binomial proof of positive integer power rule

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We will use the difference quotient in this proof of the power rule for positive integers. Let $f(x) = x^n$ for some integer $n \geq 0$. Then we have

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}.$$

We can use the binomial theorem to expand the numerator

$$f'(x) = \lim_{h \to 0} \frac{C_0^n x^0 h^n + C_1^n x^1 h^{n-1} + \cdots + C_{n-1}^n x^{n-1} h^1 + C_n^n x^n h^0 - x^n}{h}$$

where $C_k^n = \frac{n!}{k!(n-k)!}$. We can now simplify the above

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{h^n + nxh^{n-1} + \cdots + nx^{n-1}h + x^n - x^n}{h} \\
&= \lim_{h \to 0} (h^{n-1} + nxh^{n-2} + \cdots + nx^{n-1}) \\
&= nx^{n-1} \\
&= nx^{n-1}.
\end{aligned}$$