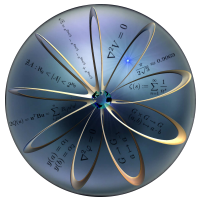

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## arc length

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|---|---|
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| Related topic | Evolute2 |
| Related topic | Cycloid |

*Arclength* is the  of a section of a differentiable curve. Finding the length of an arc is useful in many applications, for the length of a curve can represent distance traveled, work, etc. It is commonly represented as $S$ or the differential $ds$ if one is differentiating or integrating with respect to change in arclength.

If one knows the vector function or parametric equations of a curve, finding the arclength is , as it can be given by the sum of the lengths of the tangent vectors to the curve or

$$\int_a^b |\vec{F}'(t)|\, dt = S$$

Note that $t$ is an independent parameter. In Cartesian coordinates, arclength can be calculated by the formula

$$S = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

This formula is derived by viewing arclength as the Riemann sum

$$\lim_{\Delta x \to \infty} \sum_{i=1}^{n} \sqrt{1 + f'(x_i)^2}\, \Delta x$$

The term being summed is the length of an approximating secant to the curve over the distance $\Delta x$. As $\Delta x$ vanishes, the sum approaches the arclength, as desired. Arclength can also be derived for polar coordinates from the general formula for vector functions given above. The result is

$$L = \int_a^b \sqrt{r(\theta)^2 + (r'(\theta))^2}\, d\theta$$