Vitali convergence theorem

Canonical name	VitaliConvergenceTheorem
Date of creation	2013-03-22 16:17:10
Last modified on	2013-03-22 16:17:10
Owner	stevecheng (10074)
Last modified by	stevecheng (10074)
Numerical id	9
Author	stevecheng (10074)
Entry type	Theorem
Classification	msc 28A20
Synonym	uniform-integrability convergence theorem
Related topic	ModesOfConvergenceOfSequencesOfMeasurableFunctions
Related topic	UniformlyIntegrable
Related topic	DominatedConvergenceTheorem

Let $f_1, f_2, \dots$ be $\mathbf{L}^p$-integrable functions on some measure space, for $1 \leq p < \infty$.

The sequence $\{f_n\}$ converges in $\mathbf{L}^p$ to a measurable function f if and and only if

i the sequence $\{f_n\}$ converges to f in measure;

ii the functions $\{|f_n|^p\}$ are uniformly integrable; and

iii for every $\epsilon > 0$, there exists a set E of finite measure, such that $\int_{E^c} |f_n|^p < \epsilon$ for all n.

Remarks

This theorem can be used as a replacement for the more well-known dominated convergence theorem, when a dominating cannot be found for the functions f_n to be integrated. (If this theorem is known, the dominated convergence theorem can be derived as a special case.)

In a finite measure space, condition (iii) is trivial. In fact, condition (iii) is the tool used to reduce considerations in the general case to the case of a finite measure space.

In probability , the definition of "uniform integrability" is slightly different from its definition in general measure theory; either definition may be used in the statement of this theorem.

References

[1] Gerald B. Folland. *Real Analysis: Modern Techniques and Their Applications*, second ed. Wiley-Interscience, 1999.

[2] Jeffrey S. Rosenthal. *A First Look at Rigorous Probability Theory.* World Scientific, 2003.