Dirichlet kernel

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Author	mathwizard (128)
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The *Dirichlet* D_n of order n is defined as

$$D_n(t) = \sum_{k=-n}^{n} e^{ikt}.$$

It can be represented as

$$D_n(t) = \frac{\sin\left(n+\frac{1}{2}\right)t}{\sin\frac{t}{2}}.$$

Proof: It is

$$\begin{aligned}\sum_{k=-n}^{n} e^{ikt} &= e^{-int}\frac{1-e^{i(2n+1)t}}{1-e^{it}} \\ &= \frac{e^{i\left(n+\frac{1}{2}\right)t} - e^{-i\left(n+\frac{1}{2}\right)t}}{e^{i\frac{t}{2}} - e^{-i\frac{t}{2}}} \\ &= \frac{\sin\left(n+\frac{1}{2}\right)t}{\sin\frac{t}{2}}. \qquad \square\end{aligned}$$

The Dirichlet kernel arises in the analysis of periodic functions because for any function f of period 2π, the convolution of D_N and f results in the Fourier-series approximation of order n:

$$(D_N * f)(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(y)D_n(x-y)dy = \sum_{k=-n}^{n} \hat{f}(k)e^{ikx}.$$