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standard identity

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Let R be a commutative ring and X be a set of non-commuting variables over R. The *standard identity of degree* n in $R\langle X\rangle$, denoted by $[x_1, \dots x_n]$, is the polynomial

$$\sum_{\pi} \operatorname{sign}(\pi) x_{\pi(1)} \cdots x_{\pi(n)}, \text{ where } \pi \in S_n.$$

Remarks:

- A ring R satisfying the standard identity of degree 2 (i.e., $[R, R] = 0$) is commutative. In this sense, algebras satisfying a standard identity is a generalization of the class of commutative algebras.

- Two immediate properties of $[x_1, \dots x_n]$ are that it is *multilinear* over R, and it is *alternating*, in the sense that $[r_1, \dots r_n] = 0$ whenever two of the $r_i's$ are equal. Because of these two properties, one can show that an n-dimensional algebra R over a field k is a PI-algebra, satisfying the standard identity of degree $n+1$. As a corollary, $\mathbb{M}_n(k)$, the $n\times n$ matrix ring over a field k, is a PI-algebra satisfying the standard identity of degree $n^2 + 1$. In fact, Amitsur and Levitski have shown that $\mathbb{M}_n(k)$ actually satisfies the standard identity of degree $2n$.

References

[1] S. A. Amitsur and J. Levitski, *Minimal identities for algebras*, Proc. Amer. Math. Soc., 1 (1950) 449-463.