intermediate value theorem

Canonical name	IntermediateValueTheorem
Date of creation	2013-03-22 11:51:29
Last modified on	2013-03-22 11:51:29
Owner	yark (2760)
Last modified by	yark (2760)
Numerical id	15
Author	yark (2760)
Entry type	Theorem
Classification	msc 26A06
Classification	msc 70F25
Classification	msc 17B50
Classification	msc 81-00
Related topic	RollesTheorem
Related topic	MeanValueTheorem
Related topic	Continuous

If f is a real-valued continuous function on the interval $[a, b]$, and x_1 and x_2 are points with $a \leq x_1 < x_2 \leq b$ such that $f(x_1) \neq f(x_2)$, then for every y strictly between $f(x_1)$ and $f(x_2)$ there is a $c \in (x_1, x_2)$ such that $f(c) = y$.

Bolzano's theorem is a special case of this.

The theorem can be generalized as follows: If f is a real-valued continuous function on a connected topological space X, and $x_1, x_2 \in X$ with $f(x_1) \neq f(x_2)$, then for every y between $f(x_1)$ and $f(x_2)$ there is a $\xi \in X$ such that $f(\xi) = y$. (However, this "generalization" is essentially trivial, and in order to derive the intermediate value theorem from it one must first establish the less trivial fact that $[a, b]$ is connnected.) This result remains true if the codomain is an arbitrary ordered set with its order topology; see the entry `http://planetmath.org/ProofOfGeneralizedIntermediateValueTheorem`proof of generalized intermediate value theorem for a proof.