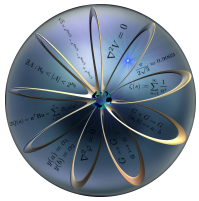

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limit examples

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Example 1. Determine the limit $\lim\limits_{x\to 0}\dfrac{\tan x}{x}$. — Using the definition of tan and the http://planetmath.org/LimitRulesOfFunctionslimit rule of product we can write

$$\lim_{x\to 0}\frac{\tan x}{x}=\lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right)=\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x}.$$

The limit in the former http://planetmath.org/Productfactor is 1 by the http://planetmath.org/LimitOfDisplaystyleFracsinXxAsXApproaches0parent entry. Also the latter limit is 1, since $\cos x$ and thus the quotient $\dfrac{1}{\cos x}$ is continuous in the point $x=0$ (see continuity of sine and cosine). Accordingly the desired limit is 1.

Example 2. Determine the limit $\lim\limits_{x\to 0}\dfrac{\sin ax}{\sin bx}$. — As above, we can write

$$\lim_{x\to 0}\frac{\sin ax}{\sin bx}=\lim_{x\to 0}\left(\frac{\sin ax}{ax}\cdot\frac{bx}{\sin bx}\cdot\frac{a}{b}\right)=\lim_{x\to 0}\frac{\sin ax}{ax}\cdot\lim_{x\to 0}\frac{bx}{\sin bx}\cdot\lim_{x\to 0}\frac{a}{b}=1\cdot 1\cdot\frac{a}{b}=\frac{a}{b}.$$

Example 3. The perimeter of a regular n-gon, circumscribed to a circle with radius 1, is $2n\tan\frac{\pi}{n}$. Determine the limit of this perimeter as n tends to infinity. — Utilising the example 1 we can calculate

$$\lim_{n\to\infty}2n\tan\frac{\pi}{n}=\lim_{n\to\infty}2\pi\frac{\tan\frac{\pi}{n}}{\frac{\pi}{n}}=2\pi\cdot 1=2\pi,$$

which is the circumference of the circle.

Example 4. Determine the limit $\lim\limits_{x\to 0}\dfrac{\arcsin x}{x}$. — If we denote

$$\arcsin x:=y,$$

the monotonicity of the http://planetmath.org/CyclometricFunctionsarcus sine function on $[-1,\,1]$ implies that "$x\to 0$" is http://planetmath.org/Equivalent3equivalent to "$y\to 0$". Then we can calculate:

$$\lim_{x\to 0}\frac{\arcsin x}{x}=\lim_{y\to 0}\frac{y}{\sin y}=\lim_{y\to 0}\frac{1}{\frac{\sin y}{y}}=\frac{1}{1}=1.$$

Example 5. One may use the definition of derivative in

$$\lim_{x\to 0}\frac{\arctan x}{x}=\lim_{x\to 0}\frac{\arctan x-\arctan 0}{x-0}=\left[\frac{d}{dx}\arctan x\right]_{x=0}=\frac{1}{1+0^2}=1.$$