Lipschitz function

Canonical name	LipschitzFunction
Date of creation	2013-03-22 14:01:42
Last modified on	2013-03-22 14:01:42
Owner	bwebste (988)
Last modified by	bwebste (988)
Numerical id	12
Author	bwebste (988)
Entry type	Definition
Classification	msc 26A16
Defines	Lipschitz

Let $W \subseteq X \subseteq \mathbb{C}$ and $f\colon X \to \mathbb{C}$. Then f is on W if there exists an $M \in \mathbb{R}$ such that, for all $x, y \in W$, $x \neq y$

$$|f(x) - f(y)| \leq M|x - y|$$

If $a, b \in \mathbb{R}$ with $a < b$ and $f\colon [a, b] \to \mathbb{R}$ is Lipschitz on (a, b), then f is absolutely continuous on $[a, b]$.

Example: Is

$$f(x) = \frac{1}{\sqrt{x}}, \quad x \in [0, 1]$$

a Lipschitz function.

We need to estimate the constant M.

$$|f(x)-f(y)| = \left|\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{y}}\right| = \left|\frac{\sqrt{x}-\sqrt{y}}{\sqrt{xy}}\right| = \left|\frac{x-y}{\sqrt{xy}(\sqrt{x}+\sqrt{y})}\right| = \frac{1}{|\sqrt{xy}(\sqrt{x}+\sqrt{y})|}|x-y|.$$

It follows that

$$M = \frac{1}{|\sqrt{xy}(\sqrt{x}+\sqrt{y})|}$$

and $f(x)$ is not Lipschitz at $x = 0$.