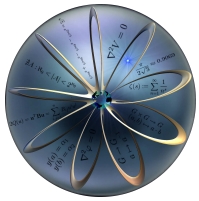

planetmath.org

Math for the people, by the people.

an Artinian integral domain is a field

Canonical name	AnArtinianIntegralDomainIsAField
Date of creation	2013-03-22 12:49:37
Last modified on	2013-03-22 12:49:37
Owner	yark (2760)
Last modified by	yark (2760)
Numerical id	13
Author	yark (2760)
Entry type	Theorem
Classification	msc 16P20
Classification	msc 13G05
Related topic	AFiniteIntegralDomainIsAField

Let R be an integral domain, and assume that R is Artinian.

Let $a \in R$ with $a \neq 0$. Then $R \supseteq aR \supseteq a^2R \supseteq \cdots$.

As R is Artinian, there is some $n \in \mathbb{N}$ such that $a^nR = a^{n+1}R$. There exists $r \in R$ such that $a^n = a^{n+1}r$, that is, $a^n1 = a^n(ar)$. But $a^n \neq 0$ (as R is an integral domain), so we have $1 = ar$. Thus a is a unit.

Therefore, every Artinian integral domain is a field.