# discrete category

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| Canonical name | DiscreteCategory |
| Date of creation | 2013-03-22 16:15:09 |
| Last modified on | 2013-03-22 16:15:09 |
| Owner | CWoo (3771) |
| Last modified by | CWoo (3771) |
| Numerical id | 8 |
| Author | CWoo (3771) |
| Entry type | Definition |
| Classification | msc 18A05 |
| Defines | trivial category |
| Defines | empty category |
| Defines | empty functor |

A category $\mathcal{C}$ is said to be a *discrete category* if the only morphisms in $\mathcal{C}$ are the identity morphisms associated with each of the objects in $\mathcal{C}$.

For example, every set can be regarded as a discrete category. The objects are just the elements of the set. Furthermore, $\hom(a, a)$ is identified with $\{a\}$, and $\hom(a, b) = \varnothing$ if $a \neq b$.

**Remarks**.

- A discrete category with one object is called a *trivial category*. For every category $\mathcal{C}$, there is only one functor from $\mathcal{C}$ to a trivial category. Hence, any trivial category is a terminal object in **Cat**, the category of small categories.

- A discrete category with no objects is called the *empty category*. For every category $\mathcal{C}$, there is only one functor from the empty category to $\mathcal{C}$. This functor is called the *empty functor*, where both the object and morphism functions are the empty set $\varnothing$. Thus, the empty category is the initial object in **Cat**.

- Given any category $\mathcal{C}$, the smallest subcategory consisting of all objects in $\mathcal{C}$ is discrete, which is also the largest discrete subcategory in $\mathcal{C}$ (largest in the sense that it contains all objects of $\mathcal{C}$). For every object $X \in \mathcal{C}$, we can associate the trivial category $\mathcal{C}_X$ consisting of one object, $X$, and one morphism $1_X$.