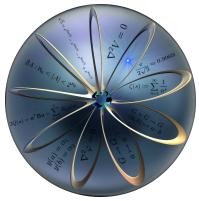

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integral over a period interval

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Theorem. If the real function f is periodic and http://planetmath.org/RiemannIntegrable over a http://planetmath.org/Periodicperiod interval, the value of integral over a period interval is always the same, i.e.

$$\int_a^{a+p} f(x)\,dx \;=\; \int_0^p f(x)\,dx \quad \forall\, a \in \mathbb{R} \tag{1}$$

where p is the period of f.

Proof. The right hand side of the equation (1) is manipulated, with one http://planetmath.org/ChangeOfVariableInDefiniteIntegralsubstitution $x = t+p$:

$$\begin{aligned}
\int_0^p f(x)\,dx &= \int_0^a f(x)\,dx + \int_a^p f(x)\,dx \\
&= \int_0^a f(x)\,dx + \int_a^{a+p} f(x)\,dx - \int_p^{a+p} f(x)\,dx \\
&= \int_0^a f(x)\,dx + \int_a^{a+p} f(x)\,dx - \int_0^a f(t+p)\,dt \\
&= \int_0^a f(x)\,dx + \int_a^{a+p} f(x)\,dx - \int_0^a f(t)\,dt \\
&= \int_a^{a+p} f(x)\,dx
\end{aligned}$$

References

[1] Ernst Lindelöf: *Johdatus korkeampaan analyysiin*. Fourth edition. Werner Söderström Osakeyhtiö, Porvoo ja Helsinki (1956).

[2] *Fråga Lund om matematik*, http://www.maths.lth.se/query/here.