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exact sequence

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If we have two homomorphisms $f : A \to B$ and $g : B \to C$ in some category of modules, then we say that f and g are *exact* at B if the image of f is equal to the kernel of g.

A sequence of homomorphisms

$$\cdots \to A_{n+1} \xrightarrow{f_{n+1}} A_n \xrightarrow{f_n} A_{n-1} \to \cdots$$

is said to be *exact* if each pair of adjacent homomorphisms (f_{n+1}, f_n) is exact – in other words if $\mathrm{im} f_{n+1} = \ker f_n$ for all n.

Compare this to the notion of a chain complex.

Remark. The notion of exact sequences can be generalized to any abelian category $\mathcal{A}$, where A_i and f_i above are objects and morphisms in $\mathcal{A}$.