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Choquet capacity

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A *Choquet capacity*, or just *capacity*, on a set X is a kind of set function, mapping the power set $\mathcal{P}(X)$ to the real numbers.

Definition. *Let $\mathcal{F}$ be a collection of subsets of X. Then, an $\mathcal{F}$-capacity is an increasing set function*

$$I\colon \mathcal{P}(X) \to \mathbb{R}_+$$

satisfying the following.

1. *If $(A_n)_{n\in\mathbb{N}}$ is an increasing sequence of subsets of X then $I(A_n) \to I\left(\bigcup_m A_m\right)$ as $n \to \infty$.*
2. *If $(A_n)_{n\in\mathbb{N}}$ is a decreasing sequence of subsets of X such that $A_n \in \mathcal{F}$ for each n, then $I(A_n) \to I\left(\bigcap_m A_m\right)$ as $n \to \infty$.*

The condition that I is increasing means that $I(A) \leq I(B)$ whenever $A \subseteq B$. Note that capacities differ from the concepts of measures and outer measures, as no additivity or subadditivity conditions are imposed. However, for any finite measure, there is a `http://planetmath.org/CapacityGeneratedByAMeasure`correspond capacity. An important application to the theory of measures and analytic sets is given by the capacitability theorem.

The $(\mathcal{F}, I)$-*capacitable* sets are defined as follows. Recall that $\mathcal{F}_\delta$ denotes the collection of countable intersections of sets in the paving $\mathcal{F}$.

Definition. *Let I be an $\mathcal{F}$-capacity on a set X. Then a subset $A \subseteq X$ is* $(\mathcal{F}, I)$-capacitable *if, for each $\epsilon > 0$, there exists a $B \in \mathcal{F}_\delta$ such that $B \subseteq A$ and $I(B) \geq I(A) - \epsilon$.*

Alternatively, such sets are called I-capacitable or, simply, capacitable.