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monomorphic set

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Defines	epimorphic pair

Let $\mathcal{C}$ be a category and $M := \{f_i : A \to B_i \mid i \in I\}$ a set (indexed by a set I) of morphisms with common domain A in $\mathcal{C}$. Then M is said to be a *monomorphic set* if for any pair of morphisms $g, h : C \to A$, $f_i \circ g = f_i \circ h$ for all $i \in I$ imply that $g = h$. A *monomorphic pair* is a monomorphic set M such that the cardinality of M is 2.

Monomorphic sets are generalizations of monomorphisms. Indeed, for if $\{f : A \to B\}$ is a monomorphic set, then f is a monomorphic.

For example, in **Set**, the category of sets, let R be an n-ary relation on a set A. For each $i = 1, \ldots, n$, let p_i be the projection of the i-th coordinate of R into A. Then

$$\{p_i \mid i = 1, \ldots, n\}$$

is a monomorphic set in **Set**. To see this, observe first that, since R is a subset of A^n, any function $f : B \to R$ has n components, $f_i : B \to A$, given by $f_i = p_i \circ f$. Now, suppose $g, h : B \to R$ are functions, such that $p_i \circ g = p_i \circ h$. Then $g_i = h_i$ for all i. In other words, all components of g and h match. Therefore $g = h$.

More generally, a relation R between sets $A_1, \ldots, A_n$ is a subset of the cartesian product $A_1 \times \cdots \times A_n$. The set of projections $\{p_i : R \to A_i \mid i = 1, \ldots, n\}$ is also a monomorphic set in **Set**. Using this concept, one may generalize the notion of a relation on sets to a relation on objects in a category.

Remark. One can dually define *epimorphic sets* and *epimorphic pairs*.

References

[1] F. Borceux *Basic Category Theory, Handbook of Categorical Algebra I*, Cambridge University Press, Cambridge (1994)