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construction of an injective resolution

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The category of modules has enough injectives. Let M be a module, and let I^0 be an injective module such that

$$0 \longrightarrow M \longrightarrow I^0$$

is exact. Then, let M_0 be the image of M in I^0 , and construct the factor module I^0/M^0 . Then, since the category of modules has enough injectives, we can find a module I^1 such that

$$0 \longrightarrow I^0/M^0 \xrightarrow{\phi_0} I^1$$

is exact. ϕ_0 induces a homomorphism $\phi: I^0 \longrightarrow I^1$, whose kernel is M^0 . We thus have an exact sequence

$$0 \longrightarrow M \longrightarrow I^0 \longrightarrow I^1.$$

One can continue this process to construct injective modules I^n for any $n \in \mathbb{Z}$ (the resolution may terminate: $I^m = 0$ for some $N \in \mathbb{Z}$ with all $m > N$).