generalized matrix ring

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Let I be an indexing set. A *ring of $I \times I$ generalized matrices* is a ring R with a decompostion (as an additive group)

$$R = \bigoplus_{i,j \in I} R_{ij},$$

such that $R_{ij}R_{kl} \subseteq R_{il}$ if $j = k$ and $R_{ij}R_{kl} = 0$ if $j \neq k$.

If I is finite, then we usually replace it by its cardinal n and speak of a ring of $n \times n$ generalized matrices with components R_{ij} for $i \leq i, j \leq n$.

If we arrange the components R_{ij} as follows:

$$\begin{pmatrix} R_{11} & R_{12} & \dots & R_{1n} \\ R_{21} & R_{22} & \dots & R_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n1} & R_{n2} & \dots & R_{nn} \end{pmatrix}$$

and we write elements of R in the same fashion, then the multiplication in R follows the same pattern as ordinary matrix multiplication.

Note that R_{ij} is an R_{ii}-R_{jj}-bimodule, and the multiplication of elements induces homomorphisms $R_{ij} \otimes_{R_{jj}} R_{jk} \to R_{ik}$ for all i, j, k.

Conversely, given a collection of rings R_i, and for each $i \neq j$ an R_i-R_j-bimodule R_{ij}, and for each i, j, k with $i \neq j$ and $j \neq k$ a homomorphism $R_{ij} \otimes_{R_j} R_{jk} \to R_{ik}$, we can construct a generalized matrix ring structure on

$$R = \bigoplus_{i,j} R_{ij},$$

where we take $R_{ii} = R_i$.