average value of function

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The set of the values of a real function f defined on an interval $[a,\, b]$ is usually uncountable, and therefore for being able to speak of an *average value* of f in the sense of the average value

$$A.V. \;=\; \frac{a_1 + a_2 + \ldots + a_n}{n} \;=\; \frac{\sum_{j=1}^{n} a_j}{\sum_{j=1}^{n} 1} \tag{1}$$

of a finite list $a_1,\, a_2,\, \ldots,\, a_n$ of numbers, one has to replace the sums with integrals. Thus one could define

$$A.V.(f) \;:=\; \frac{\int_a^b f(x)\,dx}{\int_a^b 1\,dx},$$

i.e.

$$A.V.(f) \;:=\; \frac{1}{b-a}\int_a^b f(x)\,dx. \tag{2}$$

For example, the average value of x^2 on the interval $[0,\, 1]$ is $\frac{1}{3}$ and the average value of $\sin x$ on the interval $[0,\, \pi]$ is $\frac{2}{\pi}$.

The definition (2) may be extended to complex functions f on an arc γ of a rectifiable curve via the contour integral

$$A.V.(f) \;:=\; \frac{1}{l(\gamma)}\int_\gamma f(z)\,dz \tag{3}$$

where $l(\gamma)$ is the `http://planetmath.org/ArcLength`length of the arc. If especially γ is a closed curve in a simply connected domain where f is analytic, then the average value of f on γ is always 0, as the Cauchy integral theorem implies.