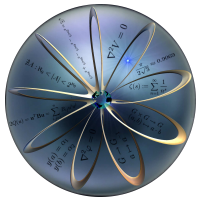

planetmath.org

Math for the people, by the people.

Helmholtz equation

Canonical name	HelmholtzEquation
Date of creation	2013-03-22 13:09:09
Last modified on	2013-03-22 13:09:09
Owner	Mathprof (13753)
Last modified by	Mathprof (13753)
Numerical id	11
Author	Mathprof (13753)
Entry type	Definition
Classification	msc 26B12
Classification	msc 35-00
Synonym	Helmholtz differential equation
Synonym	reduced wave equation
Related topic	WaveEquation
Related topic	PoissonsEquation

The *Helmholtz equation* is a partial differential equation which, in scalar form is

$$\nabla^2 f + k^2 f = 0,$$

or in vector form is

$$\nabla^2 \mathbf{A} + k^2 \mathbf{A} = 0,$$

where ∇^2 is the Laplacian. The solutions of this equation represent the solution of the wave equation, which is of great interest in physics.

Consider a wave equation

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \nabla^2 \psi$$

with wave speed c. If we look for time harmonic standing waves of frequency ω,

$$\psi(\mathbf{x}, t) = e^{-j\omega t} \phi(\mathbf{x})$$

we find that $\phi(x)$ satisfies the Helmholtz equation:

$$(\nabla^2 + k^2)\phi = 0$$

where $k = \omega/c$ is the wave number.

Usually the Helmholtz equation is solved by the separation of variables method, in Cartesian, spherical or cylindrical coordinates.