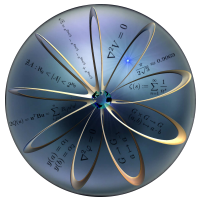

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properties of the exponential

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The exponential operation possesses the following properties.

- For $x, y \in \mathbb{R}^+, p \in \mathbb{R}$ we have

$$(xy)^p = x^p y^p$$

- For $x \in \mathbb{R}^+$ we have

$$x^0 = 1, \quad x^1 = x, \quad x^{p+q} = x^p x^q, \quad (x^p)^q = x^{pq} \qquad p, q \in \mathbb{R}.$$

- `http://planetmath.org/TotalOrder`**Monotonicity.** For $x, y \in \mathbb{R}^+$ with $x < y$ and $p \in \mathbb{R}^+$ we have

$$x^p < y^p, \qquad x^{-p} > y^{-p}.$$

- **Continuity.** The exponential operation is continuous with respect to its arguments. To be more precise, the following function is continuous:

$$P : \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}, \qquad P(x, y) = x^y.$$

Let us also note that the exponential operation is characterized (in the sense of existence and uniqueness) by the *additivity* and *continuity* properties. [**Author's note**: One can probably get away with substantially less, but I haven't given this enough thought.]