binomial formula

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The binomial formula gives the power series expansion of the p^{th} power function. The power p can be an integer, rational, real, or even a complex number. The formula is

$$\begin{aligned}(1+x)^p &= \sum_{n=0}^{\infty} \frac{p^{\underline{n}}}{n!}\, x^n \\ &= \sum_{n=0}^{\infty} \binom{p}{n} x^n\end{aligned}$$

where $p^{\underline{n}} = p(p-1)\ldots(p-n+1)$ denotes the falling factorial, and where $\binom{p}{n}$ denotes the generalized binomial coefficient.

For $p = 0, 1, 2, \ldots$ the power series reduces to a polynomial, and we obtain the usual binomial theorem. For other values of p, the radius of convergence of the series is 1; the right-hand series converges pointwise for all complex $|x| < 1$ to the value on the left side. Also note that the binomial formula is valid at $x = \pm 1$, but for certain values of p only. Of course, we have convergence if p is a natural number. Furthermore, for $x = 1$ and real p, we have absolute convergence if $p > 0$, and conditional convergence if $-1 < p < 0$. For $x = -1$ we have absolute convergence for $p > 0$.