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example of Klein 4-ring

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The Klein 4-ring K can be represented by as a left ideal of 2×2-matrices over the field with two elements $\mathbb{Z}_2$. Doing so helps to explain some of the unnatural properties of this nonunital ring and is an example of how many nonunital rings can often be understood as very natural subobjects of unital rings.

$$K = \left\{ \begin{bmatrix} x & 0 \\ y & 0 \end{bmatrix} : x, y \in \mathbb{Z}_2 \right\} \tag{1}$$

To match the product with with the table, use

$$a := \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$b := \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},$$
$$c := \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}.$$

Here the properties of the abstract multiplication table of K (given `http://planetmath.org/Kl`
can be seen as rather natural properties of unital rings. That is, the elements a and c are idempotents in the ring $M_2(\mathbb{Z}_2)$ and so they behave similar to identities, and b is nilpotent so that its annihilating property is expected as well.

The second noncommutative nonunital ring of order 4 is the transpose of these matrices, that is, a right ideal of $M_2(\mathbb{Z}_2)$.

Viewed in this way we recognize the Klein 4-ring as part of an infinite family of similar nonunital rings of left/right ideals of a unital ring. Some authors prefer to treat such objects only as ideals and not as rings so that the properties are always given the background of a more familiar structure.