hereditary ring

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Defines	hereditary module

Let R be a ring. A right (left) R-module M is called right (left) *hereditary* if every submodule of M is projective over R.
Remarks.

- If M is semisimple, then M is hereditary.

- Suppose M is an external direct sum of hereditary right (left) R-modules, then M is itself hereditary.

A ring R is said to be a right (left) *hereditary ring* if all of its right (left) ideals are projective as modules over R. If R is both left and right hereditary, then R is simply called a hereditary ring.
Remarks.

- Even though the notions of left and right heredity in rings are symmetrical, one does not imply the other.

- If R is semisimple, then R is hereditary.

- If R is hereditary, then every free R-module is a hereditary module.

- A hereditary integral domain is a Dedekind domain, and conversely.

- The global dimension of a non-semisimple hereditary ring is 1.