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mutual positions of vectors

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Defines	parallelism
Defines	perpendicular
Defines	perpendicularity
Defines	diverging
Defines	normal vector

In this entry, we work within a Euclidean space E.

1. Two non-zero Euclidean vectors $\vec{a}$ and $\vec{b}$ are said to be *parallel*, denoted by $\vec{a} \parallel \vec{b}$, iff there exists a real number k such that

$$\vec{a} = k\vec{b}.$$

Since both $\vec{a}$ and $\vec{b}$ are non-zero, $k \neq 0$. So $\parallel$ is a binary relation on on $E \diagdown \{\vec{0}\}$ and called the *parallelism*. If $k > 0$, then a and b are said to be in the *same direction*, and we denote this by $\vec{a} \upuparrows \vec{b}$; if $k < 0$, then a and b are said to be in the *opposite* or *contrary directions*, and we denote this by $\vec{a} \downarrow\uparrow \vec{b}$.

Remarks

- Actually, the parallelism is an equivalence relation on $E \diagdown \{\vec{0}\}$. If the zero vector $\vec{0}$ were allowed along, then the relation were not symmetric ($\vec{0} = 0\vec{b}$ but not necessarily $\vec{b} = k\vec{0}$).
- When two vectors $\vec{a}$ and $\vec{b}$ are not parallel to one another, written $\vec{a} \nparallel \vec{b}$, they are said to be *diverging*.

2. Two Euclidean vectors $\vec{a}$ and $\vec{b}$ are *perpendicular*, denoted by $\vec{a} \perp \vec{b}$, iff

$$\vec{a} \cdot \vec{b} = 0,$$

i.e. iff their scalar product vanishes. Then $\vec{a}$ and $\vec{b}$ are *normal vectors* of each other.

Remarks

- We may say that $\vec{0}$ is perpendicular to all vectors, because its direction is and because $\vec{0} \cdot \vec{b} = 0$.
- Perpendicularity is not an equivalence relation in the set of all vectors of the space in question, since it is neither reflexive nor transitive.

3. The angle θ between two non-zero vectors $\vec{a}$ and $\vec{b}$ is obtained from

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}.$$

The angle is chosen so that $0 \leqq \theta \leqq \pi$.