Haar measure

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Defines	left Haar measure
Defines	right Haar measure
Defines	bi-invariant Haar measure

1 Definition of Haar measures

Let G be a locally compact topological group, and denote by $\mathcal{B}$ the sigma algebra generated by the closed compact subsets of G. A *left Haar measure* on G is a measure μ on $\mathcal{B}$ which is:

1. outer regular on all sets $B \in \mathcal{B}$
2. inner regular on all open sets $U \in \mathcal{B}$
3. finite on all compact sets $K \in \mathcal{B}$
4. invariant under left translation: $\mu(gB) = \mu(B)$ for all sets $B \in \mathcal{B}$
5. nontrivial: $\mu(B) > 0$ for all non-empty open sets $B \in \mathcal{B}$.

A *right Haar measure* on G is defined similarly, except with left translation invariance replaced by right translation invariance ($\mu(Bg) = \mu(B)$ for all sets $B \in \mathcal{B}$). A *bi-invariant Haar measure* is a Haar measure that is both left invariant and right invariant.

2 Existence of Haar measures

For any discrete topological group G, the counting measure on G is a bi-invariant Haar measure. More generally, every locally compact topological group G has a left Haar measure μ, which is unique up to scalar multiples. In addition, G also admits a right Haar measure, and for an abelian group G the left and right Haar measures are always equal. The Haar measure plays an important role in the development of Fourier analysis and representation theory on locally compact groups such as Lie groups and profinite groups.