weighted power mean

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Related topic	ProofOfGeneralMeansInequality
Related topic	DerivationOfHarmonicMeanAsTheLimitOfThePowerMean

If $w_1, w_2, \ldots, w_n$ are positive real numbers such that $w_1+w_2+\cdots+w_n = 1$, we define the *r-th weighted power mean* of the x_i as:

$$M_w^r(x_1, x_2, \ldots, x_n) = \left(w_1 x_1^r + w_2 x_2^r + \cdots + w_n x_n^r\right)^{1/r}.$$

When all the $w_i = \frac{1}{n}$ we get the standard power mean. The weighted power mean is a continuous function of r, and taking limit when $r \to 0$ gives us

$$M_w^0 = x_1^{w_1} x_2^{w_2} \cdots w_n^{w_n}.$$

We can weighted use power means to generalize the power means inequality: If w is a set of weights, and if $r < s$ then

$$M_w^r \leq M_w^s.$$