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duality principle

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Defines	self-dual statement

Let Σ be any statement of the elementary theory of an abstract category. We form the dual of Σ as follows:

1. Replace each occurrence of "domain" in Σ with "codomain" and conversely.

2. Replace each occurrence of $g \circ f = h$ with $f \circ g = h$

Informally, these conditions state that the dual of a statement is formed by reversing arrows and compositions. For example, consider the following statements about a category $\mathcal{C}$:

- $f : A \to B$

- f is monic, i.e. for all morphisms g, h for which composition makes sense, $f \circ g = f \circ h$ implies $g = h$.

The respective dual statements are

- $f : B \to A$

- f is epi, i.e. for all morphisms g, h for which composition makes sense, $g \circ f = h \circ f$ implies $g = h$.

The *duality principle* asserts that if a statement is a theorem, then the dual statment is also a theorem. We take "theorem" here to mean provable from the axioms of the elementary theory of an abstract category. In practice, for a valid statement about a particular category $\mathcal{C}$, the dual statement is valid in the dual category $\mathcal{C}^*$ ($\mathcal{C}^{op}$).

If the property Σ is the same as its dual, then it is called *self-dual*.