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**faithfully flat**

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Let  $A$  be a commutative ring. Then  $M$  is *faithfully flat* if for any  $A$ -modules  $P, Q$ , and  $R$ , we have

$$0 \rightarrow P \rightarrow Q \rightarrow R \rightarrow 0$$

is exact if and only if the  $M$ -tensored sequence

$$0 \rightarrow M \otimes_A P \rightarrow M \otimes_A Q \rightarrow M \otimes_A R \rightarrow 0$$

is exact. (Note that the “if and only if” clause makes this stronger than the definition of flatness).

Equivalently, an  $A$ -module  $M$  is faithfully flat iff  $M$  is flat and the functor  $- \otimes_A M$  is a faithful functor (and hence the name).