proof of mean value theorem

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Define $h(x)$ on $[a, b]$ by

$$h(x) = f(x) - f(a) - \left(\frac{f(b) - f(a)}{b - a} \right) (x - a)$$

Clearly, h is continuous on $[a, b]$, differentiable on (a, b), and

$$\begin{aligned} h(a) &= f(a) - f(a) = 0 \\ h(b) &= f(b) - f(a) - \left(\tfrac{f(b)-f(a)}{b-a} \right) (b - a) = 0 \end{aligned}$$

Notice that h satisfies the conditions of Rolle's Theorem. Therefore, by Rolle's Theorem there exists $c \in (a, b)$ such that $h'(c) = 0$.
However, from the definition of h we obtain by differentiation that

$$h'(x) = f'(x) - \frac{f(b) - f(a)}{b - a}$$

Since $h'(c) = 0$, we therefore have

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

as required.

References

[1] Michael Spivak, *Calculus*, 3rd ed., Publish or Perish Inc., 1994.