increasing/decreasing/monotone function

Canonical name	IncreasingdecreasingmonotoneFunction
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Entry type	Definition
Classification	msc 26A06
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Defines	increasing
Defines	decreasing
Defines	strictly increasing
Defines	strictly decreasing
Defines	monotone
Defines	monotonic
Defines	strictly monotone
Defines	strictly monotonic
Defines	weakly increasing
Defines	weakly decreasing
Defines	strongly increasing
Defines	strongly decreasing
Defines	strongly monotone
Defines	weakly monotone
Defines	stronly mono

Definition Let A be a subset of $\mathbb{R}$, and let f be a function from $f : A \to \mathbb{R}$. Then

1. f is *increasing* or *weakly increasing*, if $x \leq y$ implies that $f(x) \leq f(y)$ (for all x and y in A).

2. f is *strictly increasing* or *strongly increasing*, if $x < y$ implies that $f(x) < f(y)$.

3. f is *decreasing* or *weakly decreasing*, if $x \leq y$ implies that $f(x) \geq f(y)$.

4. f is *strictly decreasing* or *strongly decreasing* if $x < y$ implies that $f(x) > f(y)$.

5. f is *monotone*, if f is either increasing or decreasing.

6. f is *strictly monotone* or *strongly monotone*, if f is either strictly increasing or strictly decreasing.

Theorem Let X be a bounded or unbounded open interval of $\mathbb{R}$. In other words, let X be an interval of the form $X = (a, b)$, where $a, b \in \mathbb{R} \cup \{-\infty, \infty\}$. Futher, let $f : X \to \mathbb{R}$ be a monotone function.

1. The set of points where f is discontinuous is at most countable [**?**, **?**].

Lebesgue f is differentiable almost everywhere ([**?**], pp. 514).

References

[1] C.D. Aliprantis, O. Burkinshaw, *Principles of Real Analysis*, 2nd ed., Academic Press, 1990.

[2] W. Rudin, *Principles of Mathematical Analysis*, McGraw-Hill Inc., 1976.

[3] F. Jones, *Lebesgue Integration on Euclidean Spaces*, Jones and Barlett Publishers, 1993.