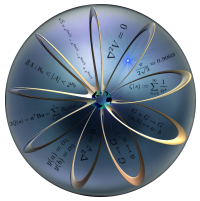

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limit rules of functions

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Theorem 1. Let f and g be two `http://planetmath.org/RealFunction`real or complex functions. Suppose that there exist the limits $\lim_{x\to x_0} f(x)$ and $\lim_{x\to x_0} g(x)$. Then there exist the limits $\lim_{x\to x_0}[f(x)\pm g(x)]$, $\lim_{x\to x_0} f(x)g(x)$ and, if $\lim_{x\to x_0} g(x) \neq 0$, also $\lim_{x\to x_0} f(x)/g(x)$, and

1. $\lim_{x\to x_0}[f(x)\pm g(x)] \;=\; \lim_{x\to x_0} f(x) \pm \lim_{x\to x_0} g(x)$,
2. $\lim_{x\to x_0} f(x)g(x) \;=\; \lim_{x\to x_0} f(x) \cdot \lim_{x\to x_0} g(x)$,
3. $\lim_{x\to x_0} \frac{f(x)}{g(x)} \;=\; \frac{\lim_{x\to x_0} f(x)}{\lim_{x\to x_0} g(x)}$,
4. $\lim_{x\to x_0} c \;=\; c$ where c is a constant.

These rules are used in limit calculations and in proving the corresponding differentiation rules (sum rule, product rule etc.).

In 1, the domains of f and g could be any topological space (not necessarily $\mathbb{R}$ or $\mathbb{C}$).

There are limit rules of `http://planetmath.org/Sequence`sequences.

As well, one often needs the

Theorem 2. If there exists the limit $\lim_{x\to x_0} f(x) = a$ and if g is continuous at the point $x = a$, then there exists the limit $\lim_{x\to x_0} g(f(x))$, and

$$\lim_{x\to x_0} g(f(x)) \;=\; g(\lim_{x\to x_0} f(x)).$$