fundamental theorem of calculus for Riemann integration

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In this entry we discuss the fundamental theorems of calculus for Riemann integration.

- Let f be a Riemann integrable function on an interval $[a,b]$ and F defined in $[a,b]$ by $F(x)=\int_a^x f(t)\,dt+k$, where $k\in\mathbb{R}$ is a constant. Then F is continuous in $[a,b]$ and $F'=f$ `http://planetmath.org/MeasureZeroInMathbbRn`almost everywhere.

- Let F be a continuous function in an interval $[a,b]$ and f a Riemann integrable function such that $F'(x)=f(x)$ except at most in a finite number of points x. Then $F(x)-F(a)=\int_a^x f(t)\,dt$.

0.1 Observations

Notice that the second fundamental theorem is not a converse of the first. In the first we conclude that $F'=f$ except in a set of `http://planetmath.org/MeasureZeroInMathbb` zero, while in the second we assume that $F'=f$ except in a finite number of points. In fact, the two theorems can never be the converse of each other as the following example shows:

Example : Let F be the devil staircase function, defined on $[0,1]$. We have that

- F is continuous in $[0,1]$,
- $F'=0$ except in a set of (this set must be contained in the Cantor set),
- $f:=0$ is clearly a Riemann integrable function and $\int_0^x 0\,dt=0$.

Thus, $F(x)\neq\int_0^x F'(t)\,dt$.

This leads to the question: what kind functions F can be expressed as $F(x)=F(a)+\int_a^x g(t)\,dt$, for some function g ? The answer to this question lies in the concept of `http://planetmath.org/AbsolutelyContinuousFunction2`absolute continuity (a which the devil staircase does not possess), but for that a more general of integration must be developed (the `http://planetmath.org/Integral2`Lebesgue integration).