stable isomorphism

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Defines	stably isomorphic
Defines	stably free

Let R be a ring with unity 1. Two left R-modules M and N are said to be *stably isomorphic* if there exists a finitely generated free R-module R^n ($n \geq 1$) such that

$$M \oplus R^n \cong N \oplus R^n.$$

A left R-module is said to be *stably free* if it is stably isomorphic to a finitely generated free R-module. In other words, M is stably free if

$$M \oplus R^m \cong R^n$$

for some positive integers m, n.

Remark In the Grothendieck group $K_0(R)$ of a ring R with 1, two finitely generated projective module representatives M and N such that $[M] = [N] \in K_0(R)$ iff they are stably isomorphic to each other. In particular, $[M]$ is the zero element in $K_0(R)$ iff it is stably free.