module

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Related topic	Module

(This is a definition of modules in terms of ring homomorphisms. You may prefer to read http://planetmath.org/Modulethe other definition instead.)

Let R be a ring, and let M be an abelian group.

We say that M is a *left R-module* if there exists a ring homomorphism $\phi\colon R \to \mathrm{End}_{\mathbb{Z}}(M)$ from R to the ring of abelian group endomorphisms on M (in which multiplication of endomorphisms is composition, using left function notation). We typically denote this function using a multiplication notation:

$$[\phi(r)](m) = r \cdot m = rm.$$

This ring homomorphism defines what is called a of R upon M.

If R is a unital ring (i.e. a ring with identity), then we typically demand that the ring homomorphism map the unit $1 \in R$ to the identity endomorphism on M, so that $1 \cdot m = m$ for all $m \in M$. In this case we may say that the module is *unital*.

Typically the abelian group structure on M is expressed in additive terms, i.e. with operator $+$, identity element 0_M (or just 0), and inverses written in the form $-m$ for $m \in M$.

Right module actions are defined similarly, only with the elements of R being written on the right sides of elements of M. In this case we either need to use an anti-homomorphism $R \to \mathrm{End}_{\mathbb{Z}}(M)$, or switch to right notation for writing functions.