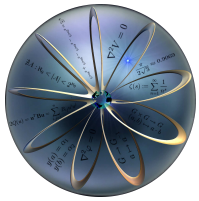

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natural equivalence

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Let $F, G : \mathcal{C} \to \mathcal{D}$ be a pair of functors from the category $\mathcal{C}$ to the category $\mathcal{D}$. A natural transformation between functors $\tau : F \to G$ is called a *natural equivalence* (or a *natural isomorphism*) if there is a natural transformation $\sigma : G \to F$ such that $\tau \bullet \sigma = \mathrm{id}_G$ and $\sigma \bullet \tau = \mathrm{id}_F$ where id_F is the identity natural transformation on F, and composition $\bullet$ is the usual (vertical) composition on natural transformations.

Equivalently, one can define a natural equivalence from functors F to G to be a natural transformation τ such that for each object A in $\mathcal{C}$, the morphism $\tau_A : F(A) \to G(A)$ is an isomorphism in $\mathcal{D}$.