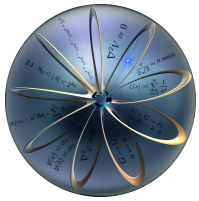

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addition and subtraction formulas for sine and cosine

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The rotation matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ will be used to obtain the addition formulas for sine and cosine.

Recall that a vector in $\mathbb{R}^2$ can be rotated θ radians in the counterclockwise direction by multiplying on the left by the rotation matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. Because rotating by $\alpha+\beta$ radians is the same as rotating by β radians followed by rotating by α radians, we obtain:

$$\begin{pmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & -\cos\alpha\sin\beta - \sin\alpha\cos\beta \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta \end{pmatrix}$$

Hence, $\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta$ and $\cos(\alpha+\beta) = \cos\alpha\cos\beta-\sin\alpha\sin\beta$.

Note that sine is an even function and that cosine is an odd function, `http://planetmath.org/Iei`.e. $\sin(-x) = -\sin x$ and $\cos(-x) = -\cos x$. These facts enable us to obtain the subtraction formulas for sine and cosine.

$$\sin(\alpha-\beta) = \sin(\alpha+(-\beta)) = \sin(\alpha)\cos(-\beta)+\cos(\alpha)\sin(-\beta) = \sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)$$

$$\cos(\alpha-\beta) = \cos(\alpha+(-\beta)) = \cos(\alpha)\cos(-\beta)-\sin(\alpha)\sin(-\beta) = \cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$$