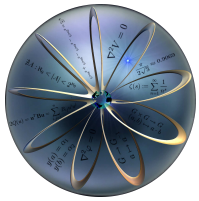

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Friedrichs' theorem

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Fix a commutative unital ring K of characteristic 0. Let X be a finite set and $K\langle X\rangle$ the free associative algebra on X. Then define the map $\delta : K\langle X\rangle \to K\langle X\rangle \otimes K\langle X\rangle$ by $x \mapsto x \otimes 1 + 1 \otimes x$.

Theorem 1 (Friedrichs)**.** *[?, Thm V.9] An element $a \in K\langle X\rangle$ is a Lie element if and only if $a\delta = a \otimes 1 + 1 \otimes a$.*

The term Lie element applies only when an element is taken from the universal enveloping algebra of a Lie algebra. Here the Lie algebra in question is the free Lie algebra on X, $FL\langle X\rangle$ whose universal enveloping algebra is $K\langle X\rangle$ by a theorem of Witt.

This characterization of Lie elements is a primary means in modern proofs of the Baker-Campbell-Hausdorff formula.

References

[1] Nathan Jacobson *Lie Algebras*, Interscience Publishers, New York, 1962.