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proof of quotient rule (using product rule)

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Suppose f and g are differentiable functions defined on some interval of \mathbb{R} , and g never vanishes. Let us prove that

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}.$$

Using the product rule $(fg)' = f'g + fg'$, and $(g^{-1})' = -g^{-2}g'$, we have

$$\begin{aligned} \left(\frac{f}{g}\right)' &= (fg^{-1})' \\ &= f'g^{-1} + f(g^{-1})' \\ &= f'g^{-1} + f(-1)g^{-2}g' \\ &= \frac{f'}{g} - \frac{fg'}{g^2} \\ &= \frac{f'g - fg'}{g^2}. \end{aligned}$$

Here $g^{-1} = 1/g$ and $g^{-2} = 1/g^2$.