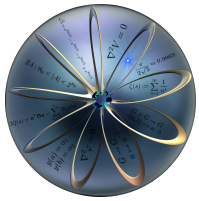

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If A and B commute so do A and B^{-1}

Canonical name	IfAAndBCommuteSoDoAAndB1
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Theorem 1. *Let A and B be commuting matrices. If B is invertible, then A and B^{-1} commute, and if A and B are invertible, then A^{-1} and B^{-1} commute.*

Proof. By assumption

$$AB = BA,$$

multiplying from the left and from the right by B^{-1} yields

$$B^{-1}A = AB^{-1}.$$

The second claim follows similarly. □

The statement and proof of this result can obviously be extended to elements of any monoid. In particular, in the case of a group, we see that two elements commute if and only if their inverses do.