composition of continuous mappings is continuous

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Theorem 1. *The composition of two continuous mappings (when defined) is continuous.*

Proof. Let X, Y, Z be topological space, and let f, g be mappings

$$f \colon X \ \to \ Y,$$
$$g \colon Y \ \to \ Z.$$

We wish to prove that $g \circ f$ is continuous. Suppose B is an open set in Z. Since g is continuous, $g^{-1}(B)$ is an open set in Y, and since f is continuous, $f^{-1}(g^{-1}(B))$ is an open set in X. Since $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$, it follows that $(g \circ f)^{-1}(B)$ is open and the composition if continuous. $\quad\square$