the characteristic embedding of the Burnside ring

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Let G be a finite group, H its subgroup and X a finite G-set. By the *H-fixed point subset* of X we understand the set

$$X^H = \{x \in X;\ \forall_{h \in H}\ hx = x\}.$$

Denote by $|X|$ the cardinality of a set X.

It is easy to see that for any G-sets X, Y we have:

$$|(X \sqcup Y)^H| = |X^H| + |Y^H|;$$

$$|(X \times Y)^H| = |X^H| \cdot |Y^H|.$$

Denote by $\mathrm{Sub}(G) = \{H \subseteq G;\ H \text{ is a subgroup of } G\}$. Recall that any $H, K \in \mathrm{Sub}(G)$ are said to be conjugate iff there exists $g \in G$ such that $H = gKg^{-1}$. Conjugation is an equivalence relation. Denote by $\mathrm{Con}(G)$ the quotient set.

One can check that for any $H, K \in \mathrm{Sub}(G)$ such that H is conjugate to K and for any finite G-set X we have

$$|X^H| = |X^K|.$$

Thus we have a well defined ring homomorphism:

$$\varphi : \Omega(G) \to \bigoplus_{(H) \in \mathrm{Con}(G)} \mathbb{Z};$$

$$\varphi([X] - [Y]) = (|X^H| - |Y^H|)_{(H) \in \mathrm{Con}(G)}.$$

This homomorphism is known as the characteristic embedding, since it is monomorphism (see [**?**] for proof).

References

[1] T. tom Dieck, *Transformation groups and representation theory*, Lecture Notes in Math. 766, Springer-Verlag, Berlin, 1979.