coalgebra isomorphisms and isomorphic coalgebras

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Let (C, Δ, ε) and $(D, \Delta', \varepsilon')$ be coalgebras.

Definition. We will say that coalgebra homomorphism $f : C \to D$ is a *coalgebra isomorphism*, if there exists a coalgebra homomorphism $g : D \to C$ such that $f \circ g = \mathrm{id}_D$ and $g \circ f = \mathrm{id}_C$.

Remark. Of course every coalgebra isomorphism is a linear isomorphism, thus it is ,,one-to-one" and ,,onto". One can show that the converse also holds, i.e. if $f : C \to D$ is a coalgebra homomorphism such that f is ,,one-to-one" and ,,onto", then f is a coalgebra isomorphism.

Definition. We will say that coalgebras (C, Δ, ε) and $(D, \Delta', \varepsilon')$ are *isomorphic* if there exists coalgebra isomorphism $f : C \to D$. In this case we often write $(C, \Delta, \varepsilon) \simeq (D, \Delta', \varepsilon')$ or simply $C \simeq D$ if structure maps are known from the context.

Remarks. Of course the relation ,,$\simeq$" is an equivalence relation. Furthermore, (from the coalgebraic point of view) isomorphic coalgebras are the same, i.e. they share all coalgebraic properties.