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Schur's inequality

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Theorem (Schur's inequality) Let A be a square $n \times n$ matrix with real (or possibly complex entries). If $\lambda_1, \dots, \lambda_n$ are the eigenvalues of A , and D is the diagonal matrix $D = \text{diag}(\lambda_1, \dots, \lambda_n)$, then

$$\|D\|_F \leq \|A\|_F,$$

where $\|\cdot\|_F$ is the Frobenius matrix norm. Equality holds if and only if A is a normal matrix.

References

- [1] V.V. Prasolov, *Problems and Theorems in Linear Algebra*, American Mathematical Society, 1994.