implicit function theorem

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Theorem. *Let Ω be an open subset of $\mathbb{R}^n \times \mathbb{R}^m$ and let $f \in C^1(\Omega, \mathbb{R}^m)$. Let $(x_0, y_0) \in \Omega \subset \mathbb{R}^n \times \mathbb{R}^m$. If the matrix $D_y f(x_0, y_0)$ defined by*

$$D_y f(x_0, y_0) = \left(\frac{\partial f_j}{\partial y_k}(x_0, y_0) \right)_{j,k} \qquad j = 1, \ldots, m \quad k = 1, \ldots, m$$

is invertible, then there exists a neighborhood $U \subset \mathbb{R}^n$ of x_0 and a function $g \in C^1(U, \mathbb{R}^m)$ such that

$$f(x, g(x)) = f(x_0, y_0) \qquad \forall x \in U.$$

Moreover

$$Dg(x) = -(D_y f(x, g(x)))^{-1} D_x f(x, g(x)).$$