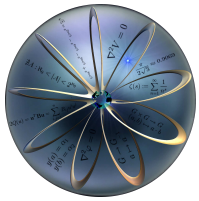

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n-torus

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The *n-torus*, denoted T^n, is a smooth orientable n dimensional manifold which is the product of n 1-spheres, i.e. $T^n = \underbrace{S^1 \times \cdots \times S^1}_{n}$.

Equivalently, the n-torus can be considered to be $\mathbb{R}^n$ modulo the action (vector addition) of the integer lattice $\mathbb{Z}^n$.

The n-torus is in addition a topological group. If we think of S^1 as the unit circle in $\mathbb{C}$ and $T^n = \underbrace{S^1 \times \cdots \times S^1}_{n}$, then S^1 is a topological group and so is T^n by coordinate-wise multiplication. That is,

$$(z_1, z_2, \ldots, z_n) \cdot (w_1, w_2, \ldots, w_n) = (z_1 w_1, z_2 w_2, \ldots, z_n w_n)$$