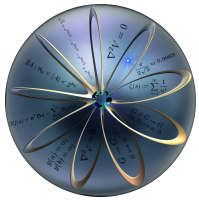

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Euclid's lemma proof

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We have $a|bc$, so $bc = na$, with n an integer. Dividing both sides by a, we have

$$\frac{bc}{a} = n$$

But $\gcd(a, b) = 1$ implies b/a is only an integer if $a = 1$. So

$$\frac{bc}{a} = b\frac{c}{a} = n$$

which means a must divide c.

Note that this proof relies on the Fundamental Theorem of Arithmetic. The alternative proof of Euclid's lemma avoids this.