planetmath.org

Math for the people, by the people.

one-sided derivatives

Canonical name	OnesidedDerivatives
Date of creation	2013-03-22 15:39:00
Last modified on	2013-03-22 15:39:00
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	9
Author	pahio (2872)
Entry type	Definition
Classification	msc 26B05
Classification	msc 26A24
Synonym	left derivative
Synonym	right derivative
Related topic	Differentiable
Related topic	OneSidedLimit
Related topic	DifferntiableFunction
Related topic	OneSidedContinuity
Related topic	SemicubicalParabola
Defines	left-sided derivative
Defines	right-sided derivative

- If the real function f is defined in the point x_0 and on some interval left from this and if the left-hand one-sided limit $\lim_{h \to 0-} \frac{f(x_0+h)-f(x_0)}{h}$ exists, then this limit is defined to be the *left-sided derivative* of f in x_0.

- If the real function f is defined in the point x_0 and on some interval right from this and if the right-hand one-sided limit $\lim_{h \to 0+} \frac{f(x_0+h)-f(x_0)}{h}$ exists, then this limit is defined to be the *right-sided derivative* of f in x_0.

It's apparent that if f has both the left-sided and the right-sided derivative in the point x_0 and these are equal, then f is differentiable in x_0 and $f'(x_0)$ equals to these one-sided derivatives. Also inversely.

Example. The real function $x \mapsto x\sqrt{x}$ is defined for $x \geqq 0$ and differentiable for $x > 0$ with $f'(x) \equiv \frac{3}{2}\sqrt{x}$. The function also has the right derivative in 0:

$$\lim_{h \to 0+} \frac{h\sqrt{h} - 0\sqrt{0}}{h} = \lim_{h \to 0+} \sqrt{h} = 0$$

Remark. For a function $f\colon [a,\, b] \to \mathbb{R}$, to have a right-sided derivative at $x = a$ with value d, is equivalent to saying that there is an extension g of f to some open interval containing $[a,\, b]$ and satisfying $g'(a) = d$. Similarly for left-sided derivatives.