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## conjugate module

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If  $M$  is a right module over a ring  $R$ , and  $\alpha$  is an endomorphism of  $R$ , we define the *conjugate module*  $M^\alpha$  to be the right  $R$ -module whose underlying set is  $\{m^\alpha \mid m \in M\}$ , with abelian group structure identical to that of  $M$  (i.e.  $(m - n)^\alpha = m^\alpha - n^\alpha$ ), and scalar multiplication given by  $m^\alpha \cdot r = (m \cdot \alpha(r))^\alpha$  for all  $m$  in  $M$  and  $r$  in  $R$ .

In other words, if  $\phi : R \rightarrow \text{End}_{\mathbb{Z}}(M)$  is the ring homomorphism that describes the right module action of  $R$  upon  $M$ , then  $\phi\alpha$  describes the right module action of  $R$  upon  $M^\alpha$ .

If  $N$  is a left  $R$ -module, we define  ${}^\alpha N$  similarly, with  $r \cdot {}^\alpha n = {}^\alpha(\alpha(r) \cdot n)$ .