



Math for the people, by the people.

## Dirichlet's function

Canonical name	DirichletsFunction
Date of creation	2013-03-22 13:11:14
Last modified on	2013-03-22 13:11:14
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	9
Author	mathcam (2727)
Entry type	Definition
Classification	msc 26A15
Related topic	FunctionContinuousAtOnlyOnePoint
Related topic	APathologicalFunctionOfRiemann

*Dirichlet's function*  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined as

$$f(x) = \begin{cases} \frac{1}{q} & \text{if } x = \frac{p}{q} \text{ is a rational number in lowest terms,} \\ 0 & \text{if } x \text{ is an irrational number.} \end{cases}$$

This function has the property that it is continuous at every irrational number and discontinuous at every rational one.

Another function that often goes by the same name is the function

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is a rational number.} \\ 0 & \text{if } x \text{ is an irrational number.} \end{cases}$$

This nowhere-continuous function has the surprising expression

$$f(x) = \lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} \cos^{2n}(m!\pi x).$$

This is often given as the (amazing!) example of a sequence of everywhere-continuous functions whose limit function is nowhere continuous.