Barbălat's lemma

Canonical name	BarbualatsLemma
Date of creation	2013-03-22 14:52:31
Last modified on	2013-03-22 14:52:31
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	7
Author	jirka (4157)
Entry type	Theorem
Classification	msc 26A06
Synonym	Barbalat's lemma

Lemma (Barbălat)**.** *Let* $f\colon (0,\infty) \to \mathbb{R}$ *be Riemann integrable and uniformly continuous then*

$$\lim_{t\to\infty} f(t) = 0.$$

Note that if f is non-negative, then Riemann integrability is the same as being L^1 in the sense of Lebesgue, but if f oscillates then the Lebesgue integral may not exist.

Further note that the uniform continuity is required to prevent sharp "spikes" that might prevent the limit from existing. For example suppose we add a spike of height 1 and area 2^{-n} at every integer. Then the function is continuous and L^1 (and thus Riemann integrable), but $f(t)$ would not have a limit at infinity.

References

[1] Hartmut Logemann, Eugene P. Ryan. . *The American Mathematical Monthly*, 111(10):864–889, 2004.