# internal direct sum of ideals

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Let $R$ be a ring and $\mathfrak{a}_1$, $\mathfrak{a}_2$, ..., $\mathfrak{a}_n$ its ideals (left, right or two-sided). We say that $R$ is the *internal direct sum* of these ideals, denoted by

$$R = \mathfrak{a}_1 \oplus \mathfrak{a}_2 \oplus \cdots \oplus \mathfrak{a}_n,$$

if both of the following conditions are true:

$$R = \mathfrak{a}_1 + \mathfrak{a}_2 + \cdots + \mathfrak{a}_n,$$

$$\mathfrak{a}_i \cap \sum_{j \neq i} \mathfrak{a}_j = \{0\} \quad \forall i.$$

**Theorem.** If $\mathfrak{a}_1$, $\mathfrak{a}_2$, ..., $\mathfrak{a}_n$ are ideals of the ring $R$, then the following two statements are equivalent:

- $R = \mathfrak{a}_1 \oplus \mathfrak{a}_2 \oplus \cdots \oplus \mathfrak{a}_n$.
- Every element $r$ of $R$ has a unique expression
  $r = a_1 + a_2 + \cdots + a_n$ with $a_i \in \mathfrak{a}_i$ $\forall i$.