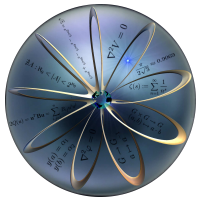

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Yoneda lemma

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If $\mathcal{C}$ is a category, write $\hat{\mathcal{C}}$ for the category of contravariant functors from $\mathcal{C}$ to **Sets**, the category of sets. The morphisms in $\hat{\mathcal{C}}$ are natural transformations of functors.

(To avoid set theoretical concerns, one can take a universe $\mathcal{U}$ and take all categories to be $\mathcal{U}$-small.)

For any object X of $\mathcal{C}$, $h_X = \mathrm{Hom}(-, X)$ is a contravariant functor from $\mathcal{C}$ to **Sets**, and therefore is an object of $\hat{\mathcal{C}}$.

Yoneda Lemma says that $X \mapsto h_X$ is a covariant functor $\mathcal{C} \to \hat{\mathcal{C}}$, which embeds $\mathcal{C}$ faithfully as a full subcategory of $\hat{\mathcal{C}}$. This embedding is called the *Yoneda embedding*.