nil and nilpotent ideals

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Entry type	Definition
Classification	msc 16N40
Related topic	KoetheConjecture
Defines	nil
Defines	nil ring
Defines	nil ideal
Defines	nil right ideal
Defines	nil left ideal
Defines	nil subring
Defines	nilpotent
Defines	nilpotent element
Defines	nilpotent ring
Defines	nilpotent ideal
Defines	nilpotent right ideal
Defines	nilpotent left ideal
Defines	nilpotent subring
Defines	locally nilpotent
Defines	locally nilpotent ring
Defines	locally nilpo

An element x of a ring is *nilpotent* if $x^n = 0$ for some positive integer n.

A ring R is *nil* if every element in R is nilpotent. Similarly, a one- or two-sided ideal is called *nil* if each of its elements is nilpotent.

A ring R [resp. a one- or two sided ideal A] is *nilpotent* if $R^n = 0$ [resp. $A^n = 0$] for some positive integer n.

A ring or an ideal is *locally nilpotent* if every finitely generated subring is nilpotent.

The following implications hold for rings (or ideals):

$$\text{nilpotent} \quad \Rightarrow \text{locally nilpotent} \quad \Rightarrow \text{nil}$$