quotient category, additive

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Defines	dense subcategory
Defines	additive quotient category

0.1 Essential data: Dense subcategory

Definition 0.1. A full subcategory $\mathcal{A}$ of an Abelian category $\mathcal{C}$ is called *dense* if for any exact sequence in $\mathcal{C}$:

$$0 \to X' \to X \to X'' \to 0,$$

X is in $\mathcal{A}$ if and only if both X' and X'' are in $\mathcal{A}$.

Remark 0.1: One can readily prove that if X is an object of the *dense subcategory* $\mathcal{A}$ of $\mathcal{C}$ as defined above, then any subobject X_Q, or quotient object of X, is also in $\mathcal{A}$.

0.1.1 System of morphisms Σ_A

Let $\mathcal{A}$ be a *dense subcategory* (as defined above) of a locally small Abelian category $\mathcal{C}$, and let us denote by Σ_A (or simply only by Σ – when there is no possibility of confusion) the system of all morphisms s of $\mathcal{C}$ such that both $kers$ and $cokers$ are in $\mathcal{A}$. One can then prove that the category of additive fractions $\mathcal{C}_\Sigma$ of $\mathcal{C}$ relative to Σ exists.

Definition 0.2. The *quotient category of* $\mathcal{C}$ *relative to* $\mathcal{A}$, denoted as $\mathcal{C}/\mathcal{A}$, is defined as the category of additive fractions $\mathcal{C}_\Sigma$ relative to a class of morphisms $\Sigma := \Sigma_A$ in $\mathcal{C}$.

Remark 0.2 In view of the restriction to additive fractions in the above definition, it may be more appropriate to call the above category $\mathcal{C}/\mathcal{A}$ an *additive quotient category.* This would be important in order to avoid confusion with the more general notion of `http://planetmath.org/QuotientCategory2`quotient category–which is defined as a category of fractions. Note however that **Remark 0.1** is also applicable in the context of the more general definition of a `http://planetmath.org/QuotientCategory2`quotient category.