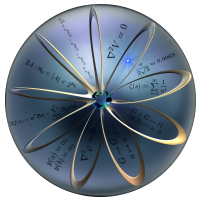

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quotient module

Canonical name	QuotientModule
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Classification	msc 16D10
Defines	quotient vector space

Let M be a module over a ring R, and let S be a submodule of M. The *quotient module* M/S is the quotient group M/S with scalar multiplication defined by $\lambda(x + S) = \lambda x + S$ for all $\lambda \in R$ and all $x \in M$.

This is a well defined operation. Indeed, if $x + S = x' + S$ then for some $s \in S$ we have $x' = x + s$ and therefore

$$\begin{aligned} \lambda x' &= \lambda(x + s) \\ &= \lambda x + \lambda s \end{aligned}$$

so that $\lambda x' + S = \lambda x + \lambda s + S = \lambda x + S$, since $\lambda s \in S$.

In the special case that R is a field this construction defines the *quotient vector space* of a vector space by a vector subspace.