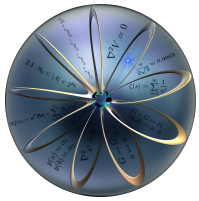

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equivalent formulations for continuity

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Suppose $f\colon X \to Y$ is a function between topological spaces X, Y. Then the following are equivalent:

1. f is continuous.
2. If B is open in Y, then $f^{-1}(B)$ is open in X.
3. If B is closed in Y, then $f^{-1}(B)$ is closed in X.
4. $f\big(\overline{A}\big) \subseteq \overline{f(A)}$ for all $A \subseteq X$.
5. If (x_i) is a net in X converging to x, then $(f(x_i))$ is a net in Y converging to $f(x)$. The concept of net can be replaced by the more familiar one of sequence if the spaces X and Y are first countable.
6. Whenever two nets S and T in X converge to the same point, then $f \circ S$ and $f \circ T$ converge to the same point in Y.
7. If $\mathbb{F}$ is a filter on X that converges to x, then $f(\mathbb{F})$ is a filter on Y that converges to $f(x)$. Here, $f(\mathbb{F})$ is the filter generated by the filter base $\{f(F) \mid F \in \mathbb{F}\}$.
8. If B is any element of a `http://planetmath.org/Subbasis`subbase $\mathcal{S}$ for the topology of Y, then $f^{-1}(B)$ is open in X.
9. If B is any element of a basis $\mathcal{B}$ for the topology of Y, then $f^{-1}(B)$ is open in X.
10. If $x \in X$, and N is any neighborhood of $f(x)$, then $f^{-1}(N)$ is a neighborhood of x.
11. f is continuous at every point in X.