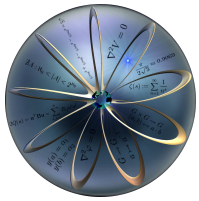

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gradient theorem

Canonical name	GradientTheorem
Date of creation	2013-03-22 19:11:16
Last modified on	2013-03-22 19:11:16
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	8
Author	pahio (2872)
Entry type	Theorem
Classification	msc 26B12
Synonym	fundamental theorem of line integrals
Related topic	LaminarField
Related topic	Gradient

If $u = u(x,\, y,\, z)$ is continuously differentiable function in a simply connected `http://planetmath.org/Domain2`domain D of $\mathbb{R}^3$ and $P = (x_0,\, y_0,\, z_0)$ and $Q = (x_1,\, y_1,\, z_1)$ lie in this domain, then

$$\int_P^Q \nabla u \cdot \vec{ds} \;=\; u(x_1,\, y_1,\, z_1) - u(x_0,\, y_0,\, z_0) \tag{1}$$

where the line integral of the left hand side is taken along an arbitrary path in D.

The equation (1) is illustrated by the fact that

$$\nabla u \cdot \vec{ds} \;=\; \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

is the total differential of u, and thus

$$\int_P^Q \nabla u \cdot \vec{ds} \;=\; \int_P^Q du.$$