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new vector spaces from old ones

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This entry list methods that give new vector spaces from old ones.

1. Changing the field (complexification, etc.)
2. vector subspace
3. Quotient vector space
4. direct product of vectors spaces
5. Cartesian product of vector spaces
6. http://planetmath.org/TensorProductClassicalTensor product of vector spaces
7. The space of linear maps from one vector space to another, also denoted by $\mathrm{Hom}_k(V,W)$, or simply $\mathrm{Hom}(V,W)$, where V and W are vector spaces over the field k
8. The space of endomorphisms of a vector space. Using the notation above, this is the space $\mathrm{Hom}_k(V,V) = \mathrm{End}(V)$
9. http://planetmath.org/DualSpacedual vector space, and bi-dual vector space. Using the notation above, this is the space $\mathrm{Hom}(V,k)$, or simply V^*.
10. The annihilator of a subspace is a subspace of the dual vector space
11. Wedge product of vector spaces
12. A field k is a vector space over itself. Consider a set B and the set V of all functions from B to k. Then V has a natural vector space structure. If B is finite, then V can be viewed as a vector space having B as a basis.

Vector spaces involving a linear map

Suppose $L\colon V \to W$ is a linear map.

1. The kernel of L is a subspace of V.
2. The image of L is a subspace of W.
3. The cokernel of L is a quotient space of W.

Topological vector spaces

Suppose V is topological vector space.

1. If W is a subspace of V then its closure $\overline{W}$ is also a subspace of V.
2. If V is a metric vector space then its completion $\widetilde{V}$ is also a (metric) vector space.
3. The direct integral of Hilbert spaces provides a new Hilbert space.

Spaces of structures and subspaces of the tensor algebra of a vector space

There are also certain spaces of interesting structures on a vector space that at least in the case of finite dimension correspond to certain subspaces of the tensor algebra of the vector space. These spaces include:

1. The space of Euclidean inner products.
2. The space of Hermitian inner products.
3. the space of symplectic structures.
4. vector bundles
5. space of connections