elimination of unknown

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Consider the simultaneous polynomial equations

$$\begin{cases} a(x,\,y) & =: \ \sum_{i=0}^{m} a_i(y)x^i \ = \ 0, \\ b(x,\,y) & =: \ \sum_{j=0}^{n} b_j(y)x^j \ = \ 0 \end{cases} \tag{1}$$

in two unknowns x and y, where e.g. $m \geq n$. It is possible to eliminate one of the unknowns from (1), i.e. derive an `http://planetmath.org/Equivalent3`equivalent pair of polynomial equations

$$\begin{cases} f(y) & = \ 0, \\ g(x,\,y) & = \ 0. \end{cases}$$

First we form the polynomial

$$c(x,\,y) \ =: \ b_n(y)\,a(x,\,y) - a_m(y)\,x^{m-n}\,b(x,\,y), \tag{2}$$

the degree of which is less than m. When $(x_0,\,y_0)$ is a solution of (1), then it satisfies

$$\begin{cases} c(x,\,y) & = \ 0 \\ b(x,\,y) & = \ 0. \end{cases} \tag{3}$$

On the other hand, when $(x_1,\,y_1)$ is a solution of (3), then (2) implies that it satisfies also (1), except possibly in the case $b_n(y_1) = 0$.
We can continue similarly until we arrive at a pair of equations

$$\begin{cases} f(y) & = \ 0, \\ g(x,\,y) & = \ 0 \end{cases} \tag{4}$$

Substituting the roots of the former of the equations (4) into the latter one, which in practice is usually of first degree with respect to x, one can get the corresponding values of x. Hence one obtains all solutions of the original system of equations (1). Since the cases $b_n(y_1) = 0$ may yield wrong solutions, one should check them by substituting into (1).

Note. One can derive from the equations (1) an equation of lower degree also by eliminating from them the constant terms; the terms of resulting equation have as common factor x or its higher power, which is removed by dividing.