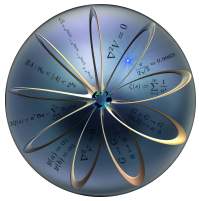

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example of a non Riemann integrable function

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Let $[a, b]$ be any closed interval and consider the Dirichlet's function $f\colon [a, b] \to \mathbb{R}$

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{otherwise.} \end{cases}$$

Then f is not Riemann integrable. In fact given any interval $[x_1, x_2] \subset [a, b]$ with $x_1 < x_2$ one has

$$\sup_{[x_1,x_2]} f(x) = 1, \qquad \inf_{[x_1,x_2]} f(x) = 0$$

because every interval contains both rational and irrational points. So all upper Riemann sums are equal to 1 and all lower Riemann sums are equal to 0.