Dynkin diagram

Canonical name	DynkinDiagram
Date of creation	2013-03-22 13:28:05
Last modified on	2013-03-22 13:28:05
Owner	bwebste (988)
Last modified by	bwebste (988)
Numerical id	5
Author	bwebste (988)
Entry type	Definition
Classification	msc 17B20

Dynkin diagrams are a combinatorial way of representing the information in a root system. Their primary advantage is that they are easier to write down, remember, and analyze than explicit representations of a root system. They are an important tool in the classification of simple Lie algebras.

Given a reduced root system $R \subset E$, with E an inner-product space, choose a base or simple roots Π (or equivalently, a set of positive roots R^+). The Dynkin diagram associated to R is a graph whose vertices are Π. If π_i and π_j are distinct elements of the root system, we add $m_{ij} = \frac{-4(\pi_i,\pi_j)^2}{(\pi_i,\pi_i)(\pi_j,\pi_j)}$ lines between them. This number is obivously positive, and an integer since it is the product of 2 quantities that the axioms of a root system require to be integers. By the Cauchy-Schwartz inequality, and the fact that simple roots are never anti-parallel (they are all strictly contained in some half space), $m_{ij} \in \{0, 1, 2, 3\}$. Thus Dynkin diagrams are finite graphs, with single, double or triple edges. Fact, the criteria are much stronger than this: if the multiple edges are counted as single edges, all Dynkin diagrams are trees, and have at most one multiple edge. In fact, all Dynkin diagrams fall into 4 infinite families, and 5 exceptional cases, in exact parallel to the classification of simple Lie algebras.

(*Does anyone have good Dynkin diagram pictures? I'd love to put some up, but am decidedly lacking.*)