local Nagano theorem

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Theorem (Local Nagano Theorem). *Let $\Omega \subset \mathbb{R}^n$ be an open neighbourhood of a point x^0. Further let $\mathfrak{g}$ be a Lie subalgebra of the Lie algebra of real analytic real vector fields on Ω which is also a $C^\omega(\Omega;\mathbb{R})$-module. Then there exists a real analytic submanifold $M \subset \Omega$ with $x^0 \in M$, such that for all $x \in M$ we have*

$$T_x(M) = \mathfrak{g}(x).$$

Furthermore the germ of M at x is the unique germ of a submanifold with this property.

Here note that $T_x(M)$ is the tangent space of M at x, $C^\omega(\Omega;\mathbb{R})$ are the real analytic real valued functions on Ω. Also real analytic real vector fields on Ω are the real analytic sections of $T(\Omega)$, the real tangent bundle of Ω.

Definition. The germ of the manifold M is called the *local Nagano leaf* of $\mathfrak{g}$ at x_0.

Definition. The union of all connected real analytic embedded submanifolds of Ω whose germ at x_0 coincides with the germ of M at x_0 is called the *global Nagano leaf.*

The global Nagano leaf turns out to be a connected immersed real analytic submanifold which may however not be an embedded submanifold of Ω.

References

[1] M. Salah Baouendi, Peter Ebenfelt, Linda Preiss Rothschild. , Princeton University Press, Princeton, New Jersey, 1999.