category of matrices

Canonical name	CategoryOfMatrices
Date of creation	2013-03-22 16:31:54
Last modified on	2013-03-22 16:31:54
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	9
Author	rspuzio (6075)
Entry type	Example
Classification	msc 18A05

The set of all matrices (rectangular as well as square) over a given field (such as real or complex numbers; more generally, one could consider matrices over a unital ring such as integers or a suitable ring of polynomials) forms a category. As objects of this category, we take the positive integers. For any two integers m and n, we take $\mathrm{Hom}(m, n)$ to be the set of $m \times n$ matrices. The composition of morphisms is taken to be matrix multiplication. It is easy to see that the defining properties of categories are satisfied:

1. The intersection of the set of $a \times b$ matrices and the set of $c \times d$ matrices is empty unless $a = c$ and $b = d$.

2. Matrix multiplication is associative.

3. For every n, there is a special element of $\mathrm{Hom}(n, n)$, the $n \times n$ identity matrix, which satisfies the properties of an identity morphism.

This example illustrates an important about the notion of category. As with groups and semigroups, categories are an algebraic structure defined by a single associative operation. Where they differ is that the closure property no longer holds — given two morphisms in a category, it is not necessarily the case that they can be composed. The reason for introducing objects is to keep track of when it is possible to compose the morphisms.