sum function of series

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Let the terms of a series be real functions f_n defined in a certain subset A_0 of $\mathbb{R}$; we can speak of a *function series*. All points x where the series

$$f_1 + f_2 + \cdots \tag{1}$$

converges form a subset A of A_0, and we have the $S: x \mapsto S(x)$ of (1) defined in A.

If the sequence $S_1, S_2, \ldots$ of the partial sums $S_n = f_1+f_2+\cdots+f_n$ of the series (1) `http://planetmath.org/LimitFunctionOfSequence`converges uniformly in the interval $[a, b] \subseteq A$ to a function $S: x \mapsto S(x)$, we say that *the series* in this interval. We may also set the direct

Definition. The function series (1), which converges in every point of the interval $[a, b]$ having sum function $S: x \mapsto S(x)$, in the interval $[a, b]$, if for every positive number ε there is an integer n_ε such that each value of x in the interval $[a, b]$ the inequality

$$|S_n(x) - S(x)| < \varepsilon$$

when $n \geqq n_\varepsilon$.

Note. One can without trouble be convinced that the term functions of a uniformly converging series converge uniformly to 0 (cf. the necessary condition of convergence).

The notion of of series can be extended to the series with complex function terms (the interval is replaced with some subset of $\mathbb{C}$). The significance of the is therein that the sum function of a series with this property and with continuous term-functions is continuous and may be integrated termwise.