existence of the essential supremum

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We state the existence of the essential supremum for a set $\mathcal{S}$ of extended real valued functions on a http://planetmath.org/SigmaFiniteσ-finite measure space $(\Omega, \mathcal{F}, \mu)$.

Theorem. *Suppose that the measure space $(\Omega, \mathcal{F}, \mu)$ is σ-finite. Then, the essential supremum of $\mathcal{S}$ exists. Furthermore, if $\mathcal{S}$ is nonempty then there exists a sequence $(f_n)_{n=1,2,\ldots}$ in $\mathcal{S}$ such that*

$$\operatorname{esssup} \mathcal{S} = \sup_n f_n. \tag{1}$$

Note that, by reversing the inequalities, this result also applies to the essential infimum, except that equation (**??**) is replaced by

$$\operatorname{essinf} \mathcal{S} = \inf_n f_n.$$