reduction formulas

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To obtain a reduction formula for $\int \sin^n x \cos^m x \, dx$:

- Split off $\sin x$ to integrate by parts

$= \int \sin^{n-1} x \cos^m x \sin x \, dx$

Take $u = \sin^{n-1} x \cos^m x$, and $dv = \sin x \, dx$

So $du = [(n-1)\sin^{n-2} x \cos^{m+1} x - m \sin^n x \cos^{m-1} x] \, dx$, and $v = -\cos x$

- Then simplify to get

$= -\sin^{n-1} x \cos^{m+1} x + (n-1) \int \sin^{n-2} x \cos^{m+2} x \, dx - m \int \sin^n x \cos^m x \, dx$

- Now use the identity $\sin^2 x + \cos^2 x = 1$ in the middle term and simplify to get

$= -\sin^{n-1} x \cos^{m+1} x + (n-1) \int \sin^{n-2} x \cos^m x \, dx - (n-1) \int \sin^n x \cos^m x \, dx - m \int \sin^n x \cos^m x \, dx$

- Take the last two integrals to the left side:

$[1+(n-1)+m] \int \sin^n x \cos^m x \, dx = -\sin^{n-1} x \cos^{m+1} x + (n-1) \int \sin^{n-2} x \cos^m x \, dx$

- Since $[1+(n-1)+m] = m+n$ divide both sides by $m+n$ and hence

$$\int \sin^n x \cos^m x \, dx = -\frac{\sin^{n-1} x \cos^{m+1} x}{m+n} + \frac{n-1}{m+n} \int \sin^{n-2} x \cos^m x \, dx$$

Using the exact same method but instead of splitting off $\sin x$, one can split off $\cos x$ and follow similar procedure to obtain another reduction formula:

$$\int \sin^n x \cos^m x \, dx = \frac{\sin^{n+1} x \cos^{m-1} x}{m+n} + \frac{m-1}{m+n} \int \sin^n x \cos^{m-2} x \, dx$$