

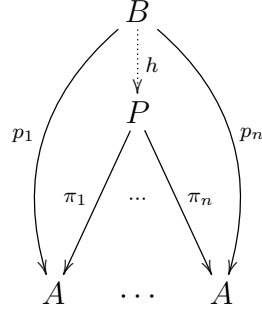


Math for the people, by the people.

power of an object

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Defines	power
Defines	copower

Let  $\mathcal{C}$  be a category and  $A$  an object in  $\mathcal{C}$ . Suppose  $n$  is a non-negative integer. The  $n$ -th power of  $A$  is defined as the direct product of  $A$  with itself  $n$  times. In other words, the  $n$ -th power of  $A$  is an object  $P$  in  $\mathcal{C}$ , together with  $n$  parallel morphisms  $\pi_1, \dots, \pi_n \in \text{hom}(P, A)$ , such that if there are  $n$  parallel morphisms  $p_1, \dots, p_n \in \text{hom}(B, A)$ , then there is a unique morphism  $h : B \rightarrow P$  such that  $\pi_i \circ h = p_i$ , where  $i = 1, \dots, n$ . The commutative diagram below illustrates the situation:



The  $n$ -th power of  $A$  is denoted by  $A^n$ .

Below are some properties of the power of an object in a category:

- Each of the projection morphisms  $\pi_i$  is a split epimorphism.
- $A^1 \cong A$ .
- $A^0$  is a terminal object in  $\mathcal{C}$ .
- $A^{m+n} \cong A^n \times A^m$ , if the product exists.

For example, in the category of sets, the  $n$ -th power of a set  $A$  is the set of  $n$ -tuples where each entry is an element of  $A$ .

**Remark.** The *copower* of an object is defined dually. All of the properties above can be dualized. For example, the 0-th copower of an object is an initial object. The  $n$ -th copower of an object  $A$  in **Set** is the disjoint union of  $n$ -copies of  $A$ .