ring

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Related topic	ExampleOfRings
Related topic	Subring
Related topic	Semiring
Related topic	Group
Related topic	Associates
Defines	multiplicative identity
Defines	multiplicative inverse
Defines	ring with unity
Defines	unit
Defines	ring addition
Defines	ring multiplication
Defines	ring sum
Defines	ring product
Defines	unital ring
Defines	unitary ring

A *ring* is a set R together with two binary operations, denoted $+ : R \times R \longrightarrow R$ and $\cdot : R \times R \longrightarrow R$, such that

1. $(a + b) + c = a + (b + c)$ and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for all $a, b, c \in R$ (associative law)

2. $a + b = b + a$ for all $a, b \in R$ (commutative law)

3. There exists an element $0 \in R$ such that $a + 0 = a$ for all $a \in R$ (additive identity)

4. For all $a \in R$, there exists $b \in R$ such that $a + b = 0$ (additive inverse)

5. $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ and $(a + b) \cdot c = (a \cdot c) + (b \cdot c)$ for all $a, b, c \in R$ (distributive law)

Equivalently, a ring is an abelian group $(R, +)$ together with a second binary operation $\cdot$ such that $\cdot$ is associative and distributes over $+$. Additive inverses are unique, and one can define *subtraction* in any ring using the formula $a - b := a + (-b)$ where $-b$ is the additive inverse of b.

We say R has a *multiplicative identity* if there exists an element $1 \in R$ such that $a \cdot 1 = 1 \cdot a = a$ for all $a \in R$. Alternatively, one may say that R is a *ring with unity*, a *unital ring*, or a *unitary ring*. Oftentimes an author will adopt the convention that all rings have a multiplicative identity. If R does have a multiplicative identity, then a *multiplicative inverse* of an element $a \in R$ is an element $b \in R$ such that $a \cdot b = b \cdot a = 1$. An element of R that has a multiplicative inverse is called a *unit* of R.

A ring R is *commutative* if $a \cdot b = b \cdot a$ for all $a, b \in R$.