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example of permutation matrix

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Consider the matrix

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

that corresponds to permuting the columns of the identity matrix under the permutation (1243) (<http://planetmath.org/Iei>.e., the first column of the identity matrix is the second column of  $P$ , the second column of the identity matrix is the fourth column of  $P$ , etc.). Then  $P$  is a permutation matrix.

We will consider what happens when we multiply a  $4 \times 4$  matrix by  $P$ . For example, let  $A$  be the matrix

$$A = \begin{pmatrix} 4 & 2 & 6 & 8 \\ 1 & 3 & 5 & 7 \\ 1 & 0 & 1 & 0 \\ -1 & -2 & -3 & -4 \end{pmatrix}.$$

Then

$$PA = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 2 & 6 & 8 \\ 1 & 3 & 5 & 7 \\ 1 & 0 & 1 & 0 \\ -1 & -2 & -3 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 5 & 7 \\ -1 & -2 & -3 & -4 \\ 4 & 2 & 6 & 8 \\ 1 & 0 & 1 & 0 \end{pmatrix}.$$

We notice that  $PA$  has the same rows as  $A$ . Moreover, the rows of  $A$  are the rows of  $PA$  permuted under (1243): The first row of  $PA$  is the second row of  $A$ , the second row of  $PA$  is the fourth row of  $A$ , etc.