complete measure

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A measure space $(X, \mathscr{S}, \mu)$ is said to be *complete* if every subset of a set of measure 0 is measurable (and consequently, has measure 0); i.e. if for all $E \in \mathscr{S}$ such that $\mu(E) = 0$ and for all $S \subset E$ we have $\mu(S) = 0$.

If a measure space is not complete, there exists a http://planetmath.org/CompletionOfAMeas of it, which is a complete measure space $(X, \overline{\mathscr{S}}, \overline{\mu})$ such that $\mathscr{S} \subset \overline{\mathscr{S}}$ and $\overline{\mu}_{|\mathscr{S}} = \mu$, where $\overline{\mathscr{S}}$ is the smallest σ-algebra containing both $\mathscr{S}$ and all subsets of elements of zero measure of $\mathscr{S}$.