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category isomorphism

Canonical name	CategoryIsomorphism
Date of creation	2013-03-22 14:22:04
Last modified on	2013-03-22 14:22:04
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	9
Author	CWoo (3771)
Entry type	Definition
Classification	msc 18A05

Let $\mathcal{C}$ and $\mathcal{D}$ be categories. An *isomorphism* $T : \mathcal{C} \to \mathcal{D}$ is a (covariant) functor which has a *two-sided inverse*. In other words, there is a (covariant) functor $S : \mathcal{D} \to \mathcal{C}$ such that $T \circ S = I_{\mathcal{D}}$ and $S \circ T = I_{\mathcal{C}}$, where $I_{\mathcal{D}}$ and $I_{\mathcal{C}}$ are the identity functors of $\mathcal{D}$ and $\mathcal{C}$ respectively. Two categories $\mathcal{C}$ and $\mathcal{D}$ are *isomorphic* if there exists a functor $T : \mathcal{C} \to \mathcal{D}$ that is an isomorphism.

Remarks

1. An isomorphism (functor) from $\mathcal{C}$ to $\mathcal{D}$ is just an `http://planetmath.org/Isomorphism2`isom (in the sense of morphism) in the functor category $\mathcal{D}^{\mathcal{C}}$.

2. Two isomorphic categories are `http://planetmath.org/EquivalenceOfCategories`equivale The converse is not true. For example, the category of all finite sets is to its subcategory of all finite ordinals. But clearly these two categories are not isomorphic. Isomorphism has a "size" restriction, whereas natural equivalence does not.