thin equivalence relation

Canonical name	ThinEquivalenceRelation
Date of creation	2013-03-22 18:15:04
Last modified on	2013-03-22 18:15:04
Owner	bci1 (20947)
Last modified by	bci1 (20947)
Numerical id	11
Author	bci1 (20947)
Entry type	Definition
Classification	msc 18D05
Classification	msc 55N33
Classification	msc 55N20
Classification	msc 55U40
Synonym	homotopy equivalence
Related topic	HomotopyAdditionLemma
Related topic	HomotopyDoubleGroupoidOfAHausdorffSpace
Related topic	WeakHomotopyAdditionLemma
Related topic	FEquivalenceInCategory
Defines	thin equivalence
Defines	semitrack

1 Thin equivalence relation

Definition 1.1. Let $a, a' : x \simeq y$ be paths in X. Then a is *thinly equivalent* to a', denoted $a \sim_T a'$, if there is a thin relative homotopy between a and a'.

We note that $\sim_T$ is an equivalence relation, see [**?**]. We use $\langle a \rangle : x \simeq y$ to denote the $\sim_T$ class of a path $a : x \simeq y$ and call $\langle a \rangle$ the *semitrack* of a. The groupoid structure of $\boldsymbol{\rho}_1^{\square}(X)$ is induced by concatenation, $+$, of paths. Here one makes use of the fact that if $a : x \simeq x'$, $a' : x' \simeq x''$, $a'' : x'' \simeq x'''$ are paths then there are canonical thin relative homotopies

$$\begin{aligned} (a + a') + a'' \simeq a + (a' + a'') : x \simeq x''' \ (\textit{rescale}) \\ a + e_{x'} \simeq a : x \simeq x';\ e_x + a \simeq a : x \simeq x' \ (\textit{dilation}) \\ a + (-a) \simeq e_x : x \simeq x \ (\textit{cancellation}). \end{aligned}$$

The source and target maps of $\boldsymbol{\rho}_1^{\square}(X)$ are given by

$$\partial_1^- \langle a \rangle = x, \ \ \partial_1^+ \langle a \rangle = y,$$

if $\langle a \rangle : x \simeq y$ is a semitrack. Identities and inverses are given by

$$\varepsilon(x) = \langle e_x \rangle \quad \text{resp.} - \langle a \rangle = \langle -a \rangle.$$

References

[1] K.A. Hardie, K.H. Kamps and R.W. Kieboom, A homotopy 2-groupoid of a Hausdorff space, *Applied Cat. Structures*, **8** (2000): 209-234.

[2] R. Brown, K.A. Hardie, K.H. Kamps and T. Porter, A homotopy double groupoid of a Hausdorff space, *Theory and Applications of Categories* **10**,(2002): 71-93.