nabla

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Defines	∇

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a $C^1(\mathbb{R}^n)$ function, that is, a partially differentiable function in all its coordinates. The symbol ∇, named *nabla*, represents the gradient operator, whose action on $f(x_1, x_2, \ldots, x_n)$ is given by

$$\begin{aligned} \nabla f &= (f_{x_1}, f_{x_2}, \ldots, f_{x_n}) \\ &= \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right) \end{aligned}$$

Properties

1. If f, g are functions, then

$$\nabla(fg) = (\nabla f)g + f\nabla g.$$

2. For any scalars α and β and functions f and g,

$$\nabla(\alpha f + \beta g) = \alpha \nabla f + \beta \nabla g.$$

The ∇ symbolism

Using the ∇ formalism, the divergence operator can be expressed as $\nabla\cdot$, the curl operator as $\nabla\times$, and the Laplacian operator as ∇^2. To wit, for a given vector field

$$\mathbf{A} = A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\,\mathbf{k},$$

and a given function f we have

$$\begin{aligned} \nabla \cdot \mathbf{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\ \nabla \times \mathbf{A} &= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \mathbf{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \mathbf{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \mathbf{k} \\ \nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}. \end{aligned}$$