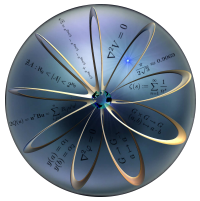

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## substitution for integration

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|---|---|
| Canonical name | SubstitutionForIntegration |
| Date of creation | 2013-03-22 14:33:38 |
| Last modified on | 2013-03-22 14:33:38 |
| Owner | pahio (2872) |
| Last modified by | pahio (2872) |
| Numerical id | 21 |
| Author | pahio (2872) |
| Entry type | Theorem |
| Classification | msc 26A36 |
| Synonym | variable changing for integration |
| Synonym | integration by substitution |
| Synonym | substitution rule |
| Related topic | IntegrationOfRationalFunctionOfSineAndCosine |
| Related topic | IntegrationOfFractionPowerExpressions |
| Related topic | ChangeOfVariableInDefiniteIntegral |

For determining the antiderivative $F(x)$ of a given real function $f(x)$ in a "closed form", i.e. for integrating $f(x)$, the result is often obtained by using the

**Theorem.** If

$$\int f(x)\,dx = F(x) + C$$

and $x = x(t)$ is a differentiable function, then

$$F(x(t)) = \int f(x(t))\,x'(t)\,dt + c. \tag{1}$$

*Proof.* By virtue of the chain rule,

$$\frac{d}{dt}F(x(t)) = F'(x(t)) \cdot x'(t),$$

and according to the supposition, $F'(x) = f(x)$. Thus we get the claimed equation (1).

**Remarks.**

- The expression $x'(t)\,dt$ in (1) may be understood as the differential of $x(t)$.
- For returning to the original variable $x$, the inverse function $t = t(x)$ of $x(t)$ must be substituted to $F(x(t))$.

**Example.** For integrating $\int \frac{x\,dx}{1+x^4}$ we take $x^2 = t$ as a new variable. Then, $2x\,dx = dt$, $x\,dx = \frac{dt}{2}$, and we get

$$\int \frac{x\,dx}{1+x^4} = \frac{1}{2}\int \frac{dt}{1+t^2} = \frac{1}{2}\arctan t + C = \frac{1}{2}\arctan x^2 + C.$$