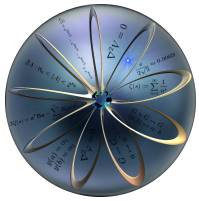

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proof of the dimension theorem for subspaces

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Let S and T be subspaces of a vector space. By the rank-nullity theorem and the second isomorphism theorem (for modules) we have

$$\begin{aligned}\dim(S+T) &= \dim S + \dim((S+T)/S) \\ &= \dim S + \dim(T/(S\cap T)).\end{aligned}$$

Therefore

$$\begin{aligned}\dim(S+T) + \dim(S\cap T) &= \dim S + \dim(T/(S\cap T)) + \dim(S\cap T) \\ &= \dim S + \dim T,\end{aligned}$$

by the rank-nullity theorem again.