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continuity of sine and cosine

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Theorem. The real functions $x \mapsto \sin x$ and $x \mapsto \cos x$ are continuous at every real number x .

Proof. Let ε be an arbitrary positive number. Denote $\Delta \sin x =: \sin z - \sin x$, $\Delta \cos x =: \cos z - \cos x$ where we suppose that $|z - x| < \frac{\pi}{2}$. We may interpret $|z - x|$ as an arc of the unit circle of the xy -plane. Let's think in the circle the right triangle with hypotenuse the chord of the arc and the catheti (i.e. the shorter sides) vertical and horizontal. Then $|\Delta \sin x|$ and $|\Delta \cos x|$ are just these catheti; so we have

$$|\Delta \sin x| \leq |z - x|, \quad |\Delta \cos x| \leq |z - x|.$$

If we make $|z - x| < \varepsilon$, then also $|\Delta \sin x|$ and $|\Delta \cos x|$ are less than ε . It means that both functions are continuous at x .

References

- [1] E. LINDELÖF: *Johdatus korkeampaan analyysiin*. Neljäs painos. Werner Söderström Osakeyhtiö, Porvoo ja Helsinki (1956).