

planetmath.org

Math for the people, by the people.

kernel pair

Canonical name KernelPair

Date of creation 2013-03-22 18:20:34 Last modified on 2013-03-22 18:20:34

Owner CWoo (3771) Last modified by CWoo (3771)

Numerical id 10

Author CWoo (3771) Entry type Definition Classification msc 18A30

 $Related\ topic \\ Kernel Of A Homomorphism Between Algebraic Systems$

Defines cokernel pair

Let $f: A \to B$ be a morphism in a category \mathcal{C} . The *kernel pair* of f is defined as the pair of morphisms $(k_1: K \to A, k_2: K \to A)$ such that

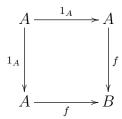
$$K \xrightarrow{k_1} A$$

$$\downarrow k_2 \qquad \qquad \downarrow f$$

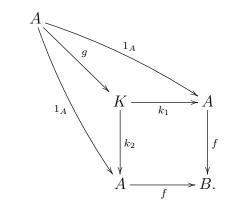
$$A \xrightarrow{f} B$$

is a pullback diagram.

Since



is a commutative diagram, we have a unique morphism $g: A \to K$ such that



is commutative. As a result, k_1 and k_2 are both monomorphisms: if $k_1 \circ h_1 = k_1 \circ h_2$, then

$$h_1 = 1_A \circ h_1 = (g \circ k_1) \circ h_1 = g \circ (k_1 \circ h_1) = g \circ (k_1 \circ h_2) = (g \circ k_1) \circ h_2 = 1_A \circ h_2 = h_2.$$

For example, in **Set**, the category of sets, the kernel pair of a function $f: A \to B$ is the pair $p_1: K \to A$ and $p_2: K \to A$, given by

$$K = \{(a, b) \in A \times A \mid f(a) = f(b)\},\$$

and p_1 and p_2 are given by

$$p_1(a, b) = a$$
 and $p_2(a, b) = b$.

This is just the kernel of a function, in the sense of universal algebra. Please see http://planetmath.org/KernelOfAHomomorphismBetweenAlgebraicSystemsthis entry for more details.

The notion of *cokernel pair* is dually defined.

Remark. $f: A \to B$ is a monomorphism iff the kernel pair of f is $(1_A, 1_A)$. Dually, f is an epimorphism iff the cokernel pair of f is $(1_A, 1_A)$.

References

[1] F. Borceux Basic Category Theory, Handbook of Categorical Algebra I, Cambridge University Press, Cambridge (1994)