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primary decomposition theorem

Canonical name	PrimaryDecompositionTheorem
Date of creation	2013-03-22 14:15:30
Last modified on	2013-03-22 14:15:30
Owner	gumau (3545)
Last modified by	gumau (3545)
Numerical id	8
Author	gumau (3545)
Entry type	Theorem
Classification	msc 15A04

This is an important theorem in linear algebra. It states the following: Let k be a field, V a vector space over k, $\dim V = n$, and $T\colon V \to V$ a linear operator, such that its minimal polynomial (or its annihilator polynomial) is m_T, which decomposes in $k[X]$ into irreducible factors as $m_T = p_1^{\alpha_1} \dots p_r^{\alpha_r}$. Then,

1. $V = \bigoplus_{i=1}^{r} \ker(p_i^{\alpha_i}(T))$
2. $\ker(p_i^{\alpha_i}(T))$ is T-invariant for every i
3. If T_i is the restriction of T to $\ker(p_i^{\alpha_i}(T))$, then $m_{T_i} = p_i^{\alpha_i}$

This is a consequence of a more general theorem: Let V, T be as above, and $f \in k[X]$ such that $f(T) = 0$, with $f = f_1 \dots f_r$ and $(f_i, f_j) = 1$ if $i \neq j$, then

1. $V = \bigoplus_{i=1}^{r} \ker(f_i(T))$
2. $\ker(f_i(T))$ is T-invariant for every i

To illustrate its importance, the primary decomposition theorem, together with the cyclic decomposition theorem, imply the existence and uniqueness of the Jordan canonical form.