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topos axioms

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Definition 0.1. The two axioms that define an *elementary topos*, or a *standard topos*, as a special category τ are:

- **i.** τ has finite limits
- **ii.** τ has power objects $\Omega(A)$ for objects A in τ .

To complete the axiomatic definition of topoi, one needs to add the ETAC axioms which allow one to define a category as an interpretation of ETAC. The above axioms imply that any topos has finite colimits, a subobject classifier (such as a Heyting logic algebra), as well as several other properties.

Alternative definitions of topoi have also been proposed, such as:

Definition 0.2. A *topos* is a category τ subject to the following axioms:

- \mathbb{T}_1 . τ is cartesian closed
- \mathbb{T}_2 . τ has a subobject classifier.

One can readily show that axioms *i.* and *ii.* also imply axioms \mathbb{T}_1 and \mathbb{T}_2 ; one notes that property \mathbb{T}_2 can also be expressed as the existence of a representable subobject functor.