extended Cartan matrix

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Let A be the Cartan matrix of a complex, semi-simple, finite dimensional, Lie algebra $\mathfrak{g}$. Recall that $A = (a_{ij})$ where $a_{ij} = \langle \alpha_i, \alpha_j^\vee \rangle$ where the α_i are simple roots for $\mathfrak{g}$ and the $\alpha_j^\vee$ are simple coroots. The *extended Cartan matrix* denoted $\hat{A}$ is obtained from A by adding a zero-th row and column corresponding to adding a new simple root $\alpha_0 := -\theta$ where θ is the maximal (relative to $\{\alpha_1, \ldots, \alpha_n\}$) root for $\mathfrak{g}$. θ can be defined as a root of $\mathfrak{g}$ such that when written in terms of simple roots $\theta = \sum_i m_i \alpha_i$ the coefficient sum $\sum_i m_i$ is maximal (i.e. it has maximal height). Such a root can be shown to be unique.

The matrix $\hat{A}$ is an example of a generalized Cartan matrix. The corresponding Kac-Moody Lie algerba is said to be of affine type.

For example if $\mathfrak{g} = \mathfrak{sl}_n\mathbb{C}$ then $\hat{A}$ is obtained from A by adding a zero-th row: $(2, -1, 0, \ldots, 0, -1)$ and zero-th column $(2, -1, 0, \ldots, 0, -1)$ simultaneously to the Cartan matrix for $\mathfrak{sl}_n\mathbb{C}$.

References

[1] Victor Kac, *Infinite Dimensional Lie Algebras*, Third edition. Cambridge University Press, Cambridge, 1990.