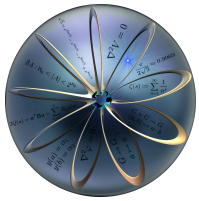

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extremum

Canonical name	Extremum
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Entry type	Definition
Classification	msc 26B12
Synonym	extrema
Related topic	Plateau
Related topic	RelationsBetweenHessianMatrixAndLocalExtrema
Related topic	LeastAndGreatestValueOfFunction
Related topic	SpeediestInclinedPlane
Defines	global minima
Defines	global maxima
Defines	local minima
Defines	local maxima
Defines	global minimum
Defines	global maximum
Defines	local minimum
Defines	local maximum
Defines	strict local minima
Defines	strict local maxima
Defines	strict local minimum
Defines	strict local maximum
Defines	saddle point

Extrema are minima and maxima. The forms of these are extremum, minimum, and maximum.

Extrema may be "global" or "local". A global minimum of a function f is the lowest value that f ever achieves. If you imagine the function as a surface, then a global minimum is the lowest point on that surface. Formally, it is said that $f\colon U \to V$ has a *global minimum* at x if $\forall u \in U, f(x) \leq f(u)$.

A local minimum of a function f is a point x which has less value than all points "next to" it. If you imagine the function as a surface, then a local minimum is the of a "valley" or "bowl" in the surface somewhere. Formally, it is said that $f\colon U \to V$ has a *local minimum* at x if $\exists$ a neighborhood N of x such that $\forall y \in N$, $f(x) \leq f(y)$.

If you flip the $\leq$ signs above to $\geq$, you get the definitions of global and local maxima.

A "strict local minima" or "strict local maxima" means that nearby points are strictly less than or strictly greater than the critical point, rather than $\leq$ or $\geq$. For instance, a strict local minima at x has a neighborhood N such that $\forall y \in N$, ($f(x) < f(y)$ or $y = x$).

A *saddle point* is a critical point which is not a local extremum.

A related concept is plateau.

Finding minima or maxima is an important task which is part of the of optimization. This task is also important in Physics where the minima correspond to equilibria.