inverses in rings

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Defines	left inverse
Defines	right inverse

Let R be a ring with unity 1 and $r \in R$. Then r is *left invertible* if there exists $q \in R$ with $qr = 1$; q is a *left inverse* of r. Similarly, r is *right invertible* if there exists $s \in R$ with $rs = 1$; s is a *right inverse* of r.

Note that, if r is left invertible, it may not have a unique left inverse, and similarly for right invertible elements. On the other hand, if r is left invertible and right invertible, then it has exactly one left inverse and one right inverse. Moreover, these two are equal, and r is a unit.