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semisimple ring

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Defines	semisimple

A ring R is (left) *semisimple* if it one of the following statements:

1. All left R-modules are semisimple.
2. All http://planetmath.org/FinitelyGeneratedRModulefinitely-generated left R-modules are semisimple.
3. All cyclic left R-modules are semisimple.
4. The left regular R-module ${}_RR$ is semisimple.
5. All short exact sequences of left R-modules http://planetmath.org/SplitShortExactSeque

The last condition offers another homological characterization of a *semisimple* ring:

- A ring R is (left) semisimple iff all of its left modules are http://planetmath.org/Projective

A more ring-theorectic characterization of a (left) semisimple ring is:

- A ring is left semisimple iff it is semiprimitive and left artinian.

In some literature, a (left) semisimple ring is defined to be a ring that is semiprimitive without necessarily being (left) artinian. Such a ring (semiprimitive) is called Jacobson semisimple, or J-semisimple, to remind us of the fact that its Jacobson radical is (0).

Relating to von Neumann regular rings, one has:

- A ring is left semisimple iff it is von Neumann regular and left noetherian.

The famous Wedderburn-Artin Theorem that a (left) semisimple ring is isomorphic to a finite direct product of matrix rings over division rings.

The theorem implies that a left semisimplicity is synonymous with right semisimplicity, so that it is safe to drop the word left or right when referring to semisimple rings.