free module

Canonical name	FreeModule
Date of creation	2013-03-22 12:10:10
Last modified on	2013-03-22 12:10:10
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	9
Author	mathcam (2727)
Entry type	Definition
Classification	msc 16D40
Related topic	FreeGroup
Defines	free module
Defines	free abelian group
Defines	free basis
Defines	rank of a free module

Let R be a commutative ring with unity. A *free module* over R is a (unital) module isomorphic to a direct sum of copies of R. In particular, as every abelian group is a $\mathbb{Z}$-module, a *free abelian group* is a direct sum of copies of $\mathbb{Z}$. This is equivalent to saying that the module has a *free basis*, i.e. a set of elements with the that every element of the module can be uniquely expressed as an linear combination over R of elements of the free basis. In the case that a free module over R is a sum of finitely many copies of R, then the number of copies is called the *rank* of the free module.

An alternative definition of a free module is via its universal property: Given a set X, the free R-module $F(X)$ on the set X is equipped with a function $i : X \to F(X)$ satisfying the property that for any other R-module A and any function $f : X \to A$, there exists a unique R-module map $h : F(X) \to A$ such that $(h \circ i) = f$.