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unitization

Canonical name	Unitization
Date of creation	2013-03-22 14:47:36
Last modified on	2013-03-22 14:47:36
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	9
Author	rspuzio (6075)
Entry type	Definition
Classification	msc 16-00
Classification	msc 13-00
Classification	msc 20-00
Synonym	minimal unitization

The operation of unitization allows one to add a unity element to an algebra. Because of this construction, one can regard any algebra as a subalgebra of an algebra with unity. If the algebra already has a unity, the operation creates a larger algebra in which the old unity is no longer the unity.

Let $\mathbf{A}$ be an algebra over a ring $\mathbf{R}$ with unity 1. Then, as a module, the unitization of $\mathbf{A}$ is the direct sum of $\mathbf{R}$ and $\mathbf{A}$:

$$\mathbf{A}^+ = \mathbf{R} \oplus \mathbf{A}$$

The product operation is defined as follows:

$$(x, a) \cdot (y, b) = (xy, ab + xb + ya)$$

The unity of $\mathbf{A}^+$ is $(1, 0)$.

It is also possible to unitize any ring using this construction if one regards the ring as an algebra over the ring of `http://planetmath.org/Integer`integers. (See the entry every ring is an integer algebra for details.) It is worth noting, however, that the result of unitizing a ring this way will always be a ring whose unity has zero characteristic. If one has a ring of finite characteristic k, one can instead regard it as an algebra over $\mathbb{Z}_k$ and unitize accordingly to obtain a ring of characteristic k.

The construction described above is often called "minimal unitization". It is in fact minimal, in the sense that every other unitization contains this unitization as a subalgebra.