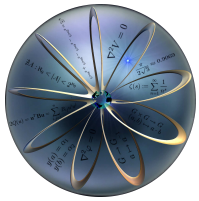

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minus one times an element is the additive inverse in a ring

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Lemma 1. *Let R be a ring (with unity 1) and let a be an element of R. Then*

$$(-1) \cdot a = -a$$

where -1 is the additive inverse of 1 and $-a$ is the additive inverse of a.

Proof. Note that for any a in R there exists a unique "$-a$" by the uniqueness of additive inverse in a ring. We check that $(-1)\cdot a$ equals the additive inverse of a.

$$\begin{aligned} a + (-1) \cdot a &= 1 \cdot a + (-1) \cdot a, \quad \text{by the definition of 1} \\ &= (1 + (-1)) \cdot a, \quad \text{by the distributive law} \\ &= 0 \cdot a, \quad \text{by the definition of } -1 \\ &= 0, \quad \text{as a result of the properties of zero} \end{aligned}$$

Hence $(-1)\cdot a$ is "an" additive inverse for a, and by uniqueness $(-1)\cdot a = -a$, *the* additive inverse of a. Analogously, we can prove that $a \cdot (-1) = -a$ as well. □