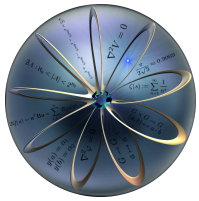

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upper nilradical

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The *upper nilradical* $\mathrm{Nil}^*(R)$ of R is the http://planetmath.org/SumOfIdealssum of all (two-sided) nil ideals in R. In other words, $a \in Nil^* R$ iff a can be expressed as a (finite) sum of nilpotent elements.

It is not hard to see that $\mathrm{Nil}^*(R)$ is the largest nil ideal in R. Furthermore, we have that $\mathrm{Nil}_*(R) \subseteq \mathrm{Nil}^*(R) \subseteq J(R)$, where $\mathrm{Nil}_*(R)$ is the lower radical or prime radical of R, and $J(R)$ is the Jacobson radical of R.

Remarks.

- If R is commutative, then $\mathrm{Nil}_*(R) = \mathrm{Nil}^*(R) = \mathrm{Nil}(R)$, the nilradical of R, consisting of all nilpotent elements.
- If R is left (or right) artinian, then $\mathrm{Nil}_*(R) = \mathrm{Nil}^*(R) = J(R)$.