dual category

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Let $\mathcal{C}$ be a category. The *dual category* $\mathcal{C}^*$ of $\mathcal{C}$ is the category which has the same objects as $\mathcal{C}$, but in which all morphisms are "reversed". That is to say if A, B are objects of $\mathcal{C}$ and we have a morphism $f : A \to B$, then we formally define an arrow $f^* : B \to A$ in $\mathcal{C}^*$. f^* is called the *opposite arrow*, or *opposite morphism* of f. The composition $f^* \circ g^*$ is then defined to be $(g \circ f)^*$. The dual category is sometimes called the *opposite category* and is denoted $\mathcal{C}^{\mathrm{op}}$.

The category of Hopf algebras over a field k is (equivalent to) the opposite category of affine group schemes over $\operatorname{spec} k$.

Categorical properties of $\mathcal{C}$ lead directly to categorical properties of $\mathcal{C}^{\mathrm{op}}$; constructions on $\mathcal{C}$ become constructions on $\mathcal{C}^{\mathrm{op}}$. Usually such a construction is indicated with the prefix "co-". For example, a coproduct is a product on the opposite category; this can be seen by looking at the commutative diagram that completely specifies a coproduct, and noting that it is the same as the diagram specifying a product with the arrows reversed. More generally, an inverse limit is a direct limit on the opposite category; for this reason, it is sometimes called a colimit. A cokernel is a kernel in the opposite category. Many other similar concepts exist.

If F is a covariant functor from $\mathcal{C}$ to some other category $\mathcal{D}$, then we can define, in a natural way, a contravariant functor F^{op} from C^{op} to D, called the *opposite functor* of F. In fact, this is often how contravariant functors are defined, and it is why most categorical theorems and constructions need not explicitly consider both cases.