



Math for the people, by the people.

grouplike elements

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Let  $(C, \Delta, \varepsilon)$  be a coalgebra over a field  $k$ .

**Definition.** The element  $g \in C$  is called *grouplike* iff  $g \neq 0$  and  $\Delta(g) = g \otimes g$ . The set of all grouplike elements in a coalgebra  $C$  is denoted by  $G(C)$ .

**Properties.** 0) The set  $G(C)$  can be empty, but (for example) if  $C$  can be turned into a bialgebra, then  $G(C) \neq \emptyset$ . In particular Hopf algebras always have grouplike elements.

1) If  $g \in G(C)$ , then it follows from the counit property that  $\varepsilon(g) = 1$ .

2) It can be shown that the set  $G(C)$  is linearly independent.