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differentiable function

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Entry type	Definition
Classification	msc 26A24
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Synonym	smooth function
Synonym	differentiable mapping
Synonym	differentiable map
Synonym	smooth mapping
Synonym	smooth map
Synonym	continuously differentiable
Related topic	OneSidedDerivatives
Related topic	RoundFunction
Related topic	ConverseTheorem
Related topic	WeierstrassFunction
Defines	differentiable
Defines	smooth

Let $f \colon V \to W$ be a function, where V and W are Banach spaces. For $x \in V$, the function f is said to be *differentiable* at x if its derivative exists at that point. Differentiability at $x \in V$ implies continuity at x. If $S \subset V$, then f is said to be differentiable on S if f is differentiable at every point $x \in S$.

For the most common example, a real function $f \colon \mathbb{R} \to \mathbb{R}$ is differentiable if its derivative $\frac{df}{dx}$ exists for every point in the region of interest. For another common case of a real function of n variables $f(x_1, x_2, \ldots, x_n)$ (more formally $f \colon \mathbb{R}^n \to \mathbb{R}$), it is not sufficient that the partial derivatives $\frac{\partial f}{\partial x_i}$ exist for f to be differentiable. The derivative of f must exist in the original sense at every point in the region of interest, where $\mathbb{R}^n$ is treated as a Banach space under the usual Euclidean vector norm.

If the derivative of f is continuous, then f is said to be C^1. If the kth derivative of f is continuous, then f is said to be C^k. By convention, if f is only continuous but does not have a continuous derivative, then f is said to be C^0. Note the inclusion property $C^{k+1} \subset C^k$. And if the k-th derivative of f is continuous for all k, then f is said to be C^∞. In other words C^∞ is the intersection $C^\infty = \bigcap_{k=0}^\infty C^k$.

Differentiable functions are often referred to as *smooth*. If f is C^k, then f is said to be k-smooth. Most often a function is called smooth (without qualifiers) if f is C^∞ or C^1, depending on the context.