additive inverse of the zero in a ring

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In any ring R, the additive identity is unique and usually denoted by 0. It is called the zero or *neutral element* of the ring and it satisfies the zero property under multiplication. The additive inverse of the zero must be zero itself. For suppose otherwise: that there is some nonzero $c \in R$ so that $0 + c = 0$. For any element $a \in R$ we have $a + 0 = a$ since 0 is the additive identity. Now, because addition is associative we have

$$\begin{aligned} 0 &= a + 0 \\ &= a + (0 + c) \\ &= (a + 0) + c \\ &= a + c. \end{aligned}$$

Since a is any arbitrary element in the ring, this would imply that (nonzero) c is an additive identity, contradicting the uniqueness of the additive identity. And so our suppostition that 0 has a nonzero inverse cannot be true. So the additive inverse of the zero is zero itself. We can write this as $-0 = 0$, where the $-$ sign means "additive inverse".

Yes, for sure, there are other ways to come to this result, and we encourage you to have a bit of fun describing your own reasons for why the additive inverse of the zero of the ring must be zero itself.

For example, since 0 is the neutral element of the ring this means that $0 + 0 = 0$. From this it immediately follows that $-0 = 0$.