Fatou-Lebesgue theorem

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Let (X, μ) be a measure space. If $\Phi\colon X \to \mathbb{R}$ is a nonnegative function with $\int \Phi d\mu < \infty$, and if $f_1, f_2, \ldots$ is a sequence of measurable functions such that $|f_n| \leq \Phi$ for each n, then

$$g = \liminf_{n\to\infty} f_n \quad \text{and} \quad h = \limsup_{n\to\infty} f_n$$

are both integrable, and

$$-\infty < \int g d\mu \leq \liminf_{n\to\infty} \int f_n d\mu \leq \limsup_{k\to\infty} \int f_n d\mu \leq \int h d\mu < \infty.$$