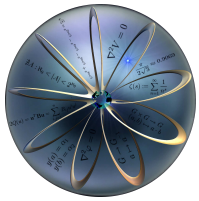

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Van der Waerden's permanent conjecture

Canonical name	VanDerWaerdensPermanentConjecture
Date of creation	2013-03-22 15:10:51
Last modified on	2013-03-22 15:10:51
Owner	marijke (8873)
Last modified by	marijke (8873)
Numerical id	5
Author	marijke (8873)
Entry type	Theorem
Classification	msc 15A15
Classification	msc 15A51
Synonym	permanent conjecture

Let A be any doubly stochastic $n \times n$ matrix (i.e. nonnegative real entries, each row sums to 1, each column too, hence square).

Let A° be the one where all entries are equal (i.e. they are $\frac{1}{n}$). Its permanent works out to

$$\operatorname{per} A^\circ \;=\; n!(\tfrac{1}{n})^n$$

and Van der Waerden conjectured in 1926 that this is the smallest value for the permanent of any doubly stochastic A, and is attained only for $A = A^\circ$:

$$\operatorname{per} A \;>\; n!(\tfrac{1}{n})^n \quad (\text{for } A \neq A^\circ).$$

It was finally proven independently by Egorychev and by Falikman, in 1979/80.

References

[Hal86] Marshall J. Hall, Jr., *Combinatorial Theory* (2nd ed.), Wiley 1986, repr. 1998, ISBN 0 471 09138 3 and 0 471 31518 4 *has a proof of the permanent conjecture.*