# example of chain rule

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Suppose we wanted to differentiate

$$h(x) = \sqrt{\sin(x)}.$$

Here, $h(x)$ is given by the composition

$$h(x) = f(g(x)),$$

where

$$f(x) = \sqrt{x} \quad \text{and} \quad g(x) = \sin(x).$$

Then chain rule says that

$$h'(x) = f'(g(x))g'(x).$$

Since

$$f'(x) = \frac{1}{2\sqrt{x}}, \quad \text{and} \quad g'(x) = \cos(x),$$

we have by chain rule

$$h'(x) = \left(\frac{1}{2\sqrt{\sin x}}\right)\cos x = \frac{\cos x}{2\sqrt{\sin x}}$$

Using the Leibniz formalism, the above calculation would have the following appearance. First we describe the functional relation as

$$z = \sqrt{\sin(x)}.$$

Next, we introduce an auxiliary variable $y$, and write

$$z = \sqrt{y}, \qquad y = \sin(x).$$

We then have

$$\frac{dz}{dy} = \frac{1}{2\sqrt{y}}, \qquad \frac{dy}{dx} = \cos(x),$$

and hence the chain rule gives

$$\begin{aligned} \frac{dz}{dx} &= \frac{1}{2\sqrt{y}}\cos(x) \\ &= \frac{1}{2}\frac{\cos(x)}{\sqrt{\sin(x)}} \end{aligned}$$