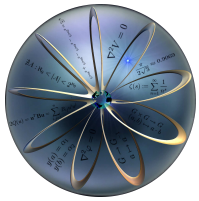

planetmath.org

Math for the people, by the people.

nerve

Canonical name	Nerve
Date of creation	2013-03-22 13:02:57
Last modified on	2013-03-22 13:02:57
Owner	mhale (572)
Last modified by	mhale (572)
Numerical id	8
Author	mhale (572)
Entry type	Definition
Classification	msc 18G30
Related topic	SimplicialCategory

Let **Set** be the category of all sets with functions as the morphisms, and let **Cat** be the category of all small categories with functors as the morphisms.

The **nerve** of a (small) category C is the simplicial set $\hom(i(-), C)$, where $i\colon \Delta \to \mathbf{Cat}$ is the fully faithful functor that takes each ordered set $[n]$ in the simplicial category, Δ, to the pre-order $\mathbf{n+1}$. The nerve is a functor $\mathbf{Cat} \to \mathbf{Set}^{\Delta^{\mathrm{op}}}$.

Example 1 (Nerve of an open covering)
Let X be a topological space with open cover $\{U_\alpha\}$. The nerve of the open covering of X is the nerve of the partially-ordered set $\{U_\alpha\}$ with relation that of inclusion. Thus, it assigns to every n the set of maps from the totally ordered set $n+1$ to the poset $\{U_\alpha\}$.