metric entropy

Canonical name	MetricEntropy
Date of creation	2013-03-22 14:31:59
Last modified on	2013-03-22 14:31:59
Owner	Koro (127)
Last modified by	Koro (127)
Numerical id	6
Author	Koro (127)
Entry type	Definition
Classification	msc 28D20
Classification	msc 37A35
Synonym	entropy
Synonym	measure theoretic entropy

Let $(X, \mathscr{B}, \mu)$ be a probability space, and $T\colon X \to X$ a measure-preserving transformation. The entropy of T with respect to a finite measurable partition $\mathcal{P}$ is

$$h_\mu(T, \mathcal{P}) = \lim_{n\to\infty} H_\mu \left(\bigvee_{k=0}^{n-1} T^{-k}\mathcal{P} \right),$$

where H_μ is the entropy of a partition and $\vee$ denotes the join of partitions. The above limit always exists, although it can be $+\infty$. The entropy of T is then defined as

$$h_\mu(T) = \sup_{\mathcal{P}} h_\mu(T, \mathcal{P}),$$

with the supremum taken over all finite measurable partitions. Sometimes $h_\mu(T)$ is called the metric or measure theoretic entropy of T, to differentiate it from topological entropy.

Remarks.

1. There is a natural correspondence between finite measurable partitions and finite sub-σ-algebras of $\mathscr{B}$. Each finite sub-σ-algebra is generated by a unique partition, and clearly each finite partition generates a finite σ-algebra. Because of this, sometimes $h_\mu(T, \mathcal{P})$ is called the entropy of T with respect to the σ-algebra $\mathscr{P}$ generated by $\mathcal{P}$, and denoted by $h_\mu(T, \mathscr{P})$. This simplifies the notation in some instances.