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**product measure**

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Let  $(E_1, \mathcal{B}_1(E_1))$  and  $(E_2, \mathcal{B}_2(E_2))$  be two measurable spaces, with measures  $\mu_1$  and  $\mu_2$ . Let  $\mathcal{B}_1 \times \mathcal{B}_2$  be the sigma algebra on  $E_1 \times E_2$  generated by subsets of the form  $B_1 \times B_2$ , where  $B_1 \in \mathcal{B}_1(E_1)$  and  $B_2 \in \mathcal{B}_2(E_2)$ .

The *product measure*  $\mu_1 \times \mu_2$  is defined to be the unique measure on the measurable space  $(E_1 \times E_2, \mathcal{B}_1 \times \mathcal{B}_2)$  satisfying the property

$$\mu_1 \times \mu_2(B_1 \times B_2) = \mu_1(B_1)\mu_2(B_2) \text{ for all } B_1 \in \mathcal{B}_1(E_1), B_2 \in \mathcal{B}_2(E_2).$$