modes of convergence of sequences of measurable functions

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Owner	Wkbj79 (1863)
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Author	Wkbj79 (1863)
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Defines	converges almost everywhere
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Defines	almost uniform convergence
Defines	converges in measure
Defines	convergence in measure
Defines	converges in $L^1(\mu)$
Defines	$L^1(\mu)$ convergence

Let $(X, \mathfrak{B}, \mu)$ be a measure space, $f_n \colon X \to [-\infty, \infty]$ be measurable functions for every positive integer n, and $f \colon X \to [-\infty, \infty]$ be a measurable function. The following are modes of convergence of $\{f_n\}$:

- $\{f_n\}$ *converges almost everywhere* to f if $\mu\left(X - \{x \in X : \lim\limits_{n\to\infty} f_n(x) = f(x)\}\right) = 0$
- $\{f_n\}$ *converges almost uniformly* to f if, for every $\varepsilon > 0$, there exists $E_\varepsilon \in \mathfrak{B}$ with $\mu(X - E_\varepsilon) < \varepsilon$ and $\{f_n\}$ converges uniformly to f on E_ε
- $\{f_n\}$ *converges in measure* to f if, for every $\varepsilon > 0$, there exists a positive integer N such that, for every positive integer $n \geq N$, $\mu\left(\{x \in X : |f_n(x) - f(x)| \geq \varepsilon\}\right) < \varepsilon$.
- If, in , f and each f_n are also Lebesgue integrable, $\{f_n\}$ *converges in* $L^1(\mu)$ to f if $\lim\limits_{n\to\infty} \int_X |f_n - f| \, d\mu = 0$.

A lot of theorems in `http://planetmath.org/BibliographyForRealAnalysis`real analysis deal with these modes of convergence. For example, Fatou's lemma, Lebesgue's monotone convergence theorem, and Lebesgue's dominated convergence theorem give conditions on sequences of measurable functions that converge almost everywhere under which they also converge in $L^1(\mu)$. Also, Egorov's theorem that, if $\mu(X) < \infty$, then convergence almost everywhere implies almost uniform convergence.