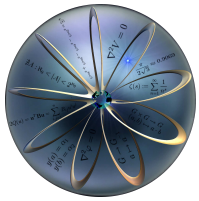

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Sorgenfrey half-open plane

Canonical name	SorgenfreyHalfopenPlane
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Classification	msc 22-00
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Synonym	Sorgenfrey's half-open square topology
Synonym	Sorgenfrey plane

The *Sorgenfrey plane* is the product of the Sorgenfrey line with itself. This topology can also be described as the topology on $\mathbb{R}^2$ which arises from the basis $\{[a,b) \times [c,d) \mid a,b,c,d \in \mathbb{R}, a < b, c < d\}$.

It is interesting to note that, even though the Sorgenfrey line enjoys the `http://planetmath.org/lindelofspace`Lindelöf property, the Sorgenfrey plane does not. To see this, one can note that the line $x + y = 0$ is a closed subset in this topology and that the induced topology on this line is the discrete topology. Since the Lindelöf property is weakly hereditary, the discrete topology on the real line would have to be Lindelöf if the Sorgenfrey plane topology were Lindelöf. However, the discrete topology on an uncountable set can never have the Lindelöf property, so the Sorgenfrey topology cannot have this property either.

Reference

Sorgenfrey, R. H. *On the Topological Product of Paracompact Spaces*, Bulletin of the American Mathematical Society, (1947) 631-632