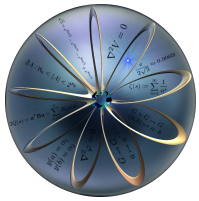

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Julia set

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Let U be an open subset of the complex plane and let $f\colon U \to U$ be analytic. Denote the n-th iterate of f by f^n, i.e. $f^1 = f$ and $f^{n+1} = f \circ f^n$. Then the *Julia set* of f is the subset J of U characterized by the following property: if $z \in J$ then the restriction of $\{f^n \mid n \in \mathbb{N}\}$ to any neighborhood of z is not a normal family.

It can also be shown that the Julia set of f is the closure of the set of repelling periodic points of f. (Repelling periodic point means that, for some n, we have $f^n(z) = z$ and $|f'(z)| > 1$.)

A simple example is afforded by the map $f(z) = z^2$; in this case, the Julia set is the unit circle. In general, however, things are much more complicated and the Julia set is a fractal.

From the definition, it follows that the Julia set is closed under f and its inverse — $f(J) = J$ and $f^{-1}(J) = J$. Topologically, Julia sets are perfect and have empty interior.