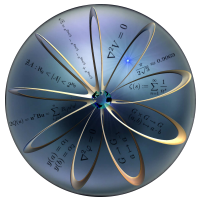

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Carathéodory's extension theorem

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In measure theory, Carathéodory's extension theorem is an important result used in the construction of measures, such as the Lebesgue measure on the real number line. The result states that a `http://planetmath.org/Additive`countably additive set function on an algebra of sets can be extended to a measure on the `http://planetmath.org/SigmaAlgebra`σ-algebra generated by that algebra.

Theorem (Carathéodory)**.** *Let X be a set, A be an algebra on X, and $\mathcal{A} \equiv \sigma(A)$ be the σ-algebra generated by A. If $\mu_0 \colon A \to \mathbb{R}_+ \cup \{\infty\}$ is a countably additive map then there exists a measure μ on $(X, \mathcal{A})$ such that $\mu = \mu_0$ on A.*

References

[1] David Williams, *Probability with martingales*, Cambridge Mathematical Textbooks, Cambridge University Press, 1991.

[2] Olav Kallenberg, *Foundations of modern probability*, Second edition. Probability and its Applications. Springer-Verlag, 2002.