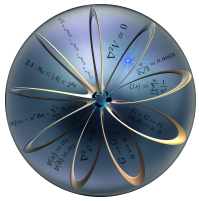

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higher order derivatives

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Related topic	TaylorSeriesOfHyperbolicFunctions
Defines	derivative function
Defines	first derivative
Defines	second derivative
Defines	order of derivative
Defines	differentiation
Defines	differentiate
Defines	twice differentiable

Let the real function f be defined and differentiable on the open interval I. Then for every $x \in I$, there exists the value $f'(x)$ as a certain real number. This means that we have a new function

$$x \mapsto f'(x), \tag{1}$$

the so-called *derivative function* of f; it is denoted by

$$f' : I \to \mathbb{R}$$

or simply f'.

Forming the derivative function of a function is called *differentiation*, the corresponding verb is *differentiate*.

If the derivative function f' is differentiable on I, then we have again a new function, the derivative function of the derivative function of f, which is denoted by f''. Then f is said to be *twice differentiable*. Formally,

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h} \quad \text{for all } x \in I.$$

The function $x \mapsto f''(x)$ is called the or *the second derivative* of f. Similarly, one can call (1) the of f.

Example. The first derivative of $x \mapsto x^3$ is $x \mapsto 3x^2$ and the second derivative is $x \mapsto 6x$, since

$$\frac{d}{dx}(3x^2) = 2 \cdot 3x^{2-1} = 6x.$$

If also f'' is a differentiable function, its derivative function is denoted by f''' and called the of f, and so on.

Generally, f can have the derivatives of first, second, third, ..., nth order, where n may be an arbitrarily big positive integer. If n is four or greater, the nth derivative of f is usually denoted by $f^{(n)}$. In , it's sometimes convenient to think that the $0th$ *order derivative* $f^{(0)}$ of f is the function f itself.

The phrase "f is infinitely differentiable" means that f has the derivatives of all .