Toeplitz matrix

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1 Toeplitz Matrix

A *Toeplitz matrix* is any $n \times n$ matrix with values constant along each (top-left to lower-right) diagonal. That is, a Toeplitz matrix has the form

$$\begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{n-1} \\ a_{-1} & a_0 & a_1 & \ddots & \vdots \\ a_{-2} & a_{-1} & a_0 & \ddots & a_2 \\ \vdots & \ddots & \ddots & \ddots & a_1 \\ a_{-(n-1)} & \cdots & a_{-2} & a_{-1} & a_0 \end{bmatrix}$$

Numerical problems involving Toeplitz matrices typically have fast solutions (only $2n-1$ distinct elements need to be solved for, as opposed to n^2). For example, the inverse of a symmetric, positive-definite $n \times n$ Toeplitz matrix can be found in $\mathcal{O}(n^2)$ `http://planetmath.org/TimeComplexity`time.

References

[1] Golub and Van Loan, *Matrix Computations*, Johns Hopkins University Press 1993