extreme value theorem

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Extreme Value Theorem. *Let a and b be real numbers with $a < b$, and let f be a continuous, real valued function on $[a, b]$. Then there exists $c, d \in [a, b]$ such that $f(c) \leq f(x) \leq f(d)$ for all $x \in [a, b]$.*

Proof. We show only the existence of d. By the boundedness theorem $f([a, b])$ is bounded above; let l be the least upper bound of $f([a, b])$. Suppose, for a contradiction, that there is no $d \in [a, b]$ such that $f(d) = l$. Then the function

$$g(x) = \frac{1}{l - f(x)}$$

is well defined and continuous on $[a, b]$. Since l is the least upper bound of $f([a, b])$, for any positive real number M we can find $\alpha \in [a, b]$ such that $f(\alpha) > l - \frac{1}{M}$, then

$$M < \frac{1}{l - f(\alpha)}.$$

So g is unbounded on $[a, b]$. But by the boundedness theorem g is bounded on $[a, b]$. This contradiction finishes the proof.