



Helmholtz decomposition

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Owner	invisiblerhino (19637)
Last modified by	invisiblerhino (19637)
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Author	invisiblerhino (19637)
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The Helmholtz theorem states that any vector \mathbf{F} may be decomposed into an irrotational (curl-free) and a solenoidal (divergence-free) part under certain conditions (given below). More precisely, it may be written in the form:

$$\mathbf{F} = -\nabla\varphi + \nabla \times \mathbf{A} \tag{1}$$

where φ is a scalar potential and \mathbf{A} is a vector potential. By the definitions of scalar and vector potentials it follows that the first term on the right-hand side is irrotational and the second is solenoidal. The general conditions for this to be true are:

1. The divergence of \mathbf{F} must vanish at infinity.
2. The curl of \mathbf{F} must also vanish at infinity.