Jacobian and chain rule

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Let u, v be differentiable functions of x, y and x, y be differentiable functions of s, t. Then the connection

$$\frac{\partial(u,v)}{\partial(s,t)} = \frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(s,t)} \tag{1}$$

between the Jacobian determinants is in .

Proof. Starting from the right hand side of (1), where one can http://planetmath.org/Determ the determinants similarly as the corresponding http://planetmath.org/MatrixMultiplication we have

$$\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} \cdot \begin{vmatrix} x_s & x_t \\ y_s & y_t \end{vmatrix} = \begin{vmatrix} u_x x_s + u_y y_s & u_x x_t + u_y y_t \\ v_x x_s + v_y y_s & v_x x_t + v_y y_t \end{vmatrix} = \begin{vmatrix} u_s & u_t \\ v_s & v_t \end{vmatrix}.$$

Here, the last stage has been written according to the http://planetmath.org/ChainRuleSeveral chain rule. But thus we have arrived at the left hand side of the equation (1), which hereby has been proved.

Remark. The rule (1) is only a visualisation of the more general one concerning the case of functions of n variables.