quadratic Lie algebra

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Owner	benjaminfjones (879)
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Author	benjaminfjones (879)
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A Lie algebra $\mathfrak{g}$ is said to be *quadratic* if $\mathfrak{g}$ as a representation (under the adjoint action) admits a non-degenerate, invariant scalar product $(\cdot \mid \cdot)$.

$\mathfrak{g}$ being quadratic implies that the adjoint and co-adjoint representations of $\mathfrak{g}$ are isomorphic.

Indeed, the non-degeneracy of $(\cdot \mid \cdot)$ implies that the induced map $\phi\colon \mathfrak{g} \to \mathfrak{g}^*$ given by $\phi(X)(Z) = (X \mid Z)$ is an isomorphism of vector spaces. Invariance of the scalar product means that $([X,Y] \mid Z) = -(Y \mid [X,Z]) = (Y \mid [Z,X])$. This implies that ϕ is a map of representations:

$$\phi(ad_X(Y))(Z) = \phi([X,Y])(Z) = ([X,Y] \mid Z) = (Y \mid [Z,X]) = ad^*_X(\phi(Y)(Z))$$