weight (Lie algebras)

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Let $\mathfrak{h}$ be an abelian Lie algebra, V a vector space and $\rho\colon \mathfrak{h} \to \operatorname{End} V$ a representation. Then the representation is said to be *diagonalisable*, if V can be written as a direct sum

$$V = \bigoplus_{\lambda \in \mathfrak{h}^*} V_\lambda$$

where $\mathfrak{h}^*$ is the dual space of $\mathfrak{h}$ and

$$V_\lambda = \{v \in V \mid \rho(h)v = \lambda(h)v \text{ for all } h \in \mathfrak{h}\}.$$

Now let $\mathfrak{g}$ be a semi-simple Lie algebra. Fix a Cartan subalgebra $\mathfrak{h}$, then $\mathfrak{h}$ is abelian. Let $\rho\colon \mathfrak{g} \to \operatorname{End} V$ be a representation whose restriction to $\mathfrak{h}$ is diagonalisable. Then for any $\lambda \in \mathfrak{h}^*$, the space V_λ is the *weight space* of λ with respect to ρ. The *multiplicity* of λ with respect to ρ is the dimension of V_λ:

$$\operatorname{mult}_\rho(\lambda) := \dim V_\lambda.$$

If the multiplicity of λ is greater than zero, then λ is called a *weight* of the representation ρ.

A representation of a semi-simple Lie algebra is determined by the multiplicities of its weights.