space-time quantization problems in quantum gravity theories

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Mathematical and Physical Problems of Quantizing Spacetime in Quantum Gravity Theories:

Beginning with Riemann there has been a prevailing tradition among mathematicians to leave space-time structure problems for theoretical physicists to solve; this tradition has been however punctuated by mathematical contributions made to the space-time structure problem by major contributors such as Minkowski, Poincaré, Weyl, viewed in conjunction with, or separate from, those of Einstein and Lorentz. This tradition is currently changing very rapidly with the major contributors being dedicated mathematicians. As there are so many excellent contributing mathematicians it is very hard to provide a short list, and thus only two will suffice here: É. Cartan (spinor theory) and A. Connes (noncommutative geometry in quantum gravity theories and SUSY). Arguably, such a trend will continue in favor of mathematicians contributing heavily to the foundation of unified physical theories of space and time as well as developing new mathematical concepts for their 'own sake', or in their own right. Examples abound to the point that there is talk not only of mathematical, or theoretical physics, but also of physical mathematics (that is, just for the sake of Abstract mathematics); such is the reaction from many in the high-energy physics, 'elementary particle' physics community. Negative attitudes aside, this is a relatively new, fertile and rather exciting borderline field at the very junction of mathematics and physics– where the 'intertwinner operators' act on both physical and mathematical spacetimes, or topoi, etc., in order to create new science that is neither 'pure' mathematics nor 'pure' physics (which has never existed anyway, as it is always expressed in a mathematical 'language', theory, model, representation or formalism).

The new trend means however much more: it means to employ deeper mathematics and mathematical tools in order to either develop or derive deeper physics. On both sides of the fence many would argue against the other side to maintain a 'status quo of pure' mathematics- that has never existed; this would be, of course, as counter-productive as it can be to sciences, in general, and especially to mathematics, computer science and informatics, for example.

0.1 Quantum fields, symmetry, space-time and connections to general relativity

As the experimental findings in high-energy physics–coupled with theoretical studies– have revealed the presence of new fields and symmetries, there appeared the need in modern physics to develop systematic procedures for generalizing/generating space-times and quantum state space (QSS) representations that reflect the existence of such new fields and symmetries. In the general relativity (GR) formulation, the local structure of space-time – which is characterized by its tensors and curvature – incorporates the gravitational fields surrounding various masses. In Einstein's own representation, the 'physical space-time of GR' has the structure of a Riemann R^4 space over large distances, although the detailed local structure of space-time – as Einstein suggested – is likely to be significantly different.

On the other hand, there is a growing consensus in theoretical physics that a valid theory of quantum gravity requires a much deeper understanding of the small (est)–scale structure of quantum space-time (QST) than currently developed. In Einstein's GR theory and his subsequent attempts at developing an unified field theory (as in the space concept advocated by Leibnitz), space-time does *not* have an *independent existence* from objects, matter or fields, but is instead an entity generated by the *continuous* transformations of fields [**?**] (Einstein, 1950, 1954). Hence, the continuous nature of space-time adopted in GR and Einstein's subsequent field theoretical developments. Furthermore, the quantum, or 'quantized', versions of space-time, *QST*, are operationally defined through local quantum measurements in general reference frames that are prescribed by GR theory. Such a definition is therefore subject to the postulates of both GR theory and the axioms of Local Quantum Physics (that are briefly summarized in Subsection 3.3). We must empasize, however, that this is *not* the usual definition of position and time observables in 'standard' QM. Therefore, the general reference frame positioning in QST is itself subject to the Heisenberg uncertainty principle, and therefore it acquires through quantum measurements a certain 'fuzziness' at the Planck scale which is intrinsic to all microphysical quantum systems, as further explained in this section. Whereas Newton, Riemann, Einstein, Weyl, Hawking, Weinberg and many other exceptional theoreticians regarded the physical space as being represented by a *continuum*, there is an increasing number of proponents for a *discrete, 'quantized'* structure of space-time. The latter view is not without its problems and advantages. The biggest problem for any discrete, 'point-set' (or discrete topology), view of physical spacetime is not only its immediate conflict with Einstein's General Relativity representation of spacetime as a *continuous Riemann* space, but also the impossibility of carrying out quantum measurements to localize precisely either quantum events or masses at singular (in the sense of disconnected, or isolated), sharply defined, geometric points in space-time. One of the proposed resolutions of this problem is Non-commutative Geometry (NCG), or 'Quantum Geometry', where QST has 'no points' (or perhaps no point!), in the sense of visualization of such a geometrical space as some kind of a distributive and commutative lattice of space-time 'points'. The quantum 'metric' of QST in NCG would be related to a certain, fundamental quantum field operator, or 'fundamental triplet (or quintet)' construction (Connes, 2004). Although quantization is standard in Quantum Mechanics (QM) for most of the quantum observables, it does run into major difficulties when applied to position and time. In standard QM, there are at least two implemented approaches to solve the problem, one of them designed 72 years ago by von Neumann (1933).

Another potential concern is the inadequacy of the long-standing model of space-time as a 4–dimensional manifold *with a Lorentz metric.* The hope of some of the earlier approaches to quantum gravity (QG) was to cope with extremely small length scales where a manifold structure may be justifiably foresaken (for instance, at the Planck length $L_p = (\frac{G\hbar}{c^3})^{\frac{1}{2}} \approx 10^{-35}m$). On the other hand, one needs to reconcile the discreteness versus continuum approach in view of space–time diffeomorphisms and that space–time may be suitably modeled as some type of 'combinatorial space' (such as a simplicial complex, a poset, or a spin network) The monumental difficulty is that to the present day, apart from a distinct lack of experimental evidence, there is no specific agreement on the kind of data, plus no agreement on the actual conceptual background to obtaining the data in the first place(!) This difficulty equates with how one can relate the approaches to QG to run the gauntlet of conceptual problems in QFT and (General Relativity) GR. To quote an example, the space–time metric tensor: $\gamma = (\gamma_{ab})$ is less a fundamental field than perhaps once thought since it leads to describing an essentially classical gravitational field. A case study in ref. [?] involves quantizing one side of Einstein's field equations by a quantum expectation value, so that a coupling of γ to quantized matter is given by an expression such as:

$$G_{\mu\nu}(\gamma) = \langle\ \psi\ | T_{\mu\nu}(g, \hat{\phi} |\ \psi\ \rangle\ ,$$

where $|\psi\rangle$ denotes a state in the Hilbert space of quantized matter variables $\hat{\phi}$, and the subsequent source of the gravitational field is given by the expectation of the corresponding energy–momentum tensor $T_{\mu\nu}$. Unfortunately, this expression is not without its ontological and 'physical' problems sufficiently serious to prevent the development of a complete QG theory that includes this expression. Three possible approaches were suggested by Butterfield and Isham in ref. [?] (cf. also an extensive survey article by Rovelli, 1997):

(1) to develop and test a quantized form of classical relativity theory;

(2) to recover GR as the low energy limit of a QFT approach which is not a quantization of a classical theory (e.g., *via quantum algebras/groups and their representations*);

(3) to develop a new theory, such as a 'quantization of topology' or 'causal' structures where, for instance, microphysical states provide amplitudes to the values of quantities whose norms squared define probabilities of occurrence for *physical*, quantum events.

We turn now to another facet of quantum measurement. Note first that QFT pure states resist description in terms of field configurations since the former are not always physically either observable or interpretable. Algebraic Quantum Field Theory (AQFT) as expounded by Roberts (2004) points to various questions raised by considering theories of (unbounded) operator–valued distributions and *quantum field nets* of von Neumann algebras. Using in part a gauge theoretic approach, the idea is to regard two field theories as equivalent when their associated nets of observables are isomorphic. More specifically, AQFT considers taking *additive nets of quantum field algebras* over subsets of Minkowski space, which among other properties, enjoy Bose–Fermi commutation relations. There may be analogs with sheaf theory in this approach, even though these analogs appear to be limited. The typical AQFT net does not seem to give rise to a presheaf because the relevant morphism orientations are in reverse. Closer then is to regard a net as a precosheaf, but the additivity does not allow proceeding to a cosheaf structure. This may be a reflection of some deeper incompatibility of AQFT with those aspects of quantum gravity (QG) where the sheaf– theoretic/topos approaches are advocated (as, for example, in [?] (Butterfield and Isham, 1999-2004).

References

[1] Butterfield, J. and C. J. Isham: 2001, Space-time and the philosophical challenges of quantum gravity., in C. Callender and N. Hugget (eds.) *Physics Meets Philosophy at the Planck scale.*, Cambridge University Press,pp.33–89.

[2] Butterfield, J. and C. J. Isham: 1998, 1999, 2000–2002, A topos perspective on the Kochen–Specker theorem I - IV, *Int. J. Theor. Phys*, **37** No 11., 2669–2733 **38** No 3., 827–859, **39** No 6., 1413–1436, **41** No 4., 613–639.