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rational function

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A real function $R(x)$ of a single variable x is called if it can be written as a quotient

$$R(x) = \frac{P(x)}{Q(x)},$$

where $P(x)$ and $Q(x)$ are polynomials in x with real coefficients. When one is only interested in algebraic properties of $R(x)$ or $P(x)$ and $Q(x)$, it is convenient to forget that they define functions and simply treat them as algebraic expressions in x. In this case $R(x)$ is referred to as a *rational expression*.

In general, a rational function (expression) $R(x_1, \ldots, x_n)$ has the form

$$R(x_1, \ldots, x_n) = \frac{P(x_1, \ldots, x_n)}{Q(x_1, \ldots, x_n)},$$

where $P(x_1, \ldots, x_n)$ and $Q(x_1, \ldots, x_n)$ are polynomials in the variables $(x_1, \ldots, x_n)$ with coefficients in some field or ring S.

In this sense, $R(x_1, \ldots, x_n)$ can be regarded as an element of the fraction field $S(x_1, \ldots, x_n)$ of the polynomial ring $S[x_1, \ldots, x_n]$.