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example of function not Lebesgue Measurable
with measurable level sets

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Consider V as in Vitali's theorem. Define the function $f : [0, 1] \rightarrow [0, +\infty[$ by:

$$f(x) = \begin{cases} x & \text{if } x \notin V \\ 2 + x & \text{if } x \in V \end{cases}$$

The level sets of f will either be the empty set, or a singleton and thus measurable.

$$f^{-1}(\{x\}) = \begin{cases} \{x\} & \text{if } 0 \leq x \leq 1 \wedge x \notin V \\ \{2 - x\} & \text{if } 2 \leq x \leq 3 \wedge x - 2 \in V \\ \{\} & \text{otherwise} \end{cases}$$

f is not a measurable function since $f^{-1}([2, +\infty[) = V$ and V is not a measurable set.