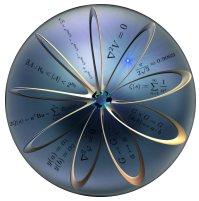

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topological $*$-algebra

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Defines	involution $*$-algebra
Defines	*-algebra

Definition (Involution) An involution on an algebra A over an http://planetmath.org/Involut field F is a map $\cdot^* : A \to A : a \mapsto a^*$ such that for every $\{a, b\} \subset A$ and $\lambda \in F$ we have

1. $a^{**} = a$,
2. $(ab)^* = b^* a^*$ and
3. $(\lambda a+b)^* = \lambda^* a^*+b^*$, where λ^* denotes the http://planetmath.org/InvolutaryRinginvoluti of λ in F.

Definition ($*$-Algebra) A $*$-algebra is an algebra with an involution.
Definition (Topological $*$-algebra) A topological $*$-algebra is a $*$-algebra which is also a topological vector space such that its algebra multiplication and involution are continuous.

0.0.1 Remarks:

- $*$-algebras are a particular of involutory rings.
- The involutory field F is often taken as $\mathbb{C}$, where the involution is given by complex conjugation. In this case, condition 3 could be rewritten as:

  3. $(\lambda a + b)^* = \overline{\lambda} a^* + b^*$
- Banach *-algebras are topological $*$-algebras.