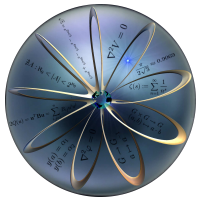

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addition and subtraction formulas for hyperbolic functions

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The addition formulas for hyperbolic sine, hyperbolic cosine, and hyperbolic tangent will be achieved via brute .

$$\begin{aligned}
\sinh(x+y) &= \frac{e^{x+y} - e^{-(x+y)}}{2} \\
&= \frac{e^x e^y - e^x e^{-y} + e^x e^{-y} - e^{-x} e^{-y}}{2} \\
&= e^x \left(\frac{e^y - e^{-y}}{2} \right) + e^{-y} \left(\frac{e^x - e^{-x}}{2} \right) \\
&= (\cosh x + \sinh x) \sinh y + (\cosh y - \sinh y) \sinh x \\
&= \cosh x \sinh y + \sinh x \sinh y + \sinh x \cosh y - \sinh x \sinh y \\
&= \sinh x \cosh y + \cosh x \sinh y
\end{aligned}$$

$$\begin{aligned}
\cosh(x+y) &= \frac{e^{x+y} + e^{-(x+y)}}{2} \\
&= \frac{e^x e^y - e^x e^{-y} + e^x e^{-y} + e^{-x} e^{-y}}{2} \\
&= e^x \left(\frac{e^y - e^{-y}}{2} \right) + e^{-y} \left(\frac{e^x + e^{-x}}{2} \right) \\
&= (\cosh x + \sinh x) \sinh y + (\cosh y - \sinh y) \cosh x \\
&= \cosh x \sinh y + \sinh x \sinh y + \cosh x \cosh y - \cosh x \sinh y \\
&= \cosh x \cosh y + \sinh x \sinh y
\end{aligned}$$

$$\begin{aligned}
\tanh(x+y) &= \frac{\sinh(x+y)}{\cosh(x+y)} \\
&= \frac{\sinh x \cosh y + \cosh x \sinh y}{\cosh x \cosh y + \sinh x \sinh y} \\
&= \frac{\dfrac{\sinh x}{\cosh x} \cdot \dfrac{\cosh y}{\cosh y} + \dfrac{\cosh x}{\cosh x} \cdot \dfrac{\sinh y}{\cosh y}}{\dfrac{\cosh x}{\cosh x} \cdot \dfrac{\cosh y}{\cosh y} + \dfrac{\sinh x}{\cosh x} \cdot \dfrac{\sinh y}{\cosh y}} \\
&= \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}
\end{aligned}$$

Note that sinh and tanh are odd functions and cosh is an even function, http://planetmath.org/Iei.e. $\sinh(-t) = -\sinh t$, $\tanh(-t) = -\tanh t$, and $\cosh(-t) = \cosh t$. These facts enable us to obtain the subtraction formulas.

$$\sinh(x-y) = \sinh(x+(-y)) = \sinh x \cosh(-y) + \cosh x \sinh(-y) = \sinh x \cosh y - \cosh x \sinh y$$

$$\cosh(x-y) = \cosh(x+(-y)) = \cosh x \cosh(-y) + \sinh x \sinh(-y) = \cosh x \cosh y - \sinh x \sinh y$$

$$\tanh(x-y) = \tanh(x+(-y)) = \frac{\tanh x + \tanh(-y)}{1 + \tanh x \tanh(-y)} = \frac{\tanh x - \tanh y}{1 - \tanh x \tanh y}$$