# Čech cohomology group

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Let $\mathcal{F}$ be a sheaf of abelian groups on a topological space $X$ and consider an $\mathcal{U} = \{U_i\}_{i \in I}$ of $X$. For the sake of simplicity denote

$$U_{i_0 i_1 \cdots i_q} = U_{i_0} \cap U_{i_1} \cap \cdots \cap U_{i_q}.$$

The group $\check{C}^q(\mathcal{U}, \mathcal{F})$ of Čech $q$-cochains is the set of families

$$c = (c_{i_0 i_1 \cdots i_q}) \in \prod_{(i_0, \ldots, i_q) \in I^{q+1}} \mathcal{F}(U_{i_0 i_1 \cdots i_q}).$$

The group on $\check{C}^q(\mathcal{U}, \mathcal{F})$ is the obvious one deduced from the addition law on sections of $\mathcal{F}$.

The Čech differential

$$\delta^q \colon \check{C}^q(\mathcal{U}, \mathcal{F}) \to \check{C}^{q+1}(\mathcal{U}, \mathcal{F})$$

is defined by the

$$(\delta^q c)_{i_0 \cdots i_{q+1}} = \sum_{0 \leq j \leq q+1} (-1)^j c_{i_0 \cdots \widehat{i_j} \cdots i_{q+1}}|_{U_{i_0 \cdots i_{q+1}}},$$

and we set $\check{C}^q(\mathcal{U}, \mathcal{F}) = 0$, $\delta^q = 0$ for $q < 0$. Easy computations show that $\delta^{q+1} \circ \delta^q = 0$. We get therefore a cochain complex $(\check{C}^\bullet(\mathcal{U}, \mathcal{F}), \delta)$, called the complex of Čech cochains relative to the $\mathcal{U}$.

The $q$-th *Čech cohomology group* of $\mathcal{F}$ relative to $\mathcal{U}$ is

$$\check{H}^q(\mathcal{U}, \mathcal{F}) = H^q(\check{C}^\bullet(\mathcal{U}, \mathcal{F})).$$