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chain rule

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Let f, g be differentiable, real-valued functions such that g is defined on an open set $I \subseteq \mathbb{R}$, and f is defined on $g(I)$. Then the derivative of the composition $f \circ g$ is given by the *chain rule*, which asserts that

$$(f \circ g)'(x) = (f' \circ g)(x)\, g'(x), \quad x \in I.$$

The chain rule has a particularly suggestive appearance in terms of the Leibniz formalism. Suppose that z depends differentiably on y, and that y in turn depends differentiably on x. Then we have

$$\frac{dz}{dx} = \frac{dz}{dy}\,\frac{dy}{dx}.$$

The apparent cancellation of the dy term is at best a formal mnemonic, and does not constitute a rigorous proof of this result. Rather, the Leibniz format is well suited to the interpretation of the chain rule in terms of related rates. To wit:

> *The instantaneous rate of change of z relative to x is equal to the rate of change of z relative to y times the rate of change of y relative to x.*