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list of common limits

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Following is a list of common limits used in elementary calculus:

- For any real numbers a and c, $lim_{x \to a} c = c$.
- For any real numbers a and n, $\lim_{x \to a} x^n = a^n$ (proven http://planetmath.org/Continuity for n a positive integer)
- $\lim_{x \to 0} \frac{\sin x}{x} = 1$ (proven http://planetmath.org/LimitOfDisplaystyleFracsinXxAsXAppr
- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$ (proven http://planetmath.org/LimitOfDisplaystyleFrac1CosXxAsXA
- $\lim_{x \to 0} \frac{\arcsin x}{x} = 1$ (proven http://planetmath.org/LimitExampleshere)
- $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$ (proven http://planetmath.org/DerivativeOfExponentialFunctionhere
- For $a > 0$, $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$ (proven http://planetmath.org/LimitOfDisplaystyleFraca
- For $b > 1$ and a any real number, $\lim_{x \to \infty} \frac{x^a}{b^x} = 0$ (proven http://planetmath.org/GrowthOf
- $\lim_{x \to 0^+} x^x = 1$ (proven http://planetmath.org/FunctionXxhere)
- $\lim_{x \to 0^+} x \ln x = 0$ (proven http://planetmath.org/GrowthOfExponentialFunctionhere)
- $\lim_{x \to \infty} \frac{\ln x}{x} = 0$ (proven http://planetmath.org/GrowthOfExponentialFunctionhere)
- $\lim_{x \to \infty} x^{\frac{1}{x}} = 1$ (proven http://planetmath.org/GrowthOfExponentialFunctionhere)
- $\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$
- $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$
- $\lim_{x \to 0} (1 + \sin x)^{\frac{1}{x}} = e$ (power of e, http://planetmath.org/LHpitalsRulel'Hôpital's rule)
- $\lim_{x \to \infty} (x - \sqrt{x^2 - a^2}) = 0$ (proven http://planetmath.org/Hyperbolahere)
- For $a > 0$ and n a positive integer, $\lim_{x \to a} \frac{x - a}{x^n - a^n} = \frac{1}{na^{n-1}}$.
- $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \frac{1}{2}$ (by http://planetmath.org/LHpitalsRulel'Hôpital's rule)
- For $q > 0$, $\lim_{x \to \infty} \frac{(\log x)^p}{x^q} = 0$

- $\tan\left(x+\frac{\pi}{2}\right) = \lim_{\xi\to\frac{\pi}{2}} \frac{\tan x+\tan\xi}{1-\tan x\tan\xi} = \lim_{\xi\to\frac{\pi}{2}} \frac{\sec^2\xi}{-\tan x\sec^2\xi} = -\cot x$ (by `http://planetmath.org/LHpitalsRule`l'Hôpital's rule)
  That is, $\tan x\tan(x+\frac{\pi}{2}) = -1$, which indicates orthogonality of the slopes represented by those functions.

- For a real or complex constant c and a variable z,
  $\lim_{n\to\infty} \frac{n^{n+1}}{z^{n+1}} \left(c+\frac{n}{z}\right)^{-(n+1)} = e^{-cz}$.

- For x real (or complex), $\lim_{n\to\infty} n(\sqrt[n]{x}-1) = \log x$ (proven `http://planetmath.org/Halleys`
  for real x).

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References

[1] Catherine Roberts & Ray McLenaghan, "Continuous Mathematics" in *Standard Mathematical Tables and Formulae* ed. Daniel Zwillinger. Boca Raton: CRC Press (1996): 333, 5.1 Differential Calculus