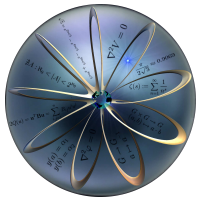

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Hahn decomposition theorem

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Defines	Hahn decomposition

Let μ be a signed measure in the measurable space $(\Omega, \mathscr{S})$. There are two measurable sets A and B such that:

1. $A \cup B = \Omega$ and $A \cap B = \emptyset$;
2. $\mu(E) \geq 0$ for each $E \in \mathscr{S}$ such that $E \subset A$;
3. $\mu(E) \leq 0$ for each $E \in \mathscr{S}$ such that $E \subset B$.

The pair (A, B) is called a *Hahn decomposition* for μ. This decomposition is not unique, but any other such decomposition (A', B') satisfies $\mu(A' \triangle A) = \mu(B \triangle B') = 0$ (where $\triangle$ denotes the symmetric difference), so the two decompositions differ in a set of measure 0.