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composition algebra over algebraically closed fields

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**Theorem 1.** *There are 4 non-isomorphic composition algebras over an algebraically closed field  $k$ : one division algebra, the field itself, and the three split algebras.*

1.  $k$ .
2. The exchange algebra:  $k \oplus k$ .
3.  $2 \times 2$  matrices over  $k$ :  $M_2(k)$ .
4. The cross-product of  $2 \times 2$ -matrices over  $k$ :  $M_2(k) \circ M_2(k)$ .

*Proof.* To see this recall that every composition algebra comes equipped with a quadratic form. Any 2-dimensional anisotropic subspace arises from a quadratic field extension. As our field is algebraically closed the quadratic form has no anisotropic subspaces and is therefore the unique quadratic form of maximal Witt index. Following Hurwitz's theorem we know the composition algebras come in dimensions 1,2,4, and 8 and arise by the Cayley-Dickson method. Thus we have the field itself and the three split composition algebras.  $\square$