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# topological space

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| Defines | open |
| Defines | closed |

A *topological space* is a set $X$ together with a set $\mathcal{T}$ whose elements are subsets of $X$, such that

- $\emptyset \in \mathcal{T}$
- $X \in \mathcal{T}$
- If $U_j \in \mathcal{T}$ for all $j \in J$, then $\bigcup_{j \in J} U_j \in \mathcal{T}$
- If $U \in \mathcal{T}$ and $V \in \mathcal{T}$, then $U \cap V \in \mathcal{T}$

Elements of $\mathcal{T}$ are called *open sets* of $X$. The set $\mathcal{T}$ is called a *topology* on $X$. A subset $C \subset X$ is called a *closed set* if the complement $X \setminus C$ is an open set.

A topology $\mathcal{T}'$ is said to be *finer* (respectively, *coarser*) than $\mathcal{T}$ if $\mathcal{T}' \supset \mathcal{T}$ (respectively, $\mathcal{T}' \subset \mathcal{T}$).

**Examples**

- The *discrete topology* is the topology $\mathcal{T} = \mathcal{P}(X)$ on $X$, where $\mathcal{P}(X)$ denotes the power set of $X$. This is the largest, or finest, possible topology on $X$.
- The *indiscrete topology* is the topology $\mathcal{T} = \{\emptyset, X\}$. It is the smallest or coarsest possible topology on $X$.
- Subspace topology
- Product topology
- Metric topology

# References


[1] J.L. Kelley, *General Topology*, D. van Nostrand Company, Inc., 1955.

[2] J. Munkres, *Topology* (2nd edition), Prentice Hall, 1999.