proof of block determinants

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If A^{-1} exists, then

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} I & O \\ CA^{-1} & I \end{pmatrix} \begin{pmatrix} A & B \\ O & D - CA^{-1}B \end{pmatrix}.$$

So

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det \begin{pmatrix} I & O \\ CA^{-1} & I \end{pmatrix} \det \begin{pmatrix} A & B \\ O & D - CA^{-1}B \end{pmatrix}.$$

Each of the first matrices in the decompositions are triangular. Hence their determinants equal 1. This means that the determinant of the original matrix equals the determinant of either of the second matrices in the decomposition. Therefore

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(A) \det(D - CA^{-1}B).$$

The second formula follows by using a similar trick.