fundamental theorem of calculus

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Let $f\colon [a,b] \to \mathbf{R}$ be a continuous function, let $c \in [a,b]$ be given and consider the integral function F defined on $[a,b]$ as

$$F(x) = \int_c^x f(t)\,dt.$$

Then F is an antiderivative of f that is, F is differentiable in $[a,b]$ and

$$F'(x) = f(x) \qquad \forall x \in [a,b].$$

The previous relation rewritten as

$$\frac{d}{dx}\int_c^x f(t)\,dt = f(x)$$

shows that the differentiation operator $\frac{d}{dx}$ is the inverse of the integration operator $\int_c^x$. This formula is sometimes called Newton-Leibniz formula.

On the other hand if $f\colon [a,b] \to \mathbf{R}$ is a continuous function and $G\colon [a,b] \to \mathbf{R}$ is any antiderivative of f, i.e. $G'(x) = f(x)$ for all $x \in [a,b]$, then

$$\int_a^b f(t)\,dt = G(b) - G(a). \tag{1}$$

This shows that up to a constant, the integration operator is the inverse of the derivative operator:

$$\int_a^x DG = G - G(a).$$

Notes

Equation (**??**) is sometimes called “Barrow’s rule” or “Barrow’s formula”.