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lamellar field

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Entry type	Definition
Classification	msc 26B12
Synonym	lamellar
Synonym	irrotational
Synonym	conservative
Synonym	laminar
Related topic	CurlFreeField
Related topic	PoincareLemma
Related topic	VectorPotential
Related topic	GradientTheorem
Defines	scalar potential
Defines	potential
Defines	rotor

A vector field $\vec{F} = \vec{F}(x,\, y,\, z)$, defined in an open set D of $\mathbb{R}^3$, is *lamellar* if the condition

$$\nabla\times\vec{F} = \vec{0}$$

is satisfied in every point $(x,\, y,\, z)$ of D.

Here, $\nabla\times\vec{F}$ is the curl or *rotor* of $\vec{F}$. The condition is equivalent with both of the following:

- The line integrals

  $$\oint_s \vec{F}\cdot d\vec{s}$$

  taken around any contractible curve s vanish.

- The vector field has a $u = u(x,\, y,\, z)$ which has continuous partial derivatives and which is up to a unique in a simply connected domain; the scalar potential means that

  $$\vec{F} = \nabla u.$$

The scalar potential has the expression

$$u = \int_{P_0}^{P} \vec{F}\cdot d\vec{s},$$

where the point P_0 may be chosen freely, $P = (x,\, y,\, z)$.

Note. In physics, u is in general replaced with $V = -u$. If the $\vec{F}$ is interpreted as a , then the potential V is equal to the work made by the when its point of application is displaced from P_0 to infinity.