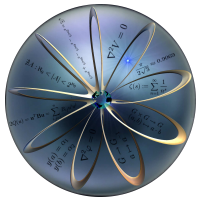

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functoriality of the Burnside ring

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We wish to show how the Burnside ring Ω can be turned into a contravariant functor from the category of finite groups into the category of commutative, unital rings.

Let G and H be finite groups. We already know how Ω acts on objects of the category of finite groups. Assume that $f : G \to H$ is a group homomorphism. Furthermore let X be a H-set. Then X can be naturally equiped with a G-set structure via function:

$$(g, x) \longmapsto f(g)x.$$

The set X equiped with this group action will be denoted by X_f.

Therefore a group homomorphism $f : G \to H$ induces a ring homomorphism

$$\Omega(f) : \Omega(H) \to \Omega(G)$$

such that

$$\Omega(f)([X] - [Y]) = [X_f] - [Y_f].$$

One can easily check that this turns Ω into a contravariant functor.