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derivative for parametric form

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Instead of the usual way $y = f(x)$ to present plane curves it is in many cases more comfortable to express both coordinates, x and y, by means of a suitable auxiliary variable, the parametre. It is true e.g. for the cycloid curve.

Suppose we have the parametric form

$$x = x(t), \quad y = y(t). \tag{1}$$

For getting now the derivative $\dfrac{dy}{dx}$ in a point P_0 of the curve, we chose another point P of the curve. If the values of the parametre t corresponding these points are t_0 and t, we thus have the points $(x(t_0),\, y(t_0))$ and $(x(t),\, y(t))$ and the slope of the secant line through the points is the difference quotient

$$\frac{y(t)-y(t_0)}{x(t)-x(t_0)} = \frac{\frac{y(t)-y(t_0)}{t-t_0}}{\frac{x(t)-x(t_0)}{t-t_0}}. \tag{2}$$

Let us assume that the functions (1) are differentiable when $t = t_0$ and that $x'(t_0) \neq 0$. As we let $t \to t_0$, the left side of (2) tends to the derivative $\frac{dy}{dx}$ and the side to the quotient $\frac{y'(t_0)}{x'(t_0)}$. Accordingly we have the result

$$\left(\frac{dy}{dx}\right)_{t=t_0} = \frac{y'(t_0)}{x'(t_0)}. \tag{3}$$

Note that the (3) may be written

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

Example. For the cycloid

$$x = a(\varphi - \sin\varphi), \quad y = a(1-\cos\varphi),$$

we obtain

$$\frac{dy}{dx} = \frac{\frac{d}{d\varphi}(1-\cos\varphi)}{\frac{d}{d\varphi}(\varphi-\sin\varphi)} = \frac{\sin\varphi}{1-\cos\varphi} = \cot\frac{\varphi}{2}.$$