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splicing together exact sequences

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This article proves a simple but very useful result about "splicing" together two exact sequences. Assume we are working in an abelian category such as groups, rings, or modules.

Proposition 1. *Let*

$$A \to B \xrightarrow{f} C$$

and

$$D \xrightarrow{g} E \to F$$

be exact, and assume that there is an isomorphism $\varphi : \operatorname{coker} f \to \ker g$. *Define* $\psi : C \to D : c \mapsto \varphi(\bar{c})$, *where* $\bar{c}$ *is the image of* c *in* $\operatorname{coker} f$. *Then the following is exact:*

$$A \to B \xrightarrow{f} C \xrightarrow{\psi} D \xrightarrow{g} E \to F$$

Proof. Exactness at C:

$$c \in \ker \psi \iff \psi(c) = \varphi(\bar{c}) = 0 \iff \bar{c} = 0 \iff c \in \operatorname{im} f.$$

Exactness at D:

$$d \in \ker g \iff d = \varphi(\bar{c}) \text{ for some } c \in C \iff d = \psi(c) \text{ for some } c \in C.$$

□