Hahn-Kolmogorov theorem

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Let $\mathscr{A}_0$ be an algebra of subsets of a set X. If a finitely additive measure $\mu_0 \colon \mathscr{A} \to \mathbb{R} \cup \{\infty\}$ satisfies

$$\mu_0\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu_0(A_n)$$

for any disjoint family $\{A_n : n \in \mathbb{N}\}$ of elements of $\mathscr{A}_0$ such that $\cup_{n=0}^{\infty} A_n \in \mathscr{A}_0$, then μ_0 extends to a measure defined on the σ-algebra $\mathscr{A}$ generated by $\mathscr{A}_0$; i.e. there exists a measure $\mu \colon \mathscr{A} \to \mathbb{R} \cup \{\infty\}$ such that its http://planetmath.org/Restrictic to $\mathscr{A}_0$ coincides with μ_0.

If μ_0 is http://planetmath.org/SigmaFiniteσ-finite, then the extension is unique.