vector space

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Let F be a field (or, more generally, a division ring). A *vector space* V over F is a set with two operations, $+ : V \times V \longrightarrow V$ and $\cdot : F \times V \longrightarrow V$, such that

1. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ for all $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$
2. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ for all $\mathbf{u}, \mathbf{v} \in V$
3. There exists an element $\mathbf{0} \in V$ such that $\mathbf{u} + \mathbf{0} = \mathbf{u}$ for all $\mathbf{u} \in V$
4. For any $\mathbf{u} \in V$, there exists an element $\mathbf{v} \in V$ such that $\mathbf{u} + \mathbf{v} = \mathbf{0}$
5. $a \cdot (b \cdot \mathbf{u}) = (a \cdot b) \cdot \mathbf{u}$ for all $a, b \in F$ and $\mathbf{u} \in V$
6. $1 \cdot \mathbf{u} = \mathbf{u}$ for all $\mathbf{u} \in V$
7. $a \cdot (\mathbf{u} + \mathbf{v}) = (a \cdot \mathbf{u}) + (a \cdot \mathbf{v})$ for all $a \in F$ and $\mathbf{u}, \mathbf{v} \in V$
8. $(a + b) \cdot \mathbf{u} = (a \cdot \mathbf{u}) + (b \cdot \mathbf{u})$ for all $a, b \in F$ and $\mathbf{u} \in V$

Equivalently, a vector space is a module V over a ring F which is a field (or, more generally, a division ring).

The elements of V are called *vectors*, and the element $\mathbf{0} \in V$ is called the *zero vector* of V.