examples of modules

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This entry is a of examples of modules over rings. Unless otherwise specified in the example, M will be a module over a ring R.

- Any abelian group is a module over the ring of integers, with action defined by $n \cdot g$ for $g \in G$ given by $n \cdot g = \sum_{i=1}^{n} g$.
- If R is a subring of a ring S, then S is an R-module, with action given by multiplication in S.
- If R is any ring, then any (left) ideal I of R is a (left) R-module, with action given by the multiplication in R.
- Let $R = \mathbb{Z}$ and let $E = \{2k \mid k \in \mathbb{Z}\}$. Then E is a module over the ring $\mathbb{Z}$ of integers. Further, define the sets $B = E \times E$ and $C = E \times \{0\}$ and $D = \{0\} \times E$. Then B, C, and D are modules over $\mathbb{Z} \times \mathbb{Z}$, with action given by $a \cdot x = (a \cdot x_1, a \cdot x_2)$ if $x = (x_1, x_2)$ even if the product is redefined as $a \cdot x_1 = 0$ and $a \cdot x_2 = 0$, but now the identity element is $(1, 1)$. However by our new product definition $a \cdot x = (a \cdot x_1, a \cdot x_2) = (0, 0)$ even if $a = (1, 1)$, the ring identity element originally In the more general definition of module which does not require an identity element $\mathbf{1}$ in the ring and does not require $\mathbf{1} \cdot m = m$ for all $m \in M$, we observe that $\mathbf{1} \cdot m \neq m$ in this example just constructed. (one of the purposes of this comment is to show that all modules need not be unital ones).
- `http://planetmath.org/QuantumDouble`Yetter-Drinfel'd module.