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dual module

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Defines	linear functional

Let R be a ring and M be a left <http://planetmath.org/node/365> R -module. The *dual module* of M is the right <http://planetmath.org/node/365> R -module consisting of all module homomorphisms from M into R .

It is denoted by M^* . The elements of M^* are called *linear functionals*.

The action of R on M^* is given by $(fr)(m) = (f(m))r$ for $f \in M^*$, $m \in M$, and $r \in R$.

If R is commutative, then every M is an <http://planetmath.org/node/987> (R, R) -bimodule with $rm = mr$ for all $r \in R$ and $m \in M$. Hence, it makes sense to ask whether M and M^* are isomorphic. Suppose that $b : M \times M \rightarrow R$ is a bilinear form. Then it is easy to check that for a fixed $m \in M$, the function $b(m, -) : M \rightarrow R$ is a module homomorphism, so is an element of M^* . Then we have a module homomorphism from M to M^* given by $m \mapsto b(m, -)$.