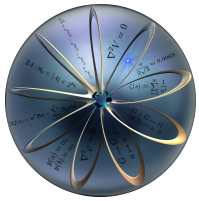

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tangent line

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Defines	tangency point

If the curve $y = f(x)$ of xy-plane is sufficiently smooth in its point $(x_0,\, y_0)$ and in a neighborhood of this, the curve may have a tangent line (or simply) in $(x_0,\, y_0)$. Then the *tangent line* of the curve $y = f(x)$ in the point $(x_0,\, y_0)$ is the limit position of the secant line through the two points $(x_0,\, y_0)$ and $(x,\, f(x))$ of the curve, when x limitlessly tends to the value x_0 (i.e. $x \to x_0$). Due to the smoothness,

$$f(x) \to f(x_0) = y_0,$$

$$(x,\, f(x)) \to (x_0,\, y_0),$$

and the slope m of the `http://planetmath.org/SecantLine`secant tends to

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0)$$

which will be the slope of the tangent line.

Note. Because the tangency is a local property on the curve, the tangent with the *tangency point* $(x_0,\, y_0)$ may intersect the curve in another point, and then the tangent is a `http://planetmath.org/SecantLine`secant, too. For example, the curve $y = x^3 - 3x^2$ has the line $y = 0$ as its tangent in the point $(0,\, 0)$ but this line the curve also in the point $(3,\, 0)$.