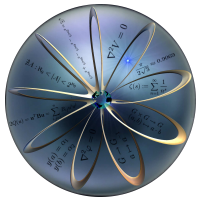

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symplectic complement

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Classification	msc 15A04
Defines	symplectic complement
Defines	isotropic subspace
Defines	coisotropic subspace
Defines	symplectic subspace
Defines	Lagrangian subspace

Definition [**?**, **?**] Let (V, ω) be a symplectic vector space and let W be a vector subspace of V. Then the *symplectic complement* of W is

$$W^\omega = \{x \in V \,|\, \omega(x, y) = 0 \text{ for all } y \in W\}.$$

It is easy to see that W^ω is also a vector subspace of V. Depending on the relation between W and W^ω, W is given different names.

1. If $W \subset W^\omega$, then W is an *isotropic subspace* (of V).
2. If $W^\omega \subset W$, then W is an *coisotropic subspace*.
3. If $W \cap W^\omega = \{0\}$, then W is an *symplectic subspace*.
4. If $W = W^\omega$, then W is an *Lagrangian subspace*.

For the symplectic complement, we have the following dimension theorem.

Theorem [**?**, **?**] Let (V, ω) be a symplectic vector space, and let W be a vector subspace of V. Then

$$\dim V = \dim W^\omega + \dim W.$$

References

[1] D. McDuff, D. Salamon, *Introduction to Symplectic Topology*, Clarendon Press, 1997.

[2] R. Abraham, J.E. Marsden, *Foundations of Mechanics*, 2nd ed., Perseus Books, 1978.