



planetmath.org

Math for the people, by the people.

category of C\*-algebras

Canonical name	CategoryOfCalgebras
Date of creation	2013-03-22 18:24:13
Last modified on	2013-03-22 18:24:13
Owner	bci1 (20947)
Last modified by	bci1 (20947)
Numerical id	23
Author	bci1 (20947)
Entry type	Definition
Classification	msc 18E05
Classification	msc 46L05
Classification	msc 18-00
Synonym	C*-algebra category
Synonym	category of $C^*$ -algebras
Related topic	CAlgebra3
Related topic	Category
Related topic	QuantumGroup
Related topic	QuantumGroups
Related topic	2CCategory
Related topic	IndexOfCategories
Defines	*-homomorphism
Defines	$C^*$ -algebra category
Defines	*-convolution

**Definition 0.1.** Let  $\mathcal{A}, \mathcal{B}$  be two [http://planetmath.org/CAgebra3C\\*-algebras](http://planetmath.org/CAgebra3C*-algebras). Then a *\*-homomorphism*  $\phi_* : \mathcal{A} \longrightarrow \mathcal{B}$  is defined as a  $C^*$ -algebra homomorphism  $\phi : \mathcal{A} \rightarrow \mathcal{B}$  which respects involutions, that is:

$$\phi(a^{*\mathcal{A}}) = \phi(a)^{*\mathcal{B}}, \quad \text{for any } a \in \mathcal{A}.$$

**Remark 0.1.** If ‘by abuse of notation’ one uses  $*$  to denote both  $*_{\mathcal{A}}$  and  $*_{\mathcal{B}}$ , then any  $*$ -homomorphism  $\phi$  commutes with  $*$ , i.e.,  $\phi* = *\phi$ . Homomorphisms between  $C^*$ -algebras are <http://planetmath.org/ContinuousLinearMapping> automatically continuous.

**Definition 0.2.** The category  $\mathcal{C}$  whose objects are  $C^*$ -algebras and whose morphisms are  $*$ -homomorphisms is called the *category of  $C^*$ -algebras* or the  *$C^*$ -algebra category*.

## References

- [1] Kustermans, J.,  $C^*$ -algebraic Quantum Groups arising from Algebraic Quantum Groups, Ph.D. Thesis, K.U.Leuven, 1997.
- [2] Sheu, A.J.L., Compact Quantum Groups and Groupoid  $C^*$ -Algebras, J. Funct. Analysis 144 (1997), 371-393.