anticommutative

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A binary operation "$\star$" is said to be *anticommutative* if it satisfies the identity

$$y \star x \;=\; -(x \star y), \tag{1}$$

where the minus denotes the element in the algebra in question. This implies that $x \star x \;=\; -(x \star x)$, i.e. $x \star x$ must be the neutral element of the addition of the algebra:

$$x \star x \;=\; \mathbf{0}. \tag{2}$$

Using the distributivity of "$\star$" over "+" we see that the indentity (2) also implies (1):

$$\mathbf{0} \;=\; (x+y) \star (x+y) \;=\; x \star x + x \star y + y \star x + y \star y \;=\; x \star y + y \star x$$

A well known example of anticommutative operations is the vector product in the algebra $(\mathbb{R}^3, +, \times)$, satisfying

$$\vec{b} \times \vec{a} \;=\; -(\vec{a} \times \vec{b}), \qquad \vec{a} \times \vec{a} \;=\; \vec{0}.$$

Also we know that the subtraction of numbers obeys identities

$$b-a \;=\; -(a-b), \qquad a-a \;=\; 0.$$

An important anticommutative operation is the Lie bracket.