and $S \cap T = \{0\}$.

Proof. Suppose that $V = S \oplus T$. Then, by definition, $V = S + T$ and $S \cap T = \{0\}$. The dimension theorem for subspaces states that

$$\dim(S + T) + \dim S \cap T = \dim S + \dim T.$$

Since the dimension of the zero vector space $\{0\}$ is zero, we have that

$$\dim V = \dim S + \dim T,$$

and the first direction of the claim follows.

For the other direction, suppose $\dim V = \dim S + \dim T$ and $S \cap T = \{0\}$. Then the dimension theorem theorem for subspaces implies that

$$\dim(S + T) = \dim V.$$

Now $S+T$ is a subspace of V with the same dimension as V so, `http://planetmath.org/VectorSub` Theorem 1 on this page, $V = S + T$. This proves the second direction. $\square$