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normal category

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| Entry type       | Definition               |
| Classification   | msc 18E10                |
| Synonym          | normal monic             |
| Synonym          | conormal epi             |
| Defines          | normal                   |
| Defines          | normal monomorphism      |
| Defines          | normal subobject         |
| Defines          | conormal                 |
| Defines          | conormal epimorphism     |
| Defines          | conormal category        |
| Defines          | conormal quotient object |

A monomorphism in a category is said to be *normal* if it is a kernel (of a morphism). A subobject of an object is *normal* if any (and hence all) of its representing monomorphisms is normal.

For example, in **Grp**, the category of groups, the inclusion of a subgroup  $H \subseteq G$  into  $G$  is normal iff  $H$  is a normal subgroup of  $G$ .

A category is said to be *normal* if every monic is a kernel. Equivalently, a normal category is a category in which every subobject of every object is normal.

Dually, an epimorphism is *conormal* if it is a cokernel (of a morphism). A quotient object of an object is *conormal* if any (and hence all) of its representing epimorphisms is conormal. A category is said to be *conormal* if every epimorphism is conormal.

The category **AbGrp** of abelian groups, and more generally, any abelian category, is normal and conormal.

## References

- [1] C. Faith *Algebra: Rings, Modules, and Categories I*, Springer-Verlag, New York (1973)