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proof of Schur's inequality

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By Schur's theorem, a unitary matrix U and an upper triangular matrix T exist such that $A = UTU^H$, T being diagonal if and only if A is normal. Then $A^H A = UT^H U^H UTU^H = UT^H TU^H$, which means $A^H A$ and $T^H T$ are similar; so they have the same trace. We have:

$\|A\|_F^2 = \mathrm{Tr}(A^H A) = \mathrm{Tr}(T^H T) = \sum_{i=1}^n |\lambda_i|^2 + \sum_{i<j} |t_{ij}|^2 =$

$= \mathrm{Tr}(D^H D) + \sum_{i<j} |t_{ij}|^2 \geq \mathrm{Tr}(D^H D) = \|D\|_F^2$.

If and only if A is normal, $T = D$ and therefore equality holds.$\square$