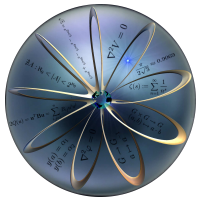

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Dedekind-finite

Canonical name	Dedekindfinite
Date of creation	2013-03-22 14:18:23
Last modified on	2013-03-22 14:18:23
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	10
Author	CWoo (3771)
Entry type	Definition
Classification	msc 16U99
Synonym	von Neumann-finite

A ring R is *Dedekind-finite* if for $a, b \in R$, whenever $ab = 1$ implies $ba = 1$.

Of course, every commutative ring is Dedekind-finite. Therefore, the theory of Dedekind finiteness is trivial in this case. Some other examples are

1. any ring of endomorphisms over a finite dimensional vector space (over a field)
2. any division ring
3. any ring of matrices over a division ring
4. finite direct product of Dedekind-finite rings
5. by the last three examples, any semi-simple ring is Dedekind-finite.
6. any ring R with the property that there is a natural number n such that $x^n = 0$ for every nilpotent element $x \in R$

The finite dimensionality in the first example can not be extended to the infinite case. Lam in [**?**] gave an example of a ring that is not Dedekind-finite arising out of the ring of endomorphisms over an infinite dimensional vector space (over a field).

References

[1] T. Y. Lam, *A First Course in Noncommutative Rings*, Springer-Verlag, New York (1991).

[2] T. Y. Lam, *Lectures on Modules and Rings*, Springer-Verlag, New York (1999).