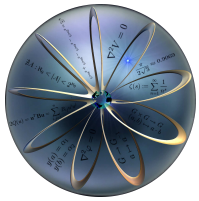

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directional derivative, derivation of

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Let $f(x, y)$ be a function where $x = x(t)$ and $y = y(t)$. Let $\vec{r} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ be the vector in the desired direction. The line through this vector is given parametrically by:

$$x = x_0 + at; \quad y = y_0 + bt$$

The derivative of f with respect to t is as follows:

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

But from the equation of the line, we know that $\frac{dx}{dt} = a$ and $\frac{dy}{dt} = b$ so the derivative becomes:

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}a + \frac{\partial f}{\partial y}b = \nabla f \cdot \vec{r}$$