Cartan matrix

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Let $R \subset E$ be a reduced root system, with E a Euclidean vector space, with inner product $(\cdot, \cdot)$, and let $\Pi = \{\alpha_1, \cdots, \alpha_n\}$ be a base of this root system. Then the Cartan matrix of the root system is the matrix

$$C_{i,j} = \left(\frac{2(\alpha_i, \alpha_j)}{(\alpha_i, \alpha_i)} \right).$$

The Cartan matrix uniquely determines the root system, and is unique up to simultaneous permutation of the rows and columns. It is also the basis change matrix from the basis of fundamental weights to the basis of simple roots in E.