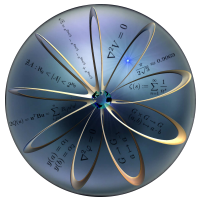

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weak global dimension

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Let R be a ring. The **(right) weak global dimension** of R is defined as

$$\mathrm{w.gl.dim}R = \sup\{\mathrm{wd}_R M \mid M \text{ is a right module}\}.$$

Unlike global dimension of R the definition of the weak global dimension is left/right symmetric. This follows from the fact that for every left module M and right module N there is an isomorphism

$$\mathrm{Tor}_n^R(M,N) \simeq \mathrm{Tor}_n^R(N,M).$$

Thus we simply say that R has the weak global dimension. Note that this does not hold for Ext functors, so (generally) the definition of global dimension is not left/right symmetric.

The following proposition is a simple consequence of the fact that every projective module is flat:

Proposition. For any ring R we have

$$\mathrm{w.gl.dim}R \leqslant \min\{\ \mathrm{l.gl.dim}R,\ \mathrm{r.gl.dim}R\ \},$$

where l.gl.dim and r.gl.dim denote the left global dimension and right global dimension respectively.