quadratic Julia set

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For each complex number c, there is an associated quadratic map $f_c\colon \mathbb{C} \to \mathbb{C}$ defined by $f_c(z) = z^2 + c$. Since polynomials are analytic, it follows that f_c has a Julia set $J(f_c)$, which we call the *quadratic Julia set* associated to c and denote by J_c.

The function can also be viewed as having $\mathbb{R}^2$ as its domain and codomain. If $c = a + ib$, then

$$f_c(x, y) = (x^2 - y^2 + a, 2xy + b).$$

The characterization of the Julia set J_c as all points z for which the collection of iterates $\{f^n\colon n \in \mathbb{N}\}$ is not a normal family can be roughly interpreted as saying that the Julia set includes only those points exhibiting chaotic behavior. In particular, points whose orbit under f goes to infinity are omitted from J_c, as well as points whose orbit converges to a point.

Sometimes for aesthetic purposes a Julia set is displayed with points of the latter type included. However, the chaoticity of the true Julia set can be exploited to plot an approximation very quickly. Given a single point z known to be in the quadratic Julia set J_c, its inverses under f, that is, the square roots of $z-c$, are also in J_c. Moreover, by the chaoticity condition the "backwards orbit" of z (selecting just one square root at each step) will be distributed fairly evenly over J_c, so this gives a computationally inexpensive method to plot Julia sets.

Before the advent of computers, the French mathematician Gaston Julia proved under what conditions a Julia set is connected or not connected. After computers became available, it became possible to make pictures displaying some of the complexity of these Julia sets, and the Mandelbrot set, a kind of index into connected quadratic Julia sets, was discovered.

In the same way that some people see recognizable shapes in clouds, some people see recognizable shapes in Julia sets, and some of them have been named accordingly. To give two examples: the San Marco dragon at $\frac{-3}{4} + 0i$ and the Douady rabbit at $\frac{-1}{8} + \frac{745}{1000}i$ (the coordinates can be varied by small values and still give very similar shapes).

Julia sets can be generalized to other iterated holomorphic functions on the complex plane or in a 3-dimensional space.

References

[1] H. Lauwerier, translated by Sophia Gill-Hoffstädt. *Fractals: Endlessly Repeated Geometric Figures* Princeton: Princeton University Press (1991): 124 - 151