tangent of halved angle

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The formulae

$$\cos 2\alpha = 1 - 2\sin^2\alpha,$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

may be solved for $\sin\alpha$ and $\cos\alpha$, respectively. One gets the equations

$$\sin\alpha = \pm\sqrt{\frac{1-\cos 2\alpha}{2}}, \quad \cos\alpha = \pm\sqrt{\frac{1+\cos 2\alpha}{2}},$$

where the signs have to be chosen according to the quadrant where the angle α is. Changing α to $\frac{x}{2}$ and dividing these equations gives us the formula

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}. \tag{1}$$

Also here one must chose the sign according to the quadrant of $\dfrac{x}{2}$.

We obtain two alternative forms of (1) when we multiply both the numerator and the denominator of the radicand the first time by $1-\cos x$ and the second time by $1+\cos x$; note that $1-\cos^2 x = \sin^2 x$:

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}, \tag{2}$$

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} \tag{3}$$

Here, $\sin x$ determines the sign of the hand sides; it can be justified that it has always the same sign as $\tan\frac{x}{2}$.