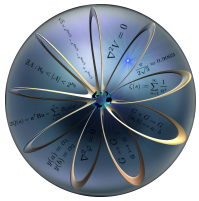

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ring-finite integral extensions are module-finite

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Theorem If B is a subring of A and $u_1, \ldots, u_s \in A$ are integral over B, then $B[u_1, \ldots, u_s]$ is module-finite over B.

Proof. If $s = 1$ then $u^n + b_1 u^{n-1} + \cdots + b_n = 0$, so $\{1, u, \ldots, u^{n-1}\}$ spans $B[u]$ over B.

If $s > 1$, use induction on $B \subset B[u_1] \subset B[u_1, u_2] \subset \ldots \subset B[u_1, \ldots, u_s]$ and multiply the spanning sets together.