snake lemma

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Let $\mathcal{A}$ be an abelian category. The *snake lemma* consists of the following two claims:

1. Suppose
$$\begin{tikzcd} 0 \arrow[r] & A_1 \arrow[r] \arrow[d, "\alpha"'] & B_1 \arrow[r] \arrow[d, "\beta"'] & C_1 \arrow[r] \arrow[d, "\gamma"'] & 0 \\ 0 \arrow[r] & A_2 \arrow[r] & B_2 \arrow[r] & C_2 \arrow[r] & 0 \end{tikzcd}$$
is a commutative diagram in $\mathcal{A}$ with exact rows. Then there is an exact sequence
$$0 \to \ker\alpha \to \ker\beta \to \ker\gamma \xrightarrow{s} \operatorname{coker}\alpha \to \operatorname{coker}\beta \to \operatorname{coker}\gamma \to 0,$$
usually called the *kernel-cokernel sequence*. The morphism s is called the *connecting morphism*.

2. Applying the previous claim inductively, for any short exact sequence
$$0 \to \mathbf{A} \to \mathbf{B} \to \mathbf{C} \to 0$$
of chain complexes in $\mathcal{A}$, there is a corresponding `http://planetmath.org/LongExactSequen` exact sequence in homology
$$\cdots \to H_n(\mathbf{A}) \to H_n(\mathbf{B}) \to H_n(\mathbf{C}) \to H_{n-1}(\mathbf{A}) \to \cdots$$