# capacity generated by a measure

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Any http://planetmath.org/SigmaFinitefinite measure can be extended to a set function on the power set of the underlying space. As the following result states, this will be a Choquet capacity.

**Theorem.** *Let $(X, \mathcal{F}, \mu)$ be a finite measure space. Then,*

$$\begin{aligned}
&\mu^*\colon \mathcal{P}(X) \to \mathbb{R}_+,\\
&\mu^*(S) = \inf\{\mu(A)\colon A \in \mathcal{F},\ A \supseteq S\}
\end{aligned}$$

*is an $\mathcal{F}$-capacity. Furthermore, a subset $S \subseteq X$ is $(\mathcal{F}, \mu^*)$-capacitable if and only if it is in the http://planetmath.org/CompleteMeasurecompletion of $\mathcal{F}$ with respect to $\mu$.*

Note that, as well as being a capacity, $\mu^*$ is also an outer measure (see http://planetmath.org/ConstructionOfOuterMeasureshere), which does not require the finiteness of $\mu$. Clearly, $\mu^*(A) = \mu(A)$ for all $A \in \mathcal{F}$, so $\mu^*$ is an extension of $\mu$ to the power set of $X$, and is referred to as the *outer measure generated by* $\mu$.

Recall that a subset $S \subseteq X$ is in the completion of $\mathcal{F}$ with respect to $\mu$ if and only if there are sets $A, B \in \mathcal{F}$ with $A \subseteq S \subseteq B$ and $\mu(B \setminus A) = 0$ which, by the above theorem, is equivalent to the capacitability of $S$.