Lebesgue integral over a subset of the measure space

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Author	Wkbj79 (1863)
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Let $(X, \mathfrak{B}, \mu)$ be a measure space and $A \in \mathfrak{B}$.

Let $s \colon X \to [0, \infty]$ be a simple function. Then $\displaystyle\int_A s\, d\mu$ is defined as $\displaystyle\int_A s\, d\mu := \int_X \chi_A s\, d\mu$, where χ_A denotes the characteristic function of A.

Let $f \colon X \to [0, \infty]$ be a measurable function and $S = \{s \colon X \to [0, \infty] \ \mid \ s \text{ is a simple function and } s \leq f\}$. Then $\displaystyle\int_A f\, d\mu$ is defined as $\displaystyle\int_A f\, d\mu := \sup_{s \in S} \int_A s\, d\mu$.

By the properties of the Lebesgue integral of nonnegative measurable functions (property 3), we have that $\displaystyle\int_A f\, d\mu = \int_X \chi_A f\, d\mu$.

Let $f \colon X \to [-\infty, \infty]$ be a measurable function such that not both of $\displaystyle\int_A f^+\, d\mu$ and $\displaystyle\int_A f^-\, d\mu$ are infinite. (Note that f^+ and f^- are defined in the entry Lebesgue integral.) Then $\displaystyle\int_A f\, d\mu$ is defined as $\displaystyle\int_A f\, d\mu := \int_A f^+\, d\mu - \int_A f^-\, d\mu$.

By the properties of the Lebesgue integral of Lebesgue integrable functions (property 3), we have that $\displaystyle\int_A f\, d\mu = \int_X \chi_A f\, d\mu$.