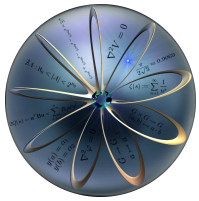

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higher order derivatives of sine and cosine

Canonical name	HigherOrderDerivativesOfSineAndCosine
Date of creation	2013-03-22 14:45:16
Last modified on	2013-03-22 14:45:16
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	13
Author	pahio (2872)
Entry type	Derivation
Classification	msc 26B05
Classification	msc 46G05
Classification	msc 26A24
Related topic	FractionalDifferentiation
Related topic	HigherOrderDerivatives
Related topic	ExampleOfTaylorPolynomialsForSinX
Related topic	CosineAtMultiplesOfStraightAngle

One may consider the sine and cosine either as http://planetmath.org/RealFunctionreal or complex functions. In both cases they are everywhere smooth, having the derivatives of all http://planetmath.org/OrderOfDerivativeorders in every point. The formulae

$$\frac{d^n}{dx^n}\sin x \;=\; \sin\,(x+n\cdot\frac{\pi}{2})$$

and

$$\frac{d^n}{dx^n}\cos x \;=\; \cos\,(x+n\cdot\frac{\pi}{2}),$$

where $n = 0, 1, 2, \ldots$ (the derivative of the 0^{th} order means the function itself), can be proven by induction on n. Another possibility is to utilize Euler's formula, obtaining

$$\frac{d^n}{dx^n}\cos x+i\frac{d^n}{dx^n}\sin x \;=\; \frac{d^n}{dx^n}e^{ix} \;=\; e^{ix}i^n \;=\; e^{ix+in\frac{\pi}{2}} \;=\; \cos\,(x+n\cdot\frac{\pi}{2})+i\sin\,(x+n\cdot\frac{\pi}{2});$$

here one has to compare the http://planetmath.org/ComplexFunctionreal and imaginary parts – supposing that x is real.