category

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Entry type	Definition
Classification	msc 18A05
Related topic	CategoryOfSets
Related topic	Monad
Related topic	GroupObject
Related topic	GroupScheme
Related topic	DirectLimit
Related topic	Small
Related topic	Endomorphism2
Related topic	Subcategory
Related topic	Precategory
Related topic	MonoidalCategory
Related topic	CategoricalDiagramsAsFunctors
Related topic	CategoryOfCAlgebras
Related topic	CategoryOfBorelSpaces
Related topic	CategoryOfPolishGroups
Related topic	CategoryOfBorelGroupoids
Related topic	Co
Defines	morphism
Defines	identity
Defines	object
Defines	large category

A *category* $\mathcal{C}$ consists of the following data:

1. a `http://planetmath.org/Class`class $\mathrm{ob}(\mathcal{C})$ of objects (of $\mathcal{C}$)

2. for each ordered pair (A, B) of objects of $\mathcal{C}$, a collection (we will assume it is a set) $\hom(A, B)$ of morphisms from the domain A to the codomain B

3. a function $\circ : \hom(A, B) \times \hom(B, C) \to \hom(A, C)$ called composition.

We normally denote $\circ(f, g)$ by $g \circ f$ for morphisms f, g. The above data must satisfy the following axioms: for objects A, B, C, D,

A1: $\hom(A, B) \cap \hom(C, D) = \emptyset$ whenever $(A, B) \neq (C, D)$, i.e. the intersection is non-trivial only when $A = C$ and $B = D$.

A2: (Associativity) if $f \in \hom(A, B)$, $g \in \hom(B, C)$ and $h \in \hom(C, D)$, $h \circ (g \circ f) = (h \circ g) \circ f$

A3: (Existence of an identity morphism) for each object A there exists an identity morphism $id_A \in \hom(A, A)$ such that for every $f \in \hom(A, B)$, $f \circ id_A = f$ and $id_A \circ g = g$ for every $g \in \hom(B, A)$.

Some examples of categories:

- **0** is the empty category with no objects or morphisms, **1** is the category with one object and one (identity) morphism.

- If we assume we have a universe U which contains all sets encountered in "everyday" mathematics, **Set** is the category of all such `http://planetmath.org/Small`small sets with morphisms being set functions

- **Top** is the category of all small topological spaces with morphisms

- **Grp** is the category of all small groups whose morphisms are group homomorphisms

Remark. If $\hom(A, B)$ in the second condition above is not required to be a set (but a class), we usually call $\mathcal{C}$ a *large category*.