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diagonal functor

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Let $\mathcal{C}$ be a category. A *diagonal functor* on $\mathcal{C}$ is a functor $\delta : \mathcal{C} \to \mathcal{C}^I$ for some set I given by

$$\delta(A) = (A)_{i\in I} \quad \text{and} \quad \delta(\alpha) = (\alpha)_{i\in I}.$$

Here, $\mathcal{C}^I$ denotes the `http://planetmath.org/ProductCategory`I-fold direct product of the category $\mathcal{C}$. For any given I, δ is unique.

δ is `http://planetmath.org/FaithfulFunctor`faithful. Its image, $\delta(\mathcal{C})$, is the subcategory of $\mathcal{C}^I$ whose objects are $(A)_{i\in I}$ and morphisms are $(\alpha)_{i\in I}$. $\delta(\mathcal{C})$ is `http://planetmath.org/CategoryIsomorphism`isomorphic to $\mathcal{C}$, and may be pictured as the great diagonal of an I-dimensional "cube".

More generally, when I is a category, then the diagonal functor is just a functor δ that sends each object $A \in \mathcal{C}$ to the constant functor $\delta(A) : I \to \mathcal{C}$ with fixed value A, and every morphism $\alpha : A \to B$ to the natural transformation $\delta(\alpha) : \delta(A) \dot{\to} \delta(B)$, which sends every object $i \in I$ to α. A routine verification shows that $\delta(\alpha)$ is indeed a natural transformation.