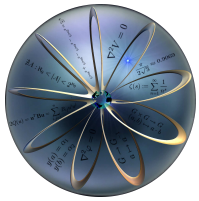

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all derivatives of sinc are bounded by 1

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Let us show that all derivatives of sinc are bounded by 1.

First of all, let us out that $\operatorname{sinc}(t) \leq 1$ is bounded by the Jordan's inequality. To the derivatives, let us write sinc as a Fourier integral,

$$\operatorname{sinc}(t) = \frac{1}{2}\int_{-1}^{1} e^{ixt}\,dx.$$

Let $k = 1, 2, \ldots$. Then

$$\frac{d^k}{dt^k}\operatorname{sinc}(t) = \frac{1}{2}\int_{-1}^{1} (ix)^k e^{ixt}\,dx.$$

and

$$\begin{aligned}
\left|\frac{d^k}{dt^k}\operatorname{sinc}(t)\right| &= \left|\frac{1}{2}\int_{-1}^{1} (ix)^k e^{ixt}\,dx\right| \\
&\leq \frac{1}{2}\int_{-1}^{1} |(ix)^k e^{ixt}|\,dx \\
&\leq \frac{1}{2}\int_{-1}^{1} |x|^k\,dx \\
&\leq \frac{1}{2}\cdot 2\int_{0}^{1} |x|^k\,dx \\
&\leq \int_{0}^{1} x^k\,dx \\
&\leq \frac{1}{k+1} \\
&< 1.
\end{aligned}$$