signed measure

Canonical name	SignedMeasure
Date of creation	2013-03-22 13:26:55
Last modified on	2013-03-22 13:26:55
Owner	Koro (127)
Last modified by	Koro (127)
Numerical id	8
Author	Koro (127)
Entry type	Definition
Classification	msc 28A12

A *signed measure* on a measurable space $(\Omega, \mathscr{S})$ is a function $\mu : \mathscr{S} \to \mathbb{R} \cup \{+\infty\}$ which is `http://planetmath.org/Additive`σ-additive and such that $\mu(\emptyset) = 0$.

Remarks.

1. The usual (positive) measure is a particular case of signed measure, in which $|\mu| = \mu$ (see Jordan decomposition.)

2. Notice that the value $-\infty$ is not allowed. For some authors, a signed measure can only take finite values (so that $+\infty$ is not allowed either). This is sometimes useful because it turns the space of all signed measures into a normed vector space, with the natural operations, and the norm given by $\|\mu\| = |\mu|(\Omega)$.

3. An important example of signed measures arises from the usual measures in the following way: Let $(\Omega, \mathscr{S}, \mu)$ be a measure space, and let f be a (real valued) measurable function such that

$$\int_{\{x\in\Omega: f(x)<0\}} |f| d\mu < \infty.$$

Then a signed measure is defined by

$$A \mapsto \int_A f d\mu.$$