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ideal generated by a subset of a ring

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Let X be a subset of a ring R . Let $S = \{I_k\}$ be the collection of all left ideals of R that contain X (note that the set is nonempty since $X \subset R$ and R is an ideal in itself). The intersection

$$I = \bigcap_{I_k \in S} I_k$$

is called the *left ideal generated by X* , and is denoted by (X) . We say that X *generates* I as an ideal.

The definition is symmetrical for right ideals.

Alternatively, we can constructively form the set of elements that constitutes this ideal: The left ideal (X) consists of finite R -linear combinations of elements of X :

$$(X) = \left\{ \sum_{\lambda} (r_{\lambda} a_{\lambda} + n_{\lambda} a_{\lambda}) \mid a_{\lambda} \in X, r_{\lambda} \in R, n_{\lambda} \in \mathbb{Z} \right\}.$$