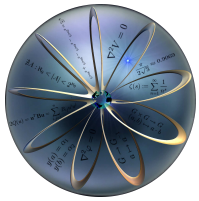

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generalized mean

Canonical name	GeneralizedMean
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Entry type	Definition
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Synonym	Kolmogorov-Nagumo function of the mean
Synonym	Hölder mean

Definition

Let $x_1, x_2, \ldots, x_n$ be real numbers, and f a continuous and strictly increasing or decreasing function on the real numbers. If each number x_i is assigned a weight p_i, with $\sum_{i=1}^{n} p_i = 1$, for $i = 1, \ldots, n$, then the *generalized mean* is defined as

$$f^{-1}\Big(\sum_{i=1}^{n} p_i f(x_i)\Big).$$

Special cases

1. $f(x) = x$, $p_i = 1/n$ for all i: arithmetic mean
2. $f(x) = x$: weighted mean
3. $f(x) = \log(x)$, $p_i = 1/n$ for all i: geometric mean
4. $f(x) = 1/x$ and $p_i = 1/n$ for all i: harmonic mean
5. $f(x) = x^2$ and $p_i = 1/n$ for all i: root-mean-square
6. $f(x) = x^d$ and $p_i = 1/n$ for all i: power mean
7. $f(x) = x^d$: weighted power mean
8. $f(x) = 2^{(1-\alpha)x}$, $\alpha \neq 1$, $x_i = -\log_2 p_i$: Rényi's α-entropy