# sum of series depends on order



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| Canonical name | SumOfSeriesDependsOnOrder |
| Date of creation | 2013-03-22 14:50:59 |
| Last modified on | 2013-03-22 14:50:59 |
| Owner | pahio (2872) |
| Last modified by | pahio (2872) |
| Numerical id | 16 |
| Author | pahio (2872) |
| Entry type | Example |
| Classification | msc 26A06 |
| Classification | msc 40A05 |
| Related topic | AbsoluteConvergence |
| Related topic | OrderOfFactorsInInfiniteProduct |
| Related topic | AlternatingHarmonicSeries |
| Related topic | ConditionallyConvergentSeries |
| Related topic | ConvergingAlternatingSeriesNotSatisfyingAllLeibnizConditions |
| Related topic | FiniteChangesInConvergentSeries |
| Related topic | FiniteChangesInConvergentSeries2 |

According to the http://planetmath.org/LeibnizEstimateForAlternatingSeriesLeibniz' test, the alternating series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\frac{1}{8}+\frac{1}{9}-\frac{1}{10}+\frac{1}{11}-\frac{1}{12}+-\ldots$$

is convergent and has a positive sum ($=\ln 2$; see the http://planetmath.org/NaturalLogarithm2 logarithm). Denote it by $S$. We can by $\frac{1}{2}$ getting the two series $S = (1-\frac{1}{2})+(\frac{1}{3}-\frac{1}{4})+(\frac{1}{5}-\frac{1}{6})+(\frac{1}{7}-\frac{1}{8})+(\frac{1}{9}-\frac{1}{10})+\ldots,$

$\frac{1}{2}S = \frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}+\frac{1}{10}-+\ldots.$

Then we add these two series termwise getting the sum

$1\frac{1}{2}S = 1+\frac{1}{3}-\frac{2}{4}+\frac{1}{5}+\frac{1}{7}-\frac{2}{8}+\frac{1}{9}+\frac{1}{11}-\frac{2}{12}+\ldots.$

Hence, this last series exactly the same as the original, but its sum is fifty percent greater. This is possible because the original series is not absolutely convergent: the series which is formed of the absolute values of its is the divergent harmonic series.

P. S. – For justification of the used manipulations of the series, see the entry.