types of morphisms

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Related topic	TypesOfHomomorphisms
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Defines	monomorphism
Defines	epimorphism
Defines	bimorphism
Defines	retraction
Defines	section
Defines	coretraction
Defines	isomorphism
Defines	inverse morphism
Defines	split monomorphism
Defines	split epimorphism
Defines	epimorphic extension
Defines	epimorphic monomorphism

Definition 1. *A morphism $f : A \to B$ is a* monomorphism*, if for any two morphisms $g, h : C \to A$ the equality $f \circ g = f \circ h$ implies $h = g$.*

http://planetmath.org/DualityPrincipleDual notion: Morphism $f : A \to B$ is an epimorphism*, if for any two morphisms $g, h : B \to C$ the equality $g \circ f = h \circ f$ implies $h = g$.*

A morphism f is a bimorphism*, if it is monomorphism and epimorphism at the same time. Also the names* epimorphic extension *and* epimorphic monomorphism *are used.*

Definition 2. *A morphism $f : A \to B$ is called* retraction *if there exists a morphism $g : B \to A$ such that $f \circ g = id_B$.*

Retractions are sometimes called split epimorphisms*.*

Dual notion: a morphism $f : A \to B$ is a section *(or* coretraction *or* split monomorphism*) if there exists a morphism $g : B \to A$ such that $g \circ f = id_A$.*

A morphism $f : A \to B$ is an http://planetmath.org/Isomorphism2isomorphism *if it is a retraction and section at the same time.*

Bimorphism and isomorphism are examples of self-dual properties. The condition that f is isomorphism is equivalent to the existence of a morphism g with $f \circ g = id_B$ and $g \circ f = id_A$ (for the proof see properties of monomorphisms and epimorphisms).

Definition 3. *If f is an isomorphism then the morphism $g : B \to A$ such that $f \circ g = id_B$ and $g \circ f = id_A$ is called* inverse *morphism of f and denoted by f^{-1}.*

Definition 4. *If there exists an isomorphism $f : A \to B$ we say that the objects A and B are* isomorphic*, denoted by $A \cong B$.*