Riemann-Stieltjes integral

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Defines	Riemann-Stieltjes sum
Defines	integrator

Let f and α be bounded, real-valued functions defined upon a closed finite interval $I = [a, b]$ of $\mathbb{R}(a \neq b)$, $P = \{x_0, ..., x_n\}$ a partition of I, and t_i a point of the subinterval $[x_{i-1}, x_i]$. A sum of the form

$$S(P, f, \alpha) = \sum_{i=1}^{n} f(t_i)(\alpha(x_i) - \alpha(x_{i-1}))$$

is called a **Riemann-Stieltjes sum** of f with respect to α. f is said to be **Riemann Stieltjes integrable with respect to** α on I if there exists $A \in \mathbb{R}$ such that given any $\epsilon > 0$ there exists a partition P_ϵ of I for which, for all P finer than P_ϵ and for every choice of points t_i, we have

$$|S(P, f, \alpha) - A| < \epsilon$$

If such an A exists, then it is unique and is known as the **Riemann-Stieltjes integral of** f **with respect to** α. f is known as the **integrand** and α the **integrator**. The integral is denoted by

$$\int_a^b f d\alpha \quad \text{or} \quad \int_a^b f(x) d\alpha(x)$$