derivation of surface area measure on sphere

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The sphere of radius r can be described parametrically by spherical coordinates (what else ;)) as follows:

$$x = r \sin u \sin v$$

$$y = r \sin u \cos v$$

$$z = r \cos u$$

Then, using trigonometric identities to simplify expressions we find that

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} r\cos u \sin v & r \sin u \cos v \\ r \cos u \cos v & -r \sin u \sin v \end{vmatrix} = -r^2 \cos u \sin u$$

$$\frac{\partial(y,z)}{\partial(u,v)} = \begin{vmatrix} r\cos u \cos v & -r \sin u \sin v \\ -r \sin u & 0 \end{vmatrix} = -r^2 \sin^2 u \sin v$$

$$\frac{\partial(z,x)}{\partial(u,v)} = \begin{vmatrix} -r \sin u & 0 \\ r\cos u \sin v & r \sin u \cos v \end{vmatrix} = r^2 \sin^2 u \cos v$$

and hence, using more trigonometric identities, we find that

$$\sqrt{\left(\frac{\partial(x,y)}{\partial(u,v)}\right)^2 + \left(\frac{\partial(y,z)}{\partial(u,v)}\right)^2 + \left(\frac{\partial(z,x)}{\partial(u,v)}\right)^2} =$$

$$\sqrt{r^4 \cos^2 u \sin^2 u + r^4 \sin^4 u \sin^2 v + r^4 \sin^4 u \cos^2 v} = r^2 \sin u.$$

This means that, on a sphere

$$d^2 A = r^2 \sin u \, du \, dv.$$

Note that in the case of a unit sphere, ($r = 1$) this agrees with the formula presented in the second paragraph of subsection 2 of the main entry.

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