subcoalgebras and coideals

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Let (C, Δ, ε) be a coalgebra over a field k.

Definition. Vector subspace $D \subseteq C$ is called *subcoalgebra* iff $\Delta(D) \subseteq D \otimes D$.

Definition. Vector subspace $I \subseteq C$ is is called *coideal* iff $\Delta(I) \subseteq I \otimes C + C \otimes I$ and $\varepsilon(I) = 0$.

One can show that if $D \subseteq C$ is a subcoalgebra, then $(D, \Delta_{|D}, \varepsilon_{|D})$ is also a coalgebra. On the other hand, if $I \subseteq C$ is a coideal, then we can cannoicaly introduce a coalgebra structure on the quotient space C/I. More precisely, if $x \in C$ and $\Delta(x) = \sum a_i \otimes b_i$, then we define

$$\Delta' : C/I \to (C/I) \otimes (C/I);$$

$$\Delta'(x + I) = \sum (a_i + I) \otimes (b_i + I)$$

and $\varepsilon' : C/I \to k$ as $\varepsilon'(x + I) = \varepsilon(x)$. One can show that these two maps are well defined and $(C/I, \Delta', \varepsilon')$ is a coalgebra.