centre of mass of half-disc

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Let E be the upper half-disc of the disc $x^2+y^2 \leqq R$ in $\mathbb{R}^2$ with a surface-density 1. By the symmetry, its centre of mass lies on its medium radius, and therefore we only have to calculate the ordinate Y of the centre of mass. For doing that, one can use the double integral

$$Y \;=\; \frac{1}{\nu(E)} \iint_E y\,dx\,dy,$$

where $\nu(E) = \frac{\pi R^2}{2}$ is the area of the half-disc. The region of integration is defined by

$$E \;=\; \{(x,\,y) \in \mathbb{R}^2 \vdots \; -R \leqq x \leqq R,\; 0 \leqq y \leqq \sqrt{R^2 - x^2}\}.$$

Accordingly we may write

$$Y \;=\; \frac{2}{\pi R^2}\int_{-R}^{R} dx \int_0^{\sqrt{R^2-x^2}} y\,dy \;=\; \frac{2}{\pi R^2}\int_{-R}^{R} \frac{R^2-x^2}{2}\,dx \;=\; \frac{2}{\pi R^2} \Bigg/_{x=-R}^{R} \left(\frac{R^2 x}{2} - \frac{x^3}{6}\right) \;=\; \frac{4R}{3\pi}.$$

Thus the centre of mass is the point $(0,\, \frac{4R}{3\pi})$.