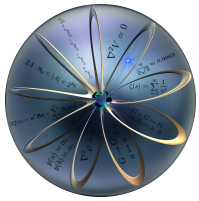

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derivative of Riemann integral

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Let f be a continuous function from an open subset A of $\mathbb{R}^2$ to $\mathbb{R}$. Suppose that also the partial derivative $f'_t(x,\, t)$ is continuous in A which contains the line segments along which the integration is performed and that $a(t)$ and $b(t)$ are real functions differentiable in some point t_0. Denote

$$F(t) = \int_{a(t)}^{b(t)} f(x,\, t)\, dx$$

and

$$G(t) = b'(t_0) \cdot f(b(t),\, t) - a'(t_0) \cdot f(a(t),\, t) + \int_{a(t)}^{b(t)} f'_t(x,\, t)\, dx.$$

Then one has the derivative

$$F'(t_0) = G(t_0)$$

in all such points $t = t_0$.