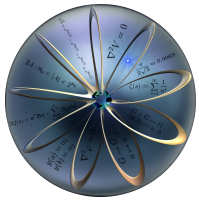

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derivative of inverse matrix

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Theorem 1. *Suppose A is a square matrix depending on a real parameter t taking values in an open set $I \subseteq \mathbb{R}$. Further, suppose all component functions in A are differentiable, and $A(t)$ is invertible for all t. Then, in I, we have*

$$\frac{dA^{-1}}{dt} = -A^{-1}\frac{dA}{dt}A^{-1},$$

where $\frac{d}{dt}$ is the derivative.

Proof. Suppose $a_{ij}(t)$ are the component functions for A, and $a^{jk}(t)$ are component functions for $A^{-1}(t)$. Then for each t we have

$$\sum_{j=1}^{n} a_{ij}(t)a^{jk}(t) = \delta_i^k$$

where n is the order of A, and δ_i^k is the Kronecker delta symbol. Hence

$$\sum_{j=1}^{n} \frac{da_{ij}}{dt}a^{jk} + a_{ij}\frac{da^{jk}}{dt} = 0,$$

that is,

$$\frac{dA}{dt}A^{-1} = -A\frac{dA^{-1}}{dt}$$

from which the claim follows. □