a special case of partial integration

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In determining the antiderivative of a http://planetmath.org/AlgebraicFunctiontranscende function U whose derivative U' is http://planetmath.org/AlgebraicFunctionalgebraic, the result can be obtained when choosing in the formula

$$\int UV'\,dx = UV - \int VU'\,dx$$

of integration by parts $V' \equiv 1$; then one has

$$\int U\,dx = \int U\cdot 1\,dx \;=\; U\cdot x - \int x\cdot U'\,dx.$$

The functions U in question are mainly the http://planetmath.org/NaturalLogarithm2logarithm the cyclometric functions and the area functions.

Examples.

1. $\displaystyle\int \ln x\,dx = x\ln x - \int x\cdot\frac{1}{x}\,dx = \; x\ln x - x + C$

2. $\displaystyle\int \arcsin x\,dx = x\arcsin x - \int x\frac{1}{\sqrt{1-x^2}}\,dx = x\arcsin x + \frac{1}{2}\int\frac{-2x}{\sqrt{1-x^2}}\,dx$
$= x\arcsin x + \sqrt{1-x^2} + C$

3. $\displaystyle\int \arctan x\,dx = x\arctan x - \int x\cdot\frac{1}{1+x^2}\,dx = x\arctan x - \frac{1}{2}\int\frac{2x}{1+x^2}\,dx$
$\displaystyle= x\arctan x - \frac{1}{2}\ln(1+x^2) + C = \; x\arctan x - \ln\sqrt{1+x^2} + C$

4. $\displaystyle\int \operatorname{arcosh} x\,dx = x\operatorname{arcosh} x - \int x\cdot\frac{1}{\sqrt{x^2-1}}\,dx$
$= x\operatorname{arcosh} x - \sqrt{x^2-1} + C$

The choice $V' \equiv 1$ works as well in such cases as $\int(\ln x)^2\,dx$ and $\int\ln(\ln x)\,dx$, giving respectively $x((\ln x)^2 - 2\ln x + 2) + C$ and $x\ln(\ln x) - \operatorname{Li} x + C$ (see logarithmic integral). Also $\int(\arcsin x)^2\,dx$, requiring two integrations by parts, and giving the result $x(\arcsin x)^2 + 2\sqrt{1-x^2}\arcsin x - 2x + C$.