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the multiplicative identity of a cyclic ring must be a generator

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Theorem. *Let R be a cyclic ring with multiplicative identity u. Then u* `http://planetmath.org/Generator`*generates the additive group of R.*

Proof. Let k be the behavior of R. Then there exists a `http://planetmath.org/Generator`generat r of the additive group of R such that $r^2 = kr$. Let $a \in \mathbb{Z}$ with $u = ar$. Then $r = ur = (ar)r = ar^2 = a(kr) = (ak)r$. If R is infinite, then $ak = 1$, causing $a = k = 1$ since k is a nonnegative integer. If R is finite, then $ak \equiv 1 \bmod |R|$. Thus, $\gcd(k, |R|) = 1$. Since k divides $|R|$, $k = 1$. Therefore, $a \equiv 1 \bmod |R|$. In either case, $u = r$. □

Note that it was also proven that, if a cyclic ring has a multiplicative identity, then it has behavior one. Its converse is also true. See `http://planetmath.org/CyclicRingsO` theorem for more details.