countable unions and intersections of analytic sets are analytic

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A property of http://planetmath.org/AnalyticSet2analytic sets which makes them particularly suited to applications in measure theory is that, in common with http://planetmath.org/SigmaAlgebraσ-algebras, they are closed under countable unions and intersections.

Theorem 1. *Let $(X,\mathcal{F})$ be a paved space and $(A_n)_{n\in\mathbb{N}}$ be a sequence of $\mathcal{F}$-analytic sets. Then, $\bigcup_n A_n$ and $\bigcap_n A_n$ are $\mathcal{F}$-analytic.*

A consequence of this is that measurable sets are analytic, as follows.

Corollary. *Let $\mathcal{F}$ be a nonempty paving on a set X such that the http://planetmath.org/Comple of any $S\in\mathcal{F}$ is a union of countably many sets in $\mathcal{F}$.*

Then, every set A in the σ-algebra generated by $\mathcal{F}$ is $\mathcal{F}$-analytic.

For example, every closed subset of a metric space X is a union of countably many open sets. Therefore, the corollary shows that all Borel sets are analytic with respect to the open subsets of X.

That the corollary does indeed follow from Theorem **??** is a simple application of the monotone class theorem. First, as the collection $a(\mathcal{F})$ of $\mathcal{F}$-analytic sets is closed under countable unions and finite intersections, it will contain all finite unions of finite intersections of sets in $\mathcal{F}$ and their complements, which is an http://planetmath.org/RingOfSetsalgebra. Then, Theorem **??** says that $a(\mathcal{F})$ is closed under taking limits of increasing and decreasing sequences of sets. So, by the monotone class theorem, it contains the σ-algebra generated by $\mathcal{F}$.