rank-nullity theorem

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The sum of the rank and the nullity of a linear mapping gives the dimension of the mapping's domain. More precisely, let $T : V \to W$ be a linear mapping. If V is a finite-dimensional, then

$$\dim V = \dim \operatorname{Ker} T + \dim \operatorname{Img} T.$$

The intuitive content of the Rank-Nullity theorem is the principle that

> *Every independent linear constraint takes away one degree of freedom.*

The rank is just the number of independent linear constraints on $v \in V$ imposed by the equation

$$T(v) = 0.$$

The dimension of V is the number of unconstrained degrees of freedom. The nullity is the degrees of freedom in the resulting space of solutions. To put it yet another way:

> *The number of variables* **minus** *the number of independent linear constraints* **equals** *the number of linearly independent solutions.*