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boundedness theorem

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Boundedness Theorem. *Let a and b be real numbers with $a < b$, and let f be a continuous, real valued function on $[a, b]$. Then f is bounded above and below on $[a, b]$.*

Proof. Suppose not. Then for all natural numbers n we can find some $x_n \in [a, b]$ such that $|f(x_n)| > n$. The sequence (x_n) is bounded, so by the Bolzano-Weierstrass theorem it has a convergent sub sequence, say (x_{n_i}). As $[a, b]$ is closed (x_{n_i}) converges to a value in $[a, b]$. By the continuity of f we should have that $f(x_{n_i})$ converges, but by construction it diverges. This contradiction finishes the proof.