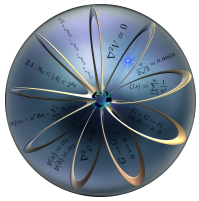

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Pauli matrices

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The Pauli matrices are a set of three Hermitian, unitary matrices used by Wolfgang Pauli in his theory of quantum-mechanical spin. They are given by:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

They satisfy the following commutation and anticommutation identities:

$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k$where ϵ_{ijk} is the Levi-Civita symbol
$\{\sigma_i, \sigma_j\} = 2\mathbf{I}\delta_{ij}$where $\mathbf{I}$ is the identity matrix and δ_{ij} is the Kronecker delta

0.1 Delta notation

With the identity matrix $\mathbf{I}$, the Pauli matrices form a group. When combined in this way, they are often given the symbols δ_i, as follows:

$$\delta_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$\delta_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\delta_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\delta_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

This choice is useful when writing the Dirac matrices.