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invertible ideals are projective

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If R is a ring and $f: M \rightarrow N$ is a homomorphism of R -modules, then a right inverse of f is a homomorphism $g: N \rightarrow M$ such that $f \circ g$ is the identity map on N . For a right inverse to exist, it is clear that f must be an epimorphism. If a right inverse exists for every such epimorphism and all modules M , then N is said to be a projective module.

For fractional ideals over an integral domain R , the property of being projective as an R -module is equivalent to being an invertible ideal.

Theorem. *Let R be an integral domain. Then a fractional ideal over R is invertible if and only if it is projective as an R -module.*

In particular, every fractional ideal over a Dedekind domain is invertible, and is therefore projective.