supplemental axioms for an Abelian category

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Defines	complete
Defines	cocomplete

These are axioms introduced by Alexandre Grothendieck for an Abelian category. The first two are satisfied by definition in an Abelian category, and others may or may not be.

(Ab1) Every morphism has a kernel and a cokernel.

(Ab2) Every monic is the kernel of its cokernel.

(Ab3) Coproducts exist. (Coproducts are also called direct sums.) If this axiom is satisfied the category is often just called cocomplete.

(Ab3*) Products exist. If this axiom is satisfied the category is often just called complete.

(Ab4) Coproducts exist and the coproduct of monics is a monic.

(Ab4*) Products exist and the product of epics is an epic.

(Ab5) Coproducts exist and filtered colimits of exact sequences are exact.

(Ab5*) Products exist and filtered inverse limits of exact sequences are exact.

Grothendieck introduced these in his homological algebra paper *Sur quelques points d'algèbre homologique* in the Tôhoku Math Journal (number 2, volume 9, 1957). They can also be found in Weibel's excellent book *An introduction to homological algebra*, Cambridge Studies in Advanced Mathematics (Cambridge University Press, 1994).