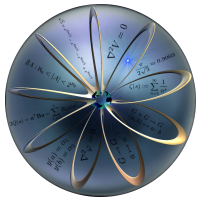

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categorical direct product is an inverse limit

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Theorem 1. *The categorical direct product can be realized as an example of an inverse limit.*

Proof. Suppose we have a direct product of $\{C_i\}_{i \in I}$ for some ($\mathscr{U}$) set I. Consider I as a category whose arrows are only the identity arrows. Then we can define a functor G by $G(i) = C_i$. It is then clear that the universal property of an inverse limit is equivalent to the universal property defining a categorical direct product. □

Reversing the arrows, it is also clear that the categorical direct sum is an example of a direct limit.

These results are of interest when one is looking to prove exactness of sums and products in a category: often it is easier to address exactness of direct and inverse limits, and the result then applies to many other constructions as well.