# alternative proof of the fundamental theorem of calculus

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An alternative proof for the first part involves the use of a formula derived by the method of exhaustion:

$$\int_a^b f(t)dt = (b-a)\sum_{n=1}^{\infty}\sum_{m=1}^{2^n-1}(-1)^{m+1}\,2^{-n}f\left(a+m(b-a)/2^n\right).$$

Given that

$$F(x)=\int_a^x f(t)dt,$$

and

$$F'(x)=\lim_{\Delta x\to 0}\frac{F(x+\Delta x)-F(x)}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\int_x^{x+\Delta x} f(t)dt,$$

the above formula leads to:

$$F'(x)=\lim_{\Delta x\to 0}\frac{(x+\Delta x-x)}{\Delta x}\sum_{n=1}^{\infty}\sum_{m=1}^{2^n-1}(-1)^{m+1}\,2^{-n}f\left(x+m\Delta x/2^n\right),$$

or

$$F'(x)=\sum_{n=1}^{\infty}\sum_{m=1}^{2^n-1}(-1)^{m+1}\,2^{-n}f\left(x\right).$$

Since it can be shown that

$$\sum_{n=1}^{\infty}\sum_{m=1}^{2^n-1}(-1)^{m+1}\,2^{-n}=\sum_{n=1}^{\infty}2^{-n}=1,$$

It follows that

$$F'(x)=f(x).$$

The second part of the proof is identical to the parent.