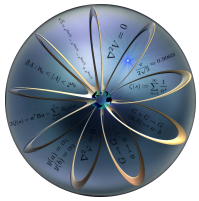

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Brauer group

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Defines	opposite algebra

1 Algebraic view

Let K be a field. The *Brauer group* $\mathrm{Br}(K)$ of K is the set of all equivalence classes of central simple algebras over K, where two central simple algebras A and B are equivalent if there exists a division ring D over K and natural numbers n, m such that A (resp. B) is isomorphic to the ring of $n \times n$ (resp. $m \times m$) matrices with coefficients in D.

The group operation in $\mathrm{Br}(K)$ is given by tensor product: for any two central simple algebras A, B over K, their product in $\mathrm{Br}(K)$ is the central simple algebra $A \otimes_K B$. The identity element in $\mathrm{Br}(K)$ is the class of K itself, and the inverse of a central simple algebra A is the *opposite algebra* A^{opp} defined by reversing the order of the multiplication operation of A.

2 Cohomological view

The Brauer group of K is naturally isomorphic to the second Galois cohomology group $H^2(\mathrm{Gal}(K^{\mathrm{sep}}/K), (K^{\mathrm{sep}})^\times)$. See `http://www.math.harvard.edu/ elkies/M250.01/inc` Theorem 12 and succeeding remarks.