Hardy-Littlewood maximal operator

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The *Hardy-Littlewood maximal operator* in $\mathbb{R}^n$ is an operator defined on $L^1_{\text{loc}}(\mathbb{R}^n)$ (the space of locally integrable functions in $\mathbb{R}^n$ with the Lebesgue measure) which maps each locally integrable function f to another function Mf, defined for each $x \in \mathbb{R}^n$ by

$$Mf(x) = \sup_Q \frac{1}{m(Q)} \int_Q |f(y)|dy,$$

where the supremum is taken over all cubes Q containing x. This function is lower semicontinuous (and hence measurable), and it is called the *Hardy-Littlewood maximal function* of f.

The operator M is sublinear, which means that

$$M(af + bg) \leq |a|Mf + |b|Mg$$

for each pair of locally integrable functions f, g and scalars a, b.