proof of Gronwall's lemma

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The inequality

$$\phi(t) \leq K + L\int_{t_0}^{t} \psi(s)\phi(s)ds \tag{1}$$

is equivalent to

$$\frac{\phi(t)}{K + L\int_{t_0}^{t} \psi(s)\phi(s)ds} \leq 1$$

Multiply by $L\psi(t)$ and integrate, giving

$$\int_{t_0}^{t} \frac{L\psi(s)\phi(s)ds}{K + L\int_{t_0}^{s} \psi(\tau)\phi(\tau)d\tau} \leq L\int_{t_0}^{t} \psi(s)ds$$

Thus

$$\ln\left(K + L\int_{t_0}^{t} \psi(s)\phi(s)ds\right) - \ln K \leq L\int_{t_0}^{t} \psi(s)ds$$

and finally

$$K + L\int_{t_0}^{t} \psi(s)\phi(s)ds \leq K\exp\left(L\int_{t_0}^{t} \psi(s)ds\right)$$

Using (**??**) in the left hand side of this inequality gives the result.