self-dual

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Definition. Let U be a finite-dimensional inner-product space over a field $\mathbb{K}$. Let $T : U \to U$ be an endomorphism, and note that the adjoint endomorphism $T^\star$ is also an endomorphism of U. It is therefore possible to add, subtract, and compare T and $T^\star$, and we are able to make the following definitions. An endomorphism T is said to be *self-dual* (a.k.a. *self-adjoint*) if

$$T = T^\star.$$

By contrast, we say that the endomorphism is *anti self-dual* if

$$T = -T^\star.$$

Exactly the same definitions can be made for an endomorphism of a complex vector space with a Hermitian inner product.

Relation to the matrix transpose. All of these definitions have their counterparts in the matrix setting. Let $M \in \mathrm{Mat}_{n,n}(\mathbb{K})$ be the matrix of T relative to an orthogonal basis of U. Then T is self-dual if and only if M is a symmetric matrix, and anti self-dual if and only if M is a skew-symmetric matrix.

In the case of a Hermitian inner product we must replace the transpose with the conjugate transpose. Thus T is self dual if and only if M is a Hermitian matrix, i.e.

$$M = \overline{M^t}.$$

It is anti self-dual if and only if

$$M = -\overline{M^t}.$$