# additive inverse of one element times another element is the additive inverse of their product

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| Author | cvalente (11260) |
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Let $R$ be a ring. For all $x, y \in R$
$(-x) \cdot y = x \cdot (-y) = -(x \cdot y)$
All we need to prove is that $(-x) \cdot y + x \cdot y = x \cdot (-y) + x \cdot y = 0$
Now: $(-x) \cdot y + x \cdot y = ((-x) + x) \cdot y$ by distributivity.
Since $(-x) + x = 0$ by definition and for all $y$, $0 \cdot y = 0$ we get:
$(-x) \cdot y + x \cdot y = 0 \cdot y = 0$ and thus $(-x) \cdot y = -(x \cdot y)$
For $x \cdot (-y)$, use the previous properties of rings to show that
$x \cdot (-y) + x \cdot y = x \cdot ((-y) + y) = x \cdot 0 = 0$
and thus $x \cdot (-y) = -(x \cdot y)$