component of identity of a topological group is a closed normal subgroup

Canonical name	ComponentOfIdentityOfATopologicalGroupIsAClosedNormalSubgroup
Date of creation	2013-03-22 18:01:42
Last modified on	2013-03-22 18:01:42
Owner	asteroid (17536)
Last modified by	asteroid (17536)
Numerical id	6
Author	asteroid (17536)
Entry type	Theorem
Classification	msc 22A05

Theorem - Let G be a topological group and e its identity element. The connected component of e is a closed normal subgroup of G.

Proof: Let F be the connected component of e. All components of a topological space are closed, so F is closed.

Let $a \in F$. Since the multiplication and inversion functions in G are continuous, the set aF^{-1} is also connected, and since $e \in aF^{-1}$ we must have $aF^{-1} \subseteq F$. Hence, for every $b \in F$ we have $ab^{-1} \in F$, i.e. F is a subgroup of G.

If g is an arbitrary element of G, then $g^{-1}Fg$ is a connected subset containing e. Hence $g^{-1}Fg \subset F$ for every $g \in G$, i.e. F is a normal subgroup. $\square$