majorization

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For any real vector $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, let $x_{(1)} \geq x_{(2)} \geq \cdots \geq x_{(n)}$ denote the components of x in non-increasing order.

For $x, y \in \mathbb{R}^n$, we say that x is majorized by y, or y majorizes x, if

$$\begin{aligned}
\sum_{i=1}^{m} x_{(i)} &\leq \sum_{i=1}^{m} y_{(i)}, \qquad \text{for } m = 1, \ldots, n-1, \text{ and} \\
\sum_{i=1}^{n} x_{(i)} &= \sum_{i=1}^{n} y_{(i)}
\end{aligned}$$

A common notation for "x is majorized by y" is $x \prec y$.

Remark:

A canonical example is that, if y_1, $y_2, \ldots, y_n$ are non-negative real numbers such that their sum is equal to 1, then

$$\left(\frac{1}{n}, \ldots, \frac{1}{n}\right) \prec (y_1, \ldots, y_n).$$

In general, $x \prec y$ vaguely means that the components of x is less spread out than are the components of y.

Reference

- G. H. Hardy, J. E. Littlewood and G. Pólya, *Inequalities*, 2nd edition, 1952, Cambridge University Press, London.
- A. W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications*, 1979, Acadamic Press, New York.