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integration by parts

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When we want to integrate a product of two functions, it is sometimes preferable to simplify the integrand by integrating one of the functions and differentiating the other. This process is called integrating by parts, and is done in the following way, where u and v are functions of x.

$$\int u \cdot v' \, dx = u \cdot v - \int v \cdot u' \, dx$$

This process may be repeated indefinitely, and in some cases it may be used to solve for the original integral algebraically. For definite integrals, the rule appears as

$$\int_a^b u(x) \cdot v'(x) \, dx = (u(b) \cdot v(b) - u(a) \cdot v(a)) - \int_a^b v(x) \cdot u'(x) \, dx$$

Proof: Integration by parts is simply the antiderivative of a product rule. Let $G(x) = u(x) \cdot v(x)$. Then,

$$G'(x) = u'(x)v(x) + u(x)v'(x)$$

Therefore,

$$G'(x) - v(x)u'(x) = u(x)v'(x)$$

We can now integrate both sides with respect to x to get

$$G(x) - \int v(x)u'(x) \, dx = \int u(x)v'(x) \, dx$$

which is just integration by parts rearranged.
Example: We integrate the function $f(x) = x \sin x$: Therefore we define $u(x) := x$ and $v'(x) = \sin x$. So integration by parts yields us:

$$\int x \sin x \, dx = -x \cos x + \int \cos x \, dx = -x \cos x + \sin x + C,$$

where C is an arbitrary constant.