# orthogonal idempotents of the group ring

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Let $G$ be a finite abelian group, let $L$ be any field containing the $|G|$-th roots of unity, and let $\hat{G}$ denote the character group of $G$ with values in $L$. For any character $\chi \in \hat{G}$, we define $\varepsilon_\chi$, the corresponding *orthogonal idempotent of the group ring* $L[G]$, by

$$\varepsilon_\chi = \frac{1}{|G|} \sum_{g \in G} \chi(g) g^{-1}.$$

The following equalities hold:

- $\varepsilon_\chi^2 = \varepsilon_\chi$ for all $\chi$
- $\varepsilon_\chi \varepsilon_\psi = 0$ for any $\chi \neq \psi$
- $\sum_{\chi \in \hat{G}} \varepsilon_\chi = 1$
- $\varepsilon_\chi \cdot g = \chi(g) \varepsilon_\chi$

These orthogonal idempotents are used to decompose modules over $L[G]$: If $M$ is such a module, then $M = \oplus_\chi (\varepsilon_\chi M)$.