



planetmath.org

Math for the people, by the people.

coalgebra homomorphism

Canonical name	CoalgebraHomomorphism
Date of creation	2013-03-22 18:49:25
Last modified on	2013-03-22 18:49:25
Owner	joking (16130)
Last modified by	joking (16130)
Numerical id	4
Author	joking (16130)
Entry type	Definition
Classification	msc 16W30

Let (C, Δ, ε) and $(D, \Delta', \varepsilon')$ be coalgebras.

Definition. Linear map $f : C \rightarrow D$ is called *coalgebra homomorphism* if $\Delta' \circ f = (f \otimes f) \circ \Delta$ and $\varepsilon' \circ f = \varepsilon$.

Examples. 1) Of course, if D is a subcoalgebra of C , then the inclusion $i : D \rightarrow C$ is a coalgebra homomorphism. In particular, the identity is a coalgebra homomorphism.

2) If (C, Δ, ε) is a coalgebra and $I \subseteq C$ is a coideal, then we have canonical coalgebra structure on C/I (please, see <http://planetmath.org/SubcoalgebrasAndCoideals> this entry for more details). Then the projection $\pi : C \rightarrow C/I$ is a coalgebra homomorphism. Furthermore, one can show that the canonical coalgebra structure on C/I is a unique coalgebra structure such that π is a coalgebra homomorphism.