weak homotopy addition lemma

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Defines	weak homotopy addition
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Weak homotopy addition lemma.
Consider $\boldsymbol{u} : I^3 \to X_{cg}$ to be a singular cube in a compactly-generated space X_{cg}. Then by restricting u to the faces of I^3 and taking the corresponding elements in $\boldsymbol{\rho}_2^{\square}(X_{cg})$, one obtains a cube in $\boldsymbol{\rho}^{\square}(X_{cg})$ which is commutative by an extension of the homotopy addition lemma for $\boldsymbol{\rho}^{\square}(X_{cg})$ ([?], Proposition 5.5) to weak homotopy *in X_{cg}. Consequently, if $f : \boldsymbol{\rho}^{\square}(X_{cg}) \to \mathsf{D}$ is a morphism of double groupoids with connections, any singular cube in X_{cg} determines a commutative 3-shell in D.*

0.1 Related concepts

Definition 0.1 ***Weak homotopy double groupoid***

Let us first define the *weak homotopy double groupoid (WHDG)* of a *compactly–generated space* X_{cg}, (weak Hausdorff space). We utilize here the construction method developed by R. Brown (ref. [?]) for the *homotopy double groupoid of a Hausdorff space*, with the key change in this construction that involves replacing the regular homotopy equivalence relation with the `http://planetmath.org/WeakHomotopyEquivalence`weak homotopy equivalence relation in the definition of the fundamental groupoid, as well as the replacement of the Hausdorff space by the compactly-generated space X_{cg}. Therefore, the weak homotopy data for the *weak homotopy double groupoid* of X_{cg}, $\boldsymbol{\rho}^{\square}(X_{cg})$, will now be:

$$(\boldsymbol{\rho}^{\square_2(X_{cg})}, \boldsymbol{\rho}_1^{\square(X_{cg}),\partial_1^-,\partial_1^+,+_1,\varepsilon_1}),\ (\boldsymbol{\rho}^{\square_2(X_{cg}),\boldsymbol{\rho}_1^{\square}(X_{cg}),\partial_2^-,\partial_2^+,+_2,\varepsilon_2})$$
$$(\boldsymbol{\rho}^{\square_1(X_{cg}),X_{cg},\partial^-,\partial^+,+,\varepsilon}),$$

where the data following the square symbol define how the construction is carried around the square as specifically explained by the concepts of *thin double track, thin equivalence relation and cubically thin homotopy.*

0.2 Path Groupoid

Definition 0.2

We have also introduced above the notation $\boldsymbol{\rho}_1(X_{cg})$ which denotes the *path groupoid* of X_{cg}, analogous to the definition in ref. [?] for a Hausdorff space. The objects of $\boldsymbol{\rho}_1(X_{cg})$ are therefore the points of X_{cg}. The morphisms of $\boldsymbol{\rho}_1^{\square}(X_{cg})$ are the *weak* homotopy equivalence classes of paths in X_{cg} with respect to the *weak* homotopy equivalence, $\sim_W$, defined on X_{cg}.

With these definitions one can now proceed as suggested next to prove the weak homotopy addition lemma with the weak homotopy equivalence defined for compactly–generated topological spaces, X_{cg}.

0.3 Remarks: a brief outline of lemma's consequences

The weak homotopy double groupoid can be expressed–at least in principle– with the help of the weak homotopy addition lemma (WHAL), and the general, categorical form of the higher dimensional, generalized Van Kampen Theorems (HDA-GVKT), by determining (*assuming that it exists!*) the categorical *colimit* of a sequence of fundamental groupoids of the spaces that define a filtering sequence; for example in the case of quantum spaces (QSS), or algebraic QFT (AQFT), of the extended Gel'fand triple $(\Phi_{cg}, \mathbf{H}, [\Phi^*]_{cg})$ representation of the generalized QSS of AQFT and/or *non-Abelian Quantum Gravity* (NAQG) theories, with Φ_{cg} and $[\Phi^*]_{cg}$ being compactly–generated topological spaces.

References

[1] K.A. Hardie, K.H. Kamps and R.W. Kieboom., A homotopy 2–groupoid of a Hausdorff space, *Applied Cat. Structures*, **8** (2000): 209-234.

[2] R. Brown, K.A. Hardie, K.H. Kamps and T. Porter., A homotopy double groupoid of a Hausdorff space., *Theory and Applications of Categories.* **10**,(2002): 71-93.