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another example of Dirac sequence

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Let $A_n = \left[\frac{-1}{2^n}, \frac{1}{2^n} \right]$ and $\delta_n = 2^{n-1} \chi_{A_n}$ for every positive integer n , where χ_S denotes the characteristic function of the set S . Then $\{\delta_n\}$ is a Dirac sequence:

1. $\delta_n(t) \geq 0$ for every positive integer n and every $t \in \mathbb{R}$.
2. Let n be a positive integer. Then $\int_{-\infty}^{\infty} \delta_n(t) dt = \int_{\frac{-1}{2^n}}^{\frac{1}{2^n}} 2^{n-1} dt = 1$.
3. Let $r > 0$. Then there exists a positive integer N such that, for every integer $k > N$, we have $\frac{1}{2^k} < r$. Thus, for every integer $k > N$, we have $\int_{\mathbb{R} \setminus [-r, r]} d_k(t) dt = 0$.