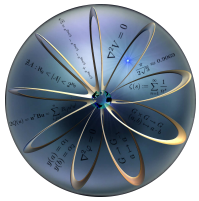

planetmath.org

Math for the people, by the people.

example of infinitesimal hyperreal number

Canonical name	ExampleOfInfinitesimalHyperrealNumber
Date of creation	2013-03-22 17:25:57
Last modified on	2013-03-22 17:25:57
Owner	asteroid (17536)
Last modified by	asteroid (17536)
Numerical id	4
Author	asteroid (17536)
Entry type	Example
Classification	msc 26E35

The hyperreal number $\{\frac{1}{n}\}_{n\in\mathbb{N}} \in {}^*\mathbb{R}$ is infinitesimal.

Proof - Let $\mathcal{F}$ be the nonprincipal ultrafilter in the `http://planetmath.org/Hyperreal`entry.

$\{n \in \mathbb{N} : 0 < \frac{1}{n}\} = \mathbb{N} \in \mathcal{F}$ so $0 < \{\frac{1}{n}\}_{n\in\mathbb{N}}$.

Given any positive $a \in \mathbb{R}$ we have that $\{n \in \mathbb{N} : a \leq \frac{1}{n}\}$ is finite, so $\{n \in \mathbb{N} : \frac{1}{n} < a\} \in \mathcal{F}$ and therefore $\{\frac{1}{n}\}_{n\in\mathbb{N}} < \{a\}_{n\in\mathbb{N}}$.

Thus $0 < \{\frac{1}{n}\}_{n\in\mathbb{N}} < \{a\}_{n\in\mathbb{N}}$ for every positive real number $\{a\}_{n\in\mathbb{N}} \in \mathbb{R}$, and so $\{\frac{1}{n}\}_{n\in\mathbb{N}}$ is infinitesimal.$\square$