proof of Yoneda lemma

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We give a proof of Yoneda's Lemma. Thus, we have to show that $\mathcal{C} \to \hat{\mathcal{C}}$ is a faithful functor. Let X and Y be two objects belonging to $\mathcal{C}$, we want to show that

$\psi : \mathrm{Hom}(X, Y) \to \mathrm{Hom}(X(.), Y(.))$

$f \mapsto (f_K : X(K) \to Y(K))_K$ is bijective.

Let's start with injectivity. Let f and g be two morphisms from X to Y which are having the same mappings for the points $f_K = g_K$ for all K. Let's show that $f = g$. What happens for the X-points? For the X-points, we have $f = g$ and the range of f_X and of g_X of the X-point of X which is Id_X is exactly the X-points of Y which are f and g. Hence $f = g$.

Now for surjectivity: let $\psi : X(.) \to Y(.)$ a morphism of functors. We need to show that this morphism comes from an arrow f which should be the range of Id_X by the map ϕ_X. Thus, let $f = \psi_X(Id_X)$. Let's verify that $f_K = \psi_K$ for all K. Let $p : K \to X$ be a K-point of X. p is a morphism between the two types of points K and X and in this case we have the following commutative diagram:

$I don't know how to do diagrams with latex, it's too hard$

If you make $Id(X)$ turn in the diagram one verifies that $\psi_K(p) = f \circ p = f_K(p)$ which proves the surjectivity.