superalgebroids and higher dimensional algebroids

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0.0.1 Definitions of double, and higher dimensional, algebroids, superalgebroids and generalized superalgebras

Double algebroids

Definition 0.1. A *double R–algebroid* consists of a http://planetmath.org/HomotopyDoubleGrou category D, as detailed in ref.[?], such that each category structure has the additional structure of an R–algebroid. More precisely, a double http://planetmath.org/RAlgebr algebroid D involves four related http://planetmath.org/RAlgebroid$R$–algebroids:

$$\begin{aligned} (D, D_1, \partial_1^0, \partial_1^1, \varepsilon_1, +_1, \circ_1, ._1), \qquad & (D, D_2, \partial_2^0, \partial_2^1, \varepsilon_2, +_2, \circ_2, ._2) \\ (D_1, D_0, \delta_1^0, \delta_1^1, \varepsilon, +, \circ, .), \qquad & (D_2, D_0, \delta_2^0, \delta_2^1, \varepsilon, +, \circ, .) \end{aligned} \tag{0.1}$$

that satisfy the following rules:

i) $\delta_2^i \partial_2^j = \delta_1^j \partial_1^i$ for $i, j \in \{0, 1\}$

ii)
$$\begin{aligned} \partial_2^i(\alpha +_1 \beta) = \partial_2^i \alpha + \partial_2^i \beta, \qquad & \partial_1^i(\alpha +_2 \beta) = \partial_1^i \alpha + \partial_1^i \beta \\ \partial_2^i(\alpha \circ_1 \beta) = \partial_2^i \alpha \circ \partial_2^i \beta, \qquad & \partial_1^i(\alpha \circ_2 \beta) = \partial_1^i \alpha \circ \partial_1^i \beta \end{aligned} \tag{0.2}$$
for $i = 0, 1, \alpha, \beta \in D$ and both sides are defined.

iii)
$$\begin{aligned} r._1(\alpha +_2 \beta) = (r._1\alpha) +_2 (r._1\beta), \qquad & r._2(\alpha +_1 \beta) = (r._2\alpha) +_1 (r._2\beta) \\ r._1(\alpha \circ_2 \beta) = (r._1\alpha) \circ_2 (r._1\beta), \qquad & r._2(\alpha \circ_1 \beta) = (r._2\alpha) \circ_1 (r._2\beta) \\ & r._1(s._2\beta) = s._2(r._1\beta) \end{aligned} \tag{0.3}$$
for all $\alpha, \beta \in D,\ r, s \in R$ and both sides are defined.

iv)
$$\begin{aligned} (\alpha +_1 \beta) +_2 (\gamma +_1 \lambda) &= (\alpha +_2 \gamma) +_1 (\beta +_2 \lambda), \\ (\alpha \circ_1 \beta) \circ_2 (\gamma \circ_1 \lambda) &= (\alpha \circ_2 \gamma) \circ_1 (\beta \circ_2 \lambda) \\ (\alpha +_i \beta) \circ_j (\gamma +_i \lambda) &= (\alpha \circ_j \gamma) +_i (\beta \circ_j \lambda) \end{aligned} \tag{0.4}$$
for $i \neq j$, whenever both sides are defined.

The definition of a double algebroid specified above was introduced by Brown and Mosa [**?**]. Two functors can be then constructed, one from the category of double algebroids to the category of crossed modules of algebroids, whereas the reverse functor is the unique adjoint (up to natural equivalence). The construction of such functors requires the following definition.

0.1 Category of Double Algebroids

A *morphism* $f : \mathsf{D} \to \mathcal{E}$ *of double algebroids* is then defined as a morphism of truncated cubical sets which commutes with all the algebroid structures. Thus, one can construct a category **DA** of double algebroids and their morphisms. The main construction in this subsection is that of two functors η, η' from this category **DA** to the category **CM** of crossed modules of algebroids.

Let D be a double algebroid. One can associate to D a crossed module $\mu : M \longrightarrow D_1$. Here $M(x, y)$ will consist of elements m of D with boundary of the form: 0 1

$$\partial m = \begin{pmatrix} & a & \\ 1_y & & 1_x \\ & 0_{xy} & \end{pmatrix} , \qquad (0.5)$$

that is $M(x, y) = \{m \in D : \partial_1^1 m = 0_{xy}, \partial_2^0 m = 1_x, \partial_2^1 m = 1_y\}$.

0.2 Cubic and Higher dimensional algebroids

One can extend the above notion of double algebroid to cubic and higher dimensional algebroids. The concepts of 2-algebroid, 3-algebroid,..., n–algebroid and superalgebroid are however quite distinct from those of double, cubic,..., n–tuple algebroid, and have technically less complicated definitions.

References

[1] R. Brown and G. H. Mosa: Double algebroids and crossed modules of algebroids, University of Wales–Bangor, Maths Preprint, 1986.

[2] R. Brown and C.B. Spencer: Double groupoids and crossed modules, *Cahiers Top. Géom.Diff.* **17**: 343–362 (1976).