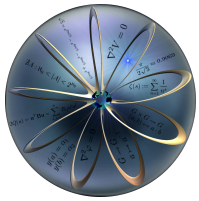

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## permutation matrix

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# 1 Permutation Matrix

Let $n$ be a positive integer. A *permutation matrix* is any $n \times n$ matrix which can be created by rearranging the rows and/or columns of the $n \times n$ identity matrix. More formally, given a permutation $\pi$ from the symmetric group $S_n$, one can define an $n \times n$ permutation matrix $P_\pi$ by $P_\pi = (\delta_{i\,\pi(j)})$, where $\delta$ denotes the Kronecker delta symbol.

Premultiplying an $n \times n$ matrix $A$ by an $n \times n$ permutation matrix results in a rearrangement of the rows of $A$. For example, if the matrix $P$ is obtained by swapping rows $i$ and $j$ of the $n \times n$ identity matrix, then rows $i$ and $j$ of $A$ will be swapped in the product $PA$.

Postmultiplying an $n \times n$ matrix $A$ by an $n \times n$ permutation matrix results in a rearrangement of the columns of $A$. For example, if the matrix $P$ is obtained by swapping rows $i$ and $j$ of the $n \times n$ identity matrix, then columns $i$ and $j$ of $A$ will be swapped in the product $AP$.

# 2 Properties

Permutation matrices have the following properties:

- They are `http://planetmath.org/OrthogonalMatrices`orthogonal.
- They are invertible.
- For a `http://planetmath.org/Fixed3`fixed positive integer $n$, the $n \times n$ permutation matrices form a group under matrix multiplication.
- Since they have a single 1 in each row *and* each column, they are doubly stochastic.
- They are the extreme points of the convex set of doubly stochastic matrices.