Lipschitz condition

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A mapping $f : X \to Y$ between metric spaces is said to satisfy the Lipschitz condition, or to be *Lipschitz continuous* or *L-Lipschitz* if there exists a real constant L such that

$$d_Y(f(p), f(q)) \leq L d_X(p, q), \quad \text{for all } p, q \in X.$$

The least constant L for which the previous inequality holds, is called the *Lipschitz constant* of f. The space of Lipschitz continuous functions is often denoted by $\mathrm{Lip}(X, Y)$.

Clearly, every Lipschitz continuous function is continuous.

Notes. More generally, one says that a mapping satisfies a Lipschitz condition of order $\alpha > 0$ if there exists a real constant C such that

$$d_Y(f(p), f(q)) \leq C d_X(p, q)^{\alpha}, \quad \text{for all } p, q \in X.$$

Functions which satisfy this condition are also called *Hölder continuous* or α-*Hölder*. The vector space of such functions is denoted by $C^{0,\alpha}(X, Y)$ and hence $\mathrm{Lip} = C^{0,1}$.