power-associative algebra

Canonical name	PowerassociativeAlgebra
Date of creation	2013-03-22 14:43:27
Last modified on	2013-03-22 14:43:27
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	15
Author	CWoo (3771)
Entry type	Definition
Classification	msc 17A05
Synonym	di-associative
Synonym	diassociative
Related topic	Associator

Let A be a non-associative algebra. A subalgebra B of A is said to be *cyclic* if it is generated by one element.

A non-associative algebra is *power-associative* if, $[B,B,B]=0$ for any cyclic subalgebra B of A, where $[-,-,-]$ is the associator.

If we inductively define the powers of an element $a\in A$ by

1. (when A is unital with $1\neq 0$) $a^0:=1$,

2. $a^1:=a$, and

3. $a^n:=a(a^{n-1})$ for $n>1$,

then power-associativity of A means that $[a^i,a^j,a^k]=0$ for any non-negative integers i,j and k, since the associator is trilinear (linear in each of the three coordinates). This implies that $a^m a^n=a^{m+n}$. In addition, $(a^m)^n=a^{mn}$.

A theorem, due to A. Albert, states that any finite power-associative division algebra over the integers of characteristic not equal to 2, 3, or 5 is a field. This is a generalization of the Wedderburn's Theorem on finite division rings.

References

[1] R. D. Schafer, *An Introduction on Nonassociative Algebras*, Dover, New York (1995).