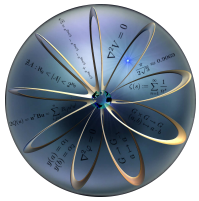

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basic results in topological groups

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The purpose of this entry is to list some and useful results concerning the topological of topological groups. We will use the following notation whenever A, B are subsets of a topological group G and r an element of G:

- $Ar := \{ar : a \in A\}$
- $rA := \{ra : a \in A\}$
- $AB := \{ab : a \in A,\ b \in B\}$
- $A^2 := \{a_1 a_2 : a_1, a_2 \in A\}$
- $A^{-1} := \{a^{-1} : a \in A\}$
- $\overline{A}$ denotes the closure of A

1 - Let G be a topological group and $r \in G$. The left multiplication $s \mapsto rs$, multiplication $s \mapsto sr$, and inversion $s \mapsto s^{-1}$, are homeomorphisms of G.

2 - Let G be a topological group and $e \in G$ the identity element. Let $\mathcal{B}$ be a neighborhood base around e. Then $\{Br\}_{B \in \mathcal{B}}$ is a neighborhood base around $r \in G$ and $\{Br : B \in \mathcal{B} \text{ and } r \in G\}$ is a `http://planetmath.org/BasisTopologicalSpace`basis for the topology of G.

3 - Let G be a topological group. If $U \subseteq G$ is open and V is any subset of G, then UV is an open set in G.

4 - Let G be a topological group and K, L compact sets in G. Then KL is also compact.

5 - Let G be a topological group and $e \in G$ the identity element. If V is a neighborhood of e then $V \subset \overline{V} \subset V^2$.

6 - Let G be a topological group, $e \in G$ the identity element and W a neighborhood around e. Then there exists a neighborhood U around e such that $U^2 \subset W$.

7 - Let G be a topological group, $e \in G$ the identity element and W a neighborhood around e. Then there exists a `http://planetmath.org/SymmetricSets`symmetric neighborhood U around e such that $U^2 \subseteq W$.

8 - Let G be a topological group. If H is a subgroup of G, then so is $\overline{H}$.

9- Let G be a topological group. If H is an open subgroup of G, then H is also closed.