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category of sets

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The *category of sets* has, as its objects, all sets and, as its morphisms, functions between sets. (This works if a category's objects are only required to be part of a class, as the class of all sets exists.) The category of sets is often denoted by **Set**.

Alternately one can specify a universe, containing all sets of interest in the situation, and take the category to contain only sets in that universe and functions between those sets.

One of the most famous endofunctors associated with the category of sets is the *powerset functor* P, which takes every set A to its power set $P(A)$, and any function $f : A \to B$ to the function $P(f) : P(A) \to P(B)$, given by

$$P(f)(S) := f(S) = \{b \in B \mid b = f(a) \text{ for some } a \in S\}.$$

If $f\colon A \to B$ and $g\colon B \to C$ are functions, then $(P(g) \circ P(f))(S) = P(g)(P(f)(S)) = P(g)(f(S)) = g(f(S)) = (g \circ f)(S) = P(g \circ f)(S)$, so that P is a covariant functor. This functor may also be defined in an "arrow theoretic" fashion as a Hom functor. Let T be a set with two elements, for instance $T = \{\{\}, \{\{\}\}\}$. (Since, by the definition of cardinality, all sets with the same number of elements are isomorphic in the category of sets, it does not matter which set with two elements we pick as T.) Then define $P(A) = \mathrm{Hom}(A, T)$; likewise, given a function $f\colon A \to B$, define $P(f)\colon P(B) \to P(A)$ by $P(f)(g) = g \circ f$.