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corner of a ring

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Does there exist a subset S of a ring R which is a ring with a multiplicative identity, but not a subring of R?

Let R be a ring without the assumption that R has a multiplicative identity. Further, assume that e is an idempotent of R. Then the subset of the form eRe is called a *corner* of the ring R.

It's not hard to see that eRe is a ring with e as its multiplicative identity:

1. $eae + ebe = e(a + b)e \in eRe$,
2. $0 = e0e \in eRe$,
3. $e(-a)e$ is the additive inverse of eae in eRe,
4. $(eae)(ebe) = e(aeb)e \in eRe$, and
5. $e = ee = eee \in eRe$, with $e(eae) = eae = (eae)e$, for any $eae \in eRe$.

If R has no multiplicative identity, then any corner of R is a proper subset of R which is a ring and not a subring of R. If R has 1 as its multiplicative identity and if $e \neq 1$ is an idempotent, then the eRe is not a subring of R as they don't share the same multiplicative identity. In this case, the corner eRe is said to be *proper*. If we set $f = 1 - e$, then fRf is also a proper corner of R.

Remark. If R has 1 with $e \neq 1$ an idempotent. Then corners $S = eRe$ and $T = fRf$, where $f = 1 - e$, are direct summands (as modules over $\mathbb{Z}$) of R via a Peirce decomposition.

References

[1] I. Kaplansky, *Rings of Operators*, W. A. Benjamin, Inc., New York, 1968.