proper generator of a Grothendieck category

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0.1 Introduction: family of generators and generator of a category

Definition 0.1. Let $\mathcal{C}$ be a category. A family of its objects $\{U_i\}_{i\in I}$ is said to be a *family of generators* of $\mathcal{C}$ if for every pair of distinct morphisms $\alpha, \beta : A \to B$ there is a morphism $u : U_i \to A$ for some index $i \in I$ such that $\alpha u \neq \beta u$.

One notes that in an additive category, $\{U_i\}_{i\in I}$ is a family of generators if and only if for each nonzero morphism α in $\mathcal{C}$ there is a morphism $u : U_i \to A$ such that $\alpha u \neq 0$.

Definition 0.2. An object U in $\mathcal{C}$ is called a *generator* for $\mathcal{C}$ if $U \in \{U_i\}_{i\in I}$ with $\{U_i\}_{i\in I}$ being a family of generators for $\mathcal{C}$.

Equivalently, (viz. Mitchell) U is a *generator* for $\mathcal{C}$ if and only if the set-valued functor H^U is an imbedding functor.

0.2 Proper generator of a Grothendieck category

Definition 0.3. A *proper generator* U_p of a Grothendieck category $\mathcal{G}$ is defined as a generator U_p which has the property that a monomorphism $i : U' \to U_p$ induces an isomorphism ι,

$$Hom_{\mathcal{G}}(U_p, U_p) \cong Hom_{\mathcal{G}}(U', U_p),$$

if and only if i is an isomorphism.