monoid as a category

Canonical name	MonoidAsACategory
Date of creation	2013-03-22 16:03:31
Last modified on	2013-03-22 16:03:31
Owner	kompik (10588)
Last modified by	kompik (10588)
Numerical id	7
Author	kompik (10588)
Entry type	Definition
Classification	msc 18B40
Defines	group as a category

For each monoid (a semigroup with an identity element) $(M, \bullet, e)$ we can define a category $\mathbf{C}(M, \bullet, e) = (\mathrm{Ob}, \mathrm{hom}, id, \circ)$ with one object by putting $\mathrm{Ob} = \{M\}$, and morphisms are elements of M: $\mathrm{hom}(M, M) = M$, where $id_M = e$, and the composition $\circ$ of morphisms is the monoidal product $\bullet$ on elements of M: $y \circ x = y \bullet x$.

Moreover, any category with a single object has a natural structure as a monoid with the binary operation given by the law of composition of morphisms.

Remark. If a monoid is a group, then the identified category again has one object, and furthermore all of its morphisms are isomorphisms. Conversely, a category with one object all of whose morphisms are isomorphisms has a natural structure as a group.