center of a Hausdorff topological group is closed

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Theorem - Let G be a Hausdorff topological group. Then the center of G is a closed normal subgroup.

Proof: Let Z be the center of G. We know that Z is a normal subgroup of G. Let us see that it is closed.

Let $s \in \overline{Z}$, the closure of Z. There exists a net $\{s_\lambda\}$ in Z converging to s. Then, for every $g \in G$, we have that

- $gs_\lambda \longrightarrow gs$

- $s_\lambda g \longrightarrow sg$

But since Z is the center of G we have that $gs_\lambda = s_\lambda g$, and as G is Hausdorff one must have $sg = gs$. This implies that $s \in Z$, i.e. Z is closed. $\square$