fundamental theorem of integral calculus

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The derivative of a real function, which has on a whole interval a http://planetmath.org/Cons
value c, vanishes in every point of this interval:

$$\frac{d}{dx}c \;=\; 0$$

The converse theorem of this is also true. Ernst Lindelöf calls it the *fundamental theorem of integral calculus* (in Finnish *integraalilaskun peruslause*). It can be formulated as

Theorem. If a real function in continuous and its derivative vanishes in all points of an interval, the value of this function does not change on this interval.

Proof. We make the antithesis that there were on the interval two distinct points x_1 and x_2 with $f(x_1) \neq f(x_2)$. Then the mean-value theorem guarantees a point ξ between x_1 and x_2 such that

$$f'(\xi) \;=\; \frac{f(x_1)-f(x_2)}{x_1-x_2},$$

which value is distinct from zero. This is, however, impossible by the assumption of the theorem. So the antithesis is wrong and the theorem .

The contents of the theorem may be expressed also such that if two functions have the same derivative on a whole interval, then the difference of the functions is constant on this interval. Accordingly, if F is an antiderivative of a function f, then any other antiderivative of f has the form $x \mapsto F(x)+C$, where C is a constant.