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proof of quotient rule

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Let $F(x) = f(x)/g(x)$. Then

$$
\begin{aligned}
F'(x) &= \lim_{h\to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{hg(x+h)g(x)}
\end{aligned}
$$

Like the product rule, the key to this proof is subtracting and adding the same quantity. We separate f and g in the above expression by subtracting and adding the term $f(x)g(x)$ in the numerator.

$$
\begin{aligned}
F'(x) &= \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{hg(x+h)g(x)} \\
&= \lim_{h\to 0} \frac{g(x)\frac{f(x+h)-f(x)}{h} - f(x)\frac{g(x+h)-g(x)}{h}}{g(x+h)g(x)} \\
&= \frac{\lim_{h\to 0} g(x) \cdot \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} - \lim_{h\to 0} f(x) \cdot \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}}{\lim_{h\to 0} g(x+h) \cdot \lim_{h\to 0} g(x)} \\
&= \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}
\end{aligned}
$$