Cartesian product of vector spaces

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Suppose $V_1, \dots, V_N$ are vector spaces over a field $\mathbb{F}$. Then the Cartesian product $V_1 \times \cdots \times V_N$ is a vector space when addition and scalar multiplication is defined as follows

$$\begin{aligned} (u_1, \dots, u_N) + (v_1, \dots, v_N) &= (u_1 + v_1, \dots, u_N + v_N), \\ k(u_1, \dots, u_N) &= (ku_1, \dots, ku_N) \end{aligned}$$

for $u_i, v_i \in V_i$, $k \in \mathbb{F}$.

For example, the vector space structure of $\mathbb{R}^n$ if defined as above.

Properties

1. If V_i are vector spaces and $W_i \subset V_i$ are subspaces, then $W_1 \times \cdots \times W_N$ is a vector subspace of $V_1 \times \cdots \times V_N$.

2. The dimension of $V_1 \times \cdots \times V_N$ is $\dim V_1 + \cdots + \dim V_N$.