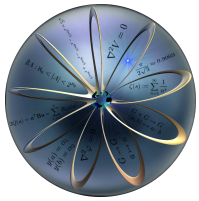

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σ-finite

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A measure space $(\Omega, \mathcal{B}, \mu)$ is a **finite measure space** if $\mu(\Omega) < \infty$; it is σ-**finite** if the total space is the union of a finite or countable family of sets of finite measure, i.e. if there exists a countable set $\mathcal{F} \subset \mathcal{B}$ such that $\mu(A) < \infty$ for each $A \in \mathcal{F}$, and $\Omega = \bigcup_{A \in \mathcal{F}} A$. In this case we also say that μ is a σ-finite measure. If μ is not σ-finite, we say that it is σ-**infinite**.

Examples. Any finite measure space is σ-finite. A more interesting example is the Lebesgue measure μ in $\mathbb{R}^n$: it is σ-finite but not finite. In fact

$$\mathbb{R}^n = \bigcup_{k \in \mathbb{N}} [-k, k]^n$$

($[-k, k]^n$ is a cube with center at 0 and side length $2k$, and its measure is $(2k)^n$), but $\mu(\mathbb{R}^n) = \infty$.