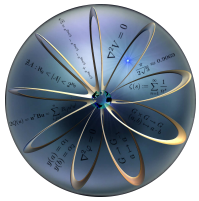

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Borel G-space

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Defines	Borel action
Defines	Borel-measurable map
Defines	standard Borel space

A (standard) Borel G-space is defined in connection with a standard Borel space which shall be specified first.

0.1 Basic definitions

- **a. Standard Borel space**

  Definition 0.1. A *standard Borel space* is defined as a measurable space, that is, a set X equipped with a σ -algebra $\mathcal{S}$, such that there exists a Polish topology on X with S its σ-algebra of Borel sets.

- **b. Borel G-space**

  Definition 0.2. Let G be a Polish group and X a (standard) Borel space. An action a of G on X is defined to be a *Borel action* if $a : G \times X \to X$ is a Borel-measurable map or a `http://planetmath.org/BorelGroupoid`Borel function. In this case, a standard Borel space X that is acted upon by a Polish group with a Borel action is called a *(standard) Borel G-space*.

- **c. Borel morphisms**

  Definition 0.3. Homomorphisms, embeddings or isomorphisms between standard Borel G-spaces are called *Borel* if they are Borel–measurable.

Remark 0.1. Borel G-spaces have the nice property that the product and sum of a countable sequence of Borel G-spaces $(X_n)_{n\in N}$ are also Borel G-spaces. Furthermore, the subspace of a Borel G-space determined by an *invariant* Borel set is also a Borel G-space.