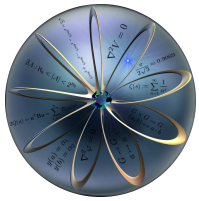

planetmath.org

Math for the people, by the people.

Gershgorin's circle theorem

Canonical name	GershgorinsCircleTheorem
Date of creation	2013-03-22 13:14:15
Last modified on	2013-03-22 13:14:15
Owner	lieven (1075)
Last modified by	lieven (1075)
Numerical id	7
Author	lieven (1075)
Entry type	Theorem
Classification	msc 15A42
Synonym	Gershgorin's disc theorem
Synonym	Gerschgorin's circle theorem
Synonym	Gerschgorin's disc theorem
Related topic	BrauersOvalsTheorem
Defines	Gershgorin disc
Defines	Gerschgorin disc

Let A be a square complex matrix. Around every element a_{ii} on the diagonal of the matrix, we draw a circle with radius the sum of the norms of the other elements on the same row $\sum_{j \neq i} |a_{ij}|$. Such circles are called *Gershgorin discs*.

Theorem: Every eigenvalue of A lies in one of these Gershgorin discs.

Proof: Let λ be an eigenvalue of A and x its corresponding eigenvector. Choose i such that $|x_i| = \max_j |x_j|$. Since x can't be 0, $|x_i| > 0$. Now $Ax = \lambda x$, or looking at the i-th component

$$(\lambda - a_{ii})x_i = \sum_{j \neq i} a_{ij} x_j.$$

Taking the norm on both sides gives

$$|\lambda - a_{ii}| = |\sum_{j \neq i} \frac{a_{ij} x_j}{x_i}| \leq \sum_{j \neq i} |a_{ij}|.$$