power mean

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Related topic	HarmonicMean
Related topic	GeneralMeansInequality
Related topic	RootMeanSquare3
Related topic	ProofOfGeneralMeansInequality
Related topic	DerivationOfZerothWeightedPowerMean
Related topic	DerivationOfHarmonicMeanAsTheLimitOfThePowerMean

The r-th power mean of the numbers $x_1, x_2, \dots, x_n$ is defined as:

$$M^r(x_1, x_2, \dots, x_n) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n} \right)^{1/r}.$$

The arithmetic mean is a special case when $r = 1$. The power mean is a continuous function of r, and taking limit when $r \to 0$ gives us the geometric mean:

$$M^0(x_1, x_2, \dots, x_n) = \sqrt[n]{x_1 x_2 \cdots x_n}.$$

Also, when $r = -1$ we get

$$M^{-1}(x_1, x_2, \dots, x_n) = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

the harmonic mean.

A generalization of power means are weighted power means.