# complete category

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| Entry type | Definition |
| Classification | msc 18A35 |
| Synonym | finitely complete |
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| Related topic | ExponentialObject |
| Related topic | CartesianClosedCategory |
| Defines | finitely complete category |
| Defines | cocomplete category |
| Defines | finitely cocomplete category |

A category $\mathcal{C}$ is said to be a *complete category* if every small diagram has a limit, that is, a limiting cone exists over every small diagram (diagram such that collections of objects and morphisms are sets).

Of course, in a complete category, a product exists for any given set of objects. Also, a set of morphisms with common domain and codomain has an equalizer. Conversely, we have

> in a category $\mathcal{C}$, if the product exists for an arbitrary set of objects, and the equalizer exists for any pair of morphisms with common domain and codomain, then $\mathcal{C}$ is complete.

**Examples**

- **Set** is complete.
- **Group** is complete.
- **Vector Space** is complete
- **R-module** is complete for a given unital ring $R$.
- **Topological Space** is complete.

A category $\mathcal{C}$ is said to be *finitely complete* if every finite diagram (sets of objects and morphisms are finite) has a limit.

A similar sufficient condition for a category $\mathcal{C}$ to be finitely complete is for $\mathcal{C}$ to possess a terminal object and that a pullback exists for every pair of morphisms with common codomain.

**Examples**

- Any complete category is clearly finitely complete.
- The subcategories of the above examples consisting of all objects with finite cardinality are finitely complete (but not complete).

**Remark**. The dual notion of a complete category is that of a *cocomplete category*, and the dual of a finitely complete category is called a *finitely cocomplete category.*