derived category

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Let $\mathscr{A}$ be an abelian category, and let $\mathcal{K}(\mathscr{A})$ be the category of chain complexes in $\mathscr{A}$, with the morphisms being chain homotopy classes of maps. Call a morphism of chain complexes a *quasi-isomorphism* if it induces an isomorphism on homology groups of the complexes. For example, any chain homotopy is a quasi-isomorphism, but not conversely. Now let the derived category $\mathcal{D}(\mathscr{A})$ be the category obtained from $\mathcal{K}(\mathscr{A})$ by adding a formal inverse to every quasi-isomorphism (technically this called a localization of the category).

Derived categories seem somewhat obscure, but in fact, many mathematicians believe they are the appropriate place to do homological algebra. One of their great advantages is that the important functors of homological algebra which are left or right exact (Hom,$N \otimes_k -$, where N is a fixed k-module, the global sections functor Γ, etc.) become exact on the of derived functors (with an appropriately modified definition of exact).

See *Methods of Homological Algebra*, by Gelfand and Manin for more details.