modules are a generalization of vector spaces

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A http://planetmath.org/node/1022module is the natural generalization of a vector space, in fact, when working over a field it is just another word for a vector space.

If M and N are R-modules then a mapping $f : M \to N$ is called an R-morphism (or homomorphism) if:

$$\forall x, y \in M : f(x+y) = f(x) + f(y) \quad \text{and} \quad \forall x \in M \forall \lambda \in R : f(\lambda x) = \lambda f(x)$$

Note as mentioned in the beginning, if R is a field, these properties are the defining properties for a linear transformation.

Similarly in vector space terminology the image $\mathrm{Im} f := \{f(x) : x \in M\}$ and kernel $\mathrm{Ker} f := \{x \in M : f(x) = 0_N\}$ are called the range and null-space respectively.