nucleus

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Defines	center of a nonassociative algebra
Defines	nuclear

Let A be an algebra, not necessarily associative multiplicatively. The *nucleus* of A is:

$$\mathcal{N}(A) := \{a \in A \mid [a, A, A] = [A, a, A] = [A, A, a] = 0\},$$

where $[\ ,\ ,\ ]$ is the associator bracket. In other words, the nucleus is the set of elements that multiplicatively associate with all elements of A. An element $a \in A$ is *nuclear* if $a \in \mathcal{N}(A)$.

$\mathcal{N}(A)$ is a Jordan subalgebra of A. To see this, let $a, b \in \mathcal{N}(A)$. Then for any $c, d \in A$,

$$\begin{aligned} [ab, c, d] &= ((ab)c)d - (ab)(cd) = (a(bc))d - (ab)(cd) && (1) \\ &= a((bc)d) - (ab)(cd) = a(b(cd)) - (ab)(cd) && (2) \\ &= a(b(cd)) - a(b(cd)) = 0 && (3) \end{aligned}$$

Similarly, $[c, ab, d] = [c, d, ab] = 0$ and so $ab \in \mathcal{N}(A)$.

Accompanying the concept of a nucleus is that of the *center of a nonassociative algebra* A (which is slightly different from the definition of the center of an associative algebra):

$$\mathcal{Z}(A) := \{a \in \mathcal{N}(A) \mid [a, A] = 0\},$$

where $[\ ,\ ]$ is the commutator bracket.

Hence elements in $\mathcal{Z}(A)$ commute *as well as* associate with all elements of A. Like the nucleus, the center of A is also a Jordan subalgebra of A.