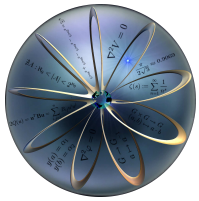

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product rule

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The *product rule* states that if $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are functions in one variable both differentiable at a point x_0, then the derivative of the product of the two functions, denoted $f \cdot g$, at x_0 is given by

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(f \cdot g\right)(x_0) = f(x_0)g'(x_0) + f'(x_0)g(x_0).$$

Proof

See the `http://planetmath.org/ProofOfProductRule`proof of the product rule.

0.1 Generalized Product Rule

More generally, for differentiable functions $f_1, f_2, \ldots, f_n$ in one variable, all differentiable at x_0, we have

$$D(f_1 \cdots f_n)(x_0) = \sum_{i=1}^{n} \left(f_i(x_0) \cdots f_{i-1}(x_0) \cdot Df_i(x_0) \cdot f_{i+1}(x_0) \cdots f_n(x_0)\right).$$

Also see `http://planetmath.org/LeibnizRule`Leibniz' rule.

Example

The derivative of $x \ln |x|$ can be found by application of this rule. Let $f(x) = x, g(x) = \ln |x|$, so that $f(x)g(x) = x \ln |x|$. Then $f'(x) = 1$ and $g'(x) = \frac{1}{x}$. Therefore, by the product rule,

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}(x \ln |x|) &= f(x)g'(x) + f'(x)g(x) \\ &= \frac{x}{x} + 1 \cdot \ln |x| \\ &= \ln |x| + 1 \end{aligned}$$