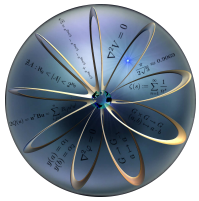

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envelope of a function

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Consider $f : \mathbb{R} \to \mathbb{R}$ a real function of real variable.

We call the upper envelope of f to the function:

$\text{env}_{\sup}(f)(x) = \inf_\epsilon\{\sup\{f(y) : \epsilon > 0,\ |y - x| < \epsilon\}\}$

similarly the lower envelope of f is the function:

$\text{env}_{\inf}(f)(x) = \sup_\epsilon\{\inf\{f(y) : \epsilon > 0,\ |y - x| < \epsilon\}\}$

The envelopes have the following properties: (in this list env_* represents either the upper or lower envelope)

- $\text{env}_{\inf}(f)(x) \leq f(x) \leq \text{env}_{\sup}(f)(x)$
- $\text{env}_{\sup}(f) = \text{env}_{\inf}(f) \iff f\text{is continuous}$
- $\text{env}_{\sup}(f)(x) - \text{env}_{\inf}(f)(x) = \text{oscillation of} f \text{at } x$
- $\text{env}_{\inf} f = -\text{env}_{\sup}(-f)$