Jacobi's theorem

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Jacobi's Theorem Any skew-symmetric matrix of odd order has determinant equal to 0.

Proof. Suppose A is an $n \times n$ square matrix. For the determinant, we then have $\det A = \det A^T$, and $\det(-A) = (-1)^n \det A$. Thus, since n is odd, and $A^T = -A$, we have $\det A = -\det A$, and the theorem follows. $\square$

0.0.1 Remarks

1. According to [?], this theorem was given by Carl Gustav Jacob Jacobi (1804-1851) [?] in 1827.

2. The 2×2 matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ shows that Jacobi's theorem does not hold for 2×2 matrices. The determinant of the $2n \times 2n$ block matrix with these 2×2 matrices on the diagonal equals $(-1)^n$. Thus Jacobi's theorem does not hold for matrices of even order.

3. For $n = 3$, any antisymmetric matrix A can be written as

$$A = \begin{pmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{pmatrix}$$

   for some real v_1, v_2, v_3, which can be written as a vector $v = (v_1, v_2, v_3)$. Then A is the matrix representing the mapping $u \mapsto v \times u$, that is, the cross product with respect to v. Since $Av = v \times v = 0$, we have $\det A = 0$.

References

[1] H. Eves, *Elementary Matrix Theory*, Dover publications, 1980.

[2] The MacTutor History of Mathematics archive, `http://www-gap.dcs.st-and.ac.uk/ history/Mathematicians/Jacobi.html`Carl Gustav Jacob Jacobi