logarithmic derivative

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Given a function f, the quantity f'/f is known as the *logarithmic derivative* of f. This name comes from the observation that, on account of the chain rule,

$$(\log f)' = f' \log'(f) = f'/f.$$

The logarithmic derivative has several basic properties which make it useful in various contexts.

The logarithmic derivative of the product of functions is the sum of their logarithmic derivatives. This follows from the product rule:

$$\frac{(fg)'}{fg} = \frac{fg' + f'g}{fg} = \frac{f'}{f} + \frac{g'}{g}$$

The logarithmic derivative of the quotient of functions is the difference of their logarithmic derivatives. This follows from the quotient rule:

$$\frac{(f/g)'}{f/g} = \frac{f'g - fg'}{g^2} \frac{g}{f} = \frac{f'}{f} - \frac{g'}{g}$$

The logarithmic derivative of the p-th power of a function is p times the logarithmic derivative of the function. This follows from the power rule:

$$\frac{(f^p)'}{f^p} = \frac{pf^{p-1}f'}{f^p} = p\,\frac{f'}{f}$$

The logarithmic derivative of the exponential of a function equals the derivative of a function. This follows from the chain rule:

$$\frac{\left(e^f\right)'}{e^f} = \frac{e^f\,f'}{e^f} = f'$$

Using these identities, it is rather easy to compute the logarithmic derivatives of expressions which are presented in factored form. For instance, suppose we want to compute the logarithmic derivative of

$$e^{x^2}\frac{(x-2)^3(x-3)}{x-1}.$$

Using our identities, we find that its logarithic derivative is

$$2x + \frac{3}{x-2} + \frac{1}{x-3} - \frac{1}{x-1}.$$