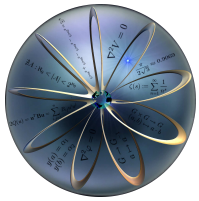

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Birkhoff-Kakutani theorem

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0.1 Birkhoff-Kakutani theorem

Theorem 0.1. *A topological group $(G, *, e)$ is metrizable if and only if G is Hausdorff and the identity e of G has a countable neighborhood basis. Here $*$ is the group composition law or operation. Furthermore, if G is* metrizable*, then G admits a compatible metric d which is left-invariant, that is,*

$$d(gx, gy) = d(x, y);$$

a right-invariant metric r also exists under these conditions.

References

[1] Howard Becker, Alexander S. Kechris. 1996. *The Descriptive Set Theory of Polish Group Actions. (London Mathematical Society Lecture Note Series)*, Cambridge University Press: Cambridge, UK, p.14.