regarding the sets A_n from the traveling hump sequence

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In this entry, $\lfloor \cdot \rfloor$ denotes the floor function.

Following is a proof that, for every positive integer n, $\left[\dfrac{n-2^{\lfloor \log_2 n \rfloor}}{2^{\lfloor \log_2 n \rfloor}}, \dfrac{n-2^{\lfloor \log_2 n \rfloor}+1}{2^{\lfloor \log_2 n \rfloor}}\right] \subseteq [0,1]$.

Proof. Note that this is `http://planetmath.org/Equivalent`equivalent to showing that, for every positive integer n,

$n-2^{\lfloor \log_2 n \rfloor} \geq 0$ and $n-2^{\lfloor \log_2 n \rfloor}+1 \leq 2^{\lfloor \log_2 n \rfloor}$. This in turn is equivalent to showing that, for every positive integer n, $2^{\lfloor \log_2 n \rfloor} \leq n$ and $n+1 \leq 2^{\lfloor \log_2 n \rfloor + 1}$.

The first inequality is easy to prove: For every positive integer n, $2^{\lfloor \log_2 n \rfloor} \leq 2^{\log_2 n} = n$.

Now for the second inequality. Let n be a positive integer. Let k be the unique positive integer such that

$2^{k-1} \leq n \leq 2^k - 1$. Then $n+1 \leq 2^k = 2^{k-1+1} = 2^{\lfloor k-1 \rfloor + 1} = 2^{\left\lfloor \log_2 2^{k-1} \right\rfloor + 1} \leq 2^{\lfloor \log_2 n \rfloor + 1}$. $\square$