example of integration with respect to surface area on a helicoid

Canonical name	ExampleOfIntegrationWithRespectToSurfaceAreaOnAHelicoid
Date of creation	2013-03-22 14:58:01
Last modified on	2013-03-22 14:58:01
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	7
Author	rspuzio (6075)
Entry type	Example
Classification	msc 28A75

In this example, we shall consider itegration with respect to surface area on the helicoid.

The helicoid may be parameterized as follows:

$$x = u \sin v$$

$$y = u \cos v$$

$$z = cv$$

(The constant c may be thought of as the "pitch of the screw".) Computing derivatives and appying trigonometric identities, we obtain

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \sin v & u \cos v \\ \cos v & -u \sin v \end{vmatrix} = -u$$

$$\frac{\partial(y,z)}{\partial(u,v)} = \begin{vmatrix} \cos v & -u \sin v \\ 0 & c \end{vmatrix} = c \cos v$$

$$\frac{\partial(z,x)}{\partial(u,v)} = \begin{vmatrix} 0 & c \\ \sin v & u \cos v \end{vmatrix} = -c \sin v.$$

From this we have

$$\sqrt{\left(\frac{\partial(x,y)}{\partial(u,v)}\right)^2 + \left(\frac{\partial(y,z)}{\partial(u,v)}\right)^2 + \left(\frac{\partial(z,x)}{\partial(u,v)}\right)^2} =$$

$$\sqrt{u^2 + c^2 \cos^2 v + c^2 \sin^2 v} = \sqrt{u^2 + c^2}$$

so we can compute area integrals over helicoids as follows

$$\int_S f(u,v)\, d^2 A = \int f(u,v) \sqrt{c^2 + u^2}\, du\, dv$$

To return to the main entry `http://planetmath.org/node/6660`click here