Generalised N-dimensional Riemann Sum

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Let $I = [a_1, b_1] \times \cdots \times [a_N, b_N]$ be an N-cell in $\mathbb{R}^N$. For each $j = 1, \ldots, N$, let $a_j = t_{j,0} < \ldots < t_{j,N} = b_j$ be a partition P_j of $[a_j, b_j]$. We define a partition P of I as

$$P := P_1 \times \cdots \times P_N$$

Each partition P of I generates a subdivision of I (denoted by $(I_\nu)_\nu$) of the form

$$I_\nu = [t_{1,j}, t_{1,j+1}] \times \cdots \times [t_{N,k}, t_{N,k+1}]$$

Let $f : U \to \mathbb{R}^M$ be such that $I \subset U$, and let $(I_\nu)_\nu$ be the corresponding subdivision of a partition P of I. For each ν, choose $x_\nu \in I_\nu$. Define

$$S(f, P) := \sum_\nu f(x_\nu) \mu(I\nu)$$

As the Riemann sum of f corresponding to the partition P.

A partition Q of I is called a refinement of P if $P \subset Q$.