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zero matrix

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The  $n \times m$  zero  $O$  over a ring  $R$  is the  $n \times m$  matrix with coefficients in  $R$  given by

$$O = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix},$$

where 0 is the [http://planetmath.org/Ringadditive identity](http://planetmath.org/Ringadditiveidentity) in  $R$ .

### 0.0.1 Properties

The zero matrix is the additive identity in the ring of  $n \times n$  matrices over  $R$ .

This is an alternative definition of  $O$  (since there's [http://planetmath.org/UniquenessOfAdditive Identity](http://planetmath.org/UniquenessOfAdditiveIdentity) one additive identity in any given ring).

The  $n \times n$  zero matrix  $O$  has the following properties:

- The determinant of  $O$  is  $\det O = 0$ , and its trace is  $\text{tr } O = 0$ .
- $O$  has only one eigenvalue  $\lambda = 0$  of multiplicity  $n$ . Any non-zero vector is an eigenvector of  $O$ , so if we're looking for a basis of eigenvectors, we could pick the standard basis  $e_1 = (1, 0, \dots, 0), \dots, e_n = (0, \dots, 0, 1)$ .
- The matrix exponential of  $O$  is  $e^O = I$ , the  $n \times n$  identity matrix.