composition series

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Let R be a ring and let M be a (right or left) R-module. A series of submodules

$$M = M_0 \supset M_1 \supset M_2 \supset \cdots \supset M_n = 0$$

in which each quotient M_i/M_{i+1} is simple is called a composition series for M.

A module need not have a composition series. For example, the ring of integers, $\mathbb{Z}$, considered as a module over itself, does not have a composition series.

A necessary and sufficient condition for a module to have a composition series is that it is both Noetherian and Artinian.

If a module does have a composition series, then all composition series are the same length. This length (the number n above) is called the *composition length* of the module.

If R is a semisimple Artinian ring, then R_R and ${}_RR$ always have composition series.