# groupoid representations induced by measure

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| Canonical name | GroupoidRepresentationsInducedByMeasure |
| Date of creation | 2013-03-22 18:16:27 |
| Last modified on | 2013-03-22 18:16:27 |
| Owner | bci1 (20947) |
| Last modified by | bci1 (20947) |
| Numerical id | 27 |
| Author | bci1 (20947) |
| Entry type | Topic |
| Classification | msc 18D05 |
| Classification | msc 55N33 |
| Classification | msc 55N20 |
| Classification | msc 55P10 |
| Classification | msc 55U40 |
| Synonym | associated Haar system |
| Related topic | Groupoids |
| Related topic | LocallyCompactGroupoids |
| Related topic | BorelGroupoid |
| Defines | associated Haar system |
| Defines | groupoid representation |
| Defines | measure induced operator |
| Defines | measure-preserving transformation |
| Defines | associated Haar system |

# 1 Associated, Left or Right, Haar System

**Definition 1.1.** A *groupoid representation induced by measure* can be defined as measure induced operator or as an operator induced by a measure preserving map in the context of Haar systems with measure that are associated with locally compact groupoids, $\mathbf{G_{lc}}$.

Thus, let us consider a locally compact groupoid $\mathbf{G_{lc}}$ endowed with an associated Haar system $\nu = \{\nu^u, u \in U_{\mathbf{G_{lc}}}\}$, and $\mu$ a quasi-invariant measure on $U_{\mathbf{G_{lc}}}$. Moreover, let $(X_1, \mathfrak{B}_1, \mu_1)$ and $(X_2, \mathfrak{B}_2, \mu_2)$ be measure spaces and denote by $L^0(X_1)$ and $L^0(X_2)$ the corresponding spaces of measurable functions (with values in $\mathbb{C}$). Let us also recall that with a measure-preserving transformation $T : X_1 \longrightarrow X_2$ one can define an *operator induced by a measure preserving map*, $U_T : L^0(X_2) \longrightarrow L^0(X_1)$ as follows:

$$(U_T f)(x) := f(Tx)\,, \qquad\qquad f \in L^0(X_2),\ x \in X_1$$

Next, let us define $\nu = \int \nu^u d\mu(u)$ and also define $\nu^{-1}$ as the mapping $x \mapsto x^{-1}$. With $f \in C_c(\mathbf{G_{lc}})$, one can now define the *measure induced operator* $\mathbf{Ind}\mu(f)$ as an operator being defined on $L^2(\nu^{-1})$ by the formula:

$$\mathbf{Ind}\mu(f)\xi(x) = \int f(y)\xi(y^{-1}x)d\nu^{r(x)}(y) = f * \xi(x)$$

**Remark 1.1.** One can readily verify that :

$$\|\mathbf{Ind}\mu(f)\| \leq \|f\|_1\,,$$

and also that $\mathbf{Ind}\mu$ is a proper representation of $C_c(\mathbf{G_{lc}})$, in the sense that the latter is usually defined for groupoids.