derivatives of hyperbolic functions

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In this entry we compute the derivative of the hyperbolic functions $\sinh(x)$ and $\cosh(x)$.

Recall that by definition:

$$\begin{aligned} \sinh(x) &:= \frac{e^x - e^{-x}}{2} \\ \cosh(x) &:= \frac{e^x + e^{-x}}{2}. \end{aligned}$$

Therefore:

$$\begin{aligned} \frac{d}{dx}\sinh(x) &= \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) \\ &= \frac{1}{2}\cdot\frac{d}{dx}\left(e^x - e^{-x}\right) \\ &= \frac{1}{2}\cdot\left(e^x - (-e^{-x})\right) \\ &= \frac{e^x + e^{-x}}{2} \\ &= \cosh(x). \end{aligned}$$

Similarly $\frac{d}{dx}\cosh(x) = \sinh(x)$. Using the quotient rule, we compute the derivative of $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$:

$$\frac{d}{dx}\tanh(x) = \frac{\cosh^2(x) - \sinh^2(x)}{\cosh^2(x)} = \frac{1}{\cosh^2(x)}$$

where we have used the equality $\cosh^2(x) - \sinh^2(x) = 1$.