annihilator

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Let R be a ring, and suppose that M is a left R-module and N a right R-module.

Annihilator of a Subset of a Module

1. If X is a subset of M, then we define the *left annihilator* of X in R:

$$\mathrm{l.ann}(X) = \{r \in R \mid rx = 0 \text{ for all } x \in X\}.$$

   If $a, b \in \mathrm{l.ann}(X)$, then so are $a - b$ and ra for all $r \in R$. Therefore, $\mathrm{l.ann}(X)$ is a left ideal of R.

2. If Y is a subset of N, then we define the *right annihilator* of Y in R:

$$\mathrm{r.ann}(Y) = \{r \in R \mid yr = 0 \text{ for all } y \in Y\}.$$

   Like above, it is easy to see that $\mathrm{r.ann}(Y)$ is a right ideal of R.

Remark. $\mathrm{l.ann}(X)$ and $\mathrm{r.ann}(Y)$ may also be written as $\mathrm{l.ann}_R(X)$ and $\mathrm{r.ann}_R(Y)$ respectively, if we want to emphasize R.

Annihilator of a Subset of a Ring

1. If Z is a subset of R, then we define the *right annihilator* of Z in M:

$$\mathrm{r.ann}_M(Z) = \{m \in M \mid zm = 0 \text{ for all } z \in Z\}.$$

   If $m, n \in \mathrm{r.ann}_M(Z)$, then so are $m-n$ and rm for all $r \in R$. Therefore, $\mathrm{r.ann}_M(Z)$ is a left R-submodule of M.

2. If Z is a subset of R, then we define the *left annihilator* of Z in N:

$$\mathrm{l.ann}_N(Z) = \{n \in N \mid nz = 0 \text{ for all } z \in Z\}.$$

   Similarly, it can be easily seen that $\mathrm{l.ann}_N(Z)$ is a right R-submodule of N.