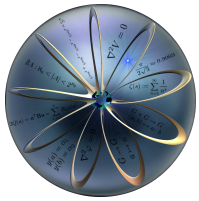

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sets where sequence of continuous functions diverge

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Related Exercise from Rudin's *Real and Complex Analysis*.
Exercise 5.20

(a) Does there exist a sequence of continuous positive functions f_n on $\mathbb{R}^1$ such that $\{f_n(x)\}$ is unbounded if and only if x is rational?

(b) Replace "rational" by irrational in (a) and answer the resulting question.

(c) Replace "$\{f_n(x)\}$ is unbounded" by "$f_n(x) \to \infty$ as $n \to \infty$" and answer the resulting analogues of (a) and (b).

Solution: The answer to (a) is negative. This by showing that the subset of points where such sequence is unbounded must be G_δ. But the rationals cannot be such, since in $\mathbb{R}$ dense G_δ sets must be of second category.

Rest of the answer not yet ready here