planetmath.org

Math for the people, by the people.

monoid bialgebra

Canonical name	MonoidBialgebra
Date of creation	2013-03-22 18:58:48
Last modified on	2013-03-22 18:58:48
Owner	joking (16130)
Last modified by	joking (16130)
Numerical id	4
Author	joking (16130)
Entry type	Example
Classification	msc 16W30

Let G be a monoid and k a field. Consider the vector space kG over k with basis G. More precisely,

$$kG = \{f : G \to k \mid f(g) = 0 \text{ for almost all } g \in G\}.$$

We identify $g \in G$ with a function $f_g : G \to k$ such that $f_g(g) = 1$ and $f_g(h) = 0$ for $h \neq g$. Thus, every element in kG is of the form

$$\sum_{g \in G} \lambda_g g,$$

for $\lambda_g \in k$. The vector space kG can be turned into a k-algebra, if we define multiplication as follows:

$$g \cdot h = gh,$$

where on the right side we have a multiplication in the monoid G. This definition extends linearly to entire kG and defines an algebra structure on kG, where neutral element of G is the identity in kG.

Furthermore, we can turn kG into a coalgebra as follows: comultiplication $\Delta : kG \to kG \otimes kG$ is defined by $\Delta(g) = g \otimes g$ and counit $\varepsilon : kG \to k$ is defined by $\varepsilon(g) = 1$. One can easily check that this defines coalgebra structure on kG.

The vector space kG is a bialgebra with with these algebra and coalgebra structures and it is called a *monoid bialgebra*.