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squaring condition for square root inequality

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Of the inequalities $\sqrt{a} \lesseqgtr b$,

- both are undefined when $a < 0$;
- both can be sidewise squared when $a \geqq 0$ and $b \geqq 0$;
- $\sqrt{a} > b$ is identically true if $a \geqq 0$ and $b < 0$.
- $\sqrt{a} < b$ is identically untrue if $b < 0$;

The above theorem may be utilised for solving inequalities involving square roots.

Example. Solve the inequality

$$\sqrt{2x+3} \;>\; x. \tag{1}$$

The reality condition $2x+3 \geqq 0$ requires that $x \geqq -1\frac{1}{2}$. For using the theorem, we distinguish two cases according to the sign of the right hand side:

1°: $-1\frac{1}{2} \leqq x < 0$. The inequality is identically true; we have for (1) the partial solution $-1\frac{1}{2} \leqq x < 0$.

2°: $x \geqq 0$. Now we can square both , obtaining

$$2x+3 \;>\; x^2$$

$$x^2-2x-3 \;<\; 0$$

The zeros of x^2-2x-3 are $x = 1 \pm 2$, i.e. -1 and 3. Since the graph of the polynomial function is a parabola opening upwards, the polynomial attains its negative values when $-1 < x < 3$ (see quadratic inequality). Thus we obtain for (1) the partial solution $0 \leqq x < 3$.

Combining both partial solutions we obtain the total solution

$$-1\frac{1}{2} \;\leqq\; x \;<\; 3.$$