fundamental lemma of calculus of variations

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The idea in the calculus of variations is to study stationary points of functionals. To derive a differential equation for such stationary points, the following theorem is needed, and hence named thereafter. It is also used in distribution theory to recover traditional calculus from distributional calculus.

Theorem 1. *Suppose $f\colon U \to \mathbb{C}$ is a locally integrable function on an open subset $U \subset \mathbb{R}^n$, and suppose that*

$$\int_U f\phi dx = 0$$

for all smooth functions with compact support $\phi \in C_0^\infty(U)$. Then $f = 0$ almost everywhere.

By linearity of the integral, it is easy to see that one only needs to prove the claim for real f. If f is continuous, this can be seen by purely geometrical arguments. A full proof based on the Lebesgue differentiation theorem is given in [**?**]. Another proof is given in [**?**].

References

[1] L. Hörmander, *The Analysis of Linear Partial Differential Operators I, (Distribution theory and Fourier Analysis)*, 2nd ed, Springer-Verlag, 1990.

[2] S. Lang, *Analysis II*, Addison-Wesley Publishing Company Inc., 1969.