ideal multiplication laws

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The http://planetmath.org/ProductOfIdealsmultiplication of the (two-sided) ideals of any ring R has following properties:

1. $(0)\mathfrak{a} = \mathfrak{a}(0) = (0)$
2. $(\mathfrak{a}\mathfrak{b})\mathfrak{c} = \mathfrak{a}(\mathfrak{b}\mathfrak{c})$
3. $\mathfrak{a}(\mathfrak{b}+\mathfrak{c}) = \mathfrak{a}\mathfrak{b}+\mathfrak{a}\mathfrak{c}, \quad (\mathfrak{a}+\mathfrak{b})\mathfrak{c} = \mathfrak{a}\mathfrak{c}+\mathfrak{b}\mathfrak{c}$
4. If R has a unity, then $R\mathfrak{a} = \mathfrak{a}R = \mathfrak{a}$
5. If R is commutative, then $\mathfrak{a}\mathfrak{b} = \mathfrak{b}\mathfrak{a}$
6. $\mathfrak{a}\mathfrak{b} \subseteq \mathfrak{a} \cap \mathfrak{b}$
7. $\mathfrak{a}(\mathfrak{b} \cap \mathfrak{c}) \subseteq \mathfrak{a}\mathfrak{b} \cap \mathfrak{a}\mathfrak{c}$
8. $\mathfrak{a} \subseteq \mathfrak{b} \quad \Rightarrow \quad \mathfrak{a}\mathfrak{c} \subseteq \mathfrak{b}\mathfrak{c}$

Remark. The properties 1, 2, 3, 4 together with the properties

$$(\mathfrak{a}+\mathfrak{b})+\mathfrak{c} = \mathfrak{a}+(\mathfrak{b}+\mathfrak{c}), \qquad \mathfrak{a}+\mathfrak{b} = \mathfrak{b}+\mathfrak{a}, \qquad \mathfrak{a}+(0) = \mathfrak{a}$$

of the ideal addition make the set A of all ideals of R to a semiring $(A,\,+,\,\cdot)$.
It is not a ring, since no non-zero ideal of R has the http://planetmath.org/Ringadditive inverse.

References

[1] M. Larsen & P. McCarthy: *Multiplicative theory of ideals.* Academic Press, New York (1971).