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thin square

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Let us consider first the concept of a *tree* that enters in the definition of a thin square. Thus, a simplified notion of thin square is that of “a continuous map from the unit square of the real plane into a Hausdorff space  $X_H$  which factors through a tree” ([?]).

**Definition 0.1.** A *tree*, is defined here as the underlying space  $|K|$  of a finite 1-connected 1-dimensional simplicial complex  $K$  and boundary  $\partial I^2$  of  $I^2 = I \times I$  (that is, a *square* (interval) defined here as the Cartesian product of the unit interval  $I := [0, 1]$  of real numbers).

**Definition 0.2.** A *square map*  $u : I^2 \longrightarrow X$  in a topological space  $X$  is *thin* if there is a factorisation of  $u$ ,

$$u : I^2 \xrightarrow{\Phi_u} J_u \xrightarrow{p_u} X,$$

where  $J_u$  is a *tree* and  $\Phi_u$  is <http://planetmath.org/GeometricallyAndorAlgebraicallyThinSquarespiecewiseLinear> (PWL) on the boundary  $\partial I^2$  of  $I^2$ .

## References

- [1] R. Brown, K.A. Hardie, K.H. Kamps and T. Porter., <http://www.tac.mta.ca/tac/volumes/10/2/10-02.pdf> A homotopy double groupoid of a Hausdorff space , *Theory and Applications of Categories* **10**,(2002): 71-93.
- [2] R. Brown and C.B. Spencer: Double groupoids and crossed modules, *Cahiers Top. Géom.Diff.*, **17** (1976), 343–362.
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- [4] K.A. Hardie, K.H. Kamps and R.W. Kieboom., A homotopy 2-groupoid of a Hausdorff *Applied Categorical Structures*, **8** (2000): 209-234.
- [5] Al-Agl, F.A., Brown, R. and R. Steiner: 2002, Multiple categories: the equivalence of a globular and cubical approach, *Adv. in Math.*, **170**: 711-118.