Étalé space

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Defines	Etale Space

The Étalé space (Espace Étalé) is a topological space associated to a presheaf $\mathcal{F}$ on a space X. The Étalé space is defined to be the disjoint union of stalks of the sheaf $\mathcal{F}$.

$$\mathcal{E}_{\mathcal{F}} \equiv \coprod_{x \in X} \mathcal{F}_x$$

Over each open set $U \subset X$, there is a set of sections $\Gamma(U, \mathcal{F})$. A basis for the topology on the Étalé space is formed by taking the open sets to be of the form $\mathcal{U}_s = \{s_x, x \in U\}$, for $s \in \Gamma(U, \mathcal{F})$ and s_x the germ of s at x. There is a natural map $\pi : \mathcal{E}_{\mathcal{F}} \to X$ which takes germs s_x in the stalk $\mathcal{F}_x$ over x to x.

Let $s \in \Gamma(U, \mathcal{F})$ and $s' \in \Gamma(U', \mathcal{F})$ with $U \cap U' \neq \emptyset$. At each point $x \in U \cap U'$ where $s_x = s'_x$, by the definition of germs there exists an open set $V \subset U \cap U'$ containing x such that s and s' restrict to the same section on V ($s|_V = s'|_V$). This verifies that $\{\mathcal{U}_s\}$ form a basis for $\mathcal{E}_{\mathcal{F}}$.

Then there is another presheaf, $\widetilde{\mathcal{F}}$, whose sections are the continuous functions from X to $\mathcal{E}_{\mathcal{F}}$ assigning an element $s(x) \in \mathcal{F}_x$ to each point $x \in X$. This presheaf forms a sheaf equivalent to the sheafification of the presheaf $\mathcal{F}$.