

planetmath.org

Math for the people, by the people.

flexible algebra

Canonical name FlexibleAlgebra
Date of creation 2013-03-22 14:43:30

Last modified on $2013-03-22 \ 14:43:30$

Owner CWoo (3771) Last modified by CWoo (3771)

Numerical id 11

Author CWoo (3771)
Entry type Definition
Classification msc 17A20
Related topic Associator

Related topic AlternativeAlgebra

Defines left power
Defines right power
Defines flexible

A non-associative algebra A is *flexible* if [a,b,a]=0 for all $a,b\in A$, where $[\ ,,]$ is the associator on A. In other words, we have (ab)a=a(ba) for all $a,b\in A$. Any associative algebra is clearly flexible. Furthermore, any alternative algebra with characteristic $\neq 2$ is flexible.

Given an element a in a flexible algebra A, define the *left power* of a iteratively as follows:

1.
$$L^1(a) = a$$
,

2.
$$L^n(a) = a \cdot L^{n-1}(a)$$
.

Similarly, we can define the *right power* of a as:

1.
$$R^1(a) = a$$
,

2.
$$R^n(a) = R^{n-1}(a) \cdot a$$
.

Then, we can show that $L^n(a) = R^n(a)$ for all positive integers n. As a result, in a flexible algebra, one can define the (multiplicative) power of an element a as a^n unambiguously.