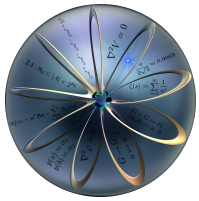

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## equivalence of categories

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Let $C$ and $D$ be two categories with functors $F\colon C \to D$ and $G\colon D \to C$. The functors $F$ and $G$ are an **equivalence of categories** if there are natural isomorphisms $FG \cong \mathrm{id}_D$ and $GF \cong \mathrm{id}_C$.

Note, $F$ is left adjoint to $G$, and $G$ is right adjoint to $F$ as

$$\hom_D(F(c), d) \xrightarrow{G} \hom_C(GF(c), G(d)) \longleftrightarrow \hom_C(c, G(d)).$$

And, $F$ is right adjoint to $G$, and $G$ is left adjoint to $F$ as

$$\hom_C(G(d), c) \xrightarrow{F} \hom_D(FG(d), F(c)) \longleftrightarrow \hom_D(d, F(c)).$$

In practical terms, two categories are equivalent if there is a `http://planetmath.org/FullFunc` faithful functor $F\colon C \to D$, such that every object $d \in D$ is isomorphic to an object $F(c)$, for some $c \in C$.