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Faà di Bruno's formula

Canonical name	FaaDiBrunosFormula
Date of creation	2013-03-22 16:38:57
Last modified on	2013-03-22 16:38:57
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	5
Author	rspuzio (6075)
Entry type	Definition
Classification	msc 16W30
Synonym	Faa di Bruno's formula
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Faà di Bruno's formula is a generalization of the chain rule to higher order derivatives which expresses the derivative of a composition of functions as a series of products of derivatives:

$$\frac{d^n}{dx^n} f(g(x)) = \sum_{\sum_{k=0}^n k m_k = n} \frac{n!}{m_1! m_2! m_3! \dots 1!^{m_1} 2!^{m_2} 3!^{m_3} \dots} f^{(m_1 + \dots + m_n)}(g(x)) \prod_{j: m_j \neq 0} (g^{(j)}(x))^{m_j}$$

This formula was discovered by Francesco Faà di Bruno in the 1850s and can be proved by induction on the order of the derivative.

References

- [1] Faà di Bruno, C. F.. “Sullo sviluppo delle funzione.” *Ann. di Scienze Matem. et Fisiche di Tortoloni* **6** (1855): 479-480
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- [3] H. Figueroa & J. M. Gracia-Bondía, “Combinatorial Hopf Algebras in Quantum Field Theory I” *Rev. Math. Phys.* **17** (2005): 881 - 975