Killing form

Canonical name	KillingForm
Date of creation	2013-03-22 13:15:47
Last modified on	2013-03-22 13:15:47
Owner	bwebste (988)
Last modified by	bwebste (988)
Numerical id	8
Author	bwebste (988)
Entry type	Definition
Classification	msc 17B05

Let $\mathfrak{g}$ be a finite dimensional Lie algebra over a field k, and $X, Y \in \mathfrak{g}$. Let $\text{ad}_X : \mathfrak{g} \to \mathfrak{g}$ be the adjoint action, $\text{ad}_X Y = [X, Y]$.

Then the Killing form on $\mathfrak{g}$ is a bilinear map

$$B_\mathfrak{g} : \mathfrak{g} \times \mathfrak{g} \to k$$

given by

$$B_\mathfrak{g}(X, Y) = \text{tr}(\text{ad}_X \circ \text{ad}_Y).$$

The Killing form is `http://planetmath.org/InvariantFormLieAlgebras`invariant under the adjoint action and symmetric (since trace is symmetric).