prime ideal

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Let R be a ring. A two-sided proper ideal $\mathfrak{p}$ of a ring R is called a prime ideal if the following equivalent conditions are met:

1. If I and J are left ideals and the product of ideals IJ satisfies $IJ \subset \mathfrak{p}$, then $I \subset \mathfrak{p}$ or $J \subset \mathfrak{p}$.
2. If I and J are right ideals with $IJ \subset \mathfrak{p}$, then $I \subset \mathfrak{p}$ or $J \subset \mathfrak{p}$.
3. If I and J are two-sided ideals with $IJ \subset \mathfrak{p}$, then $I \subset \mathfrak{p}$ or $J \subset \mathfrak{p}$.
4. If x and y are elements of R with $xRy \subset \mathfrak{p}$, then $x \in \mathfrak{p}$ or $y \in \mathfrak{p}$.

$R/\mathfrak{p}$ is a prime ring if and only if $\mathfrak{p}$ is a prime ideal. When R is commutative with identity, a proper ideal $\mathfrak{p}$ of R is prime if and only if for any $a, b \in R$, if $a \cdot b \in \mathfrak{p}$ then either $a \in \mathfrak{p}$ or $b \in \mathfrak{p}$. One also has in this case that $\mathfrak{p} \subset R$ is prime if and only if the quotient ring $R/\mathfrak{p}$ is an integral domain.