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differential graded algebra

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Author	CWoo (3771)
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Let R be a commutative ring. A *differential graded algebra* (or *DG algebra*) over R is a complex (A, ∂^A) of R -modules with an element $1 \in A$ (the unit) and a degree zero chain map

$$A \otimes_R A \rightarrow A$$

that is unitary: $a1 = a = 1a$, and is associative: $a(bc) = (ab)c$. We also will stipulate that a DG algebra is graded commutative; that is for each $x, y \in A$, we have

$$xy = (-1)^{|x||y|}yx$$

where $|x|$ means the degree of x . Also, we assume that $A_i = 0$ for $i < 0$. Without these final assumptions, we will say that A is an *associative* DG algebra.

The fact that the product is a chain map of degree zero is best described by the Leibniz Rule; that is, for each $x, y \in A$, we have

$$\partial^A(xy) = \partial^A(x)y + (-1)^{|x|}x\partial^A(y).$$