proof of product rule

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We begin with two differentiable functions $f(x)$ and $g(x)$ and show that their product is differentiable, and that the derivative of the product has the desired form.

By simply calculating, we have for all values of x in the domain of f and g that

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}[f(x)g(x)] &= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\
&= \lim_{h \to 0} \frac{f(x+h)g(x+h) + f(x+h)g(x) - f(x+h)g(x) - f(x)g(x)}{h} \\
&= \lim_{h \to 0} \left[f(x+h)\frac{g(x+h) - g(x)}{h} + g(x)\frac{f(x+h) - f(x)}{h} \right] \\
&= \lim_{h \to 0} \left[f(x+h)\frac{g(x+h) - g(x)}{h} \right] + \lim_{h \to 0} \left[g(x)\frac{f(x+h) - f(x)}{h} \right] \\
&= f(x)g'(x) + f'(x)g(x).
\end{aligned}$$

The key argument here is the next to last line, where we have used the fact that both f and g are differentiable, hence the limit can be distributed across the sum to give the desired equality.