monotonicity criterion

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Suppose that $f\colon [a, b] \to \mathbb{R}$ is a function which is continuous on $[a, b]$ and differentiable on (a, b).

Then the following relations hold.

1. $f'(x) \geq 0$ for all $x \in (a, b) \Leftrightarrow f$ is an increasing function on $[a, b]$;

2. $f'(x) \leq 0$ for all $x \in (a, b) \Leftrightarrow f$ is a decreasing function on $[a, b]$;

3. $f'(x) > 0$ for all $x \in (a, b) \Rightarrow f$ is a strictly increasing function on $[a, b]$;

4. $f'(x) < 0$ for all $x \in (a, b) \Rightarrow f$ is a strictly decreasing function on $[a, b]$.

Notice that the third and fourth statement cannot be inverted. As an example consider the function $f\colon [-1, 1] \to \mathbb{R}$, $f(x) = x^3$. This is a strictly increasing function, but $f'(0) = 0$.