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## another proof of Young inequality

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Let

$$F(x) = \int_0^x \phi(t)dt, \text{ and } G(x) = \int_0^x \phi^{-1}(t)dt.$$

Since  $\phi^{-1}$  is strictly increasing,  $G$  is strictly convex, hence lies above its supporting line, i.e. for every  $c$  and  $x \neq c$

$$G(b) > G(c) + G'(c)(b - c) = G(c) + \phi^{-1}(c)(b - c).$$

In particular, for  $c = \phi(a)$  we have

$$F(a) + G(b) > F(a) + G(\phi(a)) + a(b - \phi(a)) = ab,$$

because  $F(a) + G(\phi(a)) = a\phi(a)$ .