proof of quotient rule (using product rule)

Suppose f and g are differentiable functions defined on some interval of $\mathbb{R}$, and g never vanishes. Let us prove that

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}.$$

Using the product rule $(fg)' = f'g + fg'$, and $(g^{-1})' = -g^{-2}g'$, we have

$$
\begin{aligned}
\left(\frac{f}{g}\right)' &= (fg^{-1})' \\
&= f'g^{-1} + f(g^{-1})' \\
&= f'g^{-1} + f(-1)g^{-2}g' \\
&= \frac{f'}{g} - \frac{fg'}{g^2} \\
&= \frac{f'g - fg'}{g^2}.
\end{aligned}
$$

Here $g^{-1} = 1/g$ and $g^{-2} = 1/g^2$.