classical ring of quotients

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Let R be a ring. An element of R is called *regular* if it is not a right zero divisor or a left zero divisor in R.

A ring $Q \supset R$ is a *left classical ring of quotients* for R (resp. *right classical ring of quotients* for R) if it satisifies:

- every regular element of R is invertible in Q
- every element of Q can be written in the form $x^{-1}y$ (resp. yx^{-1}) with $x, y \in R$ and x regular.

If a ring R has a left or right classical ring of quotients, then it is unique up to isomorphism.

If R is a commutative integral domain, then the left and right classical rings of quotients always exist – they are the field of fractions of R.

For non-commutative rings, necessary and sufficient conditions are given by Ore's Theorem.

Note that the goal here is to construct a ring which is not too different from R, but in which more elements are invertible. The first condition says which elements we want to be invertible. The second condition says that Q should just enough extra elements to make the regular elements invertible.

Such rings are called classical rings of quotients, because there are other rings of quotients. These all attempt to enlarge R somehow to make more elements invertible (or sometimes to make ideals invertible).

Finally, note that a ring of quotients is not the same as a quotient ring.