nabla acting on products

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Defines	gradient of vector
Defines	divergence of dyad product
Defines	curl of dyad product

Let f, g be differentiable scalar fields and $\vec{u}$, $\vec{v}$ differentiable vector fields in some domain of $\mathbb{R}^3$. There are following formulae:

- Gradient of a product function
  $\nabla(fg) = (\nabla f)g + (\nabla g)f$
- Divergence of a scalar-multiplied vector
  $\nabla \cdot (f\vec{u}) = (\nabla f) \cdot \vec{u} + (\nabla \cdot \vec{u})f$
- Curl of a scalar-multiplied vector
  $\nabla\times(f\vec{u}) = (\nabla f) \times \vec{u} + (\nabla\times\vec{u})f$
- Divergence of a vector product
  $\nabla \cdot (\vec{u}\times\vec{v}) = (\nabla\times\vec{u}) \cdot \vec{v} - (\nabla\times\vec{v}) \cdot \vec{u}$
- Curl of a vector product
  $\nabla\times(\vec{u}\times\vec{v}) = (\vec{v} \cdot \nabla)\vec{u} - (\vec{u} \cdot \nabla)\vec{v} - (\nabla \cdot \vec{u})\vec{v} + (\nabla \cdot \vec{v})\vec{u}$
- Gradient of a scalar product
  $\nabla(\vec{u} \cdot \vec{v}) \;=\; (\vec{v} \cdot \nabla)\vec{u} + (\vec{u} \cdot \nabla)\vec{v} + \vec{v}\times(\nabla\times\vec{u}) + \vec{u}\times(\nabla\times\vec{v})$
  or, using dyads,
  $\nabla(\vec{u} \cdot \vec{v}) = (\nabla\vec{u}) \cdot \vec{v} + (\nabla\vec{v}) \cdot \vec{u}$
- Gradient of a vector product
  $\nabla(\vec{u}\times\vec{v}) = (\nabla\vec{u})\times\vec{v} - (\nabla\vec{v})\times\vec{u}$
- Divergence of a dyad product
  $\nabla \cdot (\vec{u}\,\vec{v}) = (\nabla\cdot\vec{u})\,\vec{v} + \vec{u} \cdot \nabla\vec{v}$
- Curl of a dyad product
  $\nabla\times(\vec{u}\,\vec{v}) = (\nabla\times\vec{u})\,\vec{v} - \vec{u}\times\nabla\vec{v}$

Explanations

1. $\vec{v} \cdot \nabla$ means the operator $v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}$.
2. The *gradient of a vector* $\vec{w}$ is defined as the dyad $\nabla\vec{w} := \vec{i}\,\frac{\partial\vec{w}}{\partial x} + \vec{j}\,\frac{\partial\vec{w}}{\partial y} + \vec{k}\,\frac{\partial\vec{w}}{\partial z}$.

3. The *divergence* and the *curl* of a dyad product are defined by the equation
$\nabla * (\vec{u}\vec{v}) := \vec{i} * \frac{\partial(\vec{u}\vec{v})}{\partial x} + \vec{j} * \frac{\partial(\vec{u}\vec{v})}{\partial y} + \vec{k} * \frac{\partial(\vec{u}\vec{v})}{\partial z}$, where the asterisks are dots or crosses and the partial derivatives of the dyad product the expression $\frac{\partial(\vec{u}\vec{v})}{\partial x} = \frac{\partial \vec{u}}{\partial x}\vec{v} + \vec{u}\frac{\partial \vec{v}}{\partial x}$ and so on.