skew-symmetric bilinear form

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A *skew-symmetric* (or *antisymmetric*) *bilinear form* is a special case of a bilinear form B, namely one which is skew-symmetric in the two coordinates; that is, $B(x, y) = -B(y, x)$ for all vectors x and y. Note that this definition only makes sense if B is defined over two identical vector spaces, so we must require this in the formal definition:

a bilinear form $B : V \times V \to K$ (V a vector space over a field K) is called *skew-symmetric* iff

$$B(x, y) = -B(y, x) \text{ for all vectors } x, y \in V.$$

Suppose that the characteristic of K is not 2. Set $x = y$ in the above equation. Then $B(x, x) = -B(x, x)$ for all vectors $x \in V$, which means that $2B(x, x) = 0$, or $B(x, x) = 0$. Therefore, B is an alternating form.

If, however, $\mathrm{char}(K) = 2$, then $B(x, y) = -B(y, x) = B(y, x)$; B is a symmetric bilinear form.

If V is finite-dimensional, then every bilinear form on V can be represented by a matrix. In this case the following theorem applies:

A bilinear form is skew-symmetric iff its representing matrix is skew-symmetric. (The fact that the representing matrix is skew-symmetric is independent of the choice of representing matrix).