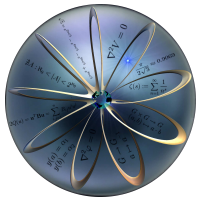

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proof of least and greatest value of function

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f is continuous, so it will transform compact sets into compact sets. Thus since $[a, b]$ is compact, $f([a, b])$ is also compact. f will thus attain on the interval $[a, b]$ a maximum and a minimum value because real compact sets are closed and bounded.

Consider the maximum and later use the same argument for $-f$ to consider the minimum.

By a known `http://planetmath.org/FermatsTheoremStationaryPoints`theorem if the maximum is attained in the interior of the domain, $c \in]a, b[$ then $f(c)$is a maximum $\implies f'(c) = 0$, since f is differentiable.

If the maximum isn't attained in $]a, b[$ and since it must be attained in $[a, b]$ either $f(a)$ or $f(b)$ is a maximum.

For the minimum consider $-f$ and note that $-f$ will verify all conditions of the theorem and that a maximum of $-f$ corresponds to a minimum of f and that $-f'(c) = 0 \iff f'(c) = 0$.