



planetmath.org

Math for the people, by the people.

examples of non-matrix Lie groups

Canonical name	ExamplesOfNonmatrixLieGroups
Date of creation	2013-03-22 13:20:48
Last modified on	2013-03-22 13:20:48
Owner	bwebste (988)
Last modified by	bwebste (988)
Numerical id	6
Author	bwebste (988)
Entry type	Example
Classification	msc 17B10
Related topic	AdosTheorem

While most well-known Lie groups are matrix groups, there do in fact exist Lie groups which are not matrix groups. That is, they have no faithful finite dimensional representations.

For example, let H be the real Heisenberg group

$$H = \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\},$$

and Γ the discrete subgroup

$$\Gamma = \left\{ \begin{pmatrix} 1 & 0 & n \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mid n \in \mathbb{Z} \right\}.$$

The subgroup Γ is central, and thus normal. The Lie group H/Γ has no faithful finite dimensional representations over \mathbb{R} or \mathbb{C} .

Another example is the universal cover of $\mathrm{SL}_2\mathbb{R}$. $\mathrm{SL}_2\mathbb{R}$ is homotopy equivalent to a circle, and thus $\pi(\mathrm{SL}_2\mathbb{R}) \cong \mathbb{Z}$, and thus has an infinite-sheeted cover. Any real or complex representation of this group factors through the projection map to $\mathrm{SL}_2\mathbb{R}$.