Lying-Over Theorem

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Let $\mathfrak{o}$ be a subring of a commutative ring $\mathfrak{O}$ with nonzero unity and integral over $\mathfrak{o}$. If $\mathfrak{a}$ is an ideal of $\mathfrak{o}$ and $\mathfrak{A}$ an ideal of $\mathfrak{O}$ such that

$$\mathfrak{A} \cap \mathfrak{o} \ = \ \mathfrak{a},$$

then $\mathfrak{A}$ is said to *lie over* $\mathfrak{a}$.

Theorem. If $\mathfrak{p}$ is a prime ideal of a ring $\mathfrak{o}$ which is a subring of a commutative ring $\mathfrak{O}$ with nonzero unity and integral over $\mathfrak{o}$, then there exists a prime ideal $\mathfrak{P}$ of $\mathfrak{O}$ lying over $\mathfrak{p}$. If the prime ideals $\mathfrak{P}$ and $\mathfrak{Q}$ both lie over $\mathfrak{p}$ and $\mathfrak{P} \subseteq \mathfrak{Q}$, then $\mathfrak{P} = \mathfrak{Q}$.

References

[1] M. Larsen & P. McCarthy: *Multiplicative theory of ideals.* Academic Press, New York (1971).

[2] P. Jaffard: *Les systèmes d'idéaux.* Dunod, Paris (1960).