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small site on a scheme

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As an example of a site, fix a scheme X and a class of morphisms E . Then take the category of schemes over X whose structure morphism is in E . Let $\{U_\alpha \rightarrow U\}$ be a covering if all the morphisms are in E and the induced map $\coprod U_\alpha \rightarrow U$ is universally surjective (if the maps are open, then this is equivalent to being surjective). This is called the small E -site over X .

Concretely, take E to be open immersions; then one obtains exactly the Zariski site, in which open sets, presheaves, sheaves, and sheaf cohomology have the usual meaning.

If we take E to be étale morphisms, then one obtains the small étale site on X . Here the open sets are étale morphisms to X . Since an étale morphism is open, one can view them as open subsets with a “twisted” embedding. This nontrivial embedding yields new behaviour from sheaves and presheaves, and the cohomology theory obtained by taking the right derived functors of the global sections functor gives étale cohomology. In particular, one can now take the cohomology of the constant sheaves $\mathbb{Z}/l^n\mathbb{Z}$ and obtain nonzero answers.