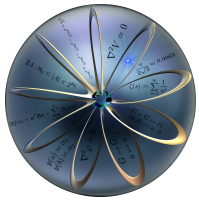

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category of additive fractions

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Definition 0.1. Let $\mathcal{C}$ be a preadditive category and also let Σ be a class of morphisms of $\mathcal{C}$. Then consider a covariant functor $T : \mathcal{C} \to \mathcal{C}_\Sigma$ from a preadditive category to an Abelian category denoted as $\mathcal{C}_\Sigma$, such that for any $s \in \Sigma$, $T(s)$ is an isomorphism in $\mathcal{C}_\Sigma$. Moreover, let us assume that if $T' : \mathcal{C} \to \mathcal{C}'$ is a covariant functor, such that for any morphism $s \in \Sigma$, $T'(s)$ is also an isomorphism; then, there exists a unique (covariant) functor $T'_* : \mathcal{C}_\Sigma \to \mathcal{C}'$ such that

$$T'_* \cdot T = T'$$

(i.e., the composition '$\cdot$' of T'_* with T yields T').

A *category of additive fractions* is defined by a couple $(T, \mathcal{C}_\Sigma)$ that satisfies all of the above conditions, with T being an `http://planetmath.org/AdditiveFunctor`additive covariant functor. (ref. [**?**]).

Remark 0.1. The above definition of a category of additive fractions is readily generalized to that of a *category of fractions* by relaxing the condition of additivity for the functor T: if T is simply a covariant functor (without being additive) in the above definition one obtains a category of fractions instead. Note, however, that in general the categories of additive fractions are essentially different from the more general categories of fractions, as for example in the particular case of rings of fractions. One also notes that the ring of fractions of a ring relative to a suitable system of elements was perhaps the first reported example of a category of additive fractions. The concepts introduced above are fundamental in the theory of additive fractions.

References

[1] Pages 150 to 180 in Ref. [279] of the `http://planetmath.org/CategoricalOntologyABibliographyOfCa`
for category theory and algebraic topology.