compact spaces with group structure

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Proposition. Assume that (G, M) is a group (with multiplication $M : G \times G \to G$) and G is also a topological space. If G is compact Hausdorff and $M : G \times G \to G$ is continuous, then (G, M) is a topological group.

Proof. Indeed, all we need to show is that function $f : G \to G$ given by $f(g) = g^{-1}$ is continuous. Note, that the following holds for the graph of f:

$$\Gamma(f) = \{(g, f(g)) \in G \times G\} = \{(g, g^{-1}) \in G \times G\} = M^{-1}(e),$$

where e denotes the neutral element in G. It follows (from continuity of M) that $\Gamma(f)$ is closed in $G \times G$. It is well known (see the parent object for details) that this implies that f is continuous, which completes the proof. $\square$