algebra formed from a category

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Given a category $\mathcal{C}$ and a ring R, one can construct an algebra $\mathcal{A}$ as follows. Let $\mathcal{A}$ be the set of all formal finite linear combinations of the form

$$\sum_i c_i e_{a_i,b_i,\mu_i},$$

where the coefficients c_i lie in R and, to every pair of objects a and b of $\mathcal{C}$ and every morphism μ from a to b, there corresponds a basis element $e_{a,b,\mu}$. Addition and scalar multiplication are defined in the usual way. Multiplication of elements of $\mathcal{A}$ may be defined by specifying how to multiply basis elements. If $b \neq c$, then set $e_{a,b,\phi} \cdot e_{c,d,\psi} = 0$; otherwise set $e_{a,b,\phi} \cdot e_{b,c,\psi} = e_{a,c,\psi\circ\phi}$. Because of the associativity of composition of morphisms, $\mathcal{A}$ will be an associative algebra over R.

Two instances of this construction are worth noting. If G is a group, we may regard G as a category with one object. Then this construction gives us the group algebra of G. If P is a partially ordered set, we may view P as a category with at most one morphism between any two objects. Then this construction provides us with the incidence algebra of P.