partial fractions of expressions

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Defines	fractional expression

Let $R(z) = \frac{P(z)}{Q(z)}$ be a *fractional expression*, i.e., a quotient of the polynomials $P(z)$ and $Q(z)$ such that $P(z)$ is not divisible by $Q(z)$. Let's restrict to the case that the coefficients are real or complex numbers.

If the distinct complex zeros of the denominator are $b_1, b_2, \ldots, b_t$ with the multiplicities $\tau_1, \tau_2, \ldots, \tau_t$ $(t \geq 1)$, and the numerator has not common zeros, then $R(z)$ can be decomposed uniquely as the sum

$$R(z) \;=\; H(z) + \sum_{j=1}^{t}\left(\frac{A_{j1}}{z-b_j} + \frac{A_{j2}}{(z-b_j)^2} + \ldots + \frac{A_{j\tau_j}}{(z-b_j)^{\tau_j}}\right),$$

where $H(z)$ is a polynomial and the A_{jk}'s are certain complex numbers.

Let us now take the special case that all coefficients of $P(z)$ and $Q(z)$ are real. Then the (i.e. non-real) zeros of $Q(z)$ are pairwise complex conjugates, with same multiplicities, and the corresponding linear http://planetmath.org/Productfactors of $Q(z)$ may be pairwise multiplied to quadratic polynomials of the form z^2+pz+q with real p's and q's and $p^2 < 4q$. Hence the above decomposition leads to the unique decomposition of the form

$$\begin{aligned} R(x) \;=\;\; & H(x) + \sum_{i=1}^{m}\left(\frac{A_{i1}}{x-b_i} + \frac{A_{i2}}{(x-b_i)^2} + \ldots + \frac{A_{i\mu_i}}{(x-b_i)^{\mu_i}}\right) \\ & + \sum_{j=1}^{n}\left(\frac{B_{j1}x+C_{j1}}{x^2+p_jx+q_j} + \frac{B_{j2}x+C_{j2}}{(x^2+p_jx+q_j)^2} + \ldots + \frac{B_{j\nu_j}x+C_{j\nu_j}}{(x^2+p_jx+q_j)^{\nu_j}}\right), \end{aligned}$$

where m is the number of the distinct real zeros and $2n$ the number of the distinct zeros of the denominator $Q(x)$ of the fractional expression $R(x) = \frac{P(x)}{Q(x)}$. The coefficients A_{ik}, B_{jk} and C_{jk} are uniquely determined real numbers.

Cf. the partial fractions of *fractional numbers*.

Example.

$$\frac{-x^5+6x^4-7x^3+15x^2-4x+3}{(x-1)^3(x^2+1)^2} \;=\; -\frac{1}{x-1} + \frac{3}{(x-1)^3} + \frac{x}{x^2+1} + \frac{2x-1}{(x^2+1)^2}$$