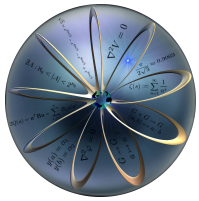

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Weierstrass product inequality

Canonical name	WeierstrassProductInequality
Date of creation	2013-03-22 13:58:23
Last modified on	2013-03-22 13:58:23
Owner	Daume (40)
Last modified by	Daume (40)
Numerical id	5
Author	Daume (40)
Entry type	Theorem
Classification	msc 26D05

For any finite family $(a_i)_{i \in I}$ of real numbers in the interval $[0, 1]$, we have

$$\prod_i (1 - a_i) \geq 1 - \sum_i a_i \ .$$

Proof: Write

$$f = \prod_i (1 - a_i) + \sum_i a_i \ .$$

For any $k \in I$, and any fixed values of the a_i for $i \neq k$, f is a polynomial of the first degree in a_k. Consequently f is minimal either at $a_k = 0$ or $a_k = 1$. That brings us down to two cases: all the a_i are zero, or at least one of them is 1. But in both cases it is clear that $f \geq 1$, QED.