alternative proof of derivative of x^n

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The typical derivative formula

$$\frac{df}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

combined with the binomial theorem yield an alternative way to prove that

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

for any positive integer n.

Proof.

$$\begin{aligned}
\frac{d}{dx}(x^n) &= \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} \\
&= \lim_{h\to 0} \frac{\left(\sum_{j=0}^{n} \binom{n}{j} x^j h^{n-j}\right) - x^n}{h} \\
&= \lim_{h\to 0} \frac{x^n + nx^{n-1}h + h^2 \left(\sum_{j=0}^{n-2} \binom{n}{j} x^j h^{n-2-j}\right) - x^n}{h} \\
&= \lim_{h\to 0} \frac{nx^{n-1}h + h^2 \sum_{j=0}^{n-2} \binom{n}{j} x^j h^{n-2-j}}{h} \\
&= \lim_{h\to 0} \left(nx^{n-1} + h \sum_{j=0}^{n-2} \binom{n}{j} x^j h^{n-2-j}\right) \\
&= nx^{n-1}
\end{aligned}$$

□