



(special) unitary Lie algebra

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Let  $V$  be a vector space over a field  $K$  admitting an involution  $\sigma : K \rightarrow K$ , and let  $B : V \times V \rightarrow \mathbb{F}$  be a <http://planetmath.org/node/SesquilinearFormsOverGeneralFields> form relative to  $\sigma$ . Then the *unitary Lie algebra*  $\mathfrak{u}(V, B)$ , or just  $\mathfrak{u}(V)$ , consists of the linear transformations  $T$  satisfying

$$B(Tx, y) + B(x, Ty) = 0,$$

for all  $x, y \in V$ . This is a Lie algebra over  $k = \{\alpha \in K \mid \alpha^\sigma = \alpha\}$ , but not over  $K$  in the case that  $K \neq k$  (because  $B$  is linear in the first, but not in the second variable).

The *special unitary Lie algebra*  $\mathfrak{su}(V, B)$ , or just  $\mathfrak{su}(V)$ , consists of those linear transformations in  $\mathfrak{u}(V, B)$  with trace zero.