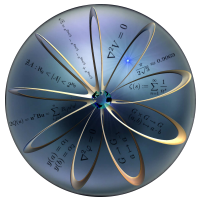

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## homology of a chain complex

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If $(\mathbf{A}, d)$ is a chain complex

$$\cdots \xleftarrow{d_{n-1}} A_{n-1} \xleftarrow{d_n} A_n \xleftarrow{d_{n+1}} A_{n+1} \xleftarrow{d_{n+2}} \cdots$$

then the $n$-th *homology group* (or *homology module*) $H_n(\mathbf{A}, d)$ of $(\mathbf{A}, d)$ is the quotient module

$$H_n(\mathbf{A}, d) = \frac{\ker d_n}{\operatorname{im} d_{n+1}}.$$

The chain complex is an `http://planetmath.org/ExactSequence`exact sequence if and only if all of the homology groups are trivial. The homology groups can therefore be thought of as measuring the extent to which the chain complex fails to be exact.

Homology groups of other objects are defined as the homology groups of an associated chain complex. (In particular, see the entry on the `http://planetmath.org/Homolog` of topological spaces.)