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Riemann integral

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Let $I = [a, b]$ be an interval of $\mathbb{R}$ and let $f\colon I \to \mathbb{R}$ be a bounded function. For any finite set of points $\{x_0, x_1, x_2, \ldots, x_n\}$ such that $a = x_0 < x_1 < x_2 \cdots < x_n = b$, there is a corresponding partition $P = \{[x_0, x_1), [x_1, x_2), \ldots, [x_{n-1}, x_n]\}$ of I.

Let $C(\epsilon)$ be the set of all partitions of I with $\max(x_{i+1} - x_i) < \epsilon$. Then let $S^*(\epsilon)$ be the infimum of the set of upper Riemann sums with each partition in $C(\epsilon)$, and let $S_*(\epsilon)$ be the supremum of the set of lower Riemann sums with each partition in $C(\epsilon)$. If $\epsilon_1 < \epsilon_2$, then $C(\epsilon_1) \subset C(\epsilon_2)$, so $S^*(\epsilon)$ is `http://planetmath.org/IncreasingdecreasingmonotoneFunction`decreasing and $S_*(\epsilon)$ is `http://planetmath.org/IncreasingdecreasingmonotoneFunction`increasing. Moreover, $|S^*(\epsilon)|$ and $|S_*(\epsilon)|$ are bounded by $(b-a)\sup_x |f(x)|$. Therefore, the limits $S^* = \lim_{\epsilon\to 0} S^*(\epsilon)$ and $S_* = \lim_{\epsilon\to 0} S_*(\epsilon)$ exist and are finite. If $S^* = S_*$, then f is Riemann-integrable over I, and the Riemann integral of f over I is defined by

$$\int_a^b f(x)dx = S^* = S_*.$$