



planetmath.org

Math for the people, by the people.

eigenvalues of a Hermitian matrix are real

Canonical name	EigenvaluesOfAHermitianMatrixAreReal
Date of creation	2013-03-22 14:23:09
Last modified on	2013-03-22 14:23:09
Owner	Andrea Ambrosio (7332)
Last modified by	Andrea Ambrosio (7332)
Numerical id	8
Author	Andrea Ambrosio (7332)
Entry type	Theorem
Classification	msc 15A57

The eigenvalues of a Hermitian (or self-adjoint) matrix are real.

Proof. Suppose λ is an eigenvalue of the self-adjoint matrix A with non-zero eigenvector v . Then $Av = \lambda v$.

$$\lambda^* v^H v = (\lambda v)^H v = (Av)^H v = v^H A^H v = v^H A v = v^H \lambda v = \lambda v^H v$$

Since v is non-zero by assumption, $v^H v$ is non-zero as well and so $\lambda^* = \lambda$, meaning that λ is real. \square