topological groupoid

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Defines	groupoid
Defines	transitive groupoid
Defines	principal groupoid
Defines	isotropy group
Defines	topological groupoid
Defines	domain map
Defines	range map
Defines	unit space
Defines	isotropy group

A *groupoid* is a set G together with a subset $G_2 \subset G^2$ of composable pairs, a *multiplication* $\mu : G_2 \to G : (a, b) \mapsto ab$ and an *inversion* $\cdot^{-1} : G \to G : a \mapsto a^{-1}$ such that

1. $\cdot^{-1} \circ \cdot^{-1} = \mathrm{id}_G$,
2. if $\{(a, b), (b, c)\} \subset G_2$ then $\{(ab, c), (a, bc)\} \subset G_2$ and $(ab)c = a(bc)$,
3. $(b, b^{-1}) \in G_2$ and if $(a, b) \in G_2$ then $abb^{-1} = a$ and
4. $(b^{-1}, b) \in G_2$ and if $(b, c) \in G_2$ then $b^{-1}bc = c$.

Furthermore we have the *source* or *domain map* $\sigma : G \to G : a \mapsto a^{-1}a$ and the *target* or *range map* $\tau : G \to G : a \mapsto aa^{-1}$. The image of these maps is called the *unit space* and denoted G_0. If the unit space is a singleton than we regain the notion of a group.

We also define $G_a := \sigma^{-1}(\{a\})$, $G^b := \tau^{-1}(\{b\})$ and $G^b_a := G_a \cap G^b$. It is not hard to see that G^a_a is a group, which is called the *isotropy group* at a.

We say that a groupoid G is *principal* and *transitive*, if the map $(\sigma, \tau) : G \to G_0 \times G_0$ is injective and surjective, respectively.

A groupoid can be more abstractly and more succinctly defined as a category whose morphisms are all isomorphisms.

A *topological groupoid* is a groupoid G which is also a topological space, such that the multiplication and inversion are continuous when G_2 is endowed with the induced product topology from G^2. Consequently also σ and τ are continuous.

References

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[2] R. Brown, *Topology and groupoids*, xxv+512pp, Booksurge 2006.

[3] R. Brown, 'Three themes in the work of Charles Ehresmann: Local-to-global; Groupoids; Higher dimensions', *Proceedings of the 7th Conference on the Geometry and Topology of Manifolds: The Mathematical Legacy of Charles Ehresmann, Bedlewo (Poland) 8.05.2005-15.05.2005*, Banach Centre Publications 76, Institute of Mathematics Polish Academy of Sciences, Warsaw, (2007) 51-63. (math.DG/0602499).