faithful group action

Canonical name	FaithfulGroupAction
Date of creation	2013-03-22 14:02:23
Last modified on	2013-03-22 14:02:23
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	8
Author	rspuzio (6075)
Entry type	Definition
Classification	msc 16W22
Classification	msc 20M30

Let A be a G-set, that is, a set acted upon by a group G with action $\psi : G \times A \to A$. Then for any $g \in G$, the map $m_g\colon A \to A$ defined by

$$m_g(x) = \psi(g, x)$$

is a permutation of A (in other words, a bijective function from A to itself) and so an element of S_A. We can even get an homomorphism from G to S_A by the rule $g \mapsto m_g$.

If for any pair $g, h \in G$ $g \neq h$ we have $m_g \neq m_h$, in other words, the homomorphism $g \to m_g$ being injective, we say that the action is faithful.