antipodal isothermic points

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Assume that the momentary temperature on any great circle of a sphere varies http://planetmath.org/Continuouscontinuously. Then there exist two diametral points (i.e. antipodal points, end points of a certain http://planetmath.org/Diameterdiametre) having the same temperature.

Proof. Denote by x the distance of any point P measured in a certain direction along the great circle from a and let $T(x)$ be the temperature in P. Then we have a continuous (and http://planetmath.org/PeriodicFunctionsperiodic) real function T defined for $x \geqq 0$ satisfying $T(x+p) = T(x)$ where p is the perimetre of the circle. Then also the function f defined by

$$f(x) \ := \ T\left(x+\frac{p}{2}\right) - T(x),$$

i.e. the temperature difference in two antipodic (diametral) points of the great circle, is continuous. We have

$$f\left(\frac{p}{2}\right) \ = \ T(p) - T\left(\frac{p}{2}\right) = T(0) - T\left(\frac{p}{2}\right) = -f(0). \tag{1}$$

If f happens to vanish in $x = 0$, then the temperature is the same in $x = \frac{p}{2}$ and the assertion proved. But if $f(0) \neq 0$, then by (1), the values of f in $x = 0$ and in $x = \frac{p}{2}$ have opposite signs. Therefore, by Bolzano's theorem, there exists a point ξ between 0 and $\frac{p}{2}$ such that $f(\xi) = 0$. Thus the temperatures in ξ and $\xi+\frac{\pi}{2}$ are the same.

Reference: http://www.maths.lth.se/query/Fråga Lund om matematik, 6 april 2006