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almost cocommutative bialgebra

Canonical name	AlmostCocommutativeBialgebra
Date of creation	2013-03-22 13:31:50
Last modified on	2013-03-22 13:31:50
Owner	bwebste (988)
Last modified by	bwebste (988)
Numerical id	5
Author	bwebste (988)
Entry type	Definition
Classification	msc 16W30

A bialgebra A is called *almost cocommutative* if there is an unit $\mathcal{R} \in A \otimes A$ such that

$$\mathcal{R}\Delta(a) = \Delta^{op}(a)\mathcal{R}$$

where Δ^{op} is the opposite comultiplication (the usual comultiplication, composed with the flip map of the tensor product $A \otimes A$). The element $\mathcal{R}$ is often called the $\mathcal{R}$-matrix of A.

The significance of the almost cocommutative condition is that $\sigma_{V,W} = \sigma \circ \mathcal{R} : V \otimes W \to W \otimes V$ gives a natural isomorphism of bialgebra representations, where V and W are A-modules, making the category of A-modules into a quasi-tensor or braided monoidal category. Note that $\sigma_{W,V} \circ \sigma_{V,W}$ is not necessarily the identity (this is the *braiding* of the category).