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## Wedderburn-Artin theorem

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If  $R$  is a left semisimple ring, then

$$R \cong \mathbb{M}_{n_1}(D_1) \times \cdots \times \mathbb{M}_{n_r}(D_r)$$

where each  $D_i$  is a division ring and  $\mathbb{M}_{n_i}(D_i)$  is the matrix ring over  $D_i$ ,  $i = 1, 2, \dots, r$ . The positive integer  $r$  is unique, and so are the division rings (up to permutation).

Some immediate consequences of this theorem:

- A <http://planetmath.org/SimpleRings> simple Artinian ring is isomorphic to a matrix ring over a division ring.
- A commutative semisimple ring is a finite direct product of fields.

This theorem is a special case of the more general theorem on semiprimitive rings.