examples of semiprimitive rings

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Examples of semiprimitive rings:

The integers $\mathbb{Z}$:
Since $\mathbb{Z}$ is commutative, any left ideal is two-sided. So the maximal left ideals of $\mathbb{Z}$ are the maximal ideals of $\mathbb{Z}$, which are the ideals $p\mathbb{Z}$ for p prime. So $J(\mathbb{Z}) = \bigcap_p p\mathbb{Z} = (0)$, as there are infinitely many primes.

A matrix ring $M_n(D)$ over a division ring D:
The ring $M_n(D)$ is simple, so the only proper ideal is (0). Thus $J(M_n(D)) = (0)$.

A polynomial ring $R[x]$ over an integral domain R:
Take $a \in J(R[x])$ with $a \neq 0$. Then $ax \in J(R[x])$, since $J(R[x])$ is an ideal, and $\deg(ax) \geq 1$. By one of the alternate characterizations of the Jacobson radical, $1 - ax$ is a unit. But $\deg(1 - ax) = \max\{\deg(1), \deg(ax)\} \geq 1$. So $1 - ax$ is not a unit, and by this contradiction we see that $J(R[x]) = (0)$.