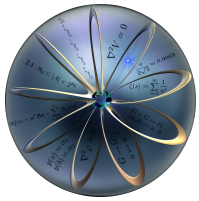

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essentially surjective

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Let $\mathcal{C}$ and $\mathcal{D}$ be categories. A functor $F\colon \mathcal{C} \to \mathcal{D}$ is *essentially surjective* if for any object $A \in \mathcal{OB}(\mathcal{D})$, there exists an object $X \in \mathcal{OB}(\mathcal{C})$, such that $F(X) \cong A$. That is, there are morphisms (in D) $f\colon F(X) \to A$ and $g\colon A \to F(X)$ such that $fg = 1_A$ and $gf = 1_{F(X)}$.

Remarks.

- Clearly, if F is surjective, it is essentially surjective. But the reverse is not true.
- A functor is an `http://planetmath.org/EquivalenceOfCategories`equivalence iff it is `http://planetmath.org/FullFunctor`full, `http://planetmath.org/FaithfulFunct` and essentially surjective.
- **isomorphism-dense subcategory**. A full subcategory $\mathcal{S}$ of a category $\mathcal{C}$ is said to be *isomorphism-dense in* $\mathcal{C}$, if the inclusion functor $\mathcal{S} \hookrightarrow \mathcal{C}$ is essentially surjective. Since $\mathcal{S}$ is full, the inclusion functor is full and faithful. As a result, $\mathcal{S}$ is isomorphism-dense if the inclusion functor is an equivalence.