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row echelon form

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A matrix is said to be in row echelon form if each non-zero row has more leading zeros than the previous row. Row-echelon form is the key idea underlying the Gaussian elimination algorithm and LU factorization.

Let us give the precise definition. Let (M_{ij}) be an $n \times m$ matrix. For each row $i = 1, \dots, n$ define the pivot position P_i to be either the minimum value of $j = 1, \dots, m$ for which $M_{ij} \neq 0$, or ∞ if the row consists entirely of zeros. A matrix is in echelon form if for all $i > 1$, either $P_i = \infty$ or $P_{i-1} < P_i$.

Examples of matrices in row echelon form include,

$$\begin{pmatrix} 0 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 5 & 0 & 1 & 3 & 2 \\ 0 & 0 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{pmatrix}$$

Note that if a matrix is an echelon form, then necessarily rows which are composed completely of zeros will be grouped at the bottom of the matrix. Also note that if several rows have the same number of leading zeros then the matrix is not in row echelon form unless the rows in question are composed entirely of zeros.