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locally finite category

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A *locally finite category* is a category \mathcal{C} such that each <http://planetmath.org/Categoryarrow> can be written as a composition of non-identity arrows in only finitely many ways. To get a picture of what this means, view \mathcal{C} as a graph, where the nodes are objects and the directed edges are all the arrows besides the identity arrows. Note that loops besides identity arrows are permitted. Then a decomposition of an arrow $f: A \rightarrow B$ is a directed path from A to B in the graph. So the statement that each arrow can be decomposed in only finitely many ways essentially means that there are only finitely many directed paths from A to B in the graph. This analogy only breaks down in the case where A and B are the same object.

The notion of a locally finite category generalizes the notion of a locally finite poset. The condition that each interval $[x, z]$ in a poset P contain only finitely many elements y of P implies that a locally finite poset is locally finite when viewed as a category, since between two objects of P there is at most one arrow.

If \mathcal{C} is a locally finite category, then an <http://planetmath.org/AlgebraFormedFromACategory> over the category arises as the dual of a coalgebra over the category.

References

- [1] S. A. Joni and G.-C. Rota, *Coalgebras and bialgebras in combinatorics*, Stud. Appl. Math., 61 (1979), pp. 93–139.