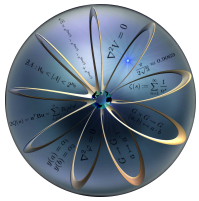

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characteristic polynomial of a orthogonal matrix is a reciprocal polynomial

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Theorem 1. *The characteristic polynomial of a orthogonal matrix is a reciprocal polynomial*

Proof. Let A be the orthogonal matrix, and let $p(\lambda) = \det(A - \lambda I)$ be its characteristic polynomial. We wish to prove that

$$p(\lambda) = \pm\lambda^n p(1/\lambda).$$

Since $A^{-1} = A^T$, we have $A - \lambda I = -\lambda A(A^T - I/\lambda)$. Taking the determinant of both sides, and using $\det A = \det A^T$ and $\det cA = c^n \det A$ ($c \in \mathbb{C}$), yields

$$\det(A - \lambda I) = \pm\lambda^n \det(A - \frac{1}{\lambda} I).$$

□

References

[1] H. Eves, *Elementary Matrix Theory*, Dover publications, 1980.