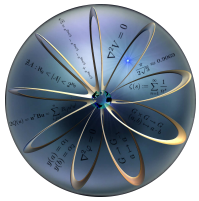

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Mitchell's embedding theorem

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Defines	Freyd-Mitchell embedding theorem

Theorem 1. *Every small abelian category admits an* *http://planetmath.org/ExactFunctorexac*
and *http://planetmath.org/FullFunctorfull* *http://planetmath.org/FaithfulFunctoremb*
into the category $\boldsymbol{Mod}_R$ *of (left) modules over some ring* R*.*

As a consequence, this theorem says that certain facts about small abelian categories can be proved in the more concrete setting of Mod_R (indeed a concrete category). For example, in order to prove that a sequence is exact in an abelian category, it is enough to prove it in the context of Mod_R, by realizing the fact that objects in Mod_R are sets (with structures) and utilizing the elements therein. In particular, the diagram chasing technique popular in homological algebra may be formulated in small abelian categories as a result of this theorem.

References

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