power of an object

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Defines	power
Defines	copower

Let $\mathcal{C}$ be a category and A an object in $\mathcal{C}$. Suppose n is a non-negative integer. The *n-th power* of A is defined as the direct product of A with itself n times. In other words, the *n-th power* of A is an object P in $\mathcal{C}$, together with n parallel morphisms $\pi_1, \ldots, \pi_n \in \hom(P, A)$, such that if there are n parallel morphisms $p_1, \ldots, p_n \in \hom(B, A)$, then there is a unique morphism $h : B \to P$ such that $\pi_i \circ h = p_i$, where $i = 1, \ldots, n$. The commutative diagram below illustrates the situation:

$$\begin{tikzcd}
& B \arrow[d, dotted, "h"] \arrow[ddl, bend right, "p_1"'] \arrow[ddr, bend left, "p_n"] & \\
& P \arrow[dl, "\pi_1"'] \arrow[dr, "\pi_n"] \arrow[d, phantom, "\cdots"] & \\
A & \cdots & A
\end{tikzcd}$$

The n-th power of A is denoted by A^n.

Below are some properties of the power of an object in a category:

- Each of the projection morphisms π_i is a split epimorphism.
- $A^1 \cong A$.
- A^0 is a terminal object in $\mathcal{C}$.
- $A^{m+n} \cong A^n \times A^m$, if the product exists.

For example, in the category of sets, the n-th power of a set A is the set of n-tuples where each entry is an element of A.

Remark. The *copower* of an object is defined dually. All of the properties above can be dualized. For example, the 0-th copower of an object is an initial object. The n-th copower of an object A in **Set** is the disjoint union of n-copies of A.