group action

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Entry type	Definition
Classification	msc 16W22
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Related topic	Group
Defines	effective
Defines	effective group action
Defines	faithful
Defines	faithful group action
Defines	transitive
Defines	transitive group action
Defines	left action
Defines	right action
Defines	faithfully
Defines	action
Defines	act on
Defines	acts on

Let G be a group and let X be a set. A left *group action* is a function $\cdot : G \times X \longrightarrow X$ such that:

1. $1_G \cdot x = x$ for all $x \in X$
2. $(g_1 g_2) \cdot x = g_1 \cdot (g_2 \cdot x)$ for all $g_1, g_2 \in G$ and $x \in X$

A right *group action* is a function $\cdot : X \times G \longrightarrow X$ such that:

1. $x \cdot 1_G = x$ for all $x \in X$
2. $x \cdot (g_1 g_2) = (x \cdot g_1) \cdot g_2$ for all $g_1, g_2 \in G$ and $x \in X$

There is a correspondence between left actions and right actions, given by associating the right action $x \cdot g$ with the left action $g \cdot x := x \cdot g^{-1}$. In many (but not all) contexts, it is useful to identify right actions with their corresponding left actions, and speak only of left actions.

Special types of group actions

A left action is said to be *effective*, or *faithful*, if the function $x \mapsto g \cdot x$ is the identity function on X only when $g = 1_G$.

A left action is said to be *transitive* if, for every $x_1, x_2 \in X$, there exists a group element $g \in G$ such that $g \cdot x_1 = x_2$.

A left action is *free* if, for every $x \in X$, the only element of G that stabilizes x is the identity; that is, $g \cdot x = x$ implies $g = 1_G$.

Faithful, transitive, and free right actions are defined similarly.