measurability of analytic sets

Canonical name	MeasurabilityOfAnalyticSets
Date of creation	2013-03-22 18:47:24
Last modified on	2013-03-22 18:47:24
Owner	gel (22282)
Last modified by	gel (22282)
Numerical id	6
Author	gel (22282)
Entry type	Theorem
Classification	msc 28A05

http://planetmath.org/AnalyticSet2Analytic subsets of a measurable space $(X,\mathcal{F})$ do not, in general, have to be measurable. See, for example, http://planetmath.org/ALebesgueMeasurableButNonBorelSeta Lebesgue measurable but non-Borel set. However, the following result is true.

Theorem. *All analytic subsets of a measurable space are universally measurable.*

Therefore for a universally complete measurable space $(X,\mathcal{F})$ all $\mathcal{F}$-analytic sets are themselves in $\mathcal{F}$ and, in particular, this applies to any complete http://planetmath.org/SigmaFiniteσ-finite measure space $(X,\mathcal{F},\mu)$. For example, analytic subsets of the real numbers $\mathbb{R}$ are Lebesgue measurable.

The proof of the theorem follows as a consequence of the capacitability theorem. Suppose that A is an $\mathcal{F}$-analytic set. Then, for any finite measure μ on $(X,\mathcal{F})$, let μ^* be the outer measure generated by μ. This is an $\mathcal{F}$-capacity and, by the capacitability theorem, A is $(\mathcal{F},\mu^*)$-capacitable, hence is in the completion of $\mathcal{F}$ with respect to μ (see http://planetmath.org/CapacityGeneratedByAMeasureca generated by a measure). As this is true for all such finite measures, A is universally measurable.