product σ-algebra

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Given measurable spaces $(E,\mathcal{F})$ and $(F,\mathcal{G})$, the *product σ-algebra* $\mathcal{F}\times\mathcal{G}$ is defined to be the σ-algebra on the Cartesian product $E\times F$ generated by sets of the form $A\times B$ for $A\in\mathcal{F}$ and $B\in\mathcal{G}$.

$$\mathcal{F}\times\mathcal{G}=\sigma\left(A\times B\colon A\in\mathcal{F},B\in\mathcal{G}\right).$$

More generally, the product σ-algebra can be defined for an arbitrary number of measurable spaces $(E_i,\mathcal{F}_i)$, where i runs over an index set I. The product $\prod_i\mathcal{F}_i$ is the σ-algebra on the generalized cartesian product $\prod_i E_i$ generated by sets of the form $\prod_i A_i$ where $A_i\in\mathcal{F}_i$ for all i, and $A_i=E_i$ for all but finitely many i. If $\pi_j\colon\prod_i E_i\rightarrow E_j$ are the projection maps, then this is the smallest σ-algebra with respect to which each π_j is `http://planetmath.org/MeasurableFunctions`