integration with respect to surface area on a helicoid

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To illustrate the result derived in http://planetmath.org/node/6666example 3, let us compute the area of a portion of helicoid of height h and radius r. (This calculation will tell us how much material is needed to make an .) The integral we need to compute in this case is

$$A = \int d^2A = \int_0^{h/c} \int_0^r \sqrt{c^2 + u^2}\, du\, dv =$$

$$\int_0^{h/c} \left(\frac{1}{2} r \sqrt{c^2 + r^2} + \frac{c^2}{2} \log \left\{ \frac{r}{c} + \sqrt{1 + \left(\frac{r}{c}\right)^2} \right\} \right) dv =$$

$$\frac{rh}{2} \sqrt{1 + \left(\frac{r}{c}\right)^2} + \frac{ch}{2} \log \left\{ \frac{r}{c} + \sqrt{1 + \left(\frac{r}{c}\right)^2} \right\}$$

As a second illustration, let us compute the second moment of a helicoid about the axis of rotation. In mechanics, this would be called the moment of inertia of the helicoid and determines how much energy is needed to make the screw rotate. This is determined as follows:

$$\int (x^2 + y^2) d^2A = \int_0^{h/c} \int_0^r u^2 \sqrt{c^2 + u^2}\, du\, dv =$$

$$\int_0^{h/c} \left(\tfrac{r(2r^2+c^2)}{8} \sqrt{c^2 + r^2} - \tfrac{c^4}{8} \log \left\{ \tfrac{r}{c} + \sqrt{1 + \left(\tfrac{r}{c}\right)^2} \right\} \right) dv =$$

$$\tfrac{rh(2r^2+c^2)}{8} \sqrt{1 + \left(\tfrac{r}{c}\right)^2} - \tfrac{hc^3}{8} \log \left\{ \tfrac{r}{c} + \sqrt{1 + \left(\tfrac{r}{c}\right)^2} \right\}$$

http://planetmath.org/node/6660main entry http://planetmath.org/node/6666previous example http://planetmath.org/node/6668next example