least and greatest value of function

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Theorem. If the real function f is

1. continuous on the closed interval $[a,\, b]$ and

2. differentiable on the open interval $(a,\, b)$,

then the function has on the interval $[a,\, b]$ a least value and a greatest value. These are always got in the end of the interval or in the zero of the derivative.

Remark 1. If the preconditions of the theorem are fulfilled by a function f, then one needs only to determine the values of f in the end points a and b of the interval and in the zeros of the derivative f' inside the interval; then the least and the greatest value are found among those values.

Remark 2. Note that the theorem does not require anything of the derivative f' in the points a and b; one needs not even the right-sided derivative in a or the left-sided derivative in b. Thus e.g. the function $f\, :\, x \mapsto \sqrt{1 - x^2}$, fulfilling the conditions of the theorem on the interval $[-1,\, 1]$ but not having such one-sided derivatives, gains its least value in the end-point $x = -1$ and its greatest value in the zero $x = 0$ of the derivative.

Remark 3. The least value of a function is also called the *absolute minimum* and the greatest value the *absolute maximum* of the function.