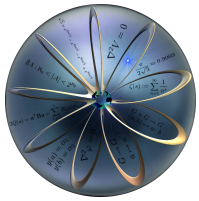

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semiprime ideal

Canonical name	SemiprimeIdeal
Date of creation	2013-03-22 12:01:23
Last modified on	2013-03-22 12:01:23
Owner	antizeus (11)
Last modified by	antizeus (11)
Numerical id	11
Author	antizeus (11)
Entry type	Definition
Classification	msc 16D25
Related topic	NSystem
Defines	semiprime ring
Defines	semiprime

Let R be a ring. An ideal I of R is a *semiprime ideal* if it satisfies the following equivalent conditions:

(a) I can be expressed as an intersection of prime ideals of R;

(b) if $x \in R$, and $xRx \subset I$, then $x \in I$;

(c) if J is a two-sided ideal of R and $J^2 \subset I$, then $J \subset I$ as well;

(d) if J is a left ideal of R and $J^2 \subset I$, then $J \subset I$ as well;

(e) if J is a right ideal of R and $J^2 \subset I$, then $J \subset I$ as well.

Here J^2 is the product of ideals $J \cdot J$.

The ring R itself satisfies all of these conditions (including being expressed as an intersection of an empty family of prime ideals) and is thus semiprime.

A ring R is said to be a *semiprime ring* if its zero ideal is a semiprime ideal.

Note that an ideal I of R is semiprime if and only if the quotient ring R/I is a semiprime ring.