support of integrable function with respect to counting measure is countable

Canonical name	SupportOfIntegrableFunctionWithRespectToCountingMeasureIsCountable
Date of creation	2013-03-22 14:59:34
Last modified on	2013-03-22 14:59:34
Owner	Wkbj79 (1863)
Last modified by	Wkbj79 (1863)
Numerical id	11
Author	Wkbj79 (1863)
Entry type	Result
Classification	msc 28A12
Related topic	UncountableSumsOfPositiveNumbers
Related topic	SupportOfIntegrableFunctionIsSigmaFinite

Let $(X, \mathfrak{B}, \mu)$ be a measure space with μ the counting measure. If f is an integrable function, $\int_X f \, d\mu < \infty$, then it has `http://planetmath.org/Countable`countable `http://planetmath.org/Support6`support.

Proof. WLOG, we assume that f is real valued and is nonnegative. Let S_0 denote the preimage of the interval $[1, \infty)$ and, for every positive integer n, let S_n denote the preimage of the interval $\left[\frac{1}{n+1}, \frac{1}{n}\right)$. Since the integral of f is bounded, each S_n can be at most finite. Taking the union of all the S_n, we get the support $\operatorname{supp} f = \displaystyle\bigcup_{n=0}^{\infty} S_n$. Thus, $\operatorname{supp} f$ is a union of countably many finite sets and hence is countable. $\square$