Clairaut's theorem

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Clairaut's Theorem. *If* $\mathbf{f}\colon \mathbb{R}^n \to \mathbb{R}^m$ *is a function whose second partial derivatives exist and are continuous on a set* $S \subseteq \mathbb{R}^n$*, then*

$$\frac{\partial^2 f}{\partial x_i \partial x_j} = \frac{\partial^2 f}{\partial x_j \partial x_i}$$

on S*, where* $1 \leq i, j \leq n$*.*

This theorem is commonly referred to as *the equality of mixed partials.* It is usually first presented in a vector calculus course, and is useful in this context for proving basic properties of the interrelations of gradient, divergence, and curl. For example, if $\mathbf{F}\colon \mathbb{R}^3 \to \mathbb{R}^3$ is a function satisfying the hypothesis, then $\nabla \cdot (\nabla \times \mathbf{F}) = 0$. Or, if $f\colon \mathbb{R}^3 \to \mathbb{R}$ is a function satisfying the hypothesis, then $\nabla \times \nabla f = \mathbf{0}$.