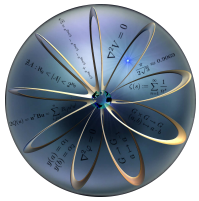

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characteristic of finite ring

Canonical name	CharacteristicOfFiniteRing
Date of creation	2013-03-22 19:10:19
Last modified on	2013-03-22 19:10:19
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	7
Author	pahio (2872)
Entry type	Theorem
Classification	msc 16B99
Related topic	Multiple
Related topic	IdealOfElementsWithFiniteOrder

The http://planetmath.org/Characteristiccharacteristic of the residue class ring $\mathbb{Z}/m\mathbb{Z}$, which contains m elements, is m, too. More generally, one has the

Theorem. The characteristic of a finite ring divides the number of the elements of the ring.

Proof. Let n be the characteristic of the ring R with m elements. Since m is the http://planetmath.org/OrderGrouporder of the group $(R, +)$, the Lagrange's theorem implies that

$$ma \;=\; 0 \quad \forall a \in R.$$

Let $m = qn+r$ where $0 \leqq r < n$. Because

$$ra \;=\; (m-qn)a \;=\; ma-q(na) \;=\; 0-0 \;=\; 0 \quad \forall a \in R$$

and n is the least positive integer ν making all $\nu a = 0$, the number r must vanish. Therefore, $m = qn$, i.e. $n \mid m$.

Remark. A ring R, the polynomial ring $R[X]$ and the ring $R[[X]]$ of formal power series have always the same characteristic.