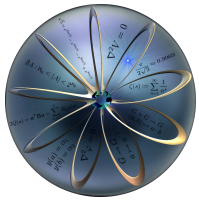

planetmath.org

Math for the people, by the people.

properties of hyperreals under field operations

Canonical name	PropertiesOfHyperrealsUnderFieldOperations
Date of creation	2013-03-22 17:26:19
Last modified on	2013-03-22 17:26:19
Owner	asteroid (17536)
Last modified by	asteroid (17536)
Numerical id	4
Author	asteroid (17536)
Entry type	Result
Classification	msc 26E35

Let ${}^*\mathbb{R}_b$ denote the set of finite (or limited) hyperreal numbers and ${}^*\mathbb{R}_0$ the set of infinitesimal hyperreal numbers.

- We have that

1. ${}^*\mathbb{R}_b$ and ${}^*\mathbb{R}_0$ are subrings of ${}^*\mathbb{R}$.
2. ${}^*\mathbb{R}_0$ is an ideal of ${}^*\mathbb{R}_b$.
3. the sum of an infinite hyperreal with a finite hyperreal is infinite.
4. the inverse of a non-zero infinitesimal hyperreal is infinite.
5. the inverse of an infinite hyperreal is infinitesimal.

The above properties can be described more informally like:

1. $\mathit{finite} + \mathit{finite} = \mathit{finite}$
2. $\mathit{infinitesimal} + \mathit{infinitesimal} = \mathit{infinitesimal}$
3. $\mathit{infinite} + \mathit{finite} = \mathit{infinite}$
4. $\mathit{finite} \times \mathit{finite} = \mathit{finite}$
5. $\mathit{infinitesimal} \times \mathit{finite} = \mathit{infinitesimal}$
6. $\mathit{infinitesimal}^{-1} = \mathit{infinite}$
7. $\mathit{infinite}^{-1} = \mathit{infinitesimal}$