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K-homology

Canonical name	Khomology
Date of creation	2013-03-22 12:57:46
Last modified on	2013-03-22 12:57:46
Owner	mhale (572)
Last modified by	mhale (572)
Numerical id	6
Author	mhale (572)
Entry type	Topic
Classification	msc 19K33
Related topic	FredholmModule
Related topic	KTheory

K-homology is a homology theory on the category of compact Hausdorff spaces. It classifies the elliptic pseudo-differential operators acting on the vector bundles over a space. In terms of C^* -algebras, it classifies the Fredholm modules over an algebra.

An operator homotopy between two Fredholm modules $(\mathcal{H}, F_0, \Gamma)$ and $(\mathcal{H}, F_1, \Gamma)$ is a norm continuous path of Fredholm modules, $t \mapsto (\mathcal{H}, F_t, \Gamma)$, $t \in [0, 1]$. Two Fredholm modules are then equivalent if they are related by unitary transformations or operator homotopies. The $K^0(A)$ group is the abelian group of equivalence classes of even Fredholm modules over A . The $K^1(A)$ group is the abelian group of equivalence classes of odd Fredholm modules over A . Addition is given by direct summation of Fredholm modules, and the inverse of (\mathcal{H}, F, Γ) is $(\mathcal{H}, -F, -\Gamma)$.

References

- [1] N. Higson and J. Roe, *Analytic K-homology*. Oxford University Press, 2000.