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power function

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A real *power function*  $f: \mathbb{R}_+ \rightarrow \mathbb{R}$  has the form

$$f(x) = x^a$$

where  $a$  is a given real number.

**Theorem.** The power function  $x \mapsto x^a$  is differentiable with the derivative  $x \mapsto ax^{a-1}$  and strictly increasing if  $a > 0$  and strictly decreasing if  $a < 0$  (and 1 if  $a = 0$ ).

The power functions comprise the *natural power functions*  $x \mapsto x^n$  with  $n = 0, 1, 2, \dots$ , the *root functions*  $x \mapsto \sqrt[n]{x} = x^{\frac{1}{n}}$  with  $n = 1, 2, 3, \dots$  and other *fraction power functions*  $x \mapsto x^a$  with  $a$  any fractional number.

**Note.** The power  $x^a$  may of course be meaningful also for other than positive values of  $x$ , if  $a$  is an integer. On the other hand, e.g.  $(-1)^{\sqrt{2}}$  has no real values — see the general power.