2-C*-category

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Defines	identity map for a 2-category
Defines	commutative monoid
Defines	$End(\rho)$

Definition 0.1. A $2 - C^*$ *-category*, $\mathcal{C^*}_2$, is defined as a (small) 2-category for which the following conditions hold:

1. for each pair of 1-arrows (ρ, σ) the space $Hom(\rho, \sigma)$ is a complex Banach space.
2. there is an anti-linear involution '$*$' acting on 2-arrows, that is,
$$* : Hom(\rho, \sigma) \to Hom(\rho, \sigma),$$
($S \mapsto S^*$) with ρ and σ being 2-arrows;
3. the Banach norm is sub-multiplicative (that is,
$$\|T \circ S\| \leq \|S\| \, \|T\| \,,$$
when the composition is defined, and satisfies the C^* -condition:
$$\|S^* \circ S\| = \left\|S^2\right\| ;$$
4. for any 2-arrow $S \in Hom(\rho, \sigma)$, $S^* \circ S$ is a positive element in $Hom(\rho, \rho)$, that is often denoted as $End(\rho)$.

Remark 0.1. With the above notations, the set of 2-arrows $End(\iota A)$ is a commutative monoid, with the identity map $\iota : {\mathcal{C^*}_2}^0 \to {\mathcal{C^*}_2}^1$ assigning to each object $A \in {\mathcal{C^*}_2}^0$ a 1-arrow ιA such that:
$$s(\iota A) = t(\iota A) = A.$$