



planetmath.org

Math for the people, by the people.

fundamental isomorphism theorem for  
coalgebras

Canonical name	FundamentalIsomorphismTheoremForCoalgebras
Date of creation	2013-03-22 18:49:30
Last modified on	2013-03-22 18:49:30
Owner	joking (16130)
Last modified by	joking (16130)
Numerical id	4
Author	joking (16130)
Entry type	Theorem
Classification	msc 16W30

Let  $(C, \Delta, \varepsilon)$  and  $(D, \Delta', \varepsilon')$  be coalgebras. Recall, that if  $D_0 \subseteq D$  is a subcoalgebra, then  $(D_0, \Delta'_{|D_0}, \varepsilon'_{|D_0})$  is a coalgebra. On the other hand, if  $I \subseteq C$  is a coideal, then there is a canonical coalgebra structure on  $C/I$  (please, see <http://planetmath.org/SubcoalgebrasAndCoideals> this entry for more details).

**Theorem.** If  $f : C \rightarrow D$  is a coalgebra homomorphism, then  $\ker(f)$  is a coideal,  $\text{im}(f)$  is a subcoalgebra and a mapping  $f' : C/\ker(f) \rightarrow \text{im}(f)$  defined by  $f'(c + \ker(f)) = f(c)$  is a well defined coalgebra isomorphism.