proof of equivalent definitions of analytic sets for measurable spaces

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Let $(X,\mathcal{F})$ be a measurable space and A be a subset of X. For any uncountable Polish space Y with http://planetmath.org/BorelSigmaAlgebraBorel σ-algebra $\mathcal{B}$, we show that the following are equivalent.

1. A is $\mathcal{F}$-http://planetmath.org/AnalyticSet2analytic.

2. A is the http://planetmath.org/GeneralizedCartesianProductprojection of a set $S\in\mathcal{F}\otimes\mathcal{B}$ onto X.

Here, $\mathcal{F}\otimes\mathcal{B}$ denotes the http://planetmath.org/ProductSigmaAlgebraproduct σ-algebra of $\mathcal{F}$ and $\mathcal{B}$.
(??) implies (??): Let $\mathcal{G}$ denote the paving consisting of the closed subsets of Y. If A is $\mathcal{F}$-analytic then there exists a set $S\in(\mathcal{F}\times\mathcal{G})_{\sigma\delta}$ such that $A=\pi_X(S)$, where $\pi_X\colon X\times Y\to X$ is the projection map (see proof of equivalent definitions of analytic sets for paved spaces). In particular, $\mathcal{G}\subseteq\mathcal{B}$ implies that S is contained in the σ-algebra $\mathcal{F}\otimes\mathcal{B}$.
(??) implies (??): This is an immediate consequence of the result that projections of analytic sets are analytic.