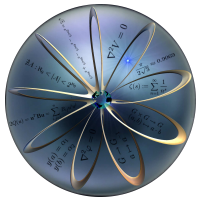

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Souslin scheme

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Synonym	Suslin scheme
Defines	regular scheme
Defines	result of a scheme

A *Souslin scheme* is a method of representing and defining analytic sets on a paved space $(X, \mathcal{F})$. Let $\mathcal{S}$ be the collection of finite sequences of positive integers. That is $\mathcal{S}$ is the disjoint union of $\mathbb{N}^n$ for $n = 1, 2, \ldots$.

A Souslin scheme on $\mathcal{F}$ is a collection $(A_s)_{s\in\mathcal{S}}$ of sets in $\mathcal{F}$. If $\mathcal{N} = \mathbb{N}^{\mathbb{N}}$ is Baire space then, for any $s \in \mathcal{N}$ and $n \in \mathbb{N}$, we write $s|_n \equiv (s_1, \ldots, s_n)$ for the restriction of s to $\{1, \ldots, n\}$. So, $s|_n \in \mathbb{N}^n$.

The *result* of the Souslin scheme (A_s) is defined to be

$$A = \bigcup_{s\in\mathcal{N}} \bigcap_{n=1}^{\infty} A_{s|_n}.$$

The set $\mathcal{S}$ can be partially ordered as follows. Say that $s \leq t$ if $s \in \mathbb{N}^r$ and $t \in \mathbb{N}^s$ for $r \leq s$, and $s_k = t_k$ for $k = 1, \ldots, r$. The scheme (A_s) is said to be *regular* if $A_s \supseteq A_t$ for all $s \leq t$.

It can be shown that the result of a Souslin scheme is $\mathcal{F}$-analytic and, conversely, any analytic set is the result of some scheme (see equivalent definitions of analytic sets).

References

[1] Jean Bourgain, *A stabilization property and its applications in the theory of sections*. Séminaire Choquet. Initiation à l'analyse, 17 no. 1 (1977).