complete set of primitive orthogonal idempotents

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Let A be a unital algebra over a field k. Recall that $e \in A$ is an idempotent iff $e^2 = e$. If $e_1, e_2 \in A$ are idempotents, then we will say that they are **orthogonal** iff $e_1 e_2 = e_2 e_1 = 0$. Furthermore an idempotent $e \in A$ is called **primitive** iff e cannot be written as a sum $e = e_1 + e_2$ where both $e_1, e_2 \in A$ are nonzero idempotents. An idempotent is called **trivial** iff it is either 0 or 1.

Now assume that A is an algebra such that

$$A = M_1 \oplus M_2$$

as right modules and $1 = m_1 + m_2$ for some $m_1 \in M_1$, $m_2 \in M_2$. Then m_1, m_2 are orthogonal idempotents in A and $M_1 = m_1 A$, $M_2 = m_2 A$. Furthermore M_i is indecomposable (as a right module) if and only if m_i is primitive. This can be easily generalized to any number (but finite) of summands.

If A is additionally finite-dimensional, then

$$A = P_1 \oplus \cdots \oplus P_n$$

for some (unique up to isomorphism) right (ideals) indecomposable modules P_i. It follows from the preceding that

$$P_i = e_i A$$

for some $e_i \in A$ and $\{e_1, \ldots, e_n\}$ is a set of pairwise orthogonal, primitive idempotents. This set is called **the complete set of primitive orthogonal idempotents of** A.