formal definition of Landau notation

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Let us consider a domain D and an accumulation point $x_0 \in \overline{D}$. Important examples are $D = \mathbb{R}$ and $x_0 \in D$ or $D = \mathbb{N}$ and $x_0 = +\infty$. Let $f\colon D \to \mathbb{R}$ be any function. We are going to define the spaces $o(f)$ and $O(f)$ which are families of real functions defined on D and which depend on the point $x_0 \in \overline{D}$.

Suppose first that there exists a neighbourhood U of x_0 such that f restricted to $U \cap D$ is always different from zero. We say that $g \in o(f)$ as $x \to x_0$ if

$$\lim_{x \to x_0} \frac{g(x)}{f(x)} = 0.$$

We say that $g \in O(f)$ as $x \to x_0$ if there exists a neighbourhood U of x_0 such that

$$\frac{g(x)}{f(x)} \text{is bounded if restricted to } D \cap U.$$

In the case when $f \equiv 0$ in a neighbourhood of x_0, we define $o(f) = O(f)$ as the set of all functions g which are null in a neighbourhood of 0.

The families o and O are usually called "small-o" and "big-o" or, sometimes, "small ordo", "big ordo".