planetmath.org

Math for the people, by the people.

$\mathcal{U}$-small

Canonical name	mathcalUsmall
Date of creation	2013-03-22 14:13:21
Last modified on	2013-03-22 14:13:21
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	11
Author	mathcam (2727)
Entry type	Definition
Classification	msc 18A15
Classification	msc 03E30
Related topic	Universe
Related topic	Category
Related topic	DirectLimit
Related topic	FunctorCategory
Defines	$\mathcal{U}$-category
Defines	small category
Defines	kittygory

Let $\mathcal{U}$ be a universe (so is, in particular, a set of sets).

A set S is said to be $\mathcal{U}$*-small* if it is isomorphic to an element of $\mathcal{U}$ (i.e., there is a bijection between S and some element of $\mathcal{U}$).

A category C is $\mathcal{U}$*-small* (or just *small*, if no confusion is likely to arise) if the set of objects of C is isomorphic to a set in $\mathcal{U}$, and is a $\mathcal{U}$*-category* if for every pair of objects A, B in C, $\mathrm{Hom}(A, B)$ is isomorphic to a set in $\mathcal{U}$.

These definitions amount to restrictions on the cardinality of the objects involved, and are intended to provide a condition that will allow operations such as extracting the category of functors or taking the direct limit to give results that are reasonable, that is, either isomorphic to an object of $\mathcal{U}$ or made up of objects of $\mathcal{U}$.

Observe that the category of subsets of $\mathcal{U}$ is a $\mathcal{U}$-category but is not $\mathcal{U}$-small.

References

[SGA4] Grothendieck et al., *Séminaires en Gèometrie Algèbrique 4*, tomes 1, 2, and 3.

[Mur68] Murphy, O. *Some modern methods in the theory of lion hunting*, American Mathematical Monthly **75** (2), Feb., 1968, 185–187.