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surjective homomorphism between unitary rings

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Theorem. Let f be a surjective homomorphism from a unitary ring R to another unitary ring R'. Then

- $f(1) = 1'$,
- $f(a^{-1}) = (f(a))^{-1}$ for all elements a belonging to the group of units of R.

Proof. 1°. In a ring, the identity element is unique, whence it suffices to show that $f(1)$ has the properties required for the unity of the ring R'. When a' is an arbitrary element of this ring, there is by the surjectivity an element a of R such that $f(a) = a'$. Thus we have

$$f(1)a' = f(1)f(a) = f(1a) = f(a) = a', \quad a'f(1) = f(a)f(1) = f(a1) = f(a) = a'.$$

2°. Let a be a unit of R. Then

$$f(a)f(a^{-1}) = f(aa^{-1}) = f(1) = 1', \quad f(a^{-1})f(a) = f(a^{-1}a) = f(1) = 1',$$

whence $f(a^{-1})$ is a multiplicative inverse of $f(a)$.