modular ideal

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Let R be a ring. A left ideal I of R is said to be *modular* if there is an $e \in R$ such that $re - r \in I$ for all $r \in R$. In other words, e acts as a right identity element modulo I:

$$re \equiv r \pmod{I}.$$

A right modular ideal is defined similarly, with e be a left identity modulo I.

Remark. If an ideal I is modular both as a left ideal as well as a right ideal in R, then R/I is a unital ring. Furthermore, every (left, right, two-sided) ideal in a unital ring is modular, implying that the notion of modular ideals is only interesting in rings without 1.

References

[1] P. M. Cohn, *Further Algebra and Applications*, Springer (2003).