(partial) tilting module

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Let A be an associative, finite-dimensional algebra over a field k. Throughout all modules are finite-dimensional.

A right A-module T is called a **partial tilting module** if the projective dimension of T is at most 1 ($\mathrm{pd}T \leqslant 1$) and $\mathrm{Ext}^1_A(T,T) = 0$.

Recall that if M is an A-module, then by $\mathrm{add}M$ we denote the class of all A-modules which are direct sums of direct summands of M. Since Krull-Schmidt Theorem holds in the category of finite-dimensional A-modules, then this means, that if

$$M = E_1 \oplus \cdots \oplus E_n$$

for some indecomposable modules E_i, then $\mathrm{add}M$ consists of all modules which are isomorphic to

$$E_1^{a_1} \oplus \cdots \oplus E_n^{a_n}$$

for some nonnegative integers $a_1, \ldots, a_n$.

A partial tiliting module T is called a **tilting module** if there exists a short exact sequence

$$0 \to A \to T' \to T'' \to 0$$

such that both $T', T'' \in \mathrm{add}T$. Here we treat the algebra A is a right module via multiplication.

Note that every projective module is partial tilting. Also a projective module P is tilting if and only if every indecomposable direct summand of A is a direct summand of P.