general means inequality

Canonical name	GeneralMeansInequality
Date of creation	2013-03-22 12:39:49
Last modified on	2013-03-22 12:39:49
Owner	drini (3)
Last modified by	drini (3)
Numerical id	6
Author	drini (3)
Entry type	Theorem
Classification	msc 26D15
Synonym	power means inequality
Related topic	ArithmeticGeometricMeansInequality
Related topic	ArithmeticMean
Related topic	GeometricMean
Related topic	HarmonicMean
Related topic	PowerMean
Related topic	ProofOfArithmeticGeometricHarmonicMeansI
Related topic	RootMeanSquare3
Related topic	DerivationOfZerothWeightedPowerMean
Related topic	ProofOfArithmeticGeometricHarmonicMeansInequality
Related topic	ComparisonOfPythagor

The power means inequality is a generalization of arithmetic-geometric means inequality.

If $0 \neq r \in \mathbb{R}$, the r-mean (or r-th power mean) of the nonnegative numbers $a_1, \ldots, a_n$ is defined as

$$M^r(a_1, a_2, \ldots, a_n) = \left(\frac{1}{n} \sum_{k=1}^{n} a_k^r \right)^{1/r}$$

Given real numbers x, y such that $xy \neq 0$ and $x < y$, we have

$$M^x \leq M^y$$

and the equality holds if and only if $a_1 = ... = a_n$.

Additionally, if we define M^0 to be the geometric mean $(a_1 a_2 ... a_n)^{1/n}$, we have that the inequality above holds for arbitrary real numbers $x < y$.

The mentioned inequality is a special case of this one, since M^1 is the arithmetic mean, M^0 is the geometric mean and M^{-1} is the harmonic mean.

This inequality can be further generalized using weighted power means.