singular function

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Definition. A monotone, non-constant, function $f\colon [a,b] \to \mathbb{R}$ is said to be a *singular function* (or a *purely singular function*) if $f'(x) = 0$ almost everywhere.

It is easy to see that a singular function cannot be `http://planetmath.org/AbsolutelyContin` continuous: If an absolutely continuous function $f\colon [a,b] \to \mathbb{R}$ satisfies $f'(x) = 0$ almost everywhere, then it must be constant.

An example of such a function is the famous Cantor function. While this is not a strictly increasing function, there also do exist singular functions which are in fact strictly increasing, and even more amazingly functions that are quasisymmetric (see attached example).

Theorem. *Any monotone increasing function can be written as a sum of an absolutely continuous function and a singular function.*

References

[1] H. L. Royden. . Prentice-Hall, Englewood Cliffs, New Jersey, 1988