# proof of determinant of the Vandermonde matrix

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To begin, note that the determinant of the $n \times n$ Vandermonde matrix (which we shall denote as '$\Delta$') is a homogeneous polynomial of order $n(n-1)/2$ because every term in the determinant is, up to sign, the product of a zeroth power of some variable times the first power of some other variable , ..., the $n-1$-st power of some variable and $0+1+\cdots+(n-1) = n(n-1)/2$.

Next, note that if $a_i = a_j$ with $i \neq j$, then $\Delta = 0$ because two columns of the matrix would be equal. Since $\Delta$ is a polynomial, this implies that $a_i - a_j$ is a factor of $\Delta$. Hence,

$$\Delta = C \prod_{1 \leq i < j \leq n} (a_j - a_i)$$

where C is some polynomial. However, since both $\Delta$ and the product on the right hand side have the same degree, $C$ must have degree zero, i.e. $C$ must be a constant. So all that remains is the determine the value of this constant.

One way to determine this constant is to look at the coefficient of the leading diagonal, $\prod_n (a_n)^{n-1}$. Since it equals 1 in both the determinant and the product, we conclude that $C = 1$, hence

$$\Delta = \prod_{1 \leq i < j \leq n} (a_j - a_i).$$