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## Young's inequality

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Let  $\phi : \mathbb{R} \rightarrow \mathbb{R}$  be a continuous , strictly increasing function such that  $\phi(0) = 0$  . Then the following inequality holds:

$$ab \leq \int_0^a \phi(x)dx + \int_0^b \phi^{-1}(y)dy$$

Equality only holds when  $b = \phi(a)$ . This inequality can be demonstrated by drawing the graph of  $\phi(x)$  and by observing that the sum of the two areas represented by the integrals above is greater than the area of a rectangle of sides  $a$  and  $b$ , as is illustrated in <http://planetmath.org/node/5575>an attachment.