



Math for the people, by the people.

Jacobi identity interpretations

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The Jacobi identity in a Lie algebra \mathfrak{g} has various interpretations that are more transparent, whence easier to remember, than the usual form

$$[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0.$$

One is the fact that the adjoint representation ¹ $\text{ad} : \mathfrak{g} \rightarrow \mathfrak{gl}(\mathfrak{g})$ really is a representation. Yet another way to formulate the identity is

$$\text{ad}(x)[y, z] = [\text{ad}(x)y, z] + [y, \text{ad}(x)z],$$

i.e., $\text{ad}(x)$ is a derivation on \mathfrak{g} for all $x \in \mathfrak{g}$.

¹Here, “ $\mathfrak{gl}(\mathfrak{g})$ ” means the space of endomorphisms of \mathfrak{g} , viewed as a vector space, with Lie bracket on $\mathfrak{gl}(\mathfrak{g})$ being commutator.