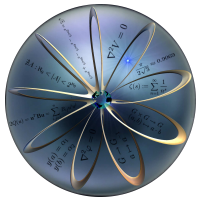

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example of a strictly increasing quasisymmetric singular function

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An example of a strictly increasing quasisymmetric function that also a purely singular function can be defined as:

$$f(x) = \lim_{k\to\infty} \int_0^x \prod_{i=1}^{k} (1 + \lambda \cos n_i s) ds,$$

where $0 < \lambda < 1$ and carefully picked n_i. We can pick the n_i such that n_{i+1} is strictly greater then $\sum_{j=1}^{i} n_j$. However if we pick the λ and n_i more carefully, we can construct functions with the quasisymmetricity constant as close to 1 as we want. That is, we can construct functions such that

$$\frac{1}{M} \leq \frac{f(x+t) - f(x)}{f(x) - f(x-t)} \leq M$$

for all x and t where M is as close to 1 as we want. If $M = 1$ note that the function must be a straight line.

It is also possible from this to construct a quasiconformal mapping of the upper half plane to itself by extending this function to the whole real line and then using the Beurling-Ahlfors quasiconformal extension. Then we'd have a quasiconformal mapping such that its boundary correspondence would be a purely singular function.

For more detailed explanation, and proof (it is too long to reproduce here) see bibliography.

Bibliography

- A. Beurling, L. V. Ahlfors. . *Acta Math.*, 96:125-142, 1956.
- J. Lebl. . . Also available at `http://www.jirka.org/thesis.pdf`http://www.jirka.org/thesi