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limit of $(1+s_n)^n$ is one when limit of ns_n is zero

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The inequalities for differences of powers may be used to show that $\lim_{n\to\infty}(1+s_n)^n = 1$ when $\lim_{n\to\infty} ns_n = 0$. This fact plays an important role in the development of the theory of the exponential function as a limit of powers.

To derive this limit, we bound $1+s_n$ using the inequalities for differences of powers.

$$ns_n \le (1+s_n)^n - 1 \le \frac{ns_n}{1-(n-1)s_n}$$

Since $\lim_{n\to\infty} ns_n = 0$, there must exist N such that $ns_n < 1/2$ when $n > N$. Hence, when $n > N$,

$$|(1+s_n)^n - 1| < 2|ns_n|$$

so, as $n \to \infty$, we have $(1+s_n)^n \to 1$.