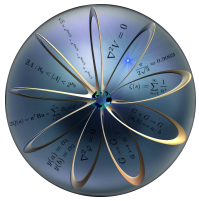

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Hurwitz's theorem on composition algebras

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Theorem 1 (Hurwitz)**.** *[?, Theorem 3.25] Given a field k of characteristic not 2, then every unital composition algebra C over k is isomorphic to one of:*

1. *k,*
2. *$\left(\frac{\alpha}{k}\right)$ for $\alpha \in k$,*
3. *$\left(\frac{\alpha,\beta}{k}\right)$ for $\alpha, \beta \in k$,*
4. *$\left(\frac{\alpha,\beta,\gamma}{k}\right)$ for $\alpha, \beta, \gamma \in k$.*

In particular, all composition algebras over k are finite dimensional and of dimension 1, 2, 4 or 8.

References

[1] Richard D. Schafer, *An introduction to nonassociative algebras*, Pure and Applied Mathematics, Vol. 22, Academic Press, New York, 1966.