symmetrizer

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Let V be a vector space over a field F. Let n be an integer, where $n < \mathrm{char}(F)$ if $\mathrm{char}(F) \neq 0$. Let S_n be the symmetric group on $\{1, \dots, n\}$. The linear operator $S : V^{\otimes n} \to V^{\otimes n}$ defined by:

$$S = \frac{1}{n!} \sum_{\sigma \in S_n} P(\sigma)$$

is called the *symmetrizer*. Here $P(\sigma)$ is the permutation operator. It is clear that $P(\sigma)S = SP(\sigma) = S$ for all $\sigma \in S_n$.

Let S be the symmetrizer for $V^{\otimes n}$. Then an order-n tensor A is `http://planetmath.org/Symmetri`
if and only $S(A) = A$.

Proof

If A is then

$$S(A) = \frac{1}{n!} \sum_{\sigma \in S_n} P(\sigma)A = \frac{1}{n!} \sum_{\sigma \in S_n} A = A.$$

If $S(A) = A$ then

$$P(\sigma)A = P(\sigma)S(A) = P(\sigma)S(A) = S(A) = A$$

for all $\sigma \in S_n$, so A is .

The theorem says that a is an eigenvector of the linear operator S corresponding to the eigenvalue 1. It is easy to verify that $S^2 = S$, so that S is a projection onto $S^n(V)$.