logarithmically convex function

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Definition. A function $f\colon [a,b] \to \mathbb{R}$ such that $f(x) > 0$ for all x is said to be *logarithmically convex* if $\log f(x)$ is a convex function.

It is easy to see that a logarithmically convex function is a convex function, but the converse is not true. For example $f(x) = x^2$ is a convex function, but $\log f(x) = \log x^2 = 2\log x$ is not a convex function and thus $f(x) = x^2$ is not logarithmically convex. On the other hand e^{x^2} is logarithmically convex since $\log e^{x^2} = x^2$ is convex. A less trivial example of a logarithmically convex function is the gamma function, if restricted to the positive reals.

The definition is easily extended to functions $f\colon U \subset \mathbb{R} \to \mathbb{R}$, for any connected set U (where still we have $f > 0$), in the obvious way. Such a function is logarithmically convex if it is logarithmically convex on all intervals $[a,b] \subset U$.

References

[1] John B. Conway. . Springer-Verlag, New York, New York, 1978.