disjoint union of categories

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Let $\{\mathcal{C}_i\}$ be a collection of categories, indexed by a set I. The $\mathcal{C}$ of these categories is defined as follows:

1. the class of objects of $\mathcal{C}$ is the disjoint union of classes of objects, $\mathrm{Ob}(\mathcal{C}_i)$, for every $i \in I$,

2. the class of morphisms of $\mathcal{C}$ is the disjoint union of classes of morphisms, $\mathrm{Mor}(\mathcal{C}_i)$, for every $i \in I$.

3. for objects A, B in $\mathcal{C}$, if they are objects of $\mathcal{C}_i$, then $\hom(A, B)$ is the set of morphisms from A to B in $\mathcal{C}_i$, otherwise, $\hom(A, B) := \varnothing$.

4. given $\hom(A, B)$ and $\hom(B, C)$, the composition of morphisms is defined so that, if A, B, C are all objects of some $\mathcal{C}_i$, the composition is the same as the composition of morphisms defined in $\mathcal{C}_i$. Otherwise, it is defined as $\varnothing$.

With the above conditions, one immediately sees that $\mathcal{C}$ is a category, as each $\hom(A, B)$ is a set, associativity of morphism composition and identity morphisms all inherit from the individual categories $\mathcal{C}_i$.

Remark. If each $\mathcal{C}_i$ is small, so is their disjoint union. In fact, in **Cat**, the category of small categories, the disjoint union of these categories is their coproduct.