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wellpowered category

Canonical name	WellpoweredCategory
Date of creation	2013-03-22 16:03:36
Last modified on	2013-03-22 16:03:36
Owner	kompik (10588)
Last modified by	kompik (10588)
Numerical id	9
Author	kompik (10588)
Entry type	Definition
Classification	msc 18A05
Synonym	wellpowered
Synonym	well-powered
Synonym	locally small
Related topic	Subobject
Defines	extremally wellpowered
Defines	regular wellpowered
Defines	cowellpowered
Defines	extremally cowellpowered
Defines	regularly cowellpowered

Wellpoweredness is a kind of smallness condition on a category.

Let  $M$  be a class of monomorphisms. A category is said to be  $M$ -wellpowered if for any object any class of pairwise non-isomorphism  $M$ -subobjects is a set. (By a  $M$ -subobject of an object  $A$  we understand a pair  $(E, e)$ , where  $e : E \rightarrow A$  is a morphism belonging to  $M$ .) In other words, if we consider isomorphic objects as the same object, the class of all  $M$ -subobjects is a set.

More precisely, for any  $A$  there exists a set of  $M$ -subobjects  $(M_i, m_i)$ ,  $i \in I$  such that for any extremal subobject  $(M, m)$  of the object  $A$  there exists  $i \in I$  and an isomorphism  $f : M_i \rightarrow M$  such that  $m_i = m \circ f$ .

If  $M$  is the class of all regular monomorphisms, extremal monomorphisms, monomorphisms, we speak about regular wellpowered, extremally wellpowered, wellpowered category.

<http://planetmath.org/DualityPrincipleDual> notions: regular cowellpowered, extremally cowellpowered, cowellpowered category.

## References

- [1] J. Adámek, H. Herrlich, and G. Strecker. *Abstract and Concrete Categories*. Wiley, New York, 1990.