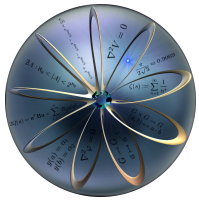

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Jacobson radical

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The *Jacobson radical* $J(R)$ of a unital ring R is the intersection of the annihilators of `http://planetmath.org/SimpleModule`simple left R-modules.

The following are alternative characterizations of the Jacobson radical $J(R)$:

1. The intersection of all left primitive ideals.
2. The intersection of all maximal left ideals.
3. The set of all $t \in R$ such that for all $r \in R$, $1 - rt$ is left invertible (i.e. there exists u such that $u(1 - rt) = 1$).
4. The largest ideal I such that for all $v \in I$, $1 - v$ is a unit in R.
5. (1) - (3) with "left" replaced by "right" and rt replaced by tr.

If R is commutative and finitely generated, then

$$J(R) = \{x \in R \mid x^n = 0 \text{ for some } n \in \mathbb{N}\} = \mathrm{Nil}(R).$$

The Jacobson radical can also be defined for non-unital rings. To do this, we first define a binary operation $\circ$ on the ring R by $x \circ y = x + y - xy$ for all $x, y \in R$. Then $(R, \circ)$ is a monoid, and the Jacobson radical is defined to be the largest ideal I of R such that $(I, \circ)$ is a group. If R is unital, this is equivalent to the definitions given earlier.