harmonic number

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Defines	harmonic number of order

The *harmonic number of order n of θ* is defined as

$$H_\theta(n) = \sum_{i=1}^{n} \frac{1}{i^\theta}$$

Note that n may be equal to ∞, provided $\theta > 1$.

If $\theta \leq 1$, while $n = \infty$, the harmonic series does not converge and hence the harmonic number does not exist.

If $\theta = 1$, we may just write $H_\theta(n)$ as H_n (this is a common notation).

- If $\Re(\theta) > 1$ and $n = \infty$ then the sum is the Riemann zeta function.
- If $\theta = 1$, then we get what is known simply as"the harmonic number", and it has many important properties. For example, it has asymptotic expansion $H_n = \ln n + \gamma + \frac{1}{2m} + \dots$ where γ is Euler's constant.
- It is possible[1] to define harmonic numbers for non-integral n. This is done by means of the series $H_n(z) = \sum_{n \geq 1}(n^{-z} - (n+x)^{-z})$.

[1]See "The Art of computer programming" vol. 2 by D. Knuth