algebraic category of quantum automata

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Defines	algebraic category of quantum automata
Defines	automorphism groupoid of quantum transitions
Defines	quantum triple
Defines	unitarity preserving mappings

1 The Algebraic Category of Quantum Automata

Definition 1.1. Let us recall that a *quantum automaton* is defined as a quantum algebraic topology object– the *quantum triple* $Q_A = (\mathcal{G}, \mathcal{H} - \Re_G, Aut(\mathcal{G}))$, where $\mathcal{G}$ is a (locally compact) quantum groupoid, $\mathcal{H} - \Re_G$ are the unitary representations of $\mathcal{G}$ on rigged Hilbert spaces $\Re_G$ of quantum states and quantum operators on the Hilbert space $\mathcal{H}$, and $Aut(\mathcal{G})$ is the transformation, or *automorphism groupoid of quantum transitions* that represents all flip-flop quantum transitions of one cubit each between the permitted quantum states of the quantum automaton.

With the data from above definition we can now define also the category of quantum automata as follows.

Definition 1.2. The *category of quantum automata* $\mathcal{Q}_A$ is defined as an algebraic category whose objects are triples $(\mathcal{H}, \Delta : \mathcal{H} \to \mathcal{H}, \mu)$ (where $\mathcal{H}$ is either a Hilbert space or a rigged Hilbert space of quantum states and operators acting on $\mathcal{H}$, and μ is a measure related to the quantum logic, LM, and (quantum) transition probabilities of this quantum system), and whose morphisms are defined between such triples by homomorphisms of Hilbert spaces, $\Omega : \mathcal{H} \to \mathcal{H}$, naturally compatible with the operators Δ, and by homomorphisms between the associated Haar measure systems.

An alternative definition is also possible based on Quantum Algebraic Topology (QAT).

Definition 1.3. A quantum algebraic topology definition of the *category of quantum algebraic automata* is in terms of the objects specified above in **Definition 0.1** as quantum automaton triples (Q_A), and quantum automata homomorphisms defined between such triples; these Q_A morphisms are defined by groupoid homomorphisms $h : \mathcal{G} \to \mathcal{G}^*$ and $\alpha : Aut(\mathcal{G}) \to Aut(\mathcal{G}^*)$, together with *unitarity preserving mappings* u that are defined between unitary representations of $\mathcal{G}$ on rigged Hilbert spaces (or Hilbert space bundles).