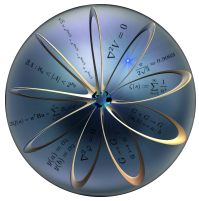

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Euclidean distance matrix

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A *Euclidean distance matrix* (EDM) is a real $m \times m$ matrix X such that for some points $y_1, \ldots, y_m$ in $\mathbb{R}^m$, $X_{ik} = \|y_i - y_k\|_2^2$, where $\|\cdot\|_2$ is the 2-norm on $\mathbb{R}^m$.

A EDM X inherits the following from the norm that defines it:

- $X_{ii} = 0$;
- $X_{ij} = X_{ji} \geq 0$;
- $\sqrt{X_{ik}} \leq \sqrt{X_{ij}} + \sqrt{X_{jk}}$.

Additionally, X is a EDM if and only if the diagonal entries of X are all 0 and for all $z \in \mathbb{R}^m$ whose components sum to 0, $z^T X z \leq 0$.

Finally, the set of $m \times m$ EDMs forms a convex `http://planetmath.org/Cone3`cone in the set of all $m \times m$ matrices.

References

[1] S. Boyd, L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.