another proof of Young inequality

Let
$$F(x) = \int_0^x \phi(t)dt, \text{ and } G(x) = \int_0^x \phi^{-1}(t)dt.$$

Since ϕ^{-1} is strictly increasing, G is strictly convex, hence lies above its supporting line, i.e. for every c and $x \neq c$

$$G(b) > G(c) + G'(c)(b - c) = G(c) + \phi^{-1}(c)(b - c).$$

In particular, for $c = \phi(a)$ we have

$$F(a) + G(b) > F(a) + G(\phi(a)) + a(b - \phi(a)) = ab,$$

because $F(a) + G(\phi(a)) = a\phi(a)$.