compact groups are unimodular

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Theorem - If G is a compact Hausdorff topological group, then G is unimodular, i.e. it's left and right Haar measures coincide.

***Proof*:**

Let Δ denote the modular function of G. It is enough to prove that Δ is constant and equal to 1, since this proves that every left Haar measure is right invariant.

Since Δ is continuous and G is compact, $\Delta(G)$ is a compact subset of $\mathbb{R}^+$. In particular, $\Delta(G)$ is a bounded subset of $\mathbb{R}^+$.

But if Δ is not identically one, then there is a $t \in G$ such that $\Delta(t) > 1$ (recall that Δ is an homomorphism). Hence, $\Delta(t^n) = \Delta(t)^n \longrightarrow \infty$ as $n \in \mathbb{N}$ increases, which is a contradiction since $\Delta(G)$ is bounded. $\square$