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## link between infinite products and sums

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Let

$$\prod_{k=1}^{\infty} p_k$$

be an infinite product such that  $p_k > 0$  for all  $k$ . Then the infinite product converges if and only if the infinite sum

$$\sum_{k=1}^{\infty} \log p_k$$

converges. Moreover

$$\prod_{k=1}^{\infty} p_k = \exp \sum_{k=1}^{\infty} \log p_k.$$

*Proof.*

Simply notice that

$$\prod_{k=1}^N p_k = \exp \sum_{k=1}^N \log p_k.$$

If the infinite sum converges then (by continuity of exp function)

$$\lim_{N \rightarrow \infty} \prod_{k=1}^N p_k = \lim_{N \rightarrow \infty} \exp \sum_{k=1}^N \log p_k = \exp \sum_{k=1}^{\infty} \log p_k$$

and also the infinite product converges.