absolutely continuous

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Let μ and ν be signed measures or complex measures on the same measurable space $(\Omega, \mathscr{S})$. We say that ν is *absolutely continuous* with respect to μ if, for each $A \in \mathscr{S}$ such that $|\mu|(A) = 0$, it holds that $\nu(A) = 0$. This is usually denoted by $\nu \ll \mu$.

Remarks.

If μ and ν are signed measures and (ν^+, ν^-) is the Jordan decomposition of ν, the following are equivalent:

1. $\nu \ll \mu$;
2. $\nu^+ \ll \mu$ and $\nu^- \ll \mu$;
3. $|\nu| \ll |\mu|$.

If ν is a finite signed or complex measure and $\nu \ll \mu$, the following useful property holds: for each $\varepsilon > 0$, there is a $\delta > 0$ such that $|\nu|(E) < \varepsilon$ whenever $|\mu|(E) < \delta$.