cochain complex

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Defines	cocycle
Defines	coboundary

Let R be a ring. A sequence of http://planetmath.org/Module$R$-modules and homomorphisms

$$\cdots \to A^{n-1} \xrightarrow{d_{n-1}} A^n \xrightarrow{d_n} A^{n+1} \xrightarrow{d_{n+1}} \cdots$$

is said to be a *cochain complex* (or *R-complex*, or just *complex*) if each pair of adjacent homomorphisms (d_{n-1}, d_n) satisfies the relation $d_n \circ d_{n-1} = 0$. This is equivalent to saying that $\operatorname{im} d_{n-1} \subseteq \ker d_n$. We often denote such a complex by $(\mathcal{A}, d)$, or simply $\mathcal{A}$.

Compare this to the notion of an exact sequence, which requires $\operatorname{im} d_{n-1} = \ker d_n$. Compare also to the notion of a chain complex, in which the arrows go in the opposite direction.

The homomorphisms d_n in the chain complex are called *coboundary operators*, or *coboundary maps*. Elements of $\ker d_n$ are known as *cocycles*; elements of $\operatorname{im} d_{n-1}$ as *coboundaries*.