nth root formulas

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In the following formulas, a is a nonnegative real number and other letters positive integers. For other formulas, see the http://planetmath.org/NthRootparent entry.

1. $\sqrt[n]{0} \;=\; 0, \qquad \sqrt[n]{1} \;=\; 1$

2. $\sqrt[1]{a} \;=\; a$

3. $\sqrt[m]{\sqrt[n]{a}} \;=\; \sqrt[mn]{a} \;=\; \sqrt[n]{\sqrt[m]{a}}$

4. $\sqrt[nk]{a^{mk}} \;=\; \sqrt[n]{a^m}$

5. $\sqrt[n]{a^m} \;=\; (\sqrt[n]{a})^m$

6. $\sqrt[m]{a} \cdot \sqrt[n]{a} \;=\; \sqrt[mn]{a^{m+n}}$

Proof. For proving, one uses the definition of http://planetmath.org/node/754nth root and the http://planetmath.org/GeneralAssociativitypower laws.

1°. $0^n = 0, \quad 1^n = 1$

2°. $a^1 = a$

3°. $(\sqrt[m]{\sqrt[n]{a}})^{mn} \;=\; ((\sqrt[m]{\sqrt[n]{a}})^m)^n \;=\; (\sqrt[n]{a})^n \;=\; a$

4°. $(\sqrt[n]{a^m})^{nk} \;=\; ((\sqrt[n]{a^m})^n)^k \;=\; (a^m)^k \;=\; a^{mk}$

5°. $((\sqrt[n]{a})^m)^n \;=\; ((\sqrt[n]{a})^n)^m \;=\; a^m$

6°. $(\sqrt[m]{a} \cdot \sqrt[n]{a})^{mn} \;=\; (\sqrt[m]{a})^{mn}(\sqrt[n]{a})^{mn} \;=\; ((\sqrt[m]{a})^m)^n((\sqrt[n]{a})^n)^m \;=\; a^n a^m \;=\; a^{m+n}$