substitution notation

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Related topic	TaylorSeriesOfArcusTangent
Related topic	PerimeterOfAstroid
Related topic	PotentialOfHollowBall
Related topic	VolumeOfEllipsoid
Related topic	Arc

The following are two commonly used *substitution notations* for calculating definite integrals with the antiderivative:

- $\int_a^b f(x)\,dx = [F(x)]_a^b$
- $\int_a^b f(x)\,dx = F(x)|_a^b$

Here, the right hand the difference $F(b) - F(a)$. For example, one has

$$\int_1^2 \frac{1}{x}\,dx \;=\; [\ln x]_1^2\,.$$

In Finland (only?) the corresponding notation is

$$\int_1^2 \frac{1}{x}\,dx \;=\; \mathop{\Big/}_{1}^{2} \ln x$$

which may be somewhat better; it is read in same manner as the definite integral notation, "*sijoitus 1:stä 2:een ln x*" (literally: "*substitution from 1 to 2 ln x*"). The position of the substitution symbol in front of the function to be substituted is perhaps more natural in the sense that the symbol has an operator (as e.g. the summing symbol). One of benefits of the Finnish notation is that one can comfortably clarify in it which is the variable to be substituted (as in the sum notation), e.g. in the case

$$\int_0^\pi \sin tx\,dt \;=\; -\frac{1}{x}\mathop{\Big/}_{t=0}^{\pi} \cos tx.$$

The notation

$$\mathop{\Big/}_{a}^{b} F(x) \;:=\; F(b) - F(a)$$

is extended also to such cases as

$$\mathop{\Big/}_{a}^{\infty} F(x) \;:=\; \lim_{b\to\infty} \mathop{\Big/}_{a}^{b} F(x).$$

Formulae

- $\Big/_{a}^{b} F(x) \; = \; -\Big/_{b}^{a} F(x)$
- $\Big/_{a}^{b} kF(x) \; = \; k\Big/_{a}^{b} F(x)$
- $\Big/_{a}^{b} [F_1(x) + \ldots + F_n(x)] \; = \; \Big/_{a}^{b} F_1(x) + \ldots + \Big/_{a}^{b} F_n(x)$
- $\int_a^b u(x)\, v'(x)\, dx \; = \; \Big/_{a}^{b} u(x)\, v(x) - \int_a^b u'(x)\, v(x)\, dx$

Note. There are in Finland also some other "national", unofficial mathematical notations used in universities, e.g.

$$-\!\!\!\ni\!\!\!-$$

which means 'such that'. For example, one may write

$$\forall\, x \in \mathbb{Z}\; \exists\, y \in \mathbb{Z} \;\; -\!\!\!\ni\!\!\!- \;\; x{+}y = 0.$$