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simplicial object

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Defines	simplicial set

A **simplicial object** in a category C is a contravariant functor from the simplicial category Δ to C . Such a functor X is uniquely specified by the morphisms $X(\delta_i^n): X([n]) \rightarrow X([n-1])$ and $X(\sigma_i^n): X([n]) \rightarrow X([n+1])$, which satisfy

$$X(\delta_i^{n-1}) X(\delta_j^n) = X(\delta_{j-1}^{n-1}) X(\delta_i^n) \quad \text{for } i < j, \quad (1)$$

$$X(\sigma_i^{n+1}) X(\sigma_j^n) = X(\sigma_{j+1}^{n+1}) X(\sigma_i^n) \quad \text{for } i \leq j, \quad (2)$$

$$X(\delta_i^{n+1}) X(\sigma_j^n) = \begin{cases} X(\sigma_{j-1}^{n+1}) X(\delta_i^n) & \text{if } i < j, \\ \text{id}_n & \text{if } i = j \text{ or } i = j + 1, \\ X(\sigma_j^{n+1}) X(\delta_{i-1}^n) & \text{if } i > j + 1. \end{cases} \quad (3)$$

In particular, a **simplicial set** is a simplicial object in **Set**. Equivalently, one could say that a simplicial set is a presheaf on Δ . The object $X([n])$ of a simplicial set is a set of n -simplices, and is called the n -skeleton.