category of pointed topological spaces

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Defines	pointed topological space
Defines	based topological space

A *pointed topological space*, written as (X, x_0), consists of a non-empty topological space X together with an element $x_0 \in X$. The terminology *based topological space* is also used often.

If (X, x_0) is a pointed space, we call X its *underlying* topological space and x_0 its *basepoint*.

A *morphism* from (X, x_0) to (Y, y_0) is a continuous map $f\colon X \to Y$ satisfying $f(x_0) = y_0$. With these morphisms, the pointed topological spaces form a category.

Two pointed topological spaces (X, x_0) and (Y, y_0) are isomorphic in this category if there exists a homeomorphism $f\colon X \to Y$ with $f(x_0) = y_0$.

Every singleton (a pointed topological space of the form $(\{x_0\}, x_0)$) is a zero object in this category.

For every pointed topological space (X, x_0), we can construct the fundamental group $\pi(X, x_0)$ and for every morphism $f\colon (X, x_0) \to (Y, y_0)$ we obtain a group homomorphism $\pi(f)\colon \pi(X, x_0) \to \pi(Y, y_0)$. This yields a functor from the category of pointed topological spaces to the category of groups.

Other interesting functors defined on the category of pointed spaces include the higher homotopy groups $\pi_i(X, x_0)$ for $i = 2, 3, \ldots$ that map into the category of abelian groups and the (based) *loop space* $\Omega(X, x_0)$ that maps into the category of topological spaces.