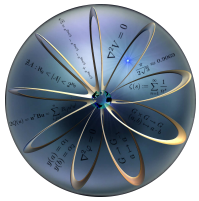

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square root of polynomial

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The f, denoted by $\sqrt{f}$, is any polynomial g having the square g^2 equal to f. For example, $\sqrt{9x^2-30x+25} = 3x-5$ or $-3x+5$.

A polynomial needs not have a square root, but if it has a square root g, then also the opposite polynomial $-g$ is its square root.

Algorithm. The idea of the squaring

$$(a+b+c+..)^2 = (a)a + (2a+b)b + (2a+2b+c)c + ..$$

(see the square of sum) gives a method for getting the square root of a polynomial:

- The .
- And so on.

In the examples below, on the .

Example 1. $\sqrt{9x^4+6x^3-11x^2-4x+4} = ?$

$$\begin{array}{rrrrrrcrrr}
\sqrt{} & (9x^4 & +6x^3 & -11x^2 & -4x & +4) & = \pm & (3x^2 & +x & -2) \\
\cline{2-2} & 9x^4 & & & & & & 3x^2 & & \\
\cline{2-3} & & 6x^3 & -11x^2 & & & & 6x^2 & +x & \\
\cline{3-4} & & 6x^3 & +x^2 & & & & & x & \\
\cline{4-6} & & & -12x^2 & -4x & +4 & & 6x^2 & +2x & -2 \\
\cline{4-6} & & & -12x^2 & -4x & +4 & & & & -2 \\
\cline{6-6} & & & & & 0 & & & & \\
\end{array}$$

Example 2. $\sqrt{x^6-2x^5-x^4+3x^2+2x+1} = ?$

$$\begin{array}{rrrrrrrrcrrrr}
\sqrt{} & (1 & +2x & +3x^2 & & -x^4 & -2x^5 & +x^6) & = \pm & (1 & +x & +x^2 & -x^3) \\
\cline{2-2} & 1 & & & & & & & & 1 & & & \\
\cline{3-4} & & 2x & +3x^2 & & & & & & 2 & +x & & \\
\cline{3-4} & & 2x & +x^2 & & & & & & & +x & & \\
\cline{4-5} & & & 2x^2 & & -x^4 & & & & 2 & +2x & +x^2 & \\
\cline{4-6} & & & 2x^2 & +2x^3 & +x^4 & & & & & & x^2 & \\
\cline{5-8} & & & & -2x^3 & -2x^4 & -2x^5 & +x^6 & & 2 & +2x & +2x^2 & -x^3 \\
\cline{5-8} & & & & -2x^3 & -2x^4 & -2x^5 & +x^6 & & & & & -x^3 \\
\cline{8-8} & & & & & & & 0 & & & & & \\
\end{array}$$

Remark. The procedure may give a Taylor series expansion of the square root, if it is not a polynomial. E.g. we get

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + -...$$

References

[1] *Meyers Rechenduden*. Erster verbesserter Neudruck. Bibliographisches Institut AG, Mannheim (1960).