ordinary quiver of an algebra

Canonical name	OrdinaryQuiverOfAnAlgebra
Date of creation	2013-03-22 19:17:41
Last modified on	2013-03-22 19:17:41
Owner	joking (16130)
Last modified by	joking (16130)
Numerical id	4
Author	joking (16130)
Entry type	Definition
Classification	msc 16S99
Classification	msc 20C99
Classification	msc 13B99

Let k be a field and A an algebra over k.

Denote by $\mathrm{rad}A$ the (Jacobson) radical of A and $\mathrm{rad}^2A = (\mathrm{rad}A)^2$ a square of radical.

Since A is finite-dimensional, then we have a `http://planetmath.org/CompleteSetOfPrimiti` set of primitive orthogonal idempotents $E = \{e_1, \ldots, e_n\}$.

Definition. The **ordinary quiver** of a finite-dimensional algebra A is defined as follows:

1. The set of vertices is equal to $Q_0 = \{1, \ldots, n\}$ which is in bijective correspondence with E.

2. If $a, b \in Q_0$, then the number of arrows from a to b is equal to the dimension of the k-vector space

$$e_a\big(\mathrm{rad}A/\mathrm{rad}^2A\big)e_b.$$

It can be shown that the ordinary quiver is well-defined, i.e. it is independent on the choice of a complete set of primitve orthogonal idempotents. Also finite dimension of A implies, then the ordinary quiver is finite.