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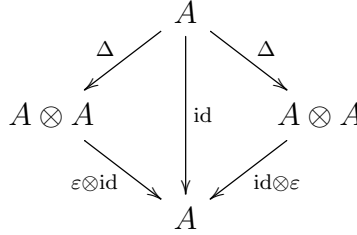
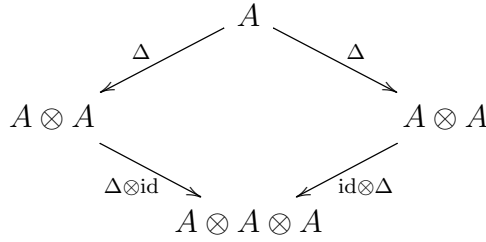
coalgebra

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Entry type	Definition
Classification	msc 16W30
Defines	comultiplication
Defines	counit
Defines	coassociative
Defines	cocommutative

A **coalgebra** is a vector space A over a field \mathbb{K} with a \mathbb{K} -linear map $\Delta: A \rightarrow A \otimes A$, called the **comultiplication**, and a (non-zero) \mathbb{K} -linear map $\varepsilon: A \rightarrow \mathbb{K}$, called the **counit**, such that

$$\begin{aligned} (\Delta \otimes \text{id}) \circ \Delta &= (\text{id} \otimes \Delta) \circ \Delta \quad (\text{coassociativity}), \\ (\varepsilon \otimes \text{id}) \circ \Delta &= \text{id} = (\text{id} \otimes \varepsilon) \circ \Delta. \end{aligned}$$

In of commutative diagrams:



Let $\sigma: A \otimes A \rightarrow A \otimes A$ be the flip map $\sigma(a \otimes b) = b \otimes a$. A coalgebra is said to be **cocommutative** if $\sigma \circ \Delta = \Delta$.

Let A and B be two coalgebras over a field \mathbb{K} . A coalgebra homomorphism is a \mathbb{K} -linear map $f: A \rightarrow B$ such that $\Delta_B \circ f = (f \otimes f) \circ \Delta_A$ and $\varepsilon_B \circ f = \varepsilon_A$.

Example 1 (Coalgebra of a set)

Let S be a set. The free vector space $\mathbb{K}S$, with basis given by the elements of S , is a coalgebra with comultiplication $\Delta(s) = s \otimes s$ and counit $\varepsilon(s) = 1$.