cyclometric functions

Canonical name	CyclometricFunctions
Date of creation	2013-03-22 14:36:00
Last modified on	2013-03-22 14:36:00
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	34
Author	pahio (2872)
Entry type	Definition
Classification	msc 26A09
Synonym	arc functions
Synonym	arcus functions
Synonym	inverse trigonometric functions
Related topic	Trigonometry
Related topic	ComplexSineAndCosine
Related topic	TaylorSeriesOfArcusSine
Related topic	TaylorSeriesOfArcusTangent
Related topic	AreaFunctions
Related topic	RamanujansFormulaForPi
Related topic	SawBladeFunction
Related topic	TerminalRay
Related topic	DerivativeOfInverseFunction
Related topic	LaplaceTransformOfFracftt
Related topic	OstensiblyDiscontinuousAntiderivative
Related topic	I
Defines	branch
Defines	principal branch
Defines	sine
Defines	cosine
Defines	arc sine
Defines	arc cosine
Defines	arc tangent
Defines	arc cotangent
Defines	inverse sine
Defines	inverse tangent

The http://planetmath.org/DefinitionsInTrigonometrytrigonometric functions are periodic, and thus get all their values infinitely many times. Therefore their inverse functions, the *cyclometric functions*, are multivalued, but the values within suitable chosen intervals are unique; they form single-valued functions, called the *branches* of the multivalued functions.

The of the most used cyclometric functions are as follows:

- $\arcsin x$ is the angle y satisfying $\sin y = x$ and $-\frac{\pi}{2} < y \leqq \frac{\pi}{2}$ (defined for $-1 \leqq x \leqq 1$)
- $\arccos x$ is the angle y satisfying $\cos y = x$ and $0 \leqq y < \pi$ (defined for $-1 \leqq x \leqq 1$)
- $\arctan x$ is the angle y satisfying $\tan y = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (defined in the whole $\mathbb{R}$)
- $\mathrm{arccot}\ x$ is the angle y satisfying $\cot y = x$ and $0 < y < \pi$ (defined in the whole $\mathbb{R}$)

Those functions are denoted also $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ and $\cot^{-1} x$. We here use these notations temporarily for giving the corresponding multivalued functions ($n = 0,\ \pm 1,\ \pm 2,\ \ldots$):

$$\sin^{-1} x = n\pi + (-1)^n \arcsin x$$

$$\cos^{-1} x = 2n\pi \pm \arccos x$$

$$\tan^{-1} x = n\pi + \arctan x$$

$$\cot^{-1} x = n\pi + \mathrm{arccot}\ x$$

Some formulae

$$\arcsin x + \arccos x = \frac{\pi}{2}$$

$$\arctan x + \mathrm{arccot}\ x = \frac{\pi}{2}$$

$$\arcsin x = \int_0^x \frac{dt}{\sqrt{1-t^2}}\, dt$$

$$\arctan x = \int_0^x \frac{dt}{1+t^2}\, dt$$

$$\arcsin x = x + \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\cdot\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{x^7}{7} + \ldots \quad (|x| \leqq 1)$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + - \ldots \quad (|x| \leqq 1)$$

$$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$$

$$\frac{d}{dx}\operatorname{arccot} x = -\frac{1}{1+x^2} \quad (\forall x \in \mathbb{R})$$

The classic abbreviations of the cyclometric functions are usually explained as follows. The values of the trigonometric functions are got from the unit circle by means of its arc (in Latin *arcus*) with starting point $(1, 0)$. The arc the angle (which may be thought as a central angle of the circle), and its end point (ξ, η) is achieved when the starting point has circulated along the circumference anticlockwise for positive angle (and clockwise for negative angle). Then the cosine of the arc (i.e. angle) is the abscissa ξ of the end point, the sine of the arc is the ordinate η of it. Correspondingly, one can get the tangent and cotangent of the arc by using certain points on the tangent lines $x = 1$ and $y = 1$ of the unit circle.

The word cosine is in Latin *cosinus*, its genitive form is *cosini*. So e.g. "arccos" of a given real number x means the 'arc of the cosine value x', in Latin *arcus cosini x*.