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chain complex

Canonical name	ChainComplex
Date of creation	2013-03-22 12:09:24
Last modified on	2013-03-22 12:09:24
Owner	yark (2760)
Last modified by	yark (2760)
Numerical id	13
Author	yark (2760)
Entry type	Definition
Classification	msc 18G35
Classification	msc 16E05
Synonym	R-complex
Related topic	HomologyChainComplex
Defines	boundary operator
Defines	boundary map

Let R be a ring. A sequence of <http://planetmath.org/ModuleR>-modules and homomorphisms

$$\cdots \rightarrow A_{n+1} \xrightarrow{d_{n+1}} A_n \xrightarrow{d_n} A_{n-1} \rightarrow \cdots$$

is said to be a *chain complex* (or *R-complex*, or just *complex*) if each pair of adjacent homomorphisms (d_{n+1}, d_n) satisfies the relation $d_n \circ d_{n+1} = 0$. This is equivalent to saying that $\text{im } d_{n+1} \subseteq \ker d_n$. We often denote such a complex by (\mathbf{A}, d) , or simply \mathbf{A} .

Compare this to the notion of an exact sequence, which requires $\text{im } d_{n+1} = \ker d_n$.

The homomorphisms d_n in the chain complex are called *boundary operators*, or *boundary maps*.