inequalities for differences of powers

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Oftentimes, one needs to estimate differences of powers of real numbers. The following inequalities are useful for this purpose:

$$n(u - v)v^{n-1} < u^n - v^n < n(u - v)u^{n-1}$$

$$nx \le (1 + x)^n - 1$$

$$(1 + x)^n - 1 \le \frac{nx}{1 - (n - 1)x}$$

Here n is an integer greater than 1. The first inequality holds when $0 < v < u$, the second inequality holds when $-1 < x$, and the third inequality holds when $-1 < x < 1/(n - 1)$. Equality can only occur in the latter two inequalities when $x = 0$.