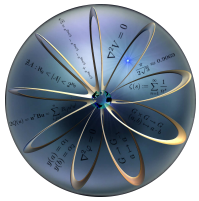

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uniform continuity of Lipschitz functions

Canonical name	UniformContinuityOfLipschitzFunctions
Date of creation	2013-03-22 15:06:16
Last modified on	2013-03-22 15:06:16
Owner	paolini (1187)
Last modified by	paolini (1187)
Numerical id	8
Author	paolini (1187)
Entry type	Theorem
Classification	msc 26A16

Proposition 1. *An Hölder continuous mapping is uniformly continuous. In particular any Lipschitz continuous mapping is uniformly continuous.*

Proof. Let $f\colon X \to Y$ be a mapping such that for some $C > 0$ and α with $0 < \alpha \leq 1$ one has

$$d_Y(f(p), f(q)) \leq C d_X(p, q)^\alpha.$$

For every given $\epsilon > 0$, choose $\delta = (\epsilon/(C+1))^{\frac{1}{\alpha}}$. If $p, q \in X$ are given points satisfying

$$d_X(p, q) < \delta$$

then

$$d_Y(f(p), f(q)) \leq C\delta^\alpha \leq C\frac{\epsilon}{C+1} < \epsilon,$$

as desired. □