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perimeter of astroid

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The *astroid*

$$\sqrt[3]{x^2} + \sqrt[3]{y^2} = \sqrt[3]{a^2}$$

can be presented in the parametric form

$$x = a\cos^3\varphi, \quad y = a\sin^3\varphi,$$

where the polar angle φ gets the values from 0 to 2π. The curve consists of four congruent arcs, one of which is obtained letting $0 \leqq \varphi \leqq \frac{\pi}{2}$. Thus the whole perimeter of the astroid is

$$s = 4\int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{d\varphi}\right)^2 + \left(\frac{dy}{d\varphi}\right)^2}\, d\varphi.$$

This expression is easily simplified to

$$s = 12a\int_0^{\frac{\pi}{2}} \sin\varphi\cos\varphi\, d\varphi$$

giving the result

$$s = 6a\int_0^{\frac{\pi}{2}} \sin 2\varphi\, d\varphi = -3a \Big/_{0}^{\frac{\pi}{2}} \cos 2\varphi = 6a.$$