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valuation domain

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An integral domain  $R$  is a *valuation domain* if for all  $a, b \in R$ , either  $a|b$  or  $b|a$ . Equivalently, an integral domain is a valuation domain if for any  $x$  in the field of fractions of  $R$ ,  $x \notin R \implies x^{-1} \in R$ .

Some properties:

- A valuation domain is a discrete valuation ring (DVR) if and only if it is a principal ideal domain (PID) if and only if it is Noetherian.
- Every valuation domain is a Bezout domain, though the converse is not true. For a partial converse, any local Bezout domain is a valuation domain.
- Valuation domains are integrally closed.