



planetmath.org

Math for the people, by the people.

nilpotent cone

Canonical name	NilpotentCone
Date of creation	2013-03-22 13:58:36
Last modified on	2013-03-22 13:58:36
Owner	rmilson (146)
Last modified by	rmilson (146)
Numerical id	9
Author	rmilson (146)
Entry type	Definition
Classification	msc 17B20
Synonym	nilcone

Let \mathfrak{g} be a finite dimensional semisimple Lie algebra. The *nilpotent cone* \mathcal{N} of \mathfrak{g} is the set of elements that act nilpotently in all representations of \mathfrak{g} . In other words,

$$\mathcal{N} = \{a \in \mathfrak{g} : \rho(a) \text{ is nilpotent for all representations } \rho : \mathfrak{g} \rightarrow \text{End}(V)\}$$

The nilpotent cone is an <http://planetmath.org/IrreducibleClosedSet> irreducible <http://planetmath.org/AffineVariety> subvariety of \mathfrak{g} (considered as a k -vector space), and is invariant under the adjoint action of \mathfrak{g} on itself.

Example: if $\mathfrak{g} = \mathfrak{sl}_2$, then the nilpotent cone is the variety of all matrices in \mathfrak{g} with rank 1.