proof of matrix inverse calculation by Gaussian elimination

Canonical name	ProofOfMatrixInverseCalculationByGaussianElimination
Date of creation	2013-03-22 14:15:10
Last modified on	2013-03-22 14:15:10
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Last modified by	rspuzio (6075)
Numerical id	9
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Entry type	Definition
Classification	msc 15A09

Let A be an invertible matrix, and A^{-1} its inverse, whose columns are $A_1^{-1}, \cdots, A_n^{-1}$. Then, by definition of matrix inverse, $AA^{-1} = I_n$. But this implies $AA_1^{-1} = e_1, \ldots, AA_n^{-1} = e_n$, with $e_1, \cdots, e_n$ being the first,…,n-th column of I_n respectively.

A being non singular (or invertible), for all $k \leq n$, $AA_k^{-1} = e_k$ has a solution for A_k^{-1}, which can be found by Gaussian elimination of $[A \mid e_k]$.

The only part that changes between the augmented matrices constructed is the last column, and these last columns, once the Gaussian elimination has been performed, correspond to the columns of A^{-1}. Because of this, the steps we need to take for the Gaussian elimination are the same for each augmented matrix.

Therefore, we can solve the matrix equation by performing Gaussian elimination on $[A \mid e_1 \cdots e_n]$, or $[A \mid I_n]$.