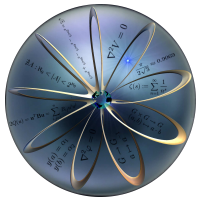

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proof of Fatou's lemma

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Let $f(x) = \liminf_{n\to\infty} f_n(x)$ and let $g_n(x) = \inf_{k\geq n} f_k(x)$ so that we have

$$f(x) = \sup_n g_n(x).$$

As g_n is an increasing sequence of measurable nonnegative functions we can apply the monotone convergence Theorem to obtain

$$\int_X f\, d\mu = \lim_{n\to\infty} \int_X g_n\, d\mu.$$

On the other hand, being $g_n \leq f_n$, we conclude by observing

$$\lim_{n\to\infty} \int_X g_n\, d\mu = \liminf_{n\to\infty} \int_X g_n\, d\mu \leq \liminf_{n\to\infty} \int_X f_n\, d\mu.$$