Jordan decomposition

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Entry type	Definition
Classification	msc 28A12
Defines	positive variation
Defines	negative variation
Defines	total variation

Let $(\Omega, \mathscr{S}, \mu)$ be a signed measure space, and let (A, B) be a Hahn decomposition for μ. We define μ^+ and μ^- by

$$\mu^+(E) = \mu(A \cap E) \quad \text{and} \quad \mu^-(E) = -\mu(B \cap E).$$

This definition is easily shown to be independent of the chosen Hahn decomposition.

It is clear that μ^+ is a positive measure, and it is called the *positive variation* of μ. On the other hand, μ^- is a positive finite measure, called the *negative variation* of μ. The measure $|\mu| = \mu^+ + \mu^-$ is called the *total variation* of μ.

Notice that $\mu = \mu^+ - \mu^-$. This decomposition of μ into its positive and negative parts is called the *Jordan decomposition* of μ.