simple and semi-simple Lie algebras

Canonical name	SimpleAndSemisimpleLieAlgebras
Date of creation	2013-03-22 13:11:28
Last modified on	2013-03-22 13:11:28
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	9
Author	mathcam (2727)
Entry type	Definition
Classification	msc 17B20
Related topic	LieAlgebra
Related topic	LieGroup
Related topic	RootSystem
Related topic	RootSystemUnderlyingASemiSimpleLieAlgebra
Defines	simple Lie algebra
Defines	semi-simple Lie algebra
Defines	semisimple Lie algebra
Defines	simple
Defines	semi-simple
Defines	semisimple

A Lie algebra is called *simple* if it has no proper ideals and is not abelian. A Lie algebra is called *semi-simple* if it has no proper solvable ideals and is not abelian.

Let $k = \mathbb{R}$ or $\mathbb{C}$. Examples of simple algebras are $\mathfrak{sl}_n k$, the Lie algebra of the special linear group (traceless matrices), $\mathfrak{so}_n k$, the Lie algebra of the special orthogonal group (skew-symmetric matrices), and $\mathfrak{sp}_{2n} k$ the Lie algebra of the symplectic group. Over $\mathbb{R}$, there are other simple Lie algebas, such as $\mathfrak{su}_n$, the Lie algebra of the special unitary group (skew-Hermitian matrices). Any semi-simple Lie algebra is a direct product of simple Lie algebras.

Simple and semi-simple Lie algebras are one of the most widely studied classes of algebras for a number of reasons. First of all, many of the most interesting Lie groups have semi-simple Lie algebras. Secondly, their representation theory is very well understood. Finally, there is a beautiful classification of simple Lie algebras.

Over $\mathbb{C}$, there are 3 infinite series of simple Lie algebras: $\mathfrak{sl}_n$, $\mathfrak{so}_n$ and $\mathfrak{sp}_{2n}$, and 5 exceptional simple Lie algebras $\mathfrak{g}_2, \mathfrak{f}_4, \mathfrak{e}_6, \mathfrak{e}_7$, and $\mathfrak{e}_8$. Over $\mathbb{R}$ the picture is more complicated, as several different Lie algebras can have the same complexification (for example, $\mathfrak{su}_n$ and $\mathfrak{sl}_n\mathbb{R}$ both have complexification $\mathfrak{sl}_n\mathbb{C}$).