logarithm

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Defines	ln
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Definition. Three real numbers x, y, p, with $x, y > 0$ and $x \neq 1$, are said to obey the logarithmic relation

$$\log_x(y) = p$$

if they obey the corresponding exponential relation:

$$x^p = y.$$

Note that by the monotonicity and continuity property of the exponential operation, for given x and y there exists a unique p satisfying the above relation. We are therefore able to says that p *is the logarithm of* y *relative to the base* x.

Properties. There are a number of basic algebraic identities involving logarithms.

$$\begin{aligned}
\log_x(yz) &= \log_x(y) + \log_x(z) \\
\log_x\left(\frac{y}{z}\right) &= \log_x(y) - \log_x(z) \\
\log_x(y^z) &= z\log_x(y) \\
\log_x(1) &= 0 \\
\log_x(x) &= 1 \\
\log_x(y)\log_y(x) &= 1 \\
\log_y(z) &= \frac{\log_x(z)}{\log_x(y)}
\end{aligned}$$

By the very first identity, any logarithm `http://planetmath.org/RestrictionOfAFunctionres` to the set of positive integers is an additive function.

Notes. In essence, logarithms convert multiplication to addition, and exponentiation to multiplication. Historically, these properties of the logarithm made it a useful tool for doing numerical calculations. Before the advent of electronic calculators and computers, tables of logarithms and the logarithmic slide rule were essential computational aids.

Scientific applications predominantly make use of logarithms whose base is the Eulerian number $e = 2.71828\ldots$. Such logarithms are called *natural logarithms* and are commonly denoted by the symbol ln, e.g.

$$\ln(e) = 1.$$

Natural logarithms naturally give rise to the natural logarithm function.

A frequent convention, seen in elementary mathematics texts and on calculators, is that logarithms that do not give a base explicitly are assumed to be base 10, e.g.

$$\log(100) = 2.$$

This is far from . In Rudin's "Real and Complex analysis", for example, we see a baseless log used to refer to the natural logarithm. By contrast, computer science and information theory texts often assume 2 as the default logarithm base. This is motivated by the fact that $\log_2(N)$ is the approximate number of bits required to encode N different messages.

The invention of logarithms is commonly credited to John Napier [http://www-groups.dcs.st-