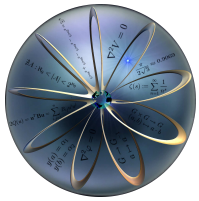

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direct images of analytic sets are analytic

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For measurable spaces $(X, \mathcal{F})$ and $(Y, \mathcal{G})$, consider a measurable function $f\colon X \to Y$. By definition, the inverse image $f^{-1}(A)$ will be in $\mathcal{F}$ whenever A is in $\mathcal{G}$. However, the situation is more complicated for `http://planetmath.org/DirectImage`direc images, which in general do not preserve measurability. However, as stated by the following theorem, the class of analytic subsets of Polish spaces is closed under direct images.

Theorem. *Let $f\colon X \to Y$ be a Borel measurable function between Polish spaces X and Y. Then, the direct image $f(A)$ is analytic whenever A is an analytic subset of X.*