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## Neumann series

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If  $A$  is a square matrix,  $\|A\| < 1$ , then  $I - A$  is nonsingular and  $(I - A)^{-1} = I + A + A^2 + \cdots = \sum_{k=0}^{\infty} A^k$ . This is the *Neumann series*. It provides approximations of  $(I - A)^{-1}$  when  $A$  has entries of small magnitude. For example, a first-order approximation is  $(I - A)^{-1} \approx I + A$ . It is obvious that this is a generalization of the geometric series.

## References

- [1] Carl D. Meyer, *Matrix Analysis and Applied Linear Algebra*.