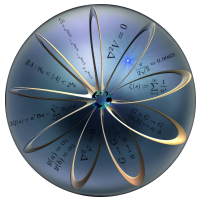

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zero vector in a vector space is unique

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Theorem The zero vector in a vector space is unique.

Proof. Suppose 0 and $\tilde{0}$ are zero vectors in a vector space V. Then both 0 and $\tilde{0}$ must satisfy `http://planetmath.org/VectorSpace`axiom 3, i.e., for all $v \in V$,

$$\begin{aligned} v + 0 &= v, \\ v + \tilde{0} &= v. \end{aligned}$$

Setting $v = \tilde{0}$ in the first equation, and $v = 0$ in the second yields $\tilde{0} + 0 = \tilde{0}$ and $0 + \tilde{0} = 0$. Thus, using `http://planetmath.org/VectorSpace`axiom 2,

$$\begin{aligned} 0 &= \tilde{0} + 0 \\ &= 0 + \tilde{0} \\ &= \tilde{0}, \end{aligned}$$

and $0 = \tilde{0}$. $\square$