reflective subcategory

Canonical name	ReflectiveSubcategory
Date of creation	2013-03-22 17:12:15
Last modified on	2013-03-22 17:12:15
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	4
Author	CWoo (3771)
Entry type	Definition
Classification	msc 18A40
Defines	reflection functor
Defines	reflection
Defines	coreflective
Defines	coreflection

Let $\mathcal{C}$ be a category and $\mathcal{D}$ a subcategory of $\mathcal{C}$. $\mathcal{D}$ is called a *reflective subcategory* of $\mathcal{C}$ if the inclusion functor $\mathrm{Inc} : \mathcal{D} \to \mathcal{C}$ has a left adjoint. More explicitly, $\mathcal{D}$ in $\mathcal{C}$ is reflective iff for every object A in $\mathcal{C}$, there is an object B in $\mathcal{D}$ and a morphism $f : A \to B$ such that any morphism $g : A \to C$ can be uniquely factored through f; that is, there is a unique morphism $h : B \to C$ such that $g = h \circ f$.

The left adjoint is called the *reflection functor* and the mapped objects and morphisms are called the *reflections* (of the objects and morphisms being mapped by the reflection functor).

Some of the most common reflective subcategories are

- The subcategory of abelian groups in the category of groups. The reflection functor is the abelianization functor.
- The subcategory of fields in the category of integral domains. The reflection of an integral domain is its field of fractions.
- The subcategory of complete lattices in the category of lattices. The reflection of a lattice is its lattice of ideals.

Remark. If the inclusion functor has a right adjoint, then the subcategory is said to be *coreflective*. In other words, $\mathcal{D}$ in $\mathcal{C}$ is coreflective iff for any object $A \in \mathcal{C}$, there is an object $B \in \mathcal{D}$ and a morphism $f : B \to A$ such that any morphism $g : C \to A$ can be uniquely factored through f (by a unique morphism $f : C \to B$). For example, the subcategory of torsion abelian groups in the category of abelian groups is coreflective. The coreflection of an abelian group is its torsion subgroup.