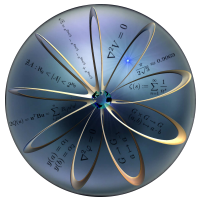

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monic

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Defines	split monomorphism
Defines	section
Defines	coretraction

A morphism $f\colon A \to B$ in a category is called a *monic* morphism, or *monomorphism*, if it can be cancelled from the left — for any object C and any morphisms $g_1, g_2\colon\ C \to A$ we have $f \circ g_1 = f \circ g_2$ if and only if $g_1 = g_2$.

A morphism $f : A \to B$ in a category is called a *split monomorphism* if there exists a morphism $g\colon B \to A$ such that $g \circ f = \mathrm{id}_A$. Note that every split monomorphism is a monomorphism; if f is a split monomorphism and $f \circ h = f \circ k$, then one has $g \circ (f \circ h) = g \circ (f \circ k)$. By associativity, $(g \circ f) \circ h = (g \circ f) \circ k$; by definition of split monomorphism, $\mathrm{id}_a \circ h = \mathrm{id}_a \circ k$; by definition of identity, $h = k$, so f is a monomorphism. Split monomorphisms are also known as *sections* and *coretractions*.

The notion of epimorphism is dual to that of monomorphism. An epimorphism of a category is a monomorphism of the dual category and vice versa.

A monomorphism in the category of sets is simply a one-to-one function. Moreover, in the category of sets all monomorphisms are split monomorphisms.