Landau kernel

Canonical name	LandauKernel
Date of creation	2013-03-22 14:11:38
Last modified on	2013-03-22 14:11:38
Owner	mathwizard (128)
Last modified by	mathwizard (128)
Numerical id	7
Author	mathwizard (128)
Entry type	Definition
Classification	msc 26A30

For $k \in \mathbb{N}$ the *Landau kernel* $L_k(t)$ is defined as

$$L_k = \begin{cases} \frac{1}{c_k}(1-t^2)^k & \text{if } t \in [-1,1] \\ 0 & \text{otherwise} \end{cases}$$

with

$$c_k := \int_{-1}^{1} (1-t^2)^k dt.$$

L_k is nonnegative and continuous on $\mathbb{R}$. Due to the choice of c_k we have

$$\int_{-\infty}^{\infty} L_k(t) dt = 1.$$

Also we have for all positive, real r:

$$\begin{aligned} \int_{\mathbb{R}\setminus[-r,r]} L_k(t) dt &\leq \frac{2}{c_k} \int_{r}^{1} (1-t^2)^k dt \\ &\leq (k+1)(1-r^2)^k. \end{aligned}$$

Therefore $(L_k)_{k\in\mathbb{N}}$ is a Dirac sequence.