# zero ring

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A ring is a *zero ring* if the product of any two elements is the additive identity (or zero).

Zero rings are commutative under multiplication. For if $Z$ is a zero ring, $0_Z$ is its additive identity, and $x, y \in Z$, then $xy = 0_Z = yx$.

Every zero ring is a nilpotent ring. For if $Z$ is a zero ring, then $Z^2 = \{0_Z\}$.

Since every subring of a ring must contain its zero element, every subring of a ring is an ideal, and a zero ring has no prime ideals.

The simplest zero ring is $\mathbb{Z}_1 = \{0\}$. Up to `http://planetmath.org/RingIsomorphism`isomorph this is the only zero ring that has a multiplicative identity.

Zero rings exist in . They can be constructed from any ring. If $R$ is a ring, then

$$\left\{ \left( \begin{array}{cc} r & -r \\ r & -r \end{array} \right) \middle| \, r \in R \right\}$$

considered as a subring of $\mathbf{M}_{2\,\mathrm{x}\,2}(R)$ (with standard matrix addition and multiplication) is a zero ring. Moreover, the cardinality of this subset of $\mathbf{M}_{2\,\mathrm{x}\,2}(R)$ is the same as that of $R$.

Moreover, zero rings can be constructed from any abelian group. If $G$ is a group with identity $e_G$, it can be made into a zero ring by declaring its addition to be its group operation and defining its multiplication by $a \cdot b = e_G$ for any $a, b \in G$.

Every finite zero ring can be written as a direct product of cyclic rings, which must also be zero rings themselves. This follows from the `http://planetmath.org/Fundamen` theorem of finite abelian groups. Thus, if $p_1, \dots, p_m$ are distinct primes, $a_1, \dots, a_m$ are positive integers, and $n = \displaystyle\prod_{j=1}^{m} {p_j}^{a_j}$, then the number of zero rings of `http://planetmath.org/Order`order $n$ is $\displaystyle\prod_{j=1}^{m} p(a_j)$, where $p$ denotes the `http://planetmath.org/PartitionFunction2`partition function.