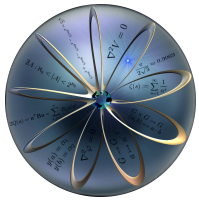

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l'Hôpital's rule

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L'Hôpital's rule states that given an unresolvable limit of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, the ratio of functions $\frac{f(x)}{g(x)}$ will have the same limit at c as the ratio $\frac{f'(x)}{g'(x)}$. In short, if the limit of a ratio of functions approaches an indeterminate form, then

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\lim_{x\to c}\frac{f'(x)}{g'(x)}$$

provided this last limit exists. L'Hôpital's rule may be applied indefinitely as long as the conditions are satisfied. However it is important to note, that the nonexistance of $\lim\frac{f'(x)}{g'(x)}$ does not prove the nonexistance of $\lim\frac{f(x)}{g(x)}$.

Example: We try to determine the value of

$$\lim_{x\to\infty}\frac{x^2}{e^x}.$$

As x approaches ∞ the expression becomes an indeterminate form $\frac{\infty}{\infty}$. By applying L'Hôpital's rule twice we get

$$\lim_{x\to\infty}\frac{x^2}{e^x}=\lim_{x\to\infty}\frac{2x}{e^x}=\lim_{x\to\infty}\frac{2}{e^x}=0.$$

Another example of the usage of L'Hôpital's rule can be found `http://planetmath.org/node/5741`