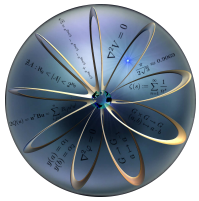

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linear equation

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Entry type	Definition
Classification	msc 15A06
Synonym	linear problem
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Related topic	HomogeneousLinearProblem
Related topic	FiniteDimensionalLinearProblem
Defines	consistent
Defines	inconsistent
Defines	particular solution

Let $L : U \to V$ be a linear mapping, and $v \in V$ an element of the codomain. A *linear equation* is a relation of the form,

$$L(u) = v,$$

where $u \in U$ is to be considered as the unknown. The solution set of a linear equation is the set of $u \in U$ that satisfy the above constraint, or to be more precise, the pre-image $L^{-1}(v)$. The equation is called inconsistent if no solutions exist, that is, if the pre-image is the empty set. Otherwise, the equation is called *consistent*.

The general solution of a linear equation has the form

$$u = u_p + u_h, \quad u_p, u_h \in U,$$

where

$$L(u_p) = v$$

is a *particular solution* and where

$$L(u_h) = 0$$

is any solution of the corresponding homogeneous problem, i.e. an element of the kernel of L.

Notes. Elementary treatments of linear algebra focus almost exclusively on finite-dimensional linear problems. They neglect to mention the underlying mapping, preferring to focus instead on "variables and equations." However, the scope of the general concept is considerably wider, e.g. linear differential equations such as

$$y'' + y = 0.$$