



Math for the people, by the people.

rearrangement inequality

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Let x_1, x_2, \dots, x_n and y_1, y_2, \dots, y_n two sequences of positive real numbers. Then the sum

$$x_1y_1 + x_2y_2 + \dots + x_ny_n$$

is maximized when the two sequences are ordered in the same way (i.e. $x_1 \leq x_2 \leq \dots \leq x_n$ and $y_1 \leq y_2 \leq \dots \leq y_n$) and is minimized when the two sequences are ordered in the opposite way (i.e. $x_1 \leq x_2 \leq \dots \leq x_n$ and $y_1 \geq y_2 \geq \dots \geq y_n$).

This can be seen intuitively as: If x_1, x_2, \dots, x_n are the prices of n kinds of items, and y_1, y_2, \dots, y_n the number of units sold of each, then the highest profit is when you sell more items with high prices and fewer items with low prices (same ordering), and the lowest profit happens when you sell more items with lower prices and less items with high prices (opposite orders).