Landau notation

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Given two functions f and g from $\mathbb{R}^+$ to $\mathbb{R}^+$, the notation

$$f = O(g)$$

means that the ratio $\frac{f(x)}{g(x)}$ stays bounded as $x \to \infty$. If moreover that ratio approaches zero, we write

$$f = o(g).$$

It is legitimate to write, say, $2x = O(x) = O(x^2)$, with the understanding that we are using the equality sign in an unsymmetric (and informal) way, in that we do not have, for example, $O(x^2) = O(x)$.

The notation

$$f = \Omega(g)$$

means that the ratio $\frac{f(x)}{g(x)}$ is bounded away from zero as $x \to \infty$, or equivalently $g = O(f)$.

If both $f = O(g)$ and $f = \Omega(g)$, we write $f = \Theta(g)$.

One more notational convention in this group is

$$f(x) \sim g(x),$$

meaning $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$.

In analysis, such notation is useful in describing error `http://planetmath.org/AsymptoticEst`
For example, the Riemann hypothesis is equivalent to the conjecture

$$\pi(x) = \operatorname{li} x + O(\sqrt{x} \log x),$$

where $\operatorname{li} x$ denotes the logarithmic integral.

Landau notation is also handy in applied mathematics, e.g. in describing the time complexity of an algorithm. It is common to say that an algorithm requires $O(x^3)$ steps, for example, without needing to specify exactly what is a step; for if $f = O(x^3)$, then $f = O(Ax^3)$ for any positive constant A.