source

Canonical name	Source
Date of creation	2013-03-22 16:03:25
Last modified on	2013-03-22 16:03:25
Owner	kompik (10588)
Last modified by	kompik (10588)
Numerical id	10
Author	kompik (10588)
Entry type	Definition
Classification	msc 18A05
Related topic	UniversalProperty
Defines	source
Defines	monosource
Defines	extremal monosource
Defines	sink
Defines	extremal episink

In the whole entry we suppose we are given a category $\mathbf{A}$. By an object we always mean an object in $\mathbf{A}$ and by a morphisms an $\mathbf{A}$-morphism.

Definition 1. *A* source *in a category* $\mathbf{A}$ *is a pair* $(A,(f_i)_{i\in I})$ *where* A *is an object and* $f_i : A \to A_i$ *are morphisms indexed by a class* I.

The object A *is called the* domain of the source *and the family* $(A_i)_{i\in I}$ *is called the codomain of the source.*

A *sink* is a pair $((f_i)_{i\in I}, A)$ where A is an object and $f_i : A_i \to A$ are morphisms.

Sources can be composed with morphisms. If $\mathcal{S} = (A,(f_i)_{i\in I}$ is a source and $f : B \to A$ is a morphism, we use the notation $(B,(f_i \circ f)_{i\in I}) = \mathcal{S} \circ f$. Similarly, for sinks, we use the notation $f \circ \mathcal{S} = ((f \circ f_i)_{i\in I}, B)$ if $\mathcal{S} = ((f_i)_{i\in I}, A)$ is a sink and $f : A \to B$ is a morphism.

Definition 2. *A source* $\mathcal{S} = (A,(f_i)_{i\in I})$ *is called a* monosource *if for any pair* $r, s : B \to A$ *of morphisms from the equality* $\mathcal{S} \circ r = \mathcal{S} \circ s$ *follows* $r = s$.

A sink $\mathcal{S} = ((f_i)_{i\in I}, A)$ *is called an* episink *if for any pair* $r, s : A \to B$ *of morphisms* $r = s$ *whenever* $r \circ \mathcal{S} = s \circ \mathcal{S}$.

A monosource $\mathcal{S}$ *is called* extremal monosource*, if the following holds: Whenever* $\mathcal{S} = \overline{\mathcal{S}} \circ e$ *for an epimorphism* e*, then* e *is an isomorphism.*

An episink $\mathcal{S}$ *is called* extremal episink *if the following holds: Whenever* $\mathcal{S} = m \circ \overline{\mathcal{S}}$ *pre for a monomorphism* m*, tak* m *is an isomorphism.*

Every limit is an extremal monosource, a colimit is an extremal episink.

References

[1] J. Adámek, H. Herrlich, and G. Strecker. *Abstract and Concrete Categories.* Wiley, New York, 1990.