sources and sinks of vector field

Canonical name	SourcesAndSinksOfVectorField
Date of creation	2013-03-22 18:47:03
Last modified on	2013-03-22 18:47:03
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	12
Author	pahio (2872)
Entry type	Definition
Classification	msc 26B15
Classification	msc 26B12
Related topic	Divergence
Related topic	SolenoidalField
Related topic	CirculationAndVorticity
Defines	source
Defines	sink
Defines	source of vector field
Defines	productivity
Defines	strength
Defines	source density

Let the vector field $\vec{U}$ of $\mathbb{R}^3$ be interpreted, as in the remark of the `http://planetmath.org/Flux`parent entry, as the velocity of a liquid. Then the flux

$$\oint_a \vec{U} \cdot d\vec{a}$$

of $\vec{U}$ through a closed surface a expresses how much more liquid per time-unit it comes from inside of a to outside than contrarily. Since for a usual incompressible liquid, the outwards flow and the inwards flow are equal, we must think in the case that the flux differs from 0 either that the flowing liquid is suitably compressible or that there are inside the surface some *sources* creating liquid and *sinks* annihilating liquid. Ordinarily, one uses the latter idea. Both the sources and the sinks may be called sources, when the sinks are *negative sources*. The flux of the vector $\vec{U}$ through a is called the *productivity* or the *strength* of the sources inside a.

For example, the sources and sinks of an electric field ($\vec{E}$) are the locations containing positive and negative charges, respectively. The gravitational field has only sinks, which are the locations containing .

The expression

$$\frac{1}{\Delta v} \oint_{\partial \Delta v} \vec{U} \cdot d\vec{a},$$

where Δv means a region in the vector field and also its volume, is the productivity of the sources in Δv per a volume-unit. When we let Δv to shrink towards a point P in it, to an infinitesimal volume-element dv, we get the limiting value

$$\varrho \;:=\; \frac{1}{dv} \oint_{\partial dv} \vec{U} \cdot d\vec{a}, \tag{1}$$

called the *source density* in P. Thus the productivity of the source in P is $\varrho\, dv$. If $\varrho = 0$, there is in P neither a source, nor a sink.

The Gauss's theorem

$$\int_v \nabla \cdot \vec{U}\, dv \;=\; \oint_a \vec{U} \cdot d\vec{a}$$

applied to dv says that

$$\nabla \cdot \vec{U} \;=\; \frac{1}{dv} \oint_{\partial dv} \vec{U} \cdot d\vec{a}. \tag{2}$$

Accordingly,

$$\varrho = \nabla \cdot \vec{U} \tag{3}$$

and

$$\oint_a \vec{U} \cdot d\vec{a} = \int_v \varrho \, dv. \tag{4}$$

This last can be read that *the flux of the vector through a closed surface equals to the total productivity of the sources inside the surface.* For example, if $\vec{U}$ is the electric flux density $\vec{D}$, (4) means that the electric flux through a closed surface equals to the total charge inside.