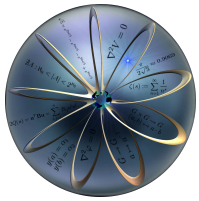

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root system

Canonical name	RootSystem
Date of creation	2013-03-22 13:11:30
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Entry type	Definition
Classification	msc 17B20
Related topic	SimpleAndSemiSimpleLieAlgebras2
Related topic	LieAlgebra
Defines	reduced root system
Defines	root
Defines	root space
Defines	root decomposition
Defines	indecomposable
Defines	reduced
Defines	crystallographic

A root system is a key notion in the classification and the representation theory of reflection groups and of semi-simple Lie algebras. Let E be a Euclidean vector space with inner product $(\cdot, \cdot)$. A root system is a finite spanning set $R \subset E$ such that for every $u \in R$, the orthogonal reflection

$$v \mapsto v - 2\frac{(u,v)}{(u,u)}u, \quad v \in E$$

preserves R.

A root system is called *crystallographic* if $2\frac{(u,v)}{(u,u)}$ is an integer for all $u, v \in R$.

A root system is called *reduced* if for all $u \in R$, we have $ku \in R$ for $k = \pm 1$ only.

We call a root system *indecomposable* if there is no proper decomposition $R = R' \cup R''$ such that every vector in R' is orthogonal to every vector in R''.