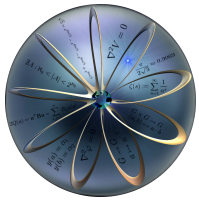

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Generalized N-dimensional Riemann Integral

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Let $I = [a_1, b_1] \times \cdots \times [a_N, b_N] \subset \mathbb{R}^N$ be a compact interval, and let $f : I \to \mathbb{R}^M$ be a function. Let $\epsilon > 0$. If there exists a $y \in \mathbb{R}^M$ and a partition P_ϵ of I such that for each refinement P of P_ϵ (and corresponding Riemann Sum $S(f, P)$),

$$\|S(f, P) - y\| < \epsilon$$

Then we say that f is Riemann integrable over I, that y is the Riemann integral of f over I, and we write

$$\int_I f := \int_I f \, d\mu := y$$

Note also that it is possible to extend this definition to more arbitrary sets; for any bounded set D, one can find a compact interval I such that $D \subset I$, and define a function

$$\tilde{f} : I \to \mathbb{R}^M \quad x \mapsto \begin{cases} f(x), & x \in D \\ 0, & x \notin D \end{cases}$$

in which case we define

$$\int_D f := \int_I \tilde{f}$$