quasi-invariant

Canonical name	Quasiinvariant
Date of creation	2013-03-22 15:56:00
Last modified on	2013-03-22 15:56:00
Owner	Mathprof (13753)
Last modified by	Mathprof (13753)
Numerical id	12
Author	Mathprof (13753)
Entry type	Definition
Classification	msc 28A12
Related topic	RepresentationsOfLocallyCompactGroupoids

Definition 1. Let $(E, \mathcal{B})$ be a measurable space, and $T : E \to E$ be a measurable map. A measure μ on $(E, \mathcal{B})$ is said to be *quasi-invariant* under T if $\mu \circ T^{-1}$ is absolutely continuous with respect to μ. That is, for all $A \in \mathcal{B}$ with $\mu(A) = 0$, we also have $\mu(T^{-1}(A)) = 0$. We also say that T leaves μ quasi-invariant.

As a example, let $E = \mathbb{R}$ with $\mathcal{B}$ the `http://planetmath.org/BorelSigmaAlgebra`Borel σ-algebra, and μ be Lebesgue measure. If $T(x) = x + 5$, then μ is quasi-invariant under T. If $S(x) = 0$, then μ is not quasi-invariant under S. (We have $\mu(\{0\}) = 0$, but $\mu(T^{-1}(\{0\})) = \mu(\mathbb{R}) = \infty$).

To give another example, take E to be the nonnegative integers and declare every subset of E to be a measurable set. Fix $\lambda > 0$. Let $\mu(\{n\}) = \frac{\lambda^n}{n!}$ and extend μ to all subsets by additivity. Let T be the shift function: $n \to n+1$. Then μ is quasi-invariant under T and not `http://planetmath.org/HaarMeasure`invariant.