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tail event

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Defines	tail sigma algebra

Definition. Let Ω be a set and $\mathcal{F}$ a sigma algebra of subsets of Ω. Given the random variables $\{X_n, n \in \mathbb{N}\}$, defined on the measurable space $(\Omega, \mathcal{F})$, the *tail events* are the events of the *tail σ-algebra*

$$\mathcal{F}_\infty = \bigcap_{n=1}^{\infty} \sigma(X_n, X_{n+1}, \cdots)$$

where $\sigma(X_n, X_{n+1}, \cdots)$ is the σ-algebra induced by $(X_n, X_{n+1}, \cdots)$.

Remark 1. One can intuitively think of tail events as those events whose ocurrence or not is not affected by altering any finite number of random variables in the sequence. Some examples are

$$\{\limsup X_n < c\}, \sum X_n \text{converges} , \lim X_n \text{ exists}$$

Remark 2. One of the most important theorems in probability theory due to Kolomogorv, is the Kolmogorov zero-one law which states that, in the case of independent random variables, the probability of any tail event is 0 or 1 (provided there is a probability measure defined on $(\Omega, \mathcal{F})$)