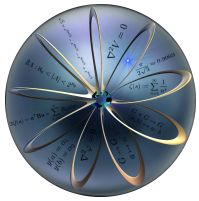

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proof of binomial formula

Canonical name	ProofOfBinomialFormula
Date of creation	2013-03-22 12:24:00
Last modified on	2013-03-22 12:24:00
Owner	rmilson (146)
Last modified by	rmilson (146)
Numerical id	6
Author	rmilson (146)
Entry type	Proof
Classification	msc 26A06

Let $p \in \mathbb{R}$ and $x \in \mathbb{R},\ |x| < 1$ be given. We wish to show that

$$(1+x)^p = \sum_{n=0}^{\infty} p^{\underline{n}}\, \frac{x^n}{n!},$$

where $p^{\underline{n}}$ denotes the n^{th} falling factorial of p.

The convergence of the series in the right-hand side of the above equation is a straight-forward consequence of the ratio test. Set

$$f(x) = (1+x)^p.$$

and note that

$$f^{(n)}(x) = p^{\underline{n}}\,(1+x)^{p-n}.$$

The desired equality now follows from Taylor's Theorem. Q.E.D.