



**planetmath.org**

Math for the people, by the people.

**cohomology of a cochain complex**

Canonical name	CohomologyOfACochainComplex
Date of creation	2013-03-22 19:03:46
Last modified on	2013-03-22 19:03:46
Owner	rm50 (10146)
Last modified by	rm50 (10146)
Numerical id	6
Author	rm50 (10146)
Entry type	Definition
Classification	msc 18G35
Related topic	Tor

If  $(\mathcal{A}, d)$  is a <http://planetmath.org/CochainComplex2cochain> complex

$$\dots \xrightarrow{d_{n-1}} A^{n-1} \xrightarrow{d_n} A^n \xrightarrow{d_{n+1}} A^{n+1} \xrightarrow{d_{n+2}} \dots$$

then the  $n^{\text{th}}$  *cohomology group* (or *cohomology module*)  $H^n(\mathcal{A}, d)$  of  $(\mathcal{A}, d)$  is the quotient module

$$H^n(\mathcal{A}, d) = \frac{\ker d_{n+1}}{\text{im } d_n}.$$

The cochain complex is an <http://planetmath.org/ExactSequenceexact> sequence if and only if all of the cohomology groups are trivial. The cohomology groups can therefore be thought of as measuring the extent to which the cochain complex fails to be exact.

Cohomology groups of other objects are defined as the cohomology groups of an associated cochain complex. (For example, see the entry on the <http://planetmath.org/Simp> of simplicial complexes.)

[Compare this entry with the entry on homology of a chain complex.]