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derivative of logarithm with respect to base

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The

$$\frac{\partial}{\partial a} \log_a x = -\frac{\ln x}{(\ln a)^2 a} \quad (1)$$

for the partial derivative of logarithm expression with respect to the base a may be derived by denoting first

$$\log_a x = y.$$

By the definition of logarithm, this equation means the same as

$$a^y = x,$$

where we can take the natural logarithms

$$y \ln a = \ln x$$

solving then

$$y = \frac{\ln x}{\ln a}.$$

Then, the differentiation is easy:

$$\frac{\partial y}{\partial a} = \frac{0 \ln a - \frac{1}{a} \ln x}{(\ln a)^2} = -\frac{\ln x}{(\ln a)^2 a}.$$