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support of integrable function with respect to counting measure is countable

 ${\bf Canonical\ name} \quad {\bf SupportOfIntegrableFunctionWithRespectToCountingMeasureIsCountable}$

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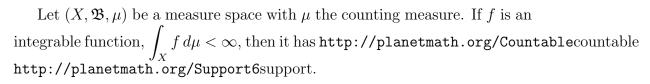
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Proof. WLOG, we assume that f is real valued and is nonnegative. Let S_0 denote the preimage of the interval $[1, \infty)$ and, for every positive integer n, let S_n denote the preimage of the interval $\left[\frac{1}{n+1}, \frac{1}{n}\right)$. Since the integral of f is bounded, each S_n can be at most finite. Taking the union of all the S_n , we get the support supp $f = \bigcup_{n=0}^{\infty} S_n$. Thus, supp f is a union of countably many finite sets and hence is countable.