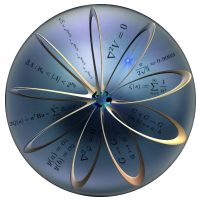

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proof of modular law

Canonical name	ProofOfModularLaw
Date of creation	2013-03-22 12:50:45
Last modified on	2013-03-22 12:50:45
Owner	yark (2760)
Last modified by	yark (2760)
Numerical id	8
Author	yark (2760)
Entry type	Proof
Classification	msc 16D10
Related topic	FirstIsomorphismTheorem

First we show $C + (B \cap A) \subseteq B \cap (C + A)$:
Note that $C \subseteq B, B \cap A \subseteq B$, and therefore $C + (B \cap A) \subseteq B$.
Further, $C \subseteq C + A$, $B \cap A \subseteq C + A$, thus $C + (B \cap A) \subseteq C + A$.

Next we show $B \cap (C + A) \subseteq C + (B \cap A)$:
Let $b \in B \cap (C + A)$. Then $b = c + a$ for some $c \in C$ and $a \in A$. Hence $a = b - c$, and so $a \in B$ since $b \in B$ and $c \in C \subseteq B$.
Hence $a \in B \cap A$, so $b = c + a \in C + (B \cap A)$.