symmetric algebra

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Let M be a module over a commutative ring R. Form the tensor algebra $T(M)$ over R. Let I be the ideal of $T(M)$ generated by elements of the form

$$u \otimes v - v \otimes u$$

where $u, v \in M$. Then the quotient algebra defined by

$$S(M) := T(M)/I$$

is called the *symmetric algebra* over the ring R.

Remark. Let R be a field, and M a finite dimensional vector space over R. Suppose $\{e_1, e_2, \ldots, e_n\}$ is a basis of M over R. Then $T(M)$ is nothing more than a free algebra on the basis elements e_i. Alternatively, the basis elements e_i can be viewed as non-commuting indeterminates in the non-commutative polynomial ring $R\langle e_1, e_2, \ldots, e_n\rangle$. This then implies that $S(M)$ is isomorphic to the "commutative" polynomial ring $R[e_1, e_2, \ldots, e_n]$, where $e_i e_j = e_j e_i$.