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proof of Jensen's inequality

Canonical name	ProofOfJensensInequality
Date of creation	2013-03-22 12:45:15
Last modified on	2013-03-22 12:45:15
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Entry type	Proof
Classification	msc 26D15
Classification	msc 39B62

We prove an equivalent, more convenient formulation: Let X be some random variable, and let $f(x)$ be a convex function (defined at least on a segment containing the range of X). Then the expected value of $f(X)$ is at least the value of f at the mean of X :

$$\mathbb{E}[f(X)] \geq f(\mathbb{E}[X]).$$

Indeed, let $c = \mathbb{E}[X]$. Since $f(x)$ is convex, there exists a supporting line for $f(x)$ at c :

$$\varphi(x) = \alpha(x - c) + f(c)$$

for some α , and $\varphi(x) \leq f(x)$. Then

$$\mathbb{E}[f(X)] \geq \mathbb{E}[\varphi(X)] = \mathbb{E}[\alpha(X - c) + f(c)] = f(c)$$

as claimed.