# example of vector potential

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If the solenoidal vector $\vec{U} = \vec{U}(x,\,y,\,z)$ is a homogeneous function of degree $\lambda$ $(\neq -2)$, then it has the vector potential

$$\vec{A} = \frac{1}{\lambda+2}\vec{U}\times\vec{r}, \tag{1}$$

where $\vec{r} = x\vec{i}+y\vec{j}+z\vec{k}$ is the position vector.

*Proof.* Using the entry nabla acting on products, we first may write

$$\nabla\times(\frac{1}{\lambda+2}\vec{U}\times\vec{r}) = \frac{1}{\lambda+2}[(\vec{r}\cdot\nabla)\vec{U}-(\vec{U}\cdot\nabla)\vec{r}-(\nabla\cdot\vec{U})\vec{r}+(\nabla\cdot\vec{r})\vec{U}].$$

In the brackets the first product is, according to Euler's theorem on homogeneous functions, equal to $\lambda\vec{U}$. The second product can be written as $U_x\frac{\partial\vec{r}}{\partial x}+U_y\frac{\partial\vec{r}}{\partial y}+U_z\frac{\partial\vec{r}}{\partial z}$, which is $U_x\vec{i}+U_y\vec{j}+U_z\vec{k}$, i.e. $\vec{U}$. The third product is, due to the sodenoidalness, equal to $0\vec{r} = \vec{0}$. The last product equals to $3\vec{U}$ (see the `http://planetmath.org/PositionVector`first formula for position vector). Thus we get the result

$$\nabla\times(\frac{1}{\lambda+2}\vec{U}\times\vec{r}) = \frac{1}{\lambda+2}[\lambda\vec{U}-\vec{U}-\vec{0}+3\vec{U}] = \vec{U}.$$

This means that $\vec{U}$ has the vector potential (1).