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identity functor

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Let $\mathcal{C}$ be a category. The *identity functor* of $\mathcal{C}$ is the unique functor, written $I_{\mathcal{C}}$, such that for every object A and every morphism α in $\mathcal{C}$, we have

$$I_{\mathcal{C}}(A) = A \quad \text{and} \quad I_{\mathcal{C}}(\alpha) = \alpha.$$

To verify that $I_{\mathcal{C}}$ is indeed a functor, we note that $I_{\mathcal{C}}(1_A) = 1_A = 1_{I_{\mathcal{C}}(A)}$, where 1_A is the identity morphism of A, and $I_{\mathcal{C}}(\alpha \circ \beta) = \alpha \circ \beta = I_{\mathcal{C}}(\alpha) \circ I_{\mathcal{C}}(\beta)$.

For any functor $F : \mathcal{C} \to \mathcal{D}$, we have $F \circ I_{\mathcal{C}} = I_{\mathcal{D}} \circ F = F$.

Since every category gives rise to its unique identity functor, we can think of *the identity functor* I as a (covariant) functor on **Cat**, the category of (small) categories. It is given by taking any category $\mathcal{C}$ to itself and any functor $F : \mathcal{C} \to \mathcal{D}$ to itself.