finitely generated module

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A module X over a ring R is said to be *finitely generated* if there is a finite subset Y of X such that Y spans X. Let us recall that the span of a (not necessarily finite) set X of vectors is the class of all (finite) linear combinations of elements of S; moreover, let us recall that the span of the empty set is defined to be the singleton consisting of only one vector, the zero vector $\vec{0}$. A module X is then called *cyclic* if it can be a singleton.

Examples. Let R be a commutative ring with 1 and x be an indeterminate.

1. $Rx = \{rx \mid r \in R\}$ is a cyclic R-module generated by $\{x\}$.

2. $R \oplus Rx$ is a finitely-generated R-module generated by $\{1, x\}$. Any element in $R \oplus Rx$ can be expressed uniquely as $r + sx$.

3. $R[x]$ is not finitely generated as an R-module. For if there is a finite set Y $R[x]$, taking d to be the largest of all degrees of polynomials in Y, then x^{d+1} would not be in the of Y, assumed to be $R[x]$, which is a contradiction. (Note, however, that $R[x]$ is finitely-generated as an R-algebra.)