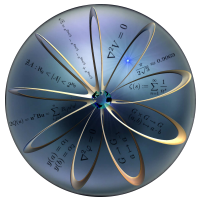

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minimal and maximal number

Canonical name	MinimalAndMaximalNumber
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Entry type	Definition
Classification	msc 26B12
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Synonym	least and greatest number
Related topic	Infimum
Related topic	Supremum
Related topic	UltrametricTriangleInequality
Related topic	GrowthOfExponentialFunction
Related topic	EstimatingTheoremOfContourIntegral
Related topic	LeastAndGreatestValueOfFunction
Related topic	FuzzyLogic2
Related topic	ZerosAndPolesOfRationalFunction
Related topic	UniformConvergenceOnUnionInterval
Related topic	Interprime
Related topic	LehmerMean
Related topic	Ab
Defines	least number
Defines	greatest number
Defines	minimal number
Defines	maximal number
Defines	set function

Let's consider a finite non-empty set $A = \{a_1, \ldots, a_n\}$ of real numbers or an infinite but compact (i.e. bounded and closed) set A of real numbers. In both cases the set has a unique least number and a unique greatest number.

- The least number of the set is denoted by $\min\{a_1, \ldots, a_n\}$ or $\min A$.
- The greatest number of the set is denoted by $\max\{a_1, \ldots, a_n\}$ or $\max A$.

In both cases we have

$$\begin{aligned} \min A &= \inf A, \\ \max A &= \sup A, \\ \min A \leqq x &\leqq \max A \quad \forall x \in A, \end{aligned}$$

where $\inf A$ and $\sup A$ are the infimum and supremum of the set A.

The min and max are *set functions*, i.e. they map subsets of a certain set to $\mathbb{R}$.

The min and max have the following distributive properties with respect to addition:

$$\begin{aligned} \min\{a_1, \ldots, a_n\} + b &= \min\{a_1 + b, \ldots, a_n + b\} \\ \max\{a_1, \ldots, a_n\} + b &= \max\{a_1 + b, \ldots, a_n + b\} \end{aligned}$$

The minimal and maximal number of a set of two real numbers obey the formulae

$$\begin{aligned} \min\{a, b\} &= \frac{a+b}{2} - \frac{|a-b|}{2}, \\ \max\{a, b\} &= \frac{a+b}{2} + \frac{|a-b|}{2}, \end{aligned}$$

$$\max\{a, b\} - \min\{a, b\} = |a-b|,$$

$$\max\{a, b\} + \min\{a, b\} = a+b,$$

$$\max\{a, -a\} = |a|$$