transpose

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The *transpose* of a matrix A is the matrix formed by "flipping" A about the diagonal line from the upper left corner. It is usually denoted A^t, although sometimes it is written as A^T or A'. So if A is an $m \times n$ matrix and

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

then

$$A^t = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nm} \end{pmatrix}$$

Note that the transpose of an $m \times n$ matrix is a $n \times m$ matrix.

Properties

Let A and B be $m \times m$ matrices, C and D be $m \times n$ matrices, E be an $n \times k$ matrix, and c be a constant. Let x and y be column vectors with n rows. Then

1. $(C^t)^t = C$
2. $(C + D)^t = C^t + D^t$
3. $(cD)^t = cD^t$
4. $(DE)^t = E^t D^t$.
5. $(AB)^t = B^t A^t$.
6. If A is invertible , then $(A^t)^{-1} = (A^{-1})^t$
7. If A is real, trace$(A^t A) \geq 0$ (where trace is the trace of a matrix).
8. The transpose is a linear mapping from the vector space of matrices to itself. That is, $(\alpha A + \beta B)^t = \alpha(A)^t + \beta(B)^t$, for same-sized matrices A and B and scalars α and β.

The familiar vector dot product can also be defined using the matrix transpose. If x and y are column vectors with n rows each,

$$x^t y = x \cdot y$$

which implies

$$x^t x = x \cdot x = ||x||_2^2$$

which is another way of defining the square of the vector Euclidean norm.