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Clarkson inequality

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The *Clarkson inequality* says that for all $f, g \in L^p$, for $2 \leq p < \infty$ we have:

$$\left\| \frac{f+g}{2} \right\|_p^p + \left\| \frac{f-g}{2} \right\|_p^p \leq \frac{1}{2} \left(\|f\|_p^p + \|g\|_p^p \right).$$

The inequality can be used to prove that L^p space is uniformly convex for $2 \leq p < \infty$.

Remark. If $1 < p < 2$, then the Clarkson inequality becomes:

$$\left\| \frac{f+g}{2} \right\|_p^q + \left\| \frac{f-g}{2} \right\|_p^q \leq \left(\frac{1}{2}\|f\|_p^p + \frac{1}{2}\|g\|_p^p \right)^{\frac{1}{p-1}}$$

.

for functions f, $g \in L^p$, where $q = \frac{p}{p-1}$.