pseudoinverse

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Related topic	MoorePenroseGeneralizedInverse

The inverse A^{-1} of a matrix A exists only if A is square and has full rank. In this case, $Ax = b$ has the solution $x = A^{-1}b$.

The *pseudoinverse* A^+ (beware, it is often denoted otherwise) is a generalization of the inverse, and exists for any $m \times n$ matrix. We assume $m > n$. If A has full rank (n) we define:

$$A^+ = (A^T A)^{-1} A^T$$

and the solution of $Ax = b$ is $x = A^+ b$.

More accurately, the above is called the *Moore-Penrose pseudoinverse*.

1 Calculation

The best way to compute A^+ is to use singular value decomposition. With $A = USV^T$, where U and V (both $n \times n$) orthogonal and S ($m \times n$) is diagonal with real, non-negative singular values σ_i, $i = 1, \ldots, n$. We find

$$A^+ = V(S^T S)^{-1} S^T U^T$$

If the rank r of A is smaller than n, the inverse of $S^T S$ does not exist, and one uses only the first r singular values; S then becomes an $r \times r$ matrix and U,V shrink accordingly. see also Linear Equations.

2 Generalization

The term "pseudoinverse" is actually used for any operator pinv satisfying

$$M \operatorname{pinv}(M) M = M$$

for a $m \times n$ matrix M. Beyond this, pseudoinverses can be defined on any reasonable matrix identity.

References

- Originally from The Data Analysis Briefbook (`http://rkb.home.cern.ch/rkb/titleA.html`