proof of complex mean-value theorem

Canonical name	ProofOfComplexMeanvalueTheorem
Date of creation	2013-03-22 14:34:39
Last modified on	2013-03-22 14:34:39
Owner	Wolfgang (5320)
Last modified by	Wolfgang (5320)
Numerical id	21
Author	Wolfgang (5320)
Entry type	Proof
Classification	msc 26A06

The function $h(t) = \mathrm{Re}\,\frac{f(a+t(b-a))-f(a)}{b-a}$ is a function defined on [0,1]. We have $h(0) = 0$ and $h(1) = \mathrm{Re}\,\frac{f(b)-f(a)}{b-a}$. By the ordinary mean-value theorem, there is a number t, $0 < t < 1$, such that $h'(t) = h(1) - h(0)$. To evaluate $h'(t)$, we use the assumption that f is complex differentiable (holomorphic). The derivative of $\frac{f(a+t(b-a))-f(a)}{b-a}$ is equal to $f'(a + t(b - a))$, then $h'(t) = \mathrm{Re}(f'(a + t(b - a)))$, so $u = a + t(b - a)$ satisfies the required equation. The proof of the second assertion can be deduced from the result just proved by applying it to the function f multiplied by i.