M-matrix

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A Z-matrix A is called an *M-matrix* if it satisfies any one of the following equivalent conditions.

1. All principal minors of A are positive.
2. The leading principal minors of A are positive.
3. A can be written in the form $A = kI - B$, where B is a non-negative matrix whose spectral radius is strictly less than k.
4. All real eigenvalues of A are positive.
5. The real part of any eigenvalue of A is positive.
6. A is non-singular and the inverse of A is non-negative.
7. $Av \geq 0$ implies $v \geq 0$.
8. There exists a vector v with non-negative entries such that $Av > 0$.
9. $A + D$ is non-singular for every non-negative diagonal matrix D.
10. $A + kI$ is non-singular for all $k \geq 0$.
11. For each nonzero vector v, $v_i(Av)_i > 0$ for some i.
12. There is a positive diagonal matrix D such that the matrix $DA + A^T D$ is positive definite.
13. A can be factorized as LU, where L is lower triangular, U is upper triangular, and the diagonal entries of both L and U are positive.
14. The diagonal entries of A are positive and AD is strictly diagonally dominant for some positive diagonal matrix D.

Reference:

M. Fiedler, *Special Matrices and Their Applications in Numerical Mathematics*, Martinus Nijhoff, Dordrecht, 1986.

R. A. Horn and C. R. Johnson, *Topics in Matrix Analysis*, Cambridge University Press, Cambridge, 1991.