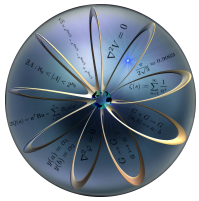

planetmath.org

Math for the people, by the people.

limit inferior

Canonical name	LimitInferior
Date of creation	2013-03-22 12:22:01
Last modified on	2013-03-22 12:22:01
Owner	rmilson (146)
Last modified by	rmilson (146)
Numerical id	10
Author	rmilson (146)
Entry type	Definition
Classification	msc 26A03
Synonym	liminf
Synonym	infimum limit
Related topic	LimitSuperior

Let $S \subset \mathbb{R}$ be a set of real numbers. Recall that a limit point of S is a real number $x \in \mathbb{R}$ such that for all $\epsilon > 0$ there exist infinitely many $y \in S$ such that

$$|x - y| < \epsilon.$$

We define $\liminf S$, pronounced the *limit inferior* of S, to be the infimum of all the limit points of S. If there are no limit points, we define the limit inferior to be $+\infty$.

The two most common notations for the limit inferior are

$$\liminf S$$

and

$$\underline{\lim}\, S\,.$$

An alternative, but equivalent, definition is available in the case of an infinite sequence of real numbers $x_0, x_1, x_2, , \ldots$. For each $k \in \mathbb{N}$, let y_k be the infimum of the k^{th} tail,

$$y_k = \inf_{j \geq k} x_j.$$

This construction produces a non-decreasing sequence

$$y_0 \leq y_1 \leq y_2 \leq \cdots,$$

which either converges to its supremum, or diverges to $+\infty$. We define the limit inferior of the original sequence to be this limit;

$$\liminf_k x_k = \lim_k y_k.$$