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group  $C^*$ -algebra

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Let  $\mathbb{C}[G]$  be the group ring of a discrete group  $G$ . It has two completions to a  $C^*$ -algebra:

**Reduced group  $C^*$ -algebra.** The reduced group  $C^*$ -algebra,  $C_r^*(G)$ , is obtained by completing  $\mathbb{C}[G]$  in the operator norm for its regular representation on  $l^2(G)$ .

**Maximal group  $C^*$ -algebra.** The maximal group  $C^*$ -algebra,  $C_{\max}^*(G)$  or just  $C^*(G)$ , is defined by the following universal property: any  $*$ -homomorphism from  $\mathbb{C}[G]$  to some  $\mathbb{B}(\mathcal{H})$  (the  $C^*$ -algebra of bounded operators on some Hilbert space  $\mathcal{H}$ ) factors through the inclusion  $\mathbb{C}[G] \hookrightarrow C_{\max}^*(G)$ .

If  $G$  is amenable then  $C_r^*(G) \cong C_{\max}^*(G)$ .