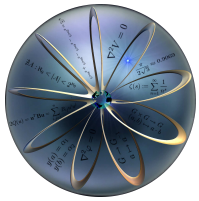

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Simpson's rule

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Simpson's rule is a method of (approximate) numerical definite integration (or quadrature). Simpson's rule is based on a parabolic model of the function to be integrated (in contrast to the trapezoidal model of the trapezoidal rule). Thus, a minimum of three points and three function values are required. Here we take three equidistant points: $x_0 x_2$ the interval endpoints, $x_1 = (x_0 + x_2)/2$ the midpoint, and let $h = |b - a|/2$ the distance between each. The definite integral is then approximated by:

$$\int_{x_0}^{x_2} f(x)dx \approx I = \frac{h}{3}(f(x_0) + 4f(x_1) + f(x_2))$$

We can extend this to greater precision by breaking our target domain into n equal-length fragments. The quadrature is then the weighted sum of the above formula for every pair of adjacent regions, which works out for even n to

$$I = \frac{h}{3}(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\cdots+4f(x_{n-3})+2f(x_{n-2})+4f(x_{n-1})+f(x_n))$$