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# length of a module

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|---|---|
| Canonical name | LengthOfAModule |
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| Entry type | Definition |
| Classification | msc 16D10 |
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| Synonym | finite-length module |
| Defines | finite length |

Let $A$ be a ring and let $M$ be an $A$-module. If there is a finite sequence of submodules of $M$

$$M = M_0 \supset M_1 \supset \cdots \supset M_n = 0$$

such that each quotient module $M_i/M_{i+1}$ is simple, then $n$ is necessarily unique by the http://planetmath.org/JordanHolderDecompositionJordan-Hölder theorem for modules. We define the above number $n$ to be the *length* of $M$. If such a finite sequence does not exist, then the length of $M$ is defined to be $\infty$.

If $M$ has finite length, then $M$ satisfies both the ascending and descending chain conditions.

A ring $A$ is said to have *finite length* if there is an $A$-module whose length is finite.