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exact functor

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Classification	msc 18A22
Synonym	left exact functor
Synonym	right exact functor
Related topic	CategoricalSequence
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A covariant functor  $F$  is said to be *left exact* if whenever

$$0 \rightarrow A \xrightarrow{\alpha} B \xrightarrow{\beta} C$$

is an exact sequence, then

$$0 \rightarrow FA \xrightarrow{F\alpha} FB \xrightarrow{F\beta} FC$$

is also an exact sequence.

A covariant functor  $F$  is said to be *right exact* if whenever

$$A \xrightarrow{\alpha} B \xrightarrow{\beta} C \rightarrow 0$$

is an exact sequence, then

$$FA \xrightarrow{F\alpha} FB \xrightarrow{F\beta} FC \rightarrow 0$$

is also an exact sequence.

A contravariant functor  $F$  is said to be *left exact* if whenever

$$A \xrightarrow{\alpha} B \xrightarrow{\beta} C \rightarrow 0$$

is an exact sequence, then

$$0 \rightarrow FC \xrightarrow{F\beta} FB \xrightarrow{F\alpha} FA$$

is also an exact sequence.

A contravariant functor  $F$  is said to be *right exact* if whenever

$$0 \rightarrow A \xrightarrow{\alpha} B \xrightarrow{\beta} C$$

is an exact sequence, then

$$FC \xrightarrow{F\beta} FB \xrightarrow{F\alpha} FA \rightarrow 0$$

is also an exact sequence.

A (covariant or contravariant) functor is said to be *exact* if it is both left exact and right exact.