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generalized Darboux function

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Recall that a function $f : I \to \mathbb{R}$, where $I \subseteq \mathbb{R}$ is an interval, is called a **Darboux function** if it satisfies the intermediate value theorem. This means, that if $a, b \in I$ and $f(a) \leqslant d \leqslant f(b)$ for some $d \in \mathbb{R}$, then there exists $c \in I$ such that $a \leqslant c \leqslant b$ and $f(c) = d$.

Darboux proved (see parent object) that if $f : [a, b] \to \mathbb{R}$ is differentiable then f' is a Darboux function. The class of Darboux functions is very wide. It can be shown that any function $f : \mathbb{R} \to \mathbb{R}$ can be written as a sum of two Darboux functions. We wish to give more general definiton of Darboux function.

Definition. Let X, Y be topological spaces. Function $f : X \to Y$ is called a **(generalized) Darboux function** if and only if whenever $C \subseteq X$ is a connected subset, then so is $f(C) \subseteq Y$.

It can be easily proved that connected subsets of intervals (in $\mathbb{R}$) are exactly intervals. Thus this definition coincides with classical definiton, when X is an interval and $Y = \mathbb{R}$.

Note that every continuous map is a Darboux function.

Also the composition of Darboux functions is again a Darboux function and thus the class of all topological spaces, together with Darboux functions forms a category. The category of topological spaces and continuous maps is its subcategory.