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## Weyl group

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The Weyl group  $W_R$  of a root system  $R \subset E$ , where  $E$  is a Euclidean vector space, is the subgroup of  $\text{GL}(E)$  generated by reflection in the hyperplanes perpendicular to the roots. The map of reflection in a root  $\alpha$  is given by

$$r_\alpha(v) = v - 2 \frac{(\alpha, v)}{(\alpha, \alpha)} \alpha.$$

The Weyl group is generated by reflections in the simple roots for any choice of a set of positive roots. There is a well-defined length function  $\ell : W_R \rightarrow \mathbb{Z}$ , where  $\ell(w)$  is the minimal number of reflections in simple roots that  $w$  can be written as. This is also the number of positive roots that  $w$  takes to negative roots.