Riemann sum

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Related topic	RiemannIntegral
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Related topic	LeftHandRule
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Related topic	MidpointRule
Defines	left Riemann sum
Defines	right Riemann sum
Defines	upper Riemann sum
Defines	lower Riemann sum

Let $I=[a,b]$ be a closed interval, $f:I\to\mathbb{R}$ be bounded on I, $n\in\mathbb{N}$, and $P=\{[x_0,x_1),[x_1,x_2),\ldots[x_{n-1},x_n]\}$ be a partition of I. The *Riemann sum* of f over I with respect to the partition P is defined as

$$S=\sum_{j=1}^{n} f(c_j)(x_j-x_{j-1})$$

where $c_j\in[x_{j-1},x_j]$ is chosen arbitrary.

If $c_j=x_{j-1}$ for all j, then S is called a *left Riemann sum*.

If $c_j=x_j$ for all j, then S is called a *Riemann sum*.

Equivalently, the Riemann sum can be defined as

$$S=\sum_{j=1}^{n} b_j(x_j-x_{j-1})$$

where $b_j\in\{f(x):x\in[x_{j-1},x_j]\}$ is chosen arbitrarily.

If $b_j=\sup_{x\in[x_{j-1},x_j]} f(x)$, then S is called an *upper Riemann sum*.

If $b_j=\inf_{x\in[x_{j-1},x_j]} f(x)$, then S is called a *lower Riemann sum*.

For some examples of Riemann sums, see the entry examples of estimating a Riemann integral.