semi-continuous

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A real function $f : A \to \mathbb{R}$, where $A \subseteq \mathbb{R}$ is said to be *lower semi-continuous* in x_0 if

$$\forall \varepsilon > 0 \; \exists \delta > 0 \; \forall x \in A \; |x - x_0| < \delta \Rightarrow f(x) > f(x_0) - \varepsilon,$$

and f is said to be *upper semi-continuous* if

$$\forall \varepsilon > 0 \; \exists \delta > 0 \; \forall x \in A \; |x - x_0| < \delta \Rightarrow f(x) < f(x_0) + \varepsilon.$$

Remark A real function is continuous in x_0 if and only if it is both upper and lower semicontinuous in x_0.

We can generalize the definition to arbitrary topological spaces as follows.

Let A be a topological space. $f : A \to \mathbb{R}$ is lower semicontinuous at x_0 if, for each $\varepsilon > 0$ there is a neighborhood U of x_0 such that $x \in U$ implies $f(x) > f(x_0) - \varepsilon$.

Theorem Let $f : [a, b] \to \mathbb{R}$ be a lower (upper) semi-continuous function. Then f has a minimum (maximum) in $[a, b]$.