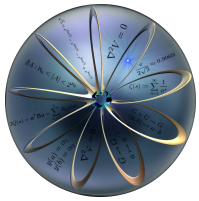

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Euclidean space

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1 Definition

Euclidean n-space is a metric space (E, d) with the property that the group of isometries is transitive and is isomorphic to an n-dimensional Euclidean vector space. To be more precise, we are saying that there exists an n-dimensional Euclidean vector space V with inner product $\langle \cdot, \cdot \rangle$ and a mapping

$$+ : E \times V \to E$$

such that the following hold:

1. For all $x, y \in E$ there exists a unique $u \in V$ satisfying

$$y = x + u, \quad d(x, y)^2 = \langle u, u \rangle,$$

2. For all $x, y \in E$ and all $u \in V$ we have

$$d(x + u, y + u) = d(x, y).$$

3. For all $x \in E$ and all $u, v \in V$ we have

$$(x + u) + v = x + (u + v).$$

Putting it more succinctly: V acts transitively and effectively on E by isometries.

Remarks.

- The difference between Euclidean space and a Euclidean vector space is one of loss of structure. Euclidean space is a Euclidean vector space that has "forgotten" its origin.
- A 2-dimensional Euclidean space is often called a *Euclidean plane*.