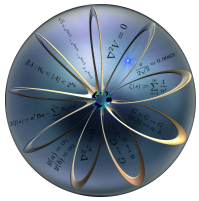

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derivatives of sine and cosine

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The of the derivatives of sine and cosine is a bit simpler by using the prosthaphaeresis formulas

$$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right), \tag{1}$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right). \tag{2}$$

Let x, t be any real numbers such that $t \neq x$. Then we obtain

$$\frac{\sin x - \sin t}{x-t} = \frac{2\sin\left(\frac{x-t}{2}\right)\cos\left(\frac{x+t}{2}\right)}{x-t} = \frac{\sin\left(\frac{x-t}{2}\right)}{\left(\frac{x-t}{2}\right)}\cdot\cos\left(\frac{x+t}{2}\right) \longrightarrow 1\cdot\cos\left(\frac{x+x}{2}\right) = \cos x,$$

as $t \to x$. Here we used the known limit $\lim_{u\to 0}\frac{\sin u}{u} = 1$ (see `http://planetmath.org/LimitOfDisp` entry).

The derivative of cosine is calculated similarly:

$$\frac{\cos x - \cos t}{x-t} = \frac{-2\sin\left(\frac{x+t}{2}\right)\sin\left(\frac{x-t}{2}\right)}{x-t} = -1\cdot\frac{\sin\left(\frac{x-t}{2}\right)}{\left(\frac{x-t}{2}\right)}\cdot\sin\left(\frac{x+t}{2}\right) \longrightarrow -1\cdot 1\cdot\sin\left(\frac{x+x}{2}\right) = -\sin x$$

as $t \to x$.