uniform dimension

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Let M be a module over a ring R, and suppose that M contains no infinite direct sums of non-zero submodules. (This is the same as saying that M is a module of finite rank.)

Then there exists an integer n such that M contains an essential submodule N where

$$N = U_1 \oplus U_2 \oplus \cdots \oplus U_n$$

is a direct sum of n uniform submodules.

This number n does not depend on the choice of N or the decomposition into uniform submodules.

We call n the *uniform dimension* of M. Sometimes this is written $\operatorname{u-dim} M = n$.

If R is a field K, and M is a finite-dimensional vector space over K, then $\operatorname{u-dim} M = \dim_K M$.

$\operatorname{u-dim} M = 0$ if and only if $M = 0$.